System Description:

LEO – A Higher Order Theorem Prover[a]

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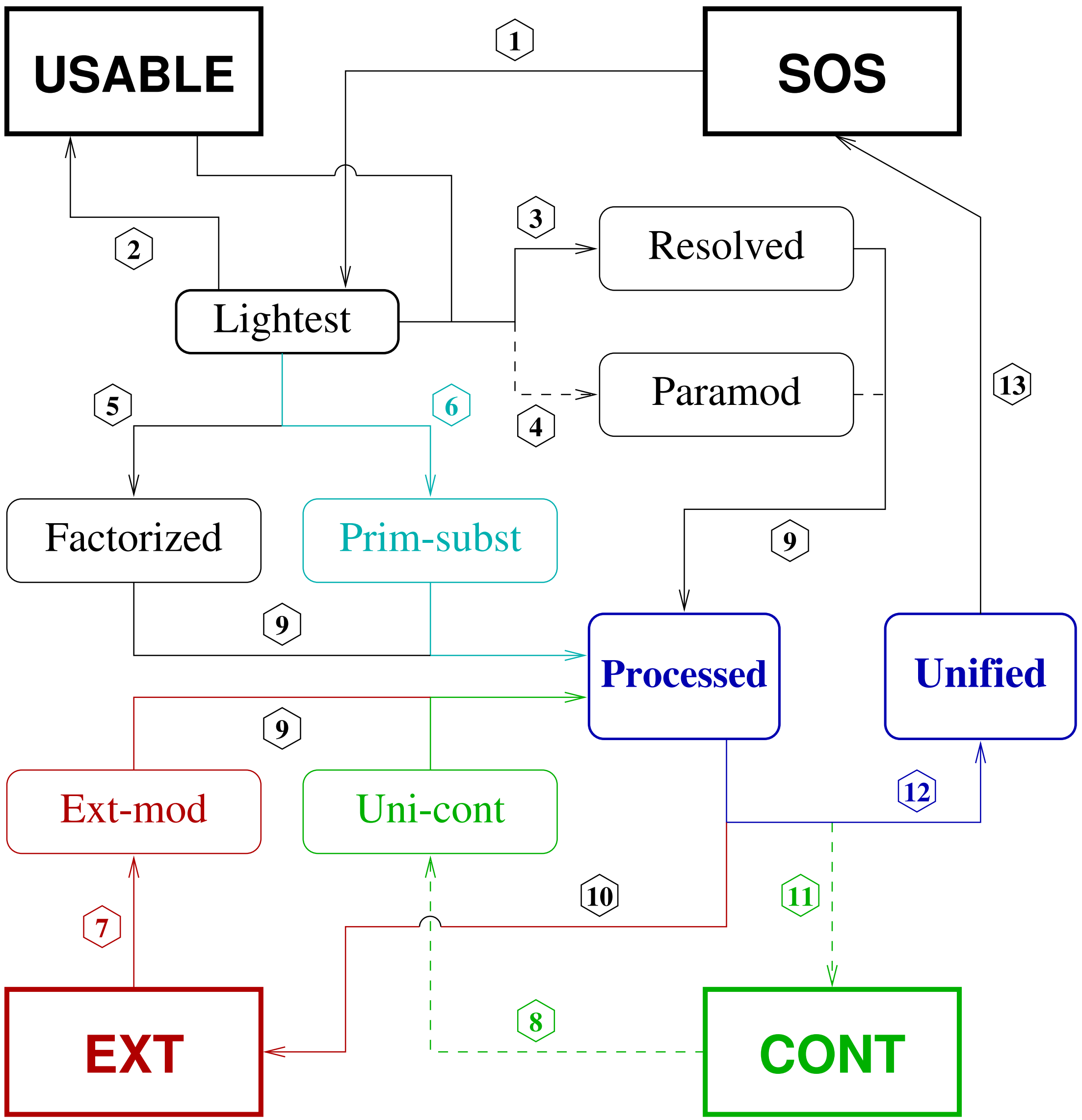
July 7, Lindau, Germany

[a]This work was supported by the Deutsche Forschungsgemeinschaft in grant HOTEL and by the Studienstiftung des Deutschen Volkes

THEORETICAL ASPECTS OF LEO

- Calculus: Extensional HO Resolution
- Built-in Extensionality Principles
- Extended SOS Architecture:
  - Extensionality Treatment
  - Interleaved HO Unification and Resolution
  - Primitive Substitution
  - Continuation of HO Unification
- Problems:
  - Leibniz-Equality or Primitive Equality
  - HO Term-Indexing is not compatible with Extensional HO Resolution
  - HO Subsumption

Architecture of LEO



→ implemented

- - → not yet implemented

1 choose lightest from SOS
2 Integrate light. to USABLE
3 resolve with USABLE
4 paramodulate with USABLE
5 factorize lightest
6 primitive substitution on lightest
7 extensionality treatment on EXT
8 pre-unification on CONT
9 process results (tautology deletion)
10 check if extensionally interesting
11 store continuation object
12 pre-unification on Processed
13 Integrate Unified into SOS

TECHNICAL ASPECTS OF LEO

- Implemented in Allegro Common Lisp
- Tested under Solaris and Linux
- Datastructures based on the KEIM-Toolbox
- Version LEO1 is available via

  `http://www.ags.uni-sb.de/projects/deduktion/projects/hot/leo/`

- Features of LEO1:
  – Automatic Mode for Extensional HO Resolution
  – Interactive Mode in a Simple Command Shell
- New Features of LEO3 (not yet available):
  – Integrated in the ΩMEGA-system
  – Graphical Proof Display and User Interface
  – Access to ΩMEGA's Knowledge Base and other Reasoning Systems

Examples about sets

LEO outperforms well known FO Theorem Provers on simple theorems about sets (e.g. Boolean Properties of Sets, Journal of Formalized Mathematics Volume 1, 1989)

Examples:

28) If $X \subseteq Y$ and $Y \subseteq X$ then $X = Y$

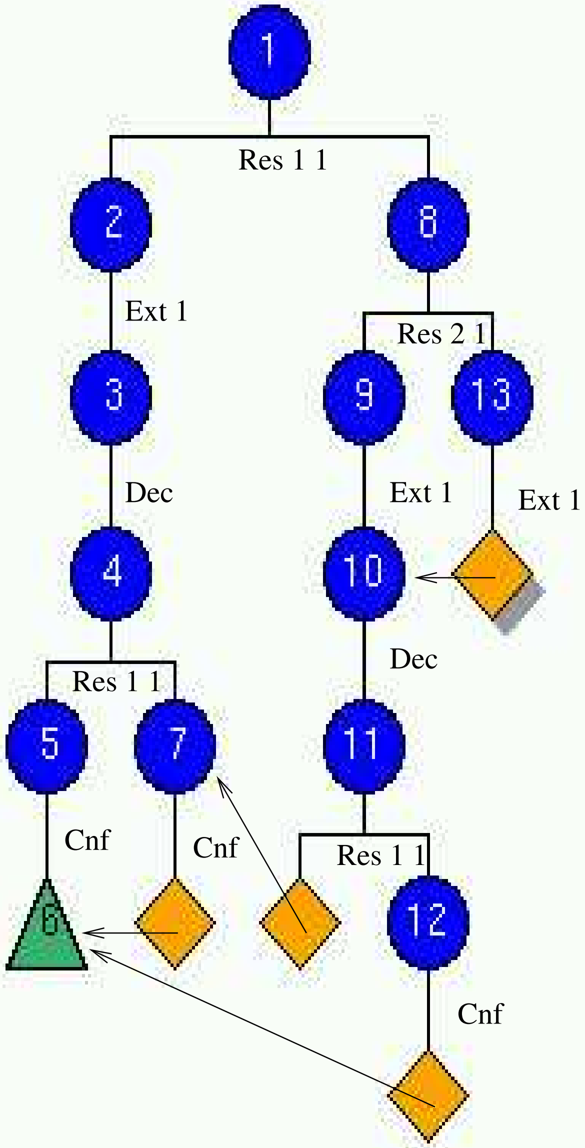
80) If $(X \cap Y) \cup (X \backslash Y) = X$

99) $(X \dot{-} Y) \dot{-} Z = X \dot{-} (Y \dot{-} Z)$

See: http://www-irm.mathematik.hu-berlin.de/ ˜ ilf/miz2atp/mizstat.html

Solved theorems (of 97)	
Waldmeister (pure equality prover, only Th 72 and 99 have been tried)	1
Spass v0.78 (on Ultra Sparc 170)	72
Setheo v3.3 (“on” PVM)	76
CM v10-15-97 (ME Prover in Prolog)	72
CM v10-15-97 (with special cost function [hdef(d1,6,1,6)])	76
CM v9-22-97 (with definition expansion in the theorem)	79
Otter (auto)	60
Gandalf v. c-1.0b	47
Spass v0.54	52
Setheo	53
All Together	94
LEO	95

EXAMPLE: $p_{oo}a_o \wedge p_{oo}b_o \Rightarrow p_{oo}(a_o \wedge b_o)$



$1: \quad \bot$

$2: \quad b$

$3: \quad \neg(b = (a \wedge b))$

$4: \quad \neg((pb) = (p(a \wedge b)))$

$5: \quad pb$

$6: \quad \neg(((pa) \wedge (pb)) \Rightarrow (p(a \wedge b)))$

$7: \quad \neg(p(a \wedge b))$

$8: \quad \neg b$

$9: \quad \neg a \vee \neg b$

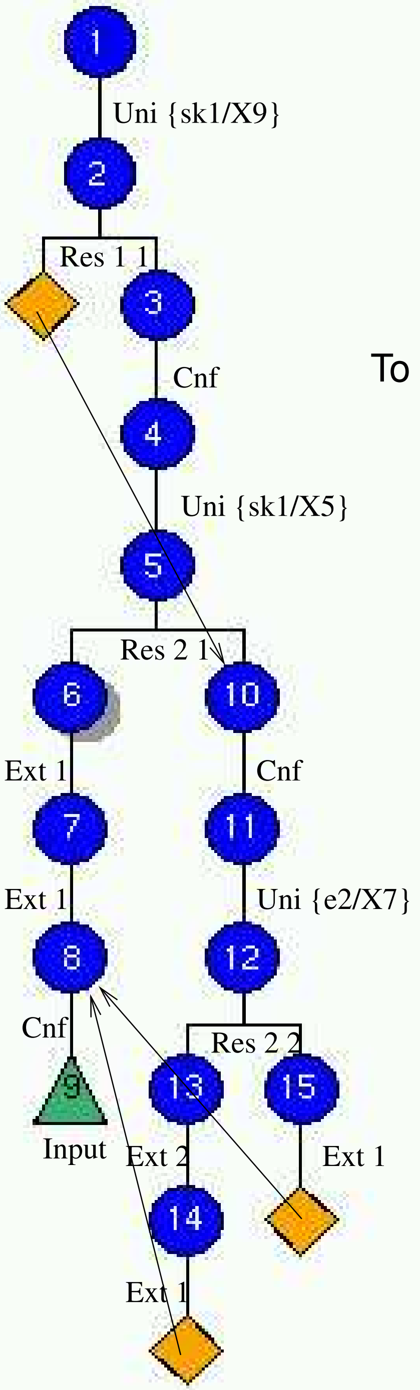
$10: \quad \neg(a = (a \wedge b))$

$11: \quad \neg((pa) = (p(a \wedge b)))$

$12: \quad pa$

$13: \quad a$

Example: $\wp(\emptyset) = \{\emptyset\}$



Def.:

$$\wp = \lambda A_{o\alpha}.\lambda B_{o\alpha}.\overbrace{\forall.X_\alpha BX \Rightarrow AX}^{B \subseteq A}$$

$$\{\} = \lambda X_\beta.\lambda Y_\beta.Y = X$$

$$\emptyset = \lambda Y_\gamma.\bot$$

To show:

$$\neg((\lambda B_{o\alpha}.(\forall X_\alpha.(BX) \Rightarrow \bot)) = (\lambda B_{o\alpha}.B = \lambda Y_\alpha.\bot))$$

$1:$	$\bot$
$2:$	$\neg(e^1_{o\alpha}X^9_\alpha = e^1_{o\alpha}sk^1)$
$3,4:$	$(e^1_{o\alpha}X^8_\alpha)$
$5:$	$\neg(e^1_{o\alpha}X^5_\alpha = e^1_{o\alpha}sk^1) \vee \neg(e^1_{o\alpha}X^6_\alpha)$
$6:$	$\neg(e^1_{o\alpha}X^2_\alpha) \vee \neg(e^1_{o\alpha}X^3_\alpha)$
$7:$	$\neg(e^1_{o\alpha}X^1_\alpha) \vee (e^1_{o\alpha} = \lambda z.\bot)$
$8,9:$	$\neg((\lambda B_{o\alpha}.(\forall X_\alpha.(BX) \Rightarrow \bot)) =$ $(\lambda B_{o\alpha}.B = \lambda Y_\alpha.\bot))$
$10,11:$	$(e^1_{o\alpha}sk^1)$
$12:$	$\neg(e^1_{o\alpha}e^2 = e^1_{o\alpha}X^7_\alpha) \vee (e^1_{o\alpha}sk^1)$
$13:$	$(e^1_{o\alpha}e^2) \vee (e^1_{o\alpha}sk^1)$
$14:$	$\neg(e^1_{o\alpha} = \lambda Y_\alpha.\bot) \vee (e^1_{o\alpha}sk^1)$
$15:$	$\neg(e^1_{o\alpha}X^4_\alpha) \vee (e^1_{o\alpha}sk^1)$

Conclusion

- LEO implements Extensional Higher Order Resolution
- Henkin-Completeness without Extensionality Axioms
- Interleaving of Resolution and Unification
- Well suited for simple theorems about sets

Current and future work

- Integration in ΩMEGA
- Primitive Equality
- Primitive Substitution
- More efficient implementation
- Cooperation with other Reasoning Systems