HOL based First-order Modal Logic Provers

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Introduction

First-order (Multi-)Modal Logics (FMLs) . . .

. . . are just fragments of Classical Higher-Order Logic (HOL)

. . . and they can be automated with HOL ATPs.

[BenzmüllerPaulson, LogicaUniversalis, 2013]

Contribution of this paper

- FMLtoHOL tool: converts FML problems in qmf-syntax [RathsOtten, IJCAR, 2012] (which extends the TPTP fol-syntax with #box and #dia), into HOL problems in TPTP `thf0`-syntax.
- FMLtoHOL tool is exemplarily applied in combination with a metaprover for HOL, called HOL-P.
- Evaluation and comparison with other (direct) FML ATPs.
- Evaluation of different options wrt. to FMLtoHOL.

HOL — Church's Simple Type Theory [Church, 1940]

Simple Types $\alpha ::= \iota \mid o \mid \alpha_1 \rightarrow \alpha_2$

HOL — Church's Simple Type Theory [Church, 1940]

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Simple Types $\alpha ::= \iota \mid o \mid \alpha_1 \rightarrow \alpha_2$

Individuals

Booleans (True and False)

Functions/Predicates

HOL — Church's Simple Type Theory [Church, 1940]

Simple Types $\alpha ::= \iota \mid \mu \mid o \mid \alpha_1 \rightarrow \alpha_2$

Possible worlds

Individuals

Booleans (True and False)

Functions/Predicates

HOL — Church's Simple Type Theory [Church, 1940]

HOL

$$s, t \quad ::= \quad c_\alpha \mid x_\alpha \mid (\lambda x_\alpha\, s_\beta)_{\alpha\to\beta} \mid (s_{\alpha\to\beta}\, t_\alpha)_\beta \mid$$
$$(\neg_{o\to o}\, s_o)_o \mid (s_o \vee_{o\to o\to o} t_o)_o \mid (\forall x_\alpha\, t_o)_o$$

HOL — Church's Simple Type Theory [Church, 1940]

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HOL

$$s, t \quad ::= \quad c_\alpha \mid x_\alpha \mid (\lambda x_\alpha\, s_\beta)_{\alpha\to\beta} \mid (s_{\alpha\to\beta}\, t_\alpha)_\beta \mid$$
$$(\neg_{o\to o}\, s_o)_o \mid (s_o \vee_{o\to o\to o} t_o)_o \mid \underbrace{(\forall x_\alpha\, t_o)_o}$$

$$\Pi_{(\alpha\to o)\to o}\lambda x_\alpha t_o$$

HOL — Church's Simple Type Theory [Church, 1940]

HOL $\quad s, t \quad ::= \quad C \mid x \mid (\lambda x\, s) \mid (s\, t) \mid (\neg s) \mid (s \vee t) \mid (\forall x\, t)$

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HOL (with Henkin semantics) is meanwhile very well understood

- Origin [Church, J.Symb.Log., 1940]
- Henkin-Semantics [Henkin, J.Symb.Log., 1950]
  [Andrews, J.Symb.Log., 1971, 1972]
- Extensionality/Intensionality [BenzmüllerBrownKohlhase, J.Symb.Log., 2004]
  [Muskens, J.Symb.Log., 2007]

Embedding of First-order Modal Logic (FML) in HOL

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Embedding of First-order Modal Logic (FML) in HOL

HOL $\quad s, t \quad ::= \quad C \mid x \mid (\lambda x\, s) \mid (s\, t) \mid (\neg s) \mid (s \vee t) \mid (\forall x\, t)$

FML $\quad \varphi, \psi \quad ::= \quad P(t_1, \ldots, t_n) \mid (\neg \varphi) \mid (\varphi \vee \psi) \mid \Box \varphi \mid (\forall x\, \varphi)$

$$
\begin{array}{lll}
M, g, s \models \neg \varphi & \text{iff} & \text{not } M, g, s \models \varphi \\
M, g, s \models \varphi \vee \psi & \text{iff} & M, g, s \models \varphi \text{ or } M, g, s \models \psi \\
M, g, s \models \Box \varphi & \text{iff} & M, g, u \models \varphi \text{ for all } u \text{ with } r(s, u) \\
M, g, s \models \forall x\, \varphi & \text{iff} & M, [d/x]g, s \models \varphi \text{ for all } d \in D
\end{array}
$$

Embedding of First-order Modal Logic (FML) in HOL

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M,g,s \models \forall x\, \varphi & \text{iff} & M,[d/x]g,s \models \varphi \text{ for all } d \in D
\end{array}
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FML in HOL:

$$
\begin{array}{lll}
\neg & = & \lambda\varphi_{\iota\to o}\, \lambda s_\iota\, \neg\varphi s \\
\vee & = & \lambda\varphi_{\iota\to o}\, \lambda\psi_{\iota\to o}\, \lambda s_\iota\, (\varphi s \vee \psi s) \\
\Box_r & = & \lambda\varphi_{\iota\to o}\, \lambda s_\iota\, \forall u_\iota\, (\neg r s u \vee \varphi u) \\
\Pi & = & \lambda h_{\mu\to(\iota\to o)}\, \lambda s_\iota\, \forall d_\mu\, h d s
\end{array}
$$

($\forall x\varphi$ stands for $\Pi\lambda x\varphi$)

Embedding of First-order Modal Logic (FML) in HOL

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\end{array}
$$

Idea: Lifting of modal formulas to predicates on worlds

Metalevel notions: $\quad \text{valid} \quad = \quad \lambda\varphi_{\iota\to o}\, \forall s_\iota\, \varphi s$

Propositional Quantification

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Propositional Quantification [Fitting, J.Symb.Log., 2002]

...

$M, g, s \models \forall^p p\, \varphi$ iff $M, [v/p]g, s \models \varphi$ for all $v \in P$

(P is a non-empty collection of sets of worlds, it includes atom sets)

Embedding in HOL

...

$\Pi^p \quad = \quad \lambda h_{(\iota\to o)\to(\iota\to o)}\, \lambda s_\iota\, \forall v_\mu\, hvs$ ($\forall \varphi \psi$ stands for $\Pi^p \lambda \varphi \psi$)

Modal logic axioms	Semantical Condition
valid $\forall^p \varphi(\Box\varphi \supset \Diamond\varphi)$	$\forall x \exists y (rxy)$

Bridge rules	Semantical Condition
valid $\forall^p \varphi(\Box_r\varphi \supset \Box_s\varphi)$	$\forall x \forall y (rxy \supset sxy)$

We get a wide range of modal logics and combinations for free!

[BenzmüllerPaulson, LogicaUniversalis, 2013]

Propositional Quantification

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$\Pi^p \quad = \quad \lambda h_{(\iota\to o)\to(\iota\to o)}\, \lambda s_\iota\, \forall v_\mu\, hvs$ $\qquad$ ($\forall\varphi\psi$ stands for $\Pi^p\lambda\varphi\psi$)

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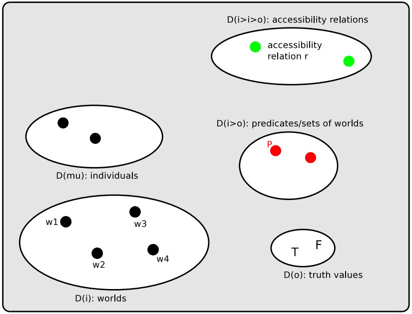
[BenzmüllerPaulson, LogicaUniversalis, 2013]

Varying and Cumulative Domain Quantification

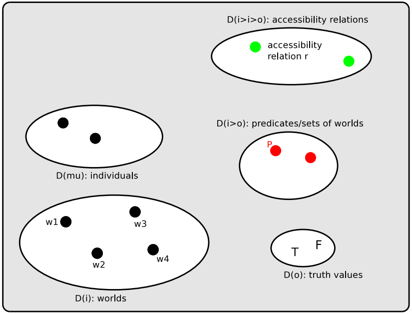
Constant Domain



$$\Pi = \lambda h \, \lambda w_\iota \, \forall x_\mu \, hxw$$

Varying and Cumulative Domain Quantification

Constant Domain



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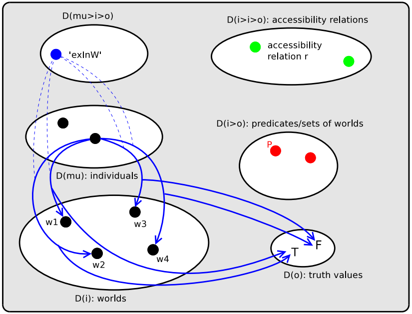
Varying and Cumulative Domain



$$\Pi^{va} = \lambda h\, \lambda w_\iota\, \forall x_\mu\, (\neg \mathbf{exInW} xw \vee hxw)$$

Varying and Cumulative Domain Quantification

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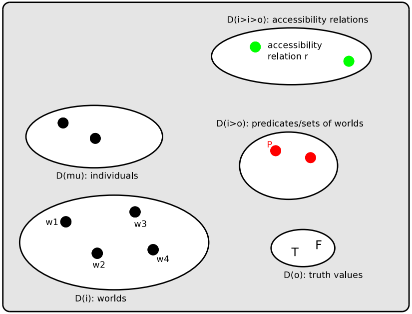
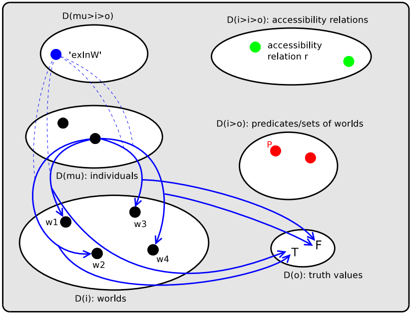
Constant Domain



Varying and Cumulative Domain



$\Pi = \lambda h\, \lambda w_\iota\, \forall x_\mu\, h x w$

$\Pi^{va} = \lambda h\, \lambda w_\iota\, \forall x_\mu\, (\neg \mathbf{exInW} x w \vee h x w)$

domains are non-empty → $\forall w_\iota\, \exists x_\mu\, \mathbf{exInW} x w$

denotation (constants & functions) →

$\forall w_\iota\, \mathbf{exInW} c w$

$\forall w_\iota\, (\mathbf{exInW} t^1 w \wedge \ldots \wedge \mathbf{exInW} t^n w \supset \mathbf{exInW} (f\, t^1 \ldots t^n) w)$

Varying and Cumulative Domain Quantification

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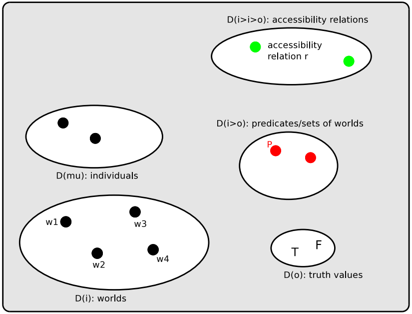
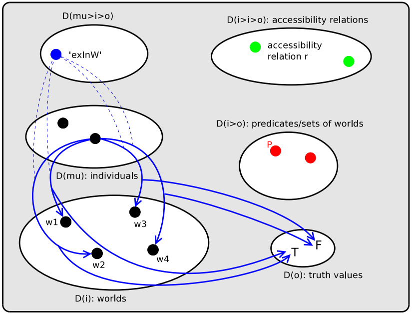
Constant Domain | **Varying and Cumulative Domain**





$$\Pi = \lambda h\, \lambda w_\iota\, \forall x_\mu\, hxw$$

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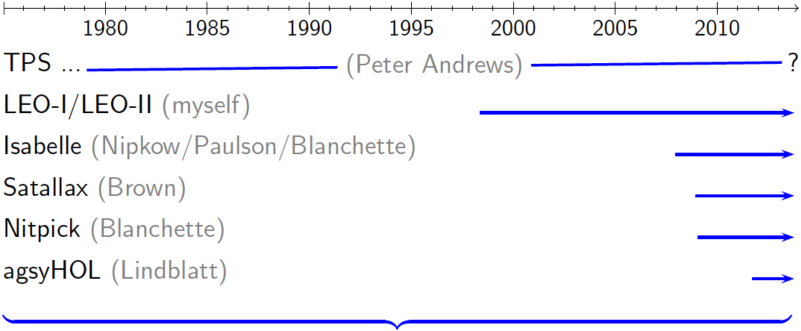
domains are non-empty → $\forall w_\iota\, \exists x_\mu\, \mathbf{exInW}\, xw$

denotation (constants & functions) → $\forall w_\iota\, \mathbf{exInW}\, cw$

$\forall w_\iota\, (\mathbf{exInW}\, t^1 w \wedge \ldots \wedge \mathbf{exInW}\, t^n w \supset \mathbf{exInW} (f\, t^1 \ldots t^n) w)$

cumulative domains → $\forall x, v, w\, (\mathbf{exInW}\, xv \wedge rvw \supset \mathbf{exInW}\, xw)$

Automated Theorem Provers and Model Finders for HOL



- all accept TPTP THF Syntax [SutcliffeBenzmüller, J.Form.Reas, 2009]
- can be called remotely via SystemOnTPTP at Miami
- they significantly gained in strength over the last years
- they can be bundled into a combined prover HOL-P

Exploit HOL with Henkin semantics as metalogic
Automate other logics (& combinations) via semantic embeddings
— HOL-P becomes a Universal Reasoner —

FMLtoHOL

- implemented as part of Sutcliffe's TPTP2X tool
- included in the QMLTP—v1.1 package available at: `http://www.iltp.de/qmltp/problems.html`
- written in Prolog, can be easily modified and extended
- invoked as

  ```
  ./tptp2X -f thf:<logic>:<domain> <qmf-file>
  ```

  where `<logic>` $\in$ {k,k4,d,d4,t,s4,s5} and `<domain>` $\in$ {*const*, *vary*, *cumul*}.
- generates TPTP thf0-files; employs `include`-mechanism
- can easily be combined (shell script) with HOL-P metaprover

FMLtoHOL combined with HOL-P

FO Modal Logic example: $(\Diamond \exists x P f x \wedge \Box \forall y (\Diamond P y \Rightarrow Q y)) \Rightarrow \Diamond \exists z Q z$

FMLtoHOL combined with HOL-P

FO Modal Logic example: $(\Diamond\exists x Pfx \wedge \Box\forall y(\Diamond Py \Rightarrow Qy)) \Rightarrow \Diamond\exists z Qz$

```
%> ./FMLtoHOL-P example.thf -timeout 20 -logic s4 -domain varying

  qmf(con,conjecture,
      ( ((#dia:  ? [X] : p(f(X))) & (#box: ! [Y]: ((#dia: p(Y)) => q(Y))))
        => #dia: ? [Z] : q(Z) )).
```

$\longrightarrow$

```
%----Include axioms for modal logic D under constant domains
include('Axioms/LCL013^0.ax.const').
include('Axioms/LCL013^2.ax').
%-------------------------------------------------------------------
thf(q_type,type,( q: mu > $i > $o )).
thf(p_type,type,( p: mu > $i > $o )).
thf(f_type,type,(f: mu > mu )).
thf(con,conjecture, ( mvalid @
    ( mimplies @
      ( mand @
        ( mdia_d @ ( mexists_ind @ ^ [X: mu] : ( p @ ( f @ X ) ) ) ) @
        ( mbox_d @ ( mforall_ind @ ^ [Y: mu] :
            ( mimplies @ ( mdia_d @ ( p @ Y ) ) @ ( q @ Y ) ) ) ) ) @
      ( mdia_d @ ( mexists_ind @ ^ [Z: mu] : ( q @ Z ) ) ) ) )).
```

FMLtoHOL combined with HOL-P

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Calling HOL Resoners remotely in Miami . . . thanks to Geoff Sutcliffe

— LEO-II says Theorem — CPU 0.08s

— Satallax says Theorem — CPU 0.03s

— Isabelle says Unknown — CPU 11.93s

— Nitpick says Unknown — CPU 10.62s

— agsyHOL says Theorem — CPU 0.55s

FMLtoHOL combined with HOL-P

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%> ./FMLtHOL-P example.thf -timeout 20 -logic k -domain constant

FMLtoHOL combined with HOL-P

FO Modal Logic example: $(\Diamond \exists x P f x \wedge \Box \forall y (\Diamond P y \Rightarrow Q y)) \Rightarrow \Diamond \exists z Q z$

```
%> ./FMLtoHOL-P example.thf -timeout 20 -logic s4 -domain varying
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```
%> ./FMLtHOL-P example.thf -timeout 20 -logic k -domain constant
```

Calling HOL Resoners remotely in Miami . . . thanks to Geoff Sutcliffe

— LEO-II says Unknown — CPU 11.93s
— Satallax says CounterSatisfiable — CPU 0.04s
— Isabelle says Unknown — CPU 16.19s
— Nitpick says CounterSatisfiable — CPU 8.19s
— agsyHOL says Unknown — CPU 10.82s

Evaluation: FML's (S4— constant/varying/cumulative)

No. of solved monomodal problems (out of 580, 600sec timeout)

	MleanSeP labelled sequents	**MleanTAP** labelled tableaux	**f2p-MSPASS** instant. & transform.	**MleanCoP** labelled connections	**HOL-P**
Logic S4, constant domains					
Theorem	197	220	111	**352**	300
Non-Theorem	1	4	36	82	**132**
Solved	198	224	147	**434**	432
Logic S4, cumulative domains					
Theorem	197	205	121	**338**	278
Non-Theorem	4	4	41	94	**146**
Solved	201	209	162	**432**	424
Logic S4, varying domains					
Theorem	-	169	-	**274**	245
Non-Theorem	-	4	-	119	**184**
Solved	-	173	-	393	**429**

Evaluation: FML's (D — constant/varying/cumulative)

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No. of solved monomodal problems (out of 580 problems, 600sec timeout, inHOL-P a timeout of 120s was given to each of the 5 subprovers.)

	MleanSeP labelled sequents	**MleanTAP** labelled tableaux	**f2p-MSPASS** instant. & transform.	**MleanCoP** labelled connections	**HOL-P**
Logic D, constant domains					
Theorem	135	134	76	**217**	208
Non-Theorem	1	4	107	209	**250**
Solved	136	138	183	426	**458**
Logic D, cumulative domains					
Theorem	130	120	79	**200**	184
Non-Theorem	4	4	108	224	**269**
Solved	134	124	187	424	**453**
Logic D, varying domains					
Theorem	-	100	-	**170**	163
Non-Theorem	-	4	-	243	**295**
Solved	-	104	-	413	**458**

Evaluation: FML's (S4— constant/varying/cumulative)

Table : Individual performances of the subprovers of HOL-P in the Theorem-category with respect to the experiments for S4. Results are presented for constant domain (const), cumulative domain (cum) and varying domain (vary) semantics.

Logic S4 **Theorem**	**Isabelle** const/cum/vary	**LEO-II** const/cum/vary	**agsyHOL** const/cum/vary	**Satallax** const/cum/vary
syn	177/126/120	213/187/163	231/192/171	**244/233/207**
sem	252/215/192	227/203/183	247/206/183	**257/239/214**
total	1082	1176	1230	**1394**

Table : Individual performances of the subprovers of HOL-P in the Non-Theorem-category with respect to the experiments for S4.

Logic S4 **Non-Theorem**	**Satallax** const/cum/vary	**Nitpick** const/cum/vary
syn	0/0/0	**132/145/185**
sem	48/56/68	**132/146/185**
total	172	**925**

Evaluation: Coverage of Logics

ATP system	supported modal logics	supported domain cond.
MleanSeP 1.2	K,K4,D,D4,T,S4	constant,cumulative
MleanTAP 1.3	D,T,S4,S5	constant,cumulative,varying
MleanCoP 1.2	D,T,S4,S5	constant,cumulative,varying
f2p-MSPASS 3.0	K,K4,K5,B,D,T,S4,S5	constant,cumulative
HOL-P	K,K4,K5,B,D,D4,T,S4,S5,...	constant,cumulative,varying

HOL-P directly applicable also for multi-modal logics.

Conclusion

Presented a HOL based ATP and Model Finder for First-order Modal Logics

- covers arbitrary logics extending base logic K
- is very competitive (strongest 'solver' to date)
- semantic axioms should be preferred over syntactic ones
- simple, lean, and elegant approach
- **most important: the presented approach**
  - is applicable to many other non-classical logics
  - supports multi-modal logics and logic combinations
  - supports meta-level reasoning
  - has many interesting applications ...

Some Recent Related Work

Automation of Kurt Ködel's Ontological Argument

in Second-order Modal Logic (KB)

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Holy Logic: Computer Scientists 'Prove' God Exists

By David Knight

picture-alliance/ Imagno/ Wiener Stadt- und Landesbibliothek

Austrian mathematician Kurt Gödel kept his proof of God's existence a secret for decades. Now two scientists say they have proven it mathematically using a computer.

Two scientists have formalized a theorem regarding the existence of God penned by mathematician Kurt Gödel. But the God angle is somewhat of a red herring -- the real step forward is the example it sets of how computers can make scientific progress simpler.