

# A Top-down Approach to Combining Logics

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## Outline/topics addressed

- A** Motivation:  
combining logics, context, expressive ontologies
- B** Example:  
first-order monomodal logic is a fragment of HOL  
constant/varying/cumulative domain  
first-order multimodal logic is a fragment of HOL  
propositional quantification  
bridge rules
- C** Many non-classical logics are natural fragments of HOL
- D** Proof automation

## Combining logics

- ▶ prominent challenge in AI (CS, Philosophy)
- ▶ epistemic, deontic, temporal, intuitionistic, relevant, linear, conditional, security . . .
- ▶ wide literature—few implementations
- ▶ some propositional systems exists: Logic Workbench, LoTREC, Tableaux Workbench, FaCT, ileanCoP, MSPASS
- ▶ no implemented systems for combinations of first-order logics
- ▶ combination is typically approached **bottom-up**

**My approach is complementary:**

works **top-down** starting from classical higher-order logic (HOL)

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- ▶ McCarthy: modeling of contexts as first-class objects

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ist(context_of("Ben's Knowledge"), likes(Sue, Bill))
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ist(context_of("Ben's Knowledge"),  
    ist(context_of(...), ...))
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- ▶ McCarthy's approach has been followed by many others
- ▶ Giunchiglia emphasizes locality aspect; structured knowledge
- ▶ McCarthy and Giunchiglia **avoid modal logics**
- ▶ they also **avoid a HOL perspective**

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### Expressive ontologies

- ▶ SUMO and Cyc
- ▶ modeling of contexts:

```
(holdsDuring (yearFn 2009 (loves Bill Mary)))
```

```
(believes Bill  
  (knows Ben  
    (forall (?X)  
      ((woman ?X) => (loves Bill ?X))))))
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- ▶ relation to McCarthy's approach is obvious
- ▶ often a questionable semantics assumed for embedded formulas and modal predicates (also in Common Logic)

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## A top-down approach to combining logics

- ▶ many non-classical logics are just natural fragments of HOL (via an elegant semantic embedding)
- ▶ they can be easily combined in HOL
- ▶ object-level reasoning enabled with off-the-shelf HOL provers and model finders
- ▶ even meta-level reasoning is feasible

### Key idea of the approach:

Bridge between the Tarski view of logics (for meta-logic HOL) and the Kripke view of logics (for the embedded logics)

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First-order Modal Logics (**FMLs**)

$$p, q ::= P(t_1, \dots, t_n) \mid (\neg p) \mid (p \vee q) \mid \Box p \mid (\forall x p)$$

are relevant for many applications, including

- ▶ planning
- ▶ natural language processing
- ▶ program verification
- ▶ modeling communication
- ▶ querying knowledge bases

Until recently, however, there has been

- ▶ a comparably large body of theory papers on FMLs
- ▶ but only **one implemented prover!** (GQML prover)

For recent progress see:

[BenzmüllerOttensRaths, ECAI, 2012]

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## B: Example — Embedding of FML in HOL

Simple Types

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Individuals

Booleans (True and False)

Functions/Predicates



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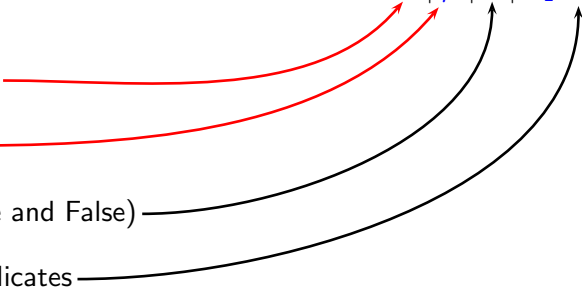
$\alpha ::= \iota \mid \mu \mid o \mid \alpha_1 \rightarrow \alpha_2$

Possible worlds

Individuals

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Functions/Predicates





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HOL  $s, t ::= C_\alpha \mid x_\alpha \mid (\lambda x_\alpha s_\beta)_{\alpha \rightarrow \beta} \mid (s_{\alpha \rightarrow \beta} t_\alpha)_\beta \mid$   
 $(\neg_{o \rightarrow o} s_o)_o \mid (s_o \vee_{o \rightarrow o \rightarrow o} t_o)_o \mid (\forall x_\alpha t_o)_o$

Constant Symbols

Variable Symbols

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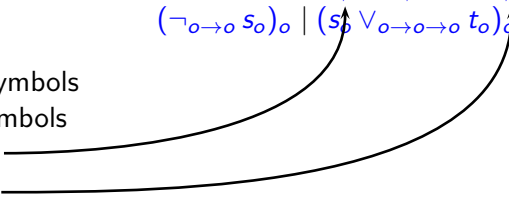
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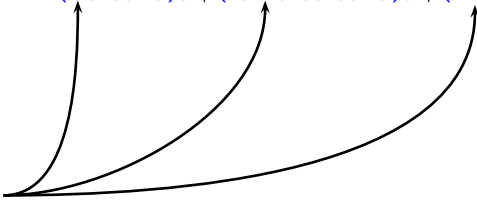
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HOL TPTP Infrastructure

HOL Provers: LEO-II, Satallax, TPS, Isabelle, Nitpick, agsyHOL



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**FML**  $p, q ::= P(t_1, \dots, t_n) \mid (\neg p) \mid (p \vee q) \mid \Box p \mid (\forall x p)$

$M, g, s \models \neg p$	iff	not $M, g, s \models p$
$M, g, s \models p \vee q$	iff	$M, g, s \models p$ or $M, g, s \models q$
$M, g, s \models \Box p$	iff	$M, g, u \models p$ for all $u$ with $R(s, u)$
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**FML** in **HOL**:

$\neg$	=	$\lambda p_{\iota \rightarrow o} \lambda w_{\iota} \neg p w$
$\vee$	=	$\lambda p_{\iota \rightarrow o} \lambda q_{\iota \rightarrow o} \lambda w_{\iota} (p w \vee q w)$
$\Box$	=	$\lambda p_{\iota \rightarrow o} \lambda w_{\iota} \forall v_{\iota} (\neg R w v \vee p v)$
$\Pi$	=	$\lambda h_{\mu \rightarrow (\iota \rightarrow o)} \lambda w_{\iota} \forall x_{\mu} h x w$ now $\forall x p$ stands for $\Pi \lambda x p$

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Meta-level notions: **valid**  $= \lambda p_{\iota \rightarrow o} \forall w_{\iota} p w$

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Meta-level notions: **valid** =  $\lambda p_{\iota \rightarrow o} \forall w_{\iota} p w$

Main idea: Lifting of modal formulas to predicates on worlds

## B: Example — Embedding of FML in HOL

$$(\Diamond \exists x P f x \wedge \Box \forall y (\Diamond P y \Rightarrow Q y)) \Rightarrow \Diamond \exists z Q z$$

*valid*  $(\Diamond \exists x P f x \wedge \Box \forall y (\Diamond P y \Rightarrow Q y)) \Rightarrow \Diamond \exists z Q z$

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### Axiomatization of properties of accessibility relation $R$

Logic K: no axioms

Logic T: (*reflexive*  $R$ ) — which expands into  $\forall x Rxx$

Logic S4: (*reflexive*  $R$ )  $\wedge$  (*symmetric*  $R$ )  $\wedge$  (*transitive*  $R$ )

Logic ... ..

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Logic ... ..

This automates **FML** with constant domain semantics in **HOL**

### To obtain varying domain semantics:

- ▶ modify quantifier:  $\Pi = \lambda q \lambda w \forall x \text{ExistsIn } W_{xw} \Rightarrow qxw$
- ▶ add non-emptiness axiom:  $\forall w \exists x \text{ExistsIn } W_{xw}$
- ▶ add designation axioms for constants  $c$ :  $\forall w \text{ExistsIn } W_{cw}$   
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### To obtain cumulative domain semantics:

- ▶ add axiom:  $\forall x \forall v \forall w \text{ExistsIn } W_{xv} \wedge Rvw \Rightarrow \text{ExistsIn } W_{xw}$

### What extras are needed?

- ▶ instead of  $\Box = \lambda p \lambda w \forall v (\neg(Rwv) \vee (pv))$   
consider  $\Box = \lambda r \lambda p \lambda w \forall v (\neg(rwv) \vee (pv))$
- ▶ now we may have:  $\Box_{knowledgeBen}, \Box_{commonKnowledge}, \dots$
- ▶ we can add quantification over propositional variables

$$\Box^P = \lambda q_{(\iota \rightarrow o) \rightarrow (\iota \rightarrow o)} \lambda w_\iota \forall p_{\iota \rightarrow o} (qp w) \quad (\forall pq \text{ stands for } \Box^P \lambda p q)$$

- ▶ and use this to explicitly encode bridge rules

$$\forall p (\Box_{commonKnowledge} p \supset \Box_{knowledgeBen} p)$$

### What can we do with that?

- ▶ actually a lot
- ▶ see e.g. the elegant modeling and effective solution of the Wise Men Puzzle as reported in the paper

## Soundness and completeness

$$\models \varphi \quad \text{iff} \quad \models^{HOL} \text{valid } \varphi_{\iota \rightarrow o}$$

### results do already exist for

- ▶ propositional multimodal logics [BenzmüllerPaulson, Log.J.IGPL, 2010]
- ▶ quantified multimodal logics [BenzmüllerPaulson, Logica Universalis, 2012]
- ▶ propositional conditional logics [BenzmüllerEtAl., AMAI, 2012]
- ▶ intuitionistic logics: [BenzmüllerPaulson, Log.J.IGPL, 2010]
- ▶ access control logics: [Benzmüller, IFIP SEC, 2009]
- ▶ combinations of logics: [Benzmüller, AMAI, 2011]
- ▶ ... more is on the way ...



# C: Why Not Throwing Things Together?

Terms:

$$m ::= C \mid x \mid (F m^1 \dots m^n)$$

Formulas:

$$s, t ::= (k m^1 \dots m^n) \mid \neg s \mid s \vee t \mid \Box_r s \mid s \Rightarrow_f t \mid \dots$$

$$\forall x s \mid \forall_{\text{vary}} x s \mid \forall_{\text{cumul}} x s \mid \forall^p p s \mid \dots$$

Embedding in HOL:

$$C = C_\mu \quad x = x_\mu \quad F = F_{\mu^n \rightarrow \mu}$$

$$k = k_{\mu^n \rightarrow \iota \rightarrow o}$$

$$r = r_{\iota \rightarrow \iota \rightarrow o} \quad (+\text{axioms for } r) \quad f = f_{\iota \rightarrow \iota \rightarrow o} \quad (+\text{axioms for } f)$$

$$\neg = \lambda s_{\iota \rightarrow o} \lambda w_{\iota} \neg s w$$

$$\vee = \lambda s_{\iota \rightarrow o} \lambda t_{\iota \rightarrow o} \lambda w_{\iota} (s w \vee t w)$$

$$\Box = \lambda r_{\iota \rightarrow \iota \rightarrow o} \lambda s_{\iota \rightarrow o} \lambda w_{\iota} \forall v_{\iota} \neg r w v \vee s v$$

$$\Rightarrow = \lambda f_{\iota \rightarrow (\iota \rightarrow o) \rightarrow o} \lambda s_{\iota \rightarrow o} \lambda t_{\iota \rightarrow o} \lambda w_{\iota} \forall v_{\iota} (\neg f w s v \vee t v)$$

$$\Pi = \lambda q_{\mu \rightarrow (\iota \rightarrow o)} \lambda w_{\iota} \forall x_{\mu} q x w$$

$$\Pi_{\text{var}/\text{cumul}} = \lambda q_{\mu \rightarrow (\iota \rightarrow o)} \lambda w_{\iota} \forall x_{\mu} \neg \text{exIn } W x w \vee q x w$$

$$\Pi^p = \lambda q_{(\iota \rightarrow o) \rightarrow (\iota \rightarrow o)} \lambda w_{\iota} \forall p_{\iota \rightarrow o} q p w$$

... further non-classical connect., quantif. over higher types,  
predicate abstraction, definite description ...

# D: Proof Automation — How Competitive is HOL?

FML Experiment: **580 problems**  $\times$  **5 logics**  $\times$  **3 domain cond.**  $\times$  **6 provers**  $\times$  **600s tmo**

**8700 problems**

Logic/ Domain	ATP system					
	f2p-MSPASS v3.0	MleanSeP v1.2	LEO-II v1.4.2	Satallax v2.2	MleanTAP v1.3	MleanCoP v1.2
K/varying	-	-	72	104	-	-
K/cumul.	70	121	89	122	-	-
K/constant	67	124	120	146	-	-
D/varying	-	-	128 81	113	100	179
D/cumul.	79	130	144 100	133	120	200
D/constant	76	134	167 135	160	135	217
T/varying	-	-	170 120	170	138	224
T/cumul.	105	163	190 139	192	160	249
T/constant	95	166	217 173	213	175	269
S4/varying	-	-	140	207	169	274
S4/cumul.	121	197	218 166	238	205	338
S4/constant	111	197	244 200	261	220	352
S5/varying	-	-	169	248	219	359
S5/cumul.	140	-	215	297	272	438
S5/constant	131	-	237	305	272	438

# D: Proof Automation — How Competitive is HOL?

FML Experiment: **580 problems** × **5 logics** × **3 domain cond.** × **6 provers** × **600s tmo**

Logic/ Domain	ATP system					
	f2p-MSPASS v3.0	MleanSeP v1.2	LEO-II v1.3.2	Satallax v2.2	MleanTAP v1.3	MleanCoP v1.2
K/varying	-	-	72	104	-	-
K/cumul.	70	121	89	122	-	-
K/constant	67	124	120	146	-	-
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**Strongest Prover!**  
A specialist system.

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K/cumul.	70	121	89	122	-	-
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HOL provers, 2nd best  
Strong recent improvements

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Results for 20 multimodal logic problems: LEO-II 15, Satallax 14

## I have argued that:

- ▶ many non-classical logics are natural fragments of HOL
- ▶ they can easily be combined in HOL
- ▶ they can be automated in HOL (object-level and meta-level)
- ▶ automation of HOL is currently making good progress
- ▶ **we get reasoners for expressive non-classical logics (and their combinations) for free**
- ▶ for many of those no practical systems are available yet
- ▶ this is relevant for: context and expressive ontologies

## Ongoing & future work:

- ▶ automation of expressive ontologies, e.g. SUMO
- ▶ proper semantics for SUMO
- ▶ further applications

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