Integrating TPS with OMEGA

OMEGA: HO theorem proving environment developed at Univ. des Saarlandes, Germany
TPS: HO theorem proving environment developed at CMU, Pittsburgh, USA

Motivation and Architecture

Motivation

- Both Systems are theorem proving environments with interactive and automatic components
- OMEGA and TPS have different priorities:
  OMEGA: proof planning, user interaction, proof presentation & verbalization, external reasoners
  TPS: automatic theorem proving with mating method and interactive theorem proving in ND
  -> Both systems can benefit from an integration.
- similarities ease the integration: HO Logic (based on Church's simply typed lambda calculus)
  Proof representation in ND–calculus.

Integration Architecture

Benefits for OMEGA

- a powerful external reasoner for HO logics
- an external reasoner that can use concepts from OMEGA's knowledge base
- the possibility to use TPS's mechanism for selectively expanding definitions
- a well tested theorem proving environment

Benefits for TPS

- a proof visualization and verbalization tool
- access to OMEGA's knowledge base of mathematical theories

In a bidirectional integration:

- access to all of OMEGA's features like proof planning or the integrated External Reasoners

Calling TPS from OMEGA: The steps A – E

A: Translation to TPS

- extract subproblem and embedded definitions from knowledge base
- straight forward syntax translation
- memorize type–symbol mapping proof line mapping

Example: Cantor's Theorem

$\forall G(oi).\text{smaller-cardinality } G \text{ (powerset } G)$

TPS input (with definitions):

B: Proof search in TPS

- either automatic proof search (mating method) or interactive proof construction in ND calculus
- clever mechanism for selectively expanding definitions
- result: (partial) proof in ND calculus

TPS proof of Cantor's Theorem:

C: Translation to OMEGA

- TPS proof becomes OMEGA proof plan
- TPS–theory in OMEGA provides one tactic for each TPS justification

D: Proof plan insertion

- insertion of original TPS proof as proof plan into OMEGA's PDS
- visualization of TPS proofs

Original TPS proof becomes a Proof Plan in OMEGA:

E: Proof transformation = proof plan expansion

- Proof plan expansion transforms the TPS ND proof into OMEGA ND proof
- different types of tactic mappings:
  a) 1:1 mappings: tps*UI –> forall1 or tps*Lambda –> lambda
  b) case distinctions: tps*Ext= –> exti or Ext=
  a) restructuring: tps*RuleC –> existse
  c) external reasoners: tps*RuleP –> propositional logic ATP

A case distinction mapping (Ext=):

Mapping RuleP to a propositional logic ATP:

Conclusion and Further Work

The two HO theorem proving environments TPS and OMEGA benefit from the integration as they have different priorities that can support each other

Some interesting aspects of the integration:
- TPS uses OMEGA's knowledge base
- integration of TPS as an interactive or automatic External Reasoner
- 1:1 translation of TPS proofs into OMEGA proof plans (Visualization of original TPS proofs)
- Proof transformation TPS <-> OMEGA = proof plan expansion/unexpansion in OMEGA

General experience:

Theorem prover can be easily combined if at least one of the systems can model the logic of the other

The combination of several interactive theorem provers gives a user the advantage of using his/her favorite tool without foregoing the benefits of others

Other external system can easify and support the integration of new ones

Further Work

Design and implementation of a fully bidirectional integration of the two systems

Usage of TPS in a proof planning scenario

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