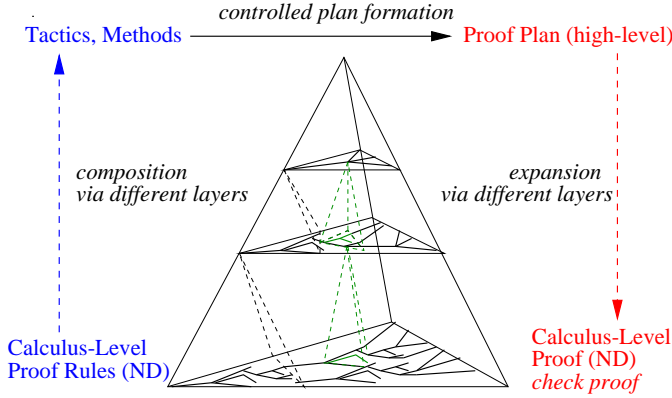# Proof Transformation and Expansion with a Parameterizable Inference Machine

Christoph Benzmüller, Andreas Meier, Martin Pollet, Volker Sorge

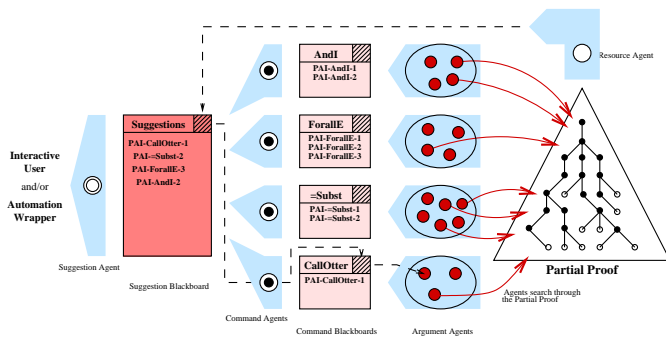## Hierarchical Proof Data Structure

controlled plan formation

Tactics, Methods → Proof Plan (high-level)

composition via different layers

expansion via different layers

Calculus-Level Proof Rules (ND)

Calculus-Level Proof (ND) *check proof*

* **Proof creation at abstract level with tactics and methods**
* **Expansion in a three dimensional data structure**

Expansion with Parameterizable Inference Machine

Goal Inferences   Control Information   ???

Abstract Step → **Parameterizable Inference Machine** → Expansion of Step
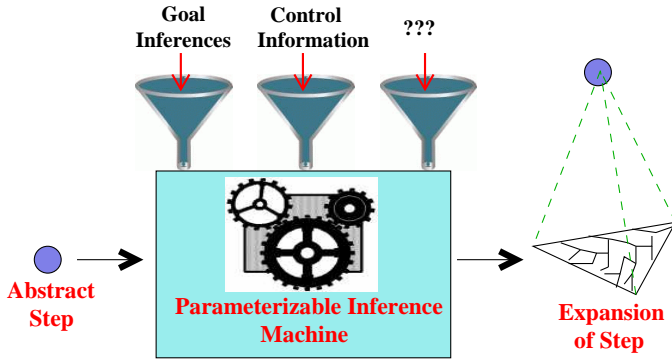
* **Instantiating a parameterizable Inference Machine with information relevant for particular expansion**
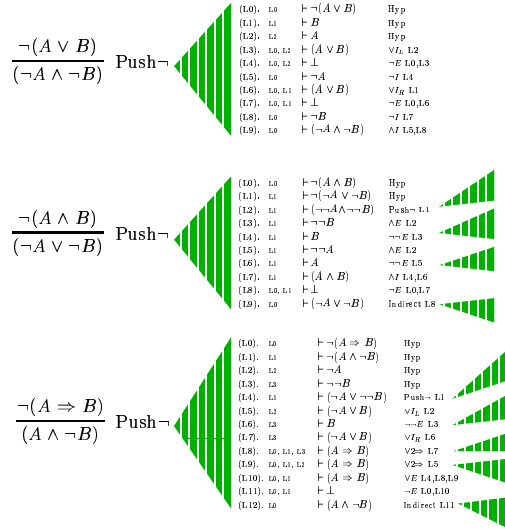   * **General expansion mechanism**

Agent Based Deduction with Ω Ants

Resource Agent

Interactive User and/or Automation Wrapper

Suggestions
PAI-CallOtter-1
PAI-=Subst-2
PAI-ForallE-3
PAI-AndI-2

Suggestion Agent

Suggestion Blackboard

AndI
PAI-AndI-1
PAI-AndI-2

ForallE
PAI-ForallE-1
PAI-ForallE-2
PAI-ForallE-3

=Subst
PAI-=Subst-1
PAI-=Subst-2

CallOtter
PAI-CallOtter-1

Command Agents

Command Blackboards

Argument Agents

Partial Proof

Agents search through the Partial Proof

* **ΩAnts is as parameterizable inference machine**
* **Agents and heuristics can be defined at run time**
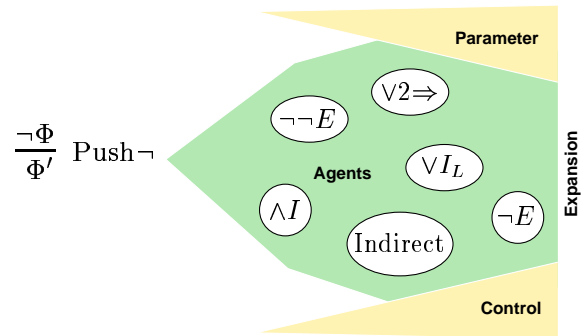* **Incorporation of control information by pre-instantiation of blackboards and heuristics**

Procedural Expansion

Expansion of tactic PUSHNEG:

$$\frac{\neg(A \vee B)}{(\neg A \wedge \neg B)} \quad \text{Push}\neg$$

(L0).	L0	$\vdash \neg(A \vee B)$	Hyp
(L1).	L1	$\vdash B$	Hyp
(L2).	L2	$\vdash A$	Hyp
(L3).	L0, L2	$\vdash (A \vee B)$	$\vee I_L$ L2
(L4).	L0, L2	$\vdash \bot$	$\neg E$ L0,L3
(L5).	L0	$\vdash \neg A$	$\neg I$ L4
(L6).	L0, L1	$\vdash (A \vee B)$	$\vee I_R$ L1
(L7).	L0, L1	$\vdash \bot$	$\neg E$ L0,L6
(L8).	L0	$\vdash \neg B$	$\neg I$ L7
(L9).	L0	$\vdash (\neg A \wedge \neg B)$	$\wedge I$ L5,L8

$$\frac{\neg(A \wedge B)}{(\neg A \vee \neg B)} \quad \text{Push}\neg$$

(L0).	L0	$\vdash \neg(A \wedge B)$	Hyp
(L1).	L1	$\vdash \neg(\neg A \vee \neg B)$	Hyp
(L2).	L1	$\vdash (\neg\neg A \wedge \neg\neg B)$	Push¬ L1
(L3).	L1	$\vdash \neg\neg B$	$\wedge E$ L2
(L4).	L1	$\vdash B$	$\neg\neg E$ L3
(L5).	L1	$\vdash \neg\neg A$	$\wedge E$ L2
(L6).	L1	$\vdash A$	$\neg\neg E$ L5
(L7).	L1	$\vdash (A \wedge B)$	$\wedge I$ L4,L6
(L8).	L0, L1	$\vdash \bot$	$\neg E$ L0,L7
(L9).	L0	$\vdash (\neg A \vee \neg B)$	Indirect L8

$$\frac{\neg(A \Rightarrow B)}{(A \wedge \neg B)} \quad \text{Push}\neg$$

(L0).	L0	$\vdash \neg(A \Rightarrow B)$	Hyp
(L1).	L1	$\vdash \neg(A \wedge \neg B)$	Hyp
(L2).	L2	$\vdash A$	Hyp
(L3).	L3	$\vdash \neg\neg B$	Hyp
(L4).	L1	$\vdash (\neg A \vee \neg\neg B)$	Push¬ L1
(L5).	L2	$\vdash (\neg A \vee B)$	$\vee I_L$ L2
(L6).	L3	$\vdash B$	$\neg\neg E$ L3
(L7).	L3	$\vdash (\neg A \vee B)$	$\vee I_R$ L6
(L8).	L0, L1, L3	$\vdash (A \Rightarrow B)$	$\vee 2 \Rightarrow$ L7
(L9).	L0, L1, L2	$\vdash (A \Rightarrow B)$	$\vee 2 \Rightarrow$ L5
(L10).	L0, L1	$\vdash (A \Rightarrow B)$	$\vee E$ L4,L8,L9
(L11).	L0, L1	$\vdash \bot$	$\neg E$ L0,L10
(L12).	L0	$\vdash (A \wedge \neg B)$	Indirect L11

* **Justification: tactic/method + parameters (for expansion)**
* **Expansion: hardwired programming code**
* **Modifications of a tactic results in reimplementing**
    * **the expansion procedure of the tactic itself**
    * **other expansion procedures employing this tactic**

Expansion with Ω Ants

$$\frac{\neg\Phi}{\Phi'} \quad \text{Push}\neg$$

Parameter

Expansion

Agents: $\neg\neg E$, $\vee 2 \Rightarrow$, $\vee I_L$, $\wedge I$, $\neg E$, Indirect

Control

* **Employing the ΩAnts mechanism with**
    * **Inferences**
    * **Control Information**

    creates expansion by proof search

Discussion:

+ **Not hardwired, reduced maintenance**
+ **Flexible and adaptable**
? **Non-determinism in:**
    Proof search
    Sub-proof structure (abstraction level)
? **Limits with respect to procedural control information**