Leibniz's "Impossible Things"

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Introduction

Within the course Computational Metaphysics W. Lenzen gave a guest lecture on one of Leibniz' ontological arguments [4].

The formalization presented here – with adjectives as modal operators – is still very experimental. There is a computerized formalization of a rectangular circle and an invalid ticket. This formalization uses the proof-assistant "Isabelle".

Leibniz's Proof of God's Existence

Leibniz' latin text is shown in Figure 1. For the complete text in printed letters see [3], N. 164. Another project group has translated and computerized Leibniz's proof (see [1]).

You may note that the phrase "falso dicitur" is translated in [1] "it is false to say". But formalization is a simple "$\neg$" so "falso dicitur non ..." becomes "$\neg\neg$". Was that Leibniz's intention or would a more adequate formalization include a form of "to say"?

On the one hand Leibniz's issue was to prove the existence of a God-like being. On the other hand Leibniz's logic includes some interesting (obscure) aspects that are hardly accessible in modern logic. The main issue of this poster is not the proof but Leibniz's example of an "Impossible Thing" .

Leibniz's note on "Impossible Things"

Translation of the footnote in Figure 1:

A conclusion which includes a contradiction is possibly true, if the conclusion is on impossible things.

Example. *A rectangular circle is not a circle.*

Proof. A rectangle is not a circle. A rectangular circle is a rectangle. Hence a rectangular circle is not a circle. □

Non-Classical Logic

Leibniz apparently did not use Classical Logic in (Classical Logic was developed 200 years later). But what kind of logic was used? Is there any logic with true contradictions $A \wedge \neg A$? In "Leibniz's Algebra of Concepts" such an "Impossible Thing" does not only contain a single contradiction but any property (see [1], *ex contradictione quodlibet*). But why should a rectangular circle include a contradiction? Similar natural language constructions apparently do not include contradictions (see). I can see two explanations: Either Leibniz was wrong or he had in mind a logic different from his algebra of concepts.

Contradiction or Paradox?

I collected some more natural language examples that have the same structure as "rectangular circle" in :

(1) An invalid ticket is not a ticket, because a ticket is valid and an invalid ticket is not valid.

(2) Melted ice is not ice: Melted ice is liquid water and liquid water is not ice.

(3) A fake pass (football) is not a pass.

We can see in these examples that formalizations like "invalid(x)$\wedge$ticket(x)" or "rectangle(x)$\wedge$circle(x)" end up in inconsistency (see [4]).

Huthmann (author of the letter on the back of Figure 1) wrote a textbook on linguistics (ca. 1687) that was not published [2]. In "Cap. XV. Daß ein iedes Wort zu einem andern Worte gesetzt entweder eben dasselbe oder auch etwas anders abbilde" (pp 78-123) he claims that in a sentence two words like "Caeci vident" can map onto "Gantz-Ebendasselbe" but not onto "Etwas-Ebendasselbe" (pp 113-117). In other words a second word ("invalid" in "invalid ticket") can change the meaning of the first one ("ticket").

Adjectives as Modal Operators

We have observed a dual standpoint on the semantics of . One standpoint is, that such an adjective-noun-construction (nominal phrase) include the name of the noun while dropping its properties. Another standpoint is, that the adjective changes the viewpoint on the noun.

The second standpoint is explained for an invalid ticket: In the context of a ticket inspection an invalid ticket x is not a ticket. The viewpoint of fakers is however, that x is a ticket (and an invalid ticket too). The adjective "invalid" kind of connects both viewpoints. Let our actual viewpoint be the ticket inspector's viewpoint. Then "an invalid ticket is not a ticket" makes sense.

So the term "invalid" behaves like a modal operator. If you take this further, you can see that even a predicate P can be converted into a modal operator $\diamond_P$ using the following equivalence for an individual x:

$$P(x) \equiv \diamond_P \, \$x \tag{1}$$

The remaining predicate $\$$ has the meaning of "to be": For $\diamond_P \, \$x$ say "x is P". There is no magic behind that conversion, $P(x)$ is just a synonym of the following: That x is in a related viewpoint (accessible with modal operator P from our actual world).

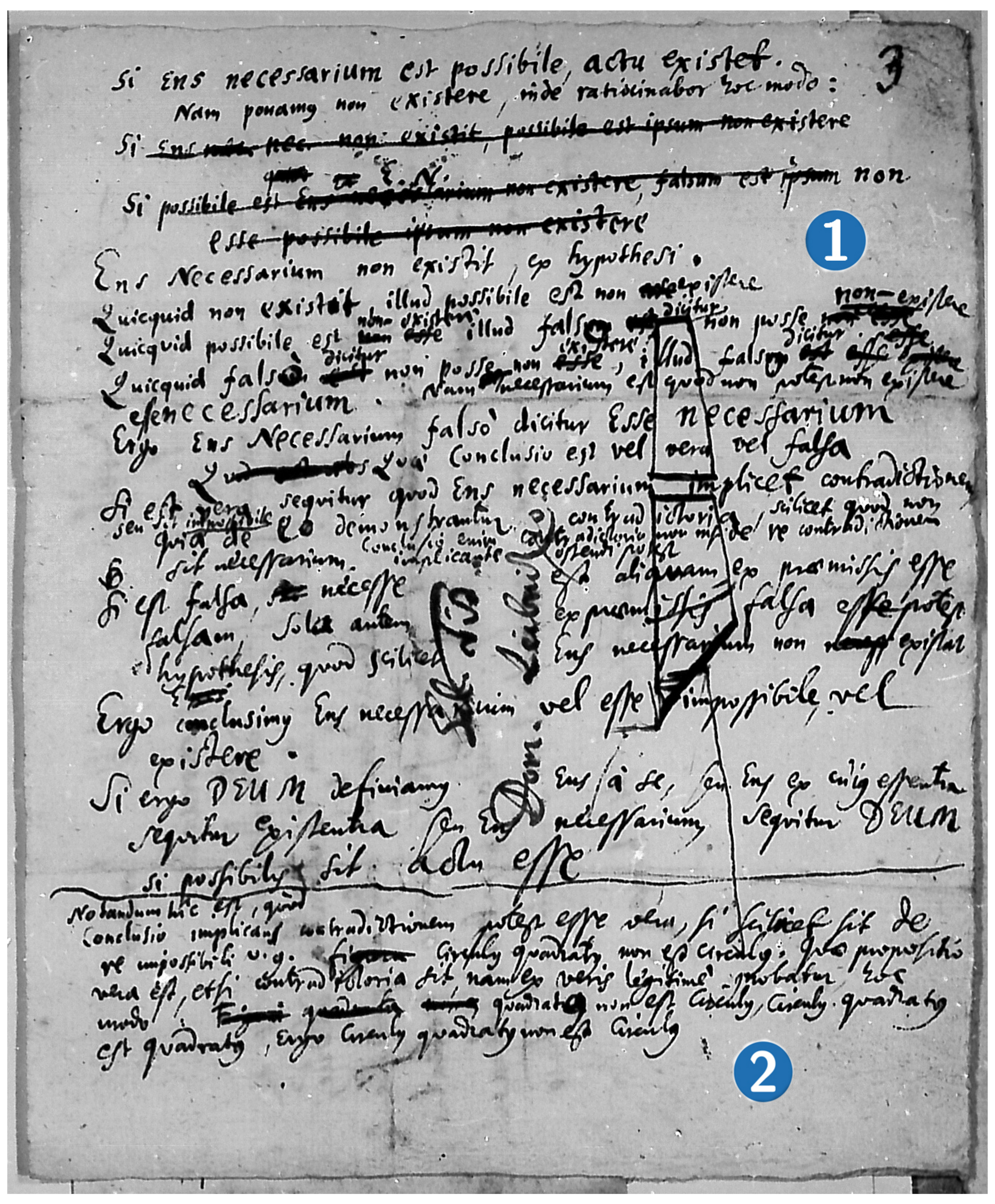

Figure 1: Version L^1 of Leibniz's proof which is the back of a letter by Huthmann to Leibniz

Formalization and Consistence

The following basic axiom for every adjective P seems evident:

$$\diamond_P \$x \implies \$x \tag{2}$$

There are more axioms for the context of a ticket inspection for every individual x:

$$\diamond_{ticket}\$x \implies \diamond_{valid}\$x \tag{3}$$

$$\diamond_{invalid}\$x \implies \neg\diamond_{valid}\,\$x \tag{4}$$

The formalization for an invalid ticket a

$$\diamond_{invalid}\diamond_{ticket}\$a \wedge \neg\diamond_{ticket}\,\$a \tag{5}$$

is consistent, as you can see in Figure 2.

Note that not any phrase of the form "AB" can be formalized as $\diamond_A\diamond_B$. E.g., a dark light is not something dark: It is light but relatively dark.

Figure 2: Model Structure with an invalid ticket a

Figure 3: Model Structure with a common circle c and a rectangular circle q

Formalization of ???

Note that the formalizition of is controversial. This new formalization is analogous with 3.

Example. $\diamond_{rectangle}\diamond_{circle}\$x \implies \neg\diamond_{circle}\,\$x.$

Proof. $\diamond_{rectangle}\,\$x \implies \neg\diamond_{circle}\,\$x.$
$\diamond_{rectangle}\diamond_{circle}\,\$x \implies \diamond_{rectangle}\,\$x.$
Hence the proposition follows. □

Results, conclusion and further work

Why did Leibniz say that a rectangular circle is impossible? As we have just shown, this is not reasonable. A specific rectangular circle x does not imply any contradiction (see Figure 3). Nevertheless we don't know what a rectangular circle looks like. It could be that never anybody gives the name "rectangular circle" to anything at any time. Hence there could impossibly be any rectangular circle. But how could that be observed?

Multimodal Logic is helpful to make use of the strength of adjectives in natural language within logic. This strength consists of a special composition of two adjectives. There are at least three types of predicate compositions in the logic presented here ($P(x)\wedge Q(x)$, $P(x)\vee Q(x)$ and $\diamond_P\diamond_Q\$x$). Another form of compositionality is provided by "Description Logics", e.g. the parent's parent is the grandparent.

On the other hand, the core part of compositionality in natural languages was described by the Indian linguist Panini more than 2000 years ago. The challenge of making formal logic more powerful than natural language is ongoing.

References

[1] Matthias Bentert, Christoph Benzmüller, David Streit, and Bruno Woltzenlogel Paleo. Analysis of an Ontological Proof Proposed by Leibniz. unpublished book chapter, URL: https://github.com/FormalTheology/GoedelGod/blob/master/Papers/2016/LeibnizProof, 2016.

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[3] Gottfried Wilhelm Leibniz. *Sämtliche Schriften und Briefe, Reihe II, Band 1.* Akademie Verlag, 2006.

[4] Wolfgang Lenzen. Leibniz's ontological proof of the existence of god and the problem of "impossible objects". In R. Silvestre, editor, *Logica Universalis(Special Issue on Logic and Religion)*. 2016.

Leibniz: *Calculemus!*

Computational Metaphysics is a interdisciplinary lecture course designed for advanced students of computer science, mathematics and philosophy. The main objective of the course is to teach the students how modern proof assistants based on expressive higher-order logic support the formal analysis of rational arguments in philosophy (and beyond). In our first course in Summer 2015 the focus has been on ontological arguments for the existence of God. However, some students picked formalisation projects also from other areas (including maths).

Computational Metaphysics was awarded the *Central Teaching Award 2015* of the FU Berlin.