OMEGA

Resource-adaptive Proof Planning

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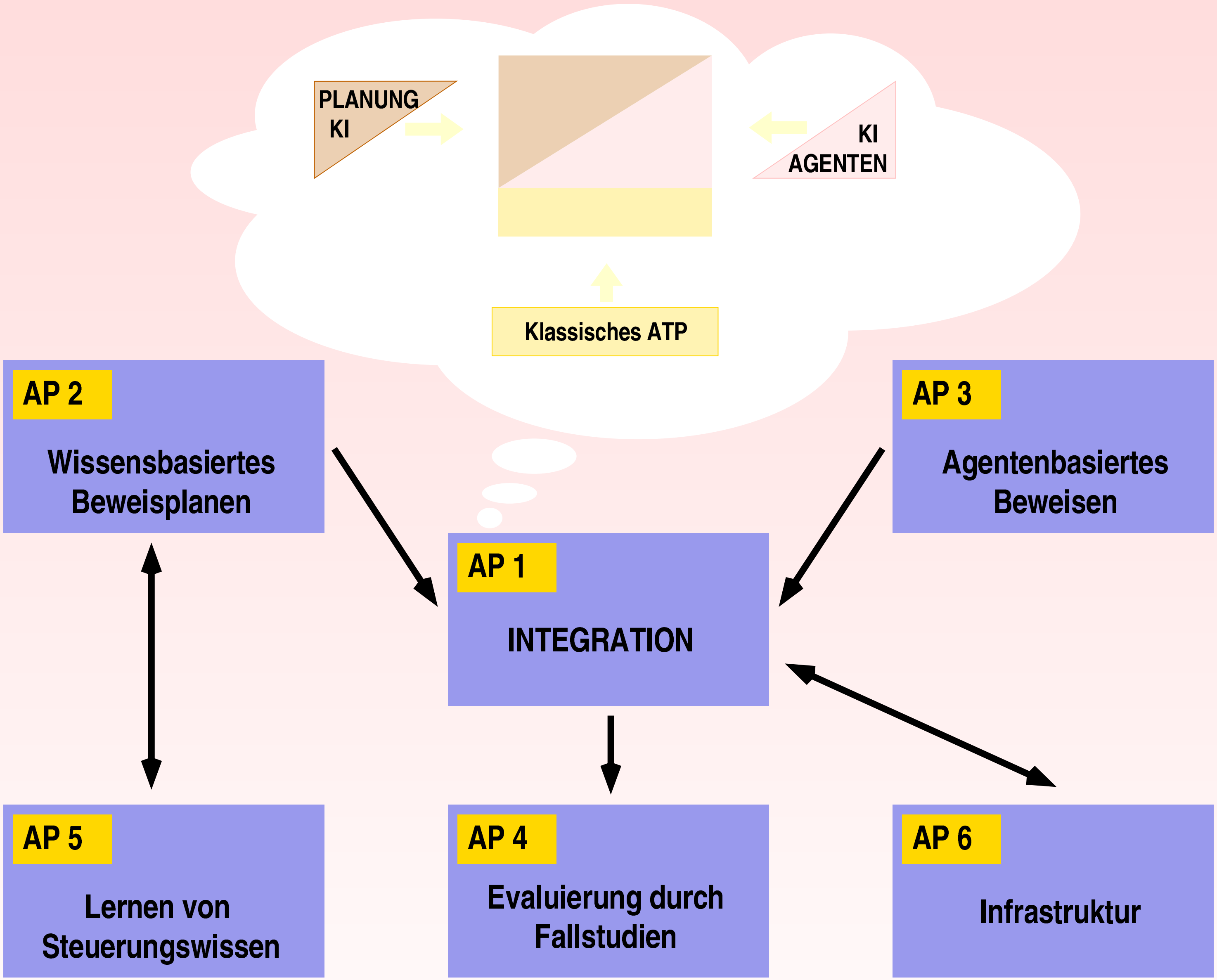
OMEGA Workplan



APs 1 & 3: Planning and Agent-based TP

Deliberative Reasoning		Pro-active Reasoning
Proof Planning (PP)	← integrate →	Agent-based Reasoning (AR)

- Framework and case studies [PhD-Sorge-01]
- AR as a means to combine and distribute complex reasoning procedures and external reasoners [Calculemus-01,KI-01]
- Expansion of proof methods via AR [ARW-01]
- Agent-based assertion retrieval [Festschrift-Siekmann-03,MKM-01]
- Theory formation and PP [Calculemus-02]
- AR and our new proof engine CORE [MSc-Huebner-03]
- International recognition: [Invited-plenary-talk-Benzmüller-at-AISB-01]

AP 2: Knowledge-based Proof Planning

- Multi-strategy proof planning with MULTI [PhD-Meier-03]
- Randomization and restarts [ECP-01]
- Critical discussion and reflection
  - Proof planning and logic layer [IJCAR-WS-01]
  - Generality of proof planning [Book-35years-of-AutoMath-03]
- Mathematical representations vs. logical representations [Australien-AI-Conf-02,FLOC-02-WS]
- Semantic guidance in proof planning [TechRep-Bham-01]
- Proof planning for permutation group problems [CADE-03]

AP 4: Evaluation by Case Studies

- Exploration of residue classes
  [Journal-of-Symbolic-Computation-02,EUROCAST-01]
- Agent-based theorem proving in naive set theory [KI-01]
- Naturalness of proof construction, interactive island planning
  [Book-35years-of-Automath-03]
- Certifying solutions to permutation group problems
  [CADE-03]

$\rightarrow$ Partial cooperation with University of Birmingham

AP 5: Learning

- Learning of proof methods

  [ECAI-02,CADE-WS-01]

- System LEARNOMATIC

  [CADE-02]

→ Cooperation with University of Birmingham

AP 6: Infrastructure

- New logic layer for OMEGA [PhD-Autexier-03,UITP-03,MSc-Hübner-03]
- Proof Presentation
  [PhD-Thesis-Fiedler,ICCS-01,NLDB-01,ICNLP-02,COLING-02,. . .]
- System *P.rex* [IJCAI-01,IJCAR-01]
- MBASE: mathematical knowledge base
  [Journal-of-Symbolic-Computation-01]
- MATHWEB-sb: mathematical software bus
  [CADE-02,Calculemus-02,Calculemus-01]
- Completeness of OMEGAs base calculus
  [Subm.-Journal-of-Symbolic-Logic]

Redesign of OMEGA Logic Layer

From	to
procedural reasoning style	declarative reasoning style
emphasis is on methods, tactics, rules	emphasis is on abstract-level applications of assertions

- Impact on
  - Interactive theorem proving
  - Proof planning
  - Agent-based theorem proving

Motivating Example

Theorem Proving with OMEGA: $\sqrt{2}$ is irrational

[Book-35years-of-Automath-03]

Theorem: $\sqrt{2}$ is irrational.

Proof: (by contradiction)
Assume $\sqrt{2}$ is rational, that is, there exist natural numbers m, n with no common divisor such that $\sqrt{2} = m/n$. Then $n\sqrt{2} = m$, and thus $2n^2 = m^2$. Hence m^2 is even and, since odd numbers square to odds, m is even; say $m = 2k$. Then $2n^2 = (2k)^2 = 4k^2$, that is, $n^2 = 2k^2$. Thus, n^2 is even too, and so is n. That means that both n and m are even, contradicting the fact that they do not have a common divisor.

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- declarative style of argumentation: from assertions A and B follows C
- logic layer (e.g. a la ND- or Sequent-Calculus) treated implicit

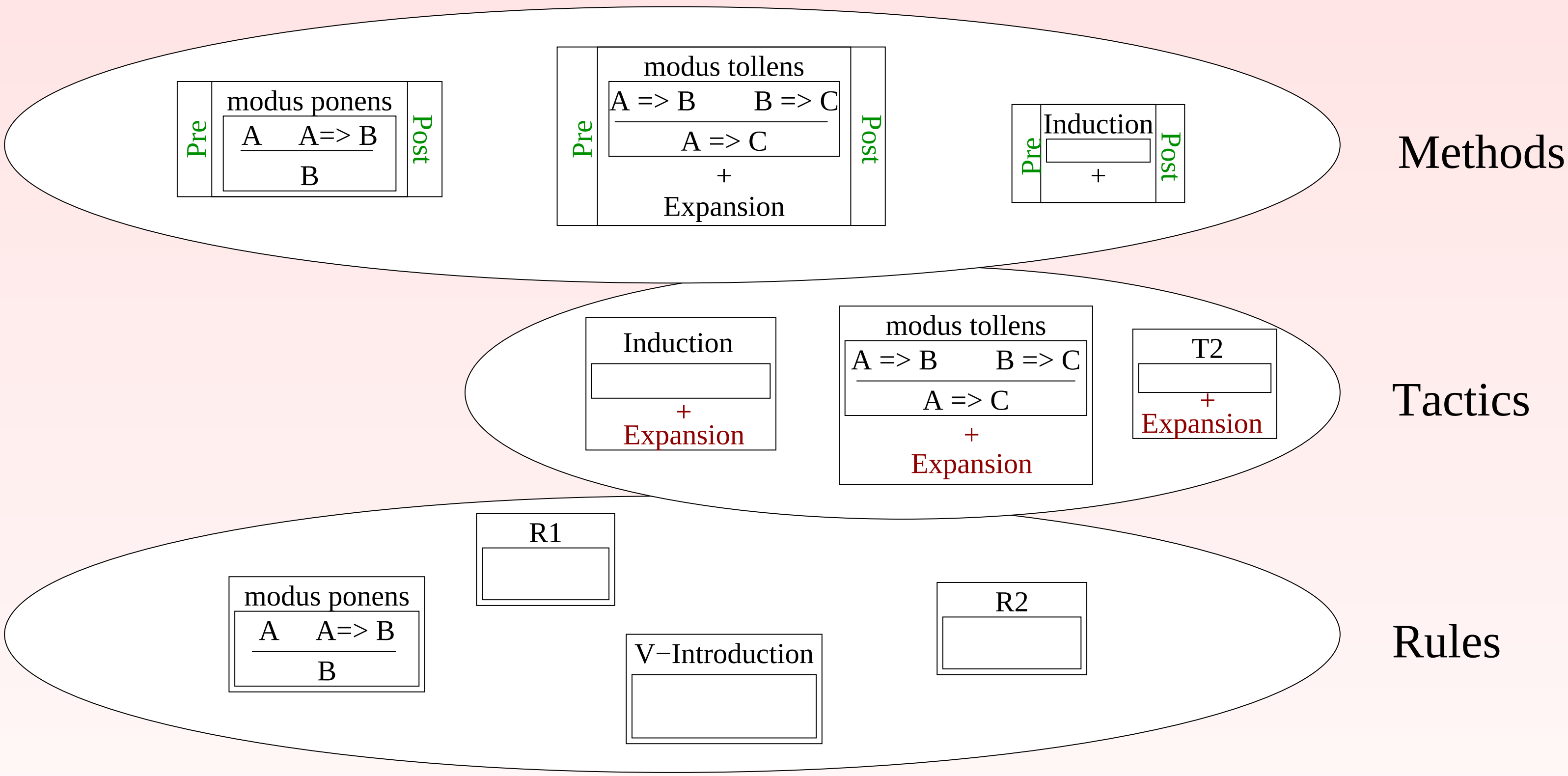
$\Rightarrow$ mismatch between procedural style logic-level reasoning as employed in todays theorem provers and declarative assertion level reasoning as typical for mathematical texts

Current OMEGA

Proof Planning: heuristically guided automated chaining of proof methods

Interactive Theorem Proving: user chains methods (tactics/rules)

$\Rightarrow$ Problem: full abstraction from logic layer is not achieved [IJCAR-WS-01]



Traditional Interactive Theorem Proving

```
Step 0:   PROVE (SQRT2-NOT-RAT)
Step 1:   DECLARE ((CONSTANTS (M NUM) (N NUM) (K NUM)))
Step 2:   NOTI default default
Step 3:   IMPORT-ASS (RAT-CRITERION)
Step 4:   FORALLE-SORT default default ((SQRT 2)) default
Step 5:   EXISTSE-SORT default default (N) default
Step 6:   ANDE default default default
Step 7:   EXISTSE-SORT (L7) default (M) default
Step 8:   ANDE* (L8) (NIL)
Step 9:   LEMMA default ((= (POWER M 2) (TIMES 2 (POWER N 2))))
Step 10:  BY-COMPUTATION (L13) ((L11))
Step 11:  LEMMA (L9) ((EVENP (POWER M 2)))
Step 12:  DEFN-CONTRACT default default default
Step 13:  LEMMA (L9) ((INT (POWER N 2)))
Step 14:  WELLSORTED default default
Step 15:  EXISTSI-SORT (L15) ((POWER N 2)) (L13) (L16) default
Step 16:  IMPORT-ASS (SQUARE-EVEN)
Step 17:  ASSERT ((EVENP M)) ((SQUARE-EVEN L10 L14)) (NIL)
Step 18:  DEFN-EXPAND (L17) default default
Step 19:  EXISTSE-SORT default default (K) default
Step 20:  ANDE (L19) default default
Step 21:  LEMMA default ((= (POWER N 2) (TIMES 2 (POWER K 2))))
Step 22:  BY-COMPUTATION (L23) ((L13 L22))
Step 23:  ...
```

$\Rightarrow$ procedural style

Traditional Island Planning

Network of proof 'islands'

$$\cfrac{\cfrac{2 * n^2 = m^2}{Even(m^2)}\ Island}{Even(m)}\ Island$$
$$\vdots$$

- Islands structure the proof in natural form
- Islands provide no argument for soundness
- Verification: expansion of island steps (automated, interactive, recursive island approach)

$\Rightarrow$ declarative style

Not solved by Island Approach:

Constructive assertion reasoning which still leaves logic level implicit

Re-design of OMEGA

Interactive Theorem Proving

Proof Planning

Agent-based Reasoning

Task Level

[MSc-Hübner-03]

Logic Engine CORE

[PhD-Autexier-03]

- supporting flexible assertion level reasoning
- complete hiding of logic layer

Future of OMEGA

- Ongoing: Integration of CORE into OMEGA
- Resource Adaptive Agentification of
  - Inference Rules and Assertions
  - Tactics and Proof Methods
  - External Services
    - FO-ATPs and HO-ATPs
    - Computer Algebra Systems
    - Mathematical Knowledge Bases
  - Agentification of the User
- Resource Adaptive Proof Planning with Agents