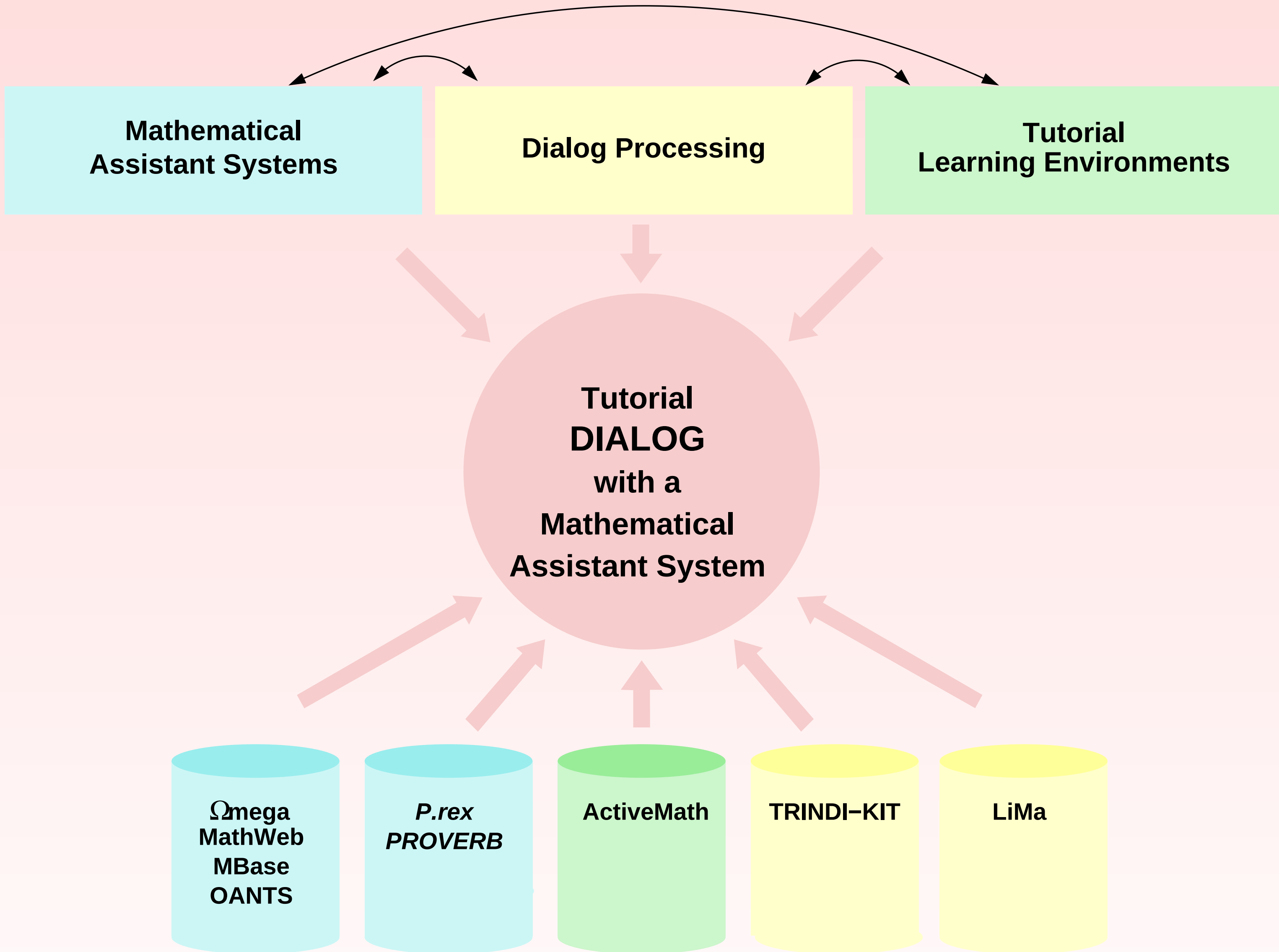
The DIALOG Project

Tutorial Dialog with a Mathematical Assistant System

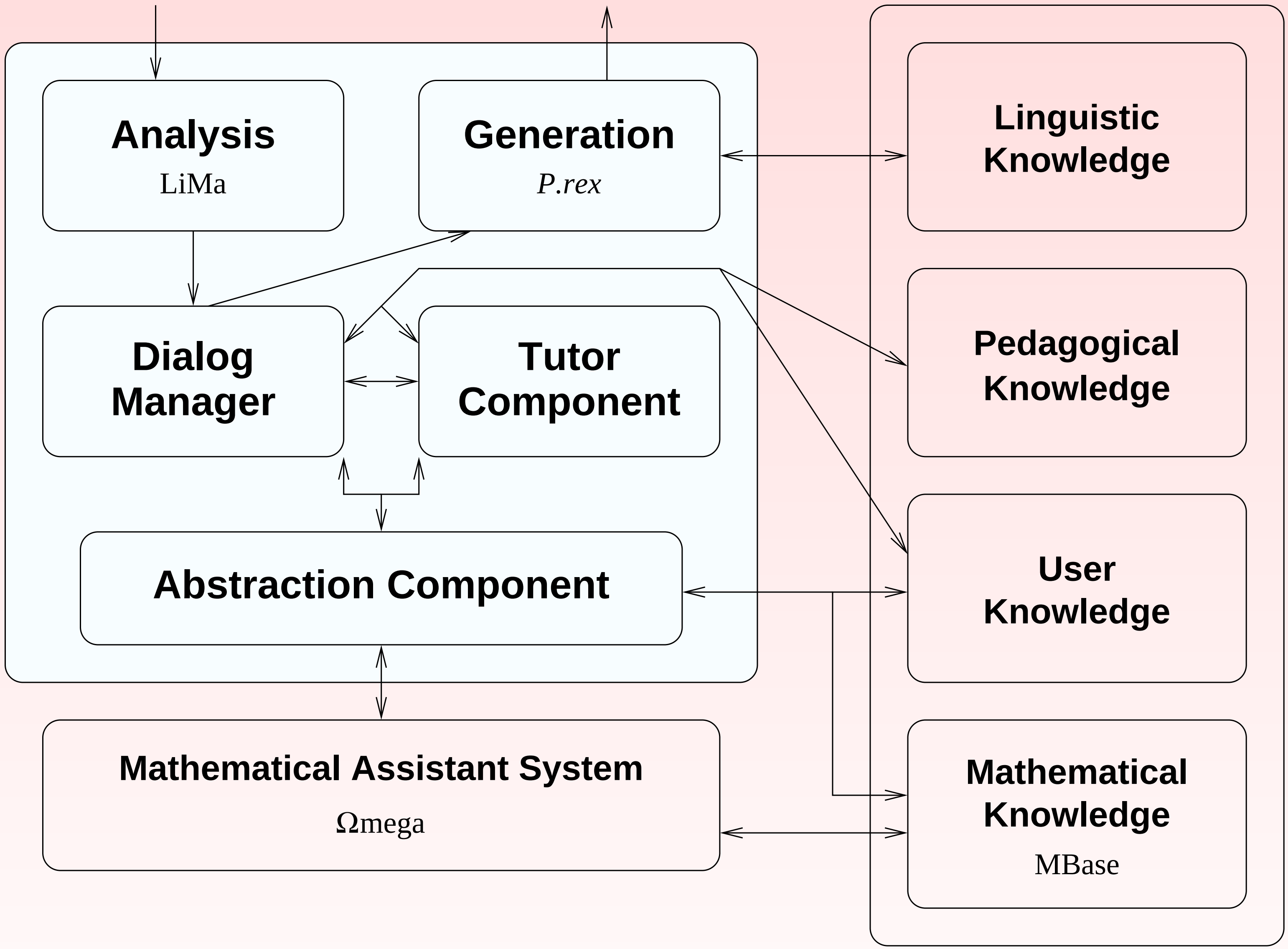
The DIALOG Group

Overview



Mathematical Assistant Systems
Dialog Processing
Tutorial Learning Environments
Tutorial
DIALOG
with a
Mathematical
Assistant System
Ωmega
MathWeb
MBase
OANTS
P.rex
PROVERB
ActiveMath
TRINDI−KIT
LiMa

Architecture for DIALOG

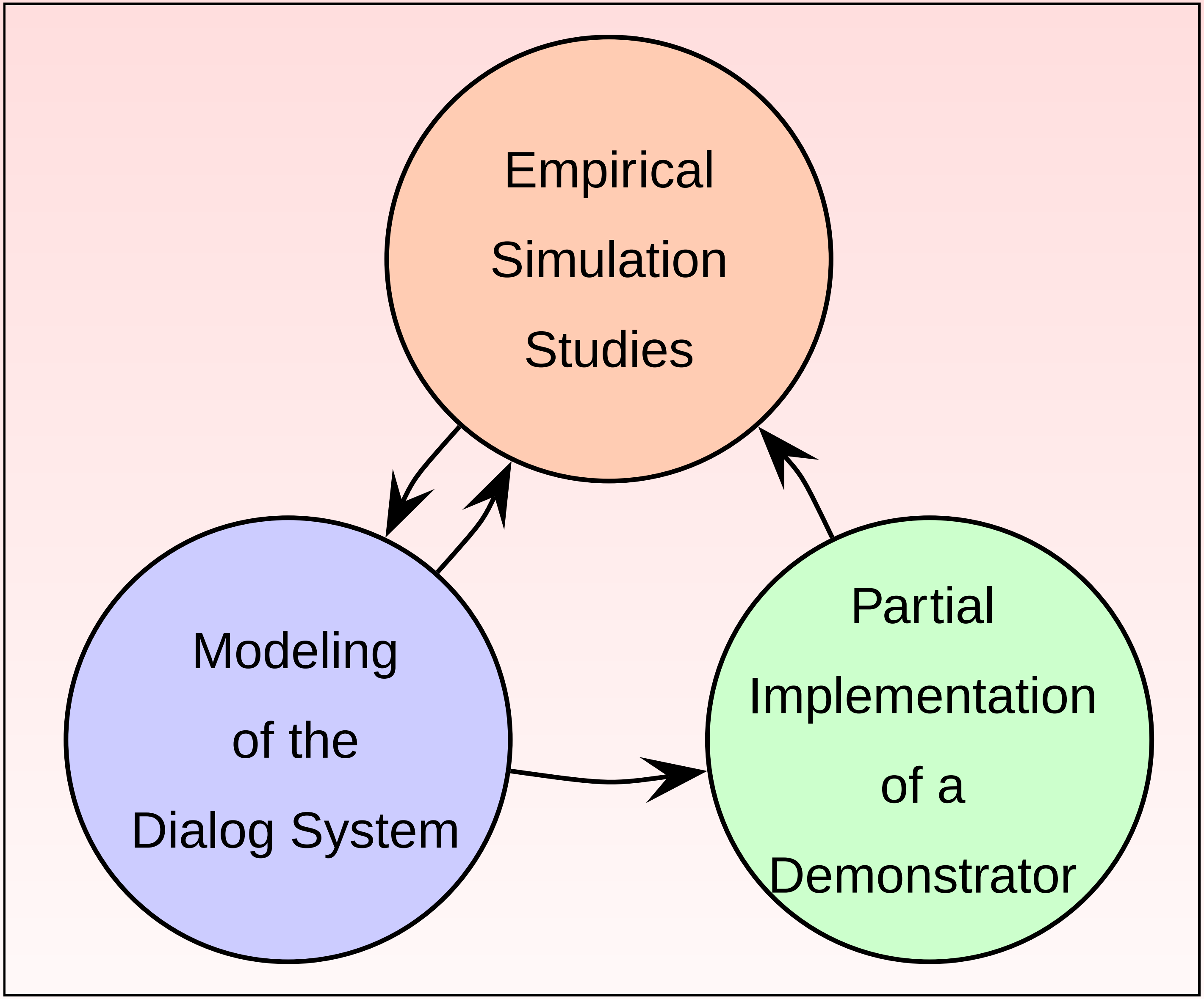


Domain: Naive Set Theory

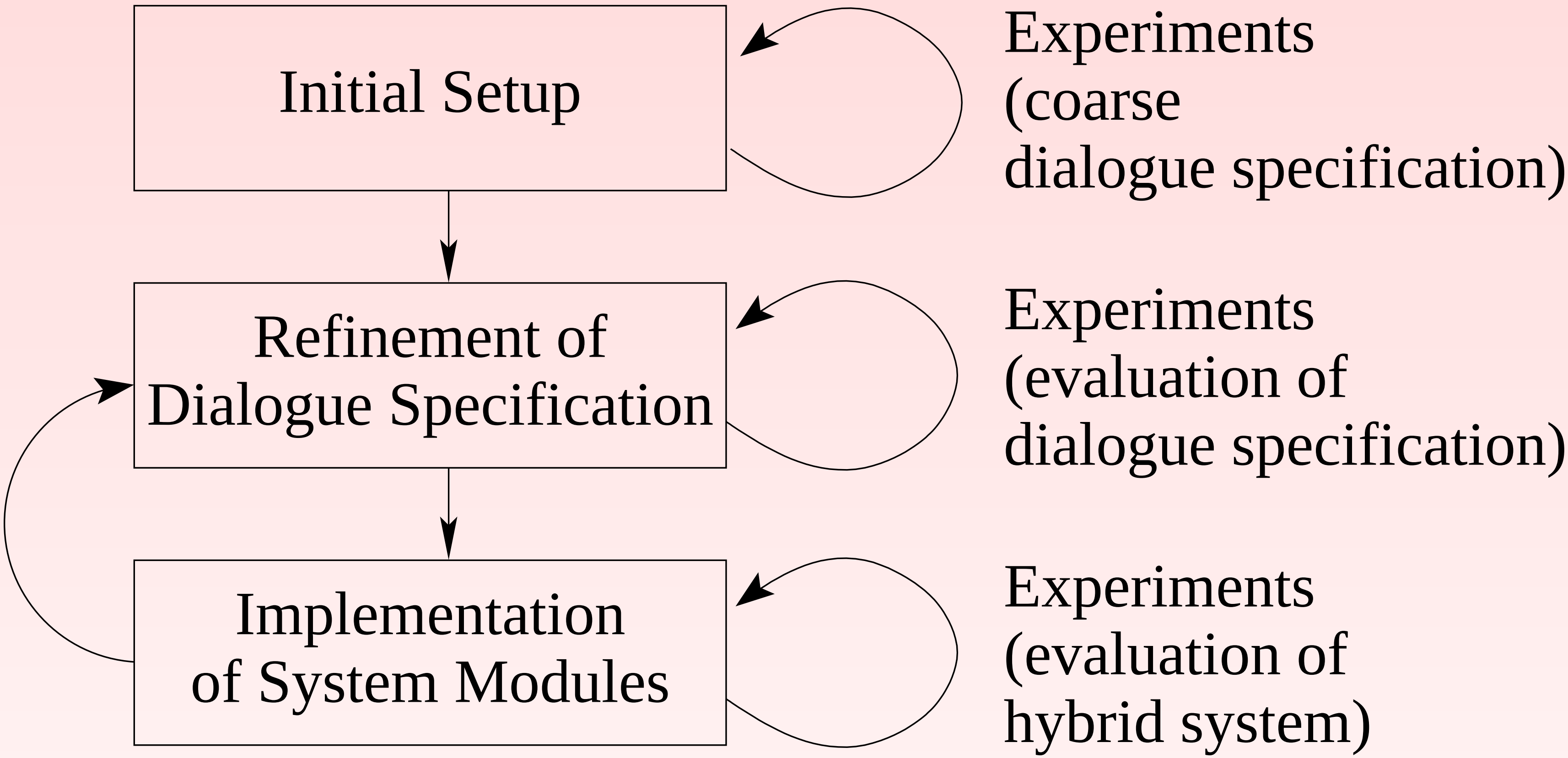
- Is domain well chosen?
  - What advantages has the domain?
  - Representative also for other domains?
  - Suitable for empirical studies?
  - Manageable by OMEGA?
  - Enough interesting structure?

Some aspects will be addressed later ...

Method: Increasing Refinement



Method: Increasing Refinement



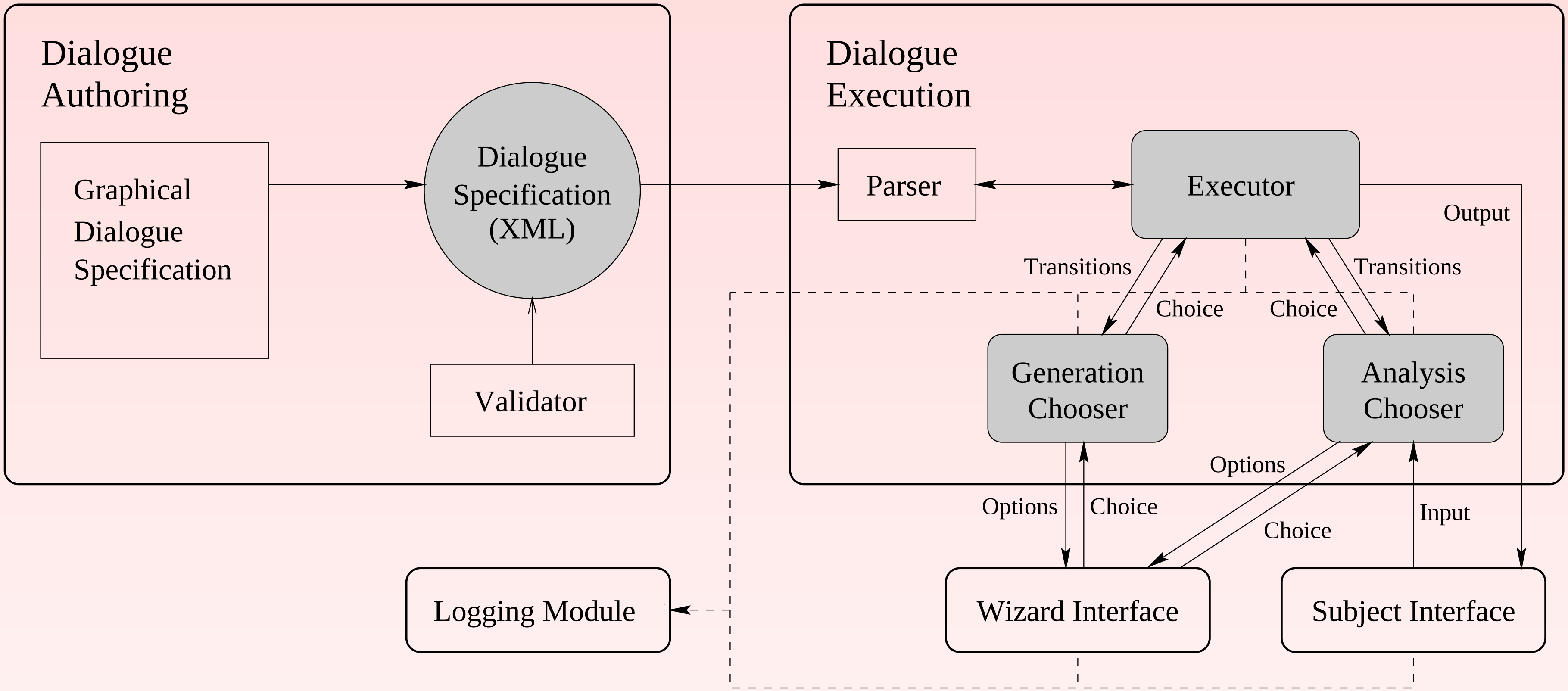
DiaWoZ

System that supports the design and execution of *Wizard-of-Oz* experiments

- Combination of finite-state automata and information-state based dialog model (TRINDI)
- Global and local variables (for subdialogs)
- *Dialog Authoring* and *Dialog Execution* components

Armin Fiedler, Malte Gabsdil, Christian Korthals, Elsa Pecourt

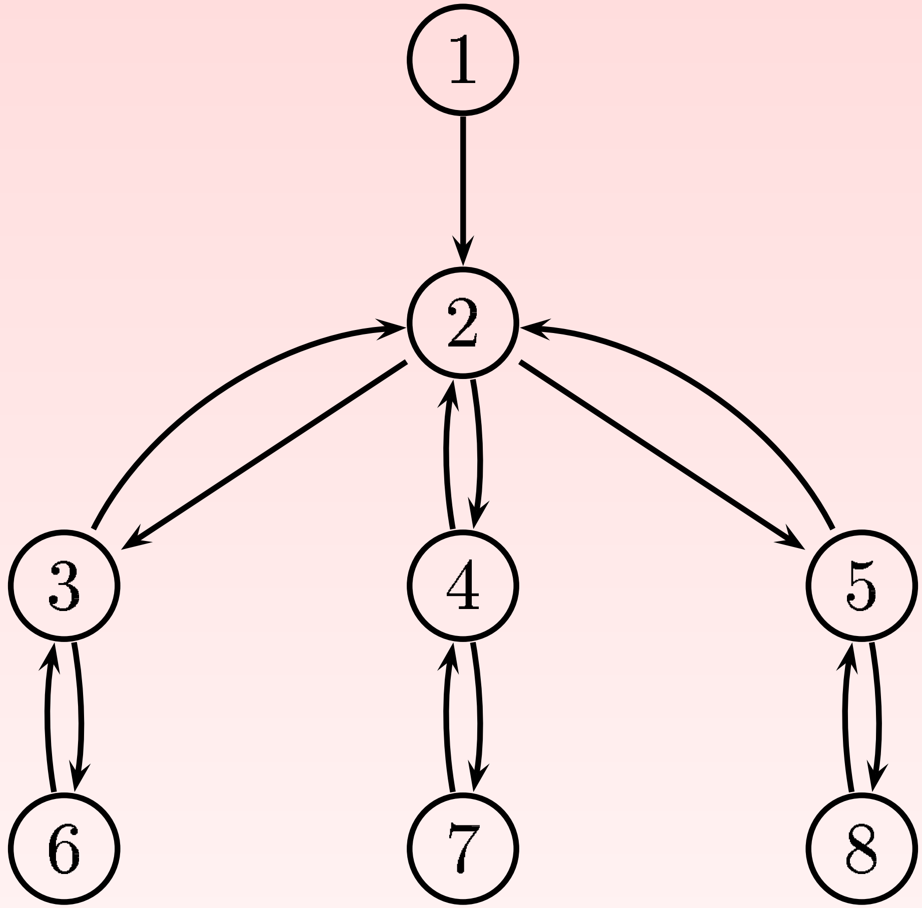
Architecture of DiaWoZ



Dialog Specification

Information State:

NEUTRAL:	**open**
INVERSE:	**open**
ASSOCIATIVE:	**open**



An Example Dialog

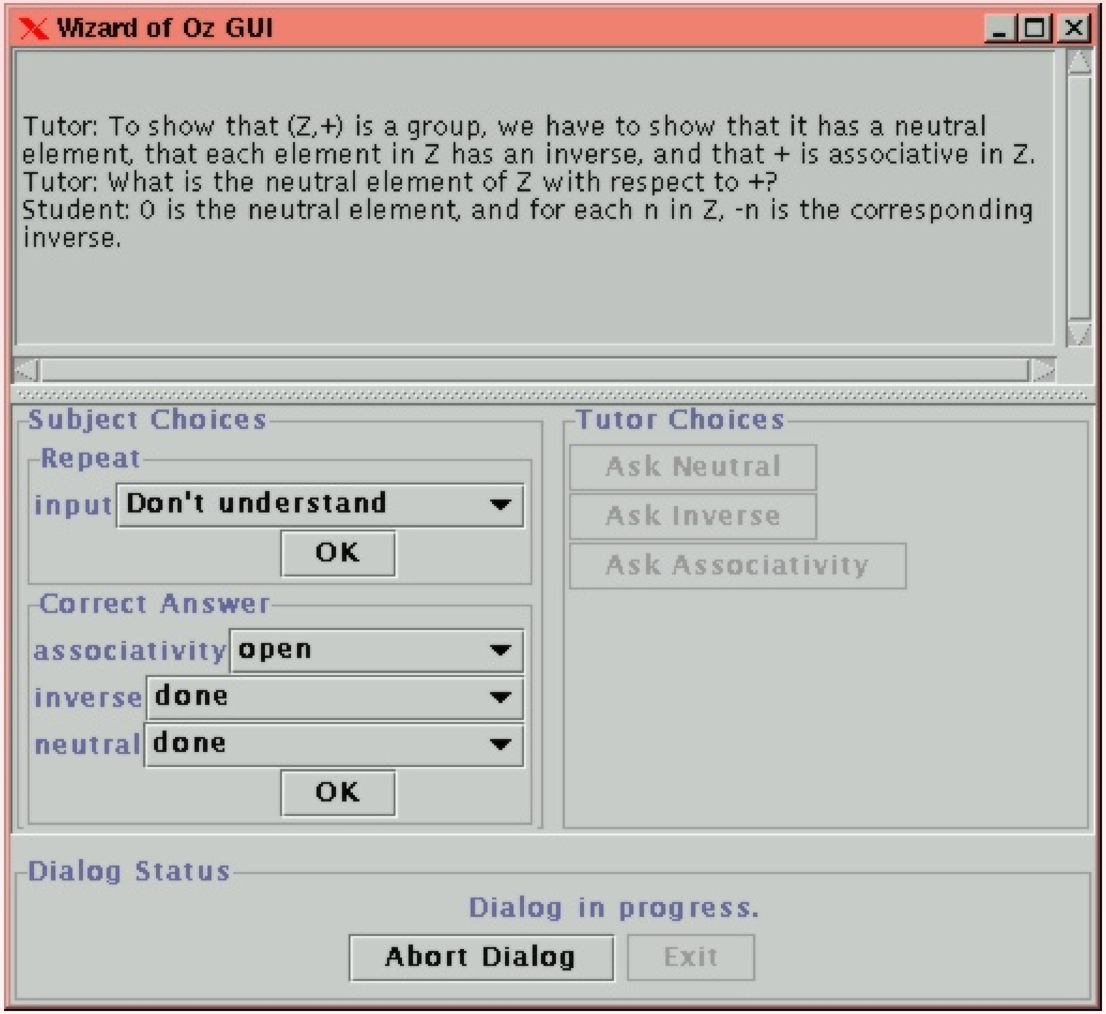
(U1) **Tutor:** To show that $(Z, +)$ is a group, we have to show that it has a neutral element, that each element in Z has an inverse, and that $+$ is associative in Z.

(U2) **Tutor:** What is the neutral element of Z with respect to $+$?

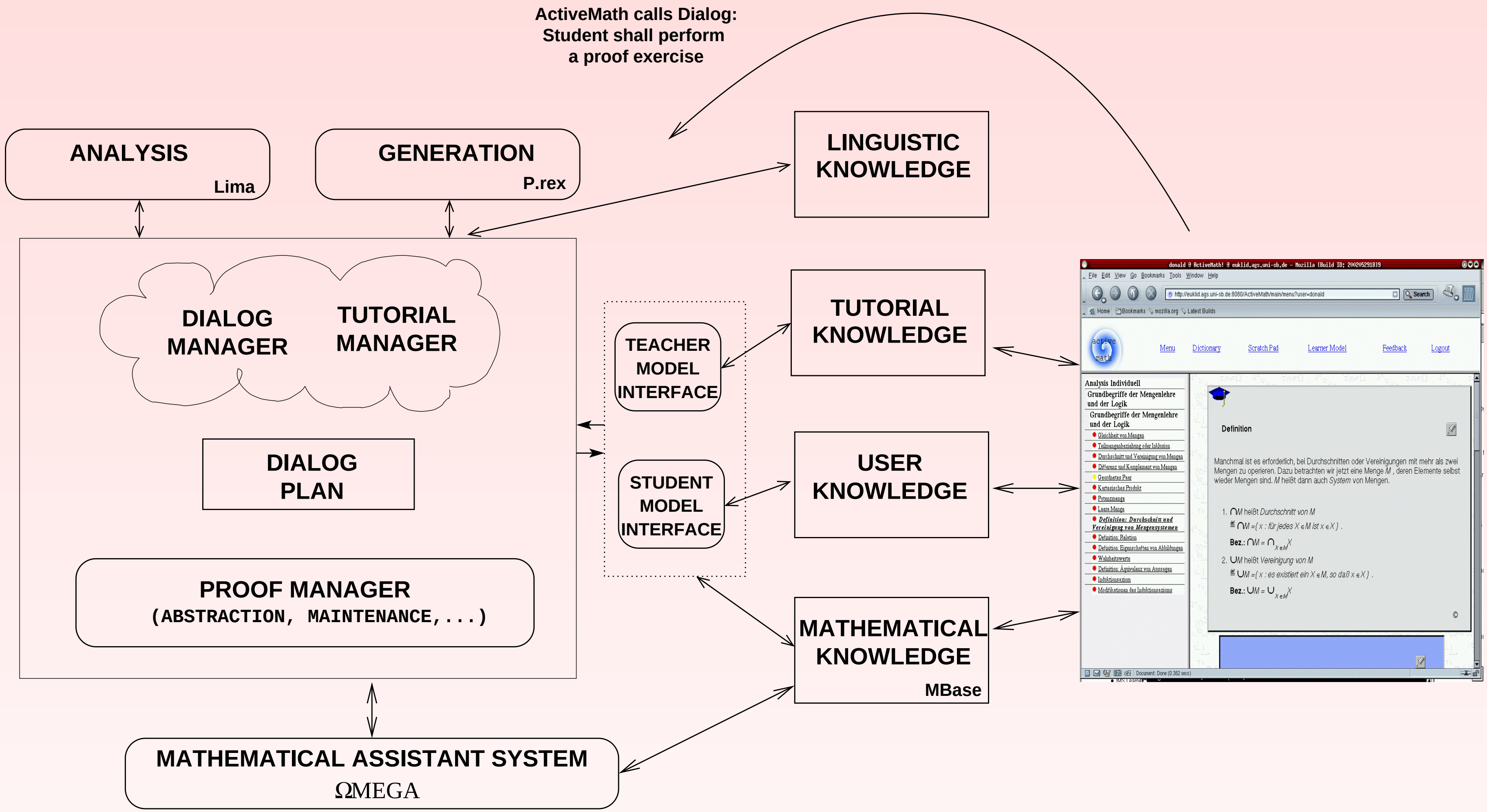
(U3) **Student:** 0 is the neutral element, and for each n in Z, $-n$ is the corresponding inverse.

(U4) **Tutor:** That leaves us to show associativity.

DiaWoZ Interface



DIALOG and ACTIVEMATH



Mathematical Knowledge

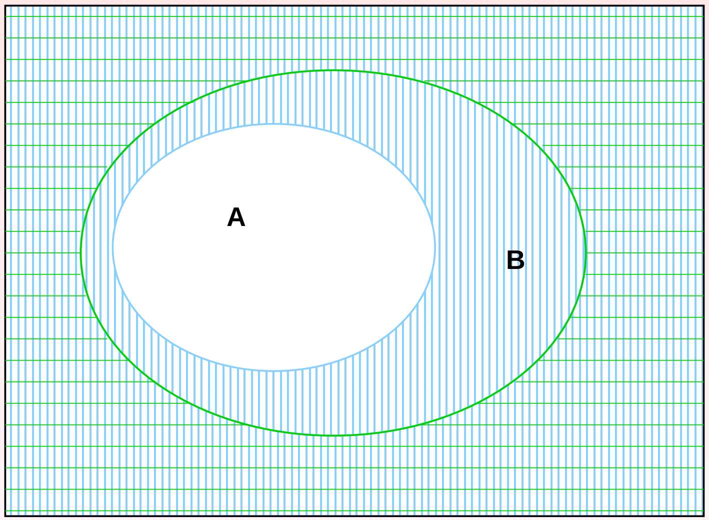
Theorem

$$(A \subseteq B) \Rightarrow (B^c \subseteq A^c)$$

Tactic

$$\frac{B \subseteq A}{A^c \subseteq B^c} \subseteq^c\text{-I}$$

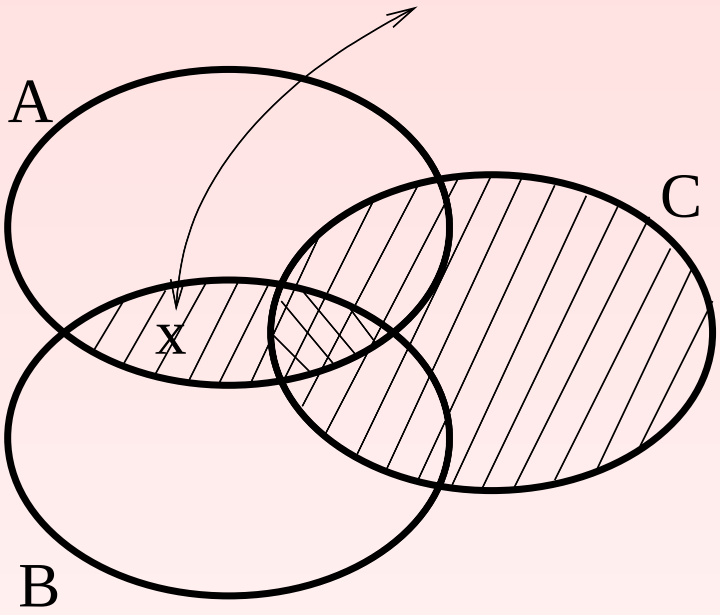
Diagram:



Venn-Diagrams for Counterexamples

Non−theorem: $(A \cap B) \cup C = (A \cap B) \cap C$

Counterexample: $x \in A \wedge x \in B \wedge \neg(x \in C)$



Element x is in

$(A \cap B) \cup C$

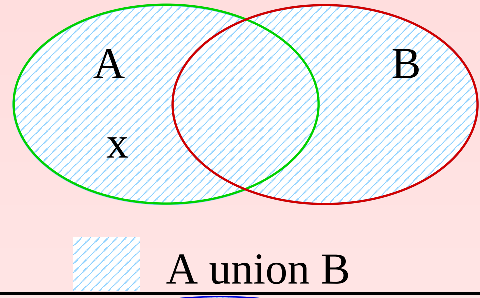
but not in

$(A \cap B) \cap C$

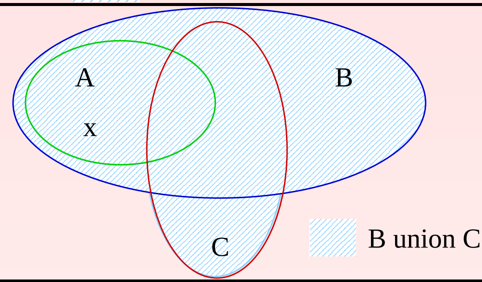
VENN Diagram

Further Mathematical Knowledge

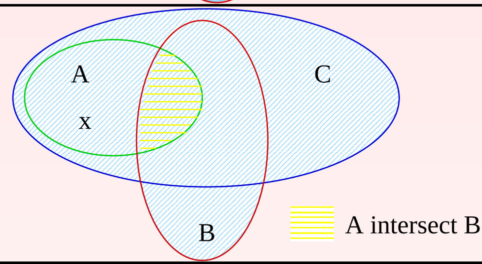
$$\in\text{-}\cup\text{-IL} : e \in A \Rightarrow (e \in A \cup B) \qquad \frac{e \in A}{e \in A \cup B}\ \in\text{-}\cup\text{-IL}$$



$$\subseteq\text{-}\cup\text{-IL} : A \subseteq B \Rightarrow (A \subseteq B \cup C) \qquad \frac{A \subseteq B}{A \subseteq B \cup C}\ \subseteq\text{-}\cup\text{-IL}$$



$$\subseteq\text{-}\cap\text{-IL} : (A \subseteq C) \Rightarrow (A \cap B \subseteq C) \qquad \frac{A \subseteq C}{A \cap B \subseteq C}\ \subseteq\text{-}\cap\text{-IL}$$



$$\wp\text{-I} : (A \subseteq B) \Rightarrow (A \in \wp(B)) \qquad \frac{A \subseteq B}{A \in \wp(B)}\ \wp\text{-I}$$

?

$\ldots$

User Knowledge

Student A:

- Novice in Set Theory
- Has studied the following concepts:
  - Definitions: $\in, \cap, \cup, \subseteq, \wp$, set-complement
  - Theorems: $\subseteq$-$\cap$-IL : $(A \subseteq C) \Rightarrow (A \cap B \subseteq C)$
  - etc.

Student B: . . .

Tutorial Knowledge

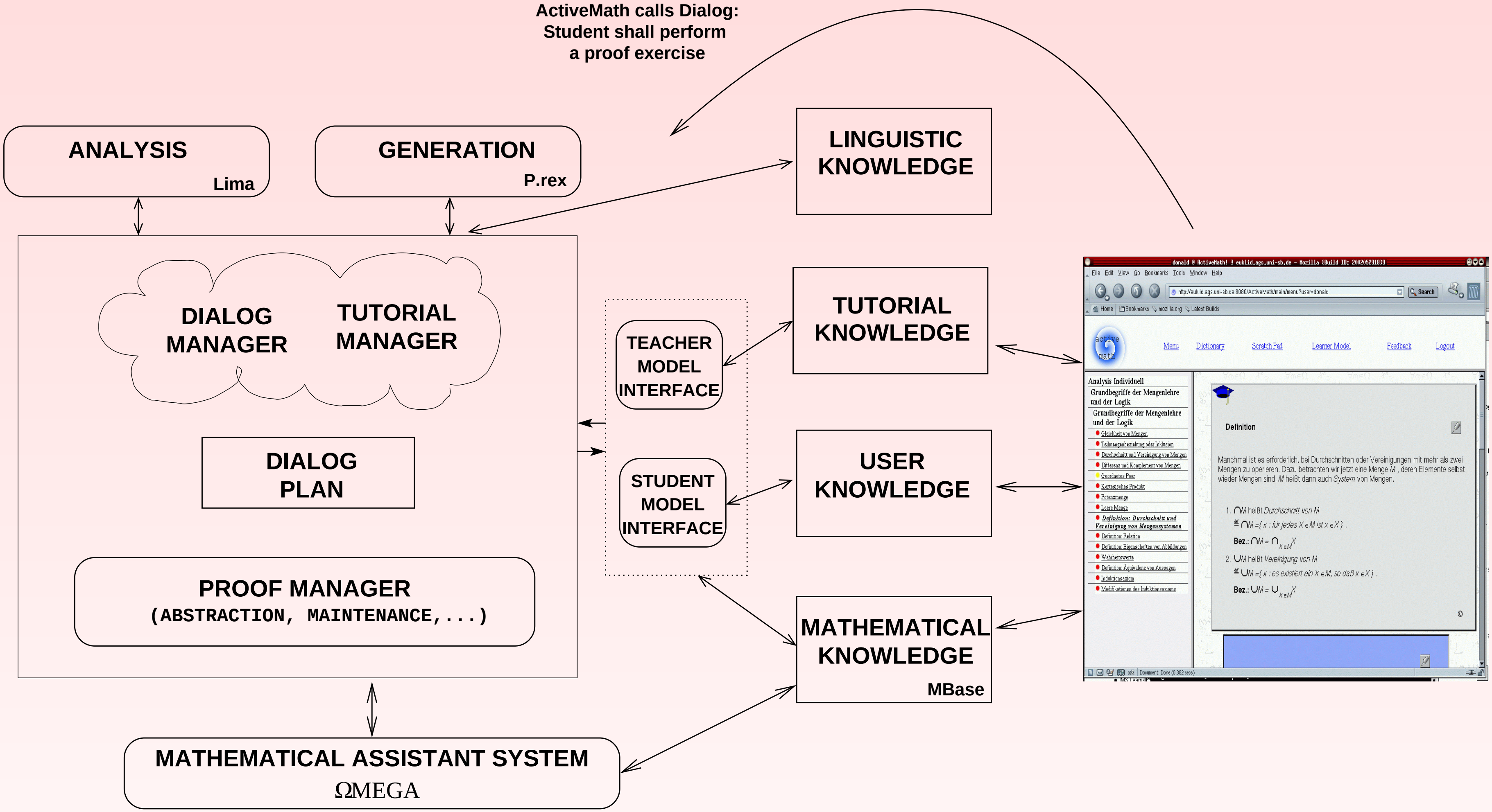
- Novice student should solve exercises with mathematical knowledge that have been taught in the ActiveMath tutorial.

  Chapter1,Ex1 $\rightarrow R = \{\subseteq\text{-REF}, \in\text{-}\cup\text{-IL}, \subseteq\text{-}\cup\text{-IL}, \subseteq\text{-}\cup\text{-IR}, \subseteq\text{-}\cap\text{-IL}, \wp\text{-I}\}$

- If novice student does not understand an application of $r \in R$ then employ one of the following

  1. show/explain the Venn-Diagram for r
  2. - explain instantiation of r
     - choose theorem r as exercise (recursion!)
  3. refer to ActiveMathtext for r
  4. ...

Architecture for DIALOG



Challenge for OMEGA & Ω-Ants

Proof Planning in OMEGA & Ω-Ants with resources:

- inference rules
- control knowledge to structure the search space

Teacher model $\rightarrow$ T = (inf-rules-1, control-1)

User model $\rightarrow$ U = (inf-rules-2, control-2)

CONSTRUCT-PROOF(T) $\rightarrow$ Teacher proof

vs.

CONSTRUCT-PROOF(U) $\rightarrow$ Predictable steps of user

Example Proof

$$A \cap B \in \wp((A \cup C) \cap (B \cup C))$$

OMEGA(

ND $\cup$ $\{$$\subseteq$-REF, $\subseteq$-$\cap$-I, $\subseteq$-$\cap$-IL, $\subseteq$-$\cap$-IR, $\subseteq$-$\cup$-IL, $\subseteq$-$\cup$-IR, $\ldots\}$,

prefer-set-tactics-over-ND$)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\top}{A \subseteq A}\ \subseteq\text{-REF}}{A \subseteq A \cup C}\ \subseteq\text{-}\cup\text{-IL}}{A \cap B \subseteq A \cup C}\ \subseteq\text{-}\cap\text{-IL} \qquad \dfrac{\dfrac{\dfrac{\top}{B \subseteq B}\ \subseteq\text{-REF}}{B \subseteq B \cup C}\ \subseteq\text{-}\cup\text{-IL}}{A \cap B \subseteq B \cup C}\ \subseteq\text{-}\cap\text{-IR}}{A \cap B \subseteq (A \cup C) \cap (B \cup C)}\ \subseteq\text{-}\cap\text{-I}}{A \cap B \in \wp((A \cup C) \cap (B \cup C))}\ \wp\text{-I}$$

Example Proof

To show: $A \cap B \in \wp((A \cup C) \cap (B \cup C))$

By $\wp$-I enough to show: $A \cap B \subseteq (A \cup C) \cap (B \cup C)$

By $\subseteq$-$\cap$-I we have to show (1) and (2):

1. $A \cap B \subseteq A \cup C$

   By $\subseteq$-$\cap$-IL enough to show $A \subseteq A \cup C$

   By $\subseteq$-$\cup$-IL enough to show $A \subseteq A$

   Follows by $\subseteq$-REF

2. $A \cap B \subseteq B \cup C$

   By $\subseteq$-$\cup$-IL enough to show $A \cap B \subseteq B$

   By $\subseteq$-$\cap$-IR enough to show $B \subseteq B$

   Follows by $\subseteq$-REF

q.e.d.

$\Rightarrow$ Note multiplicities in proof: proof abstraction is needed

Enough structure in Naive Set Theory

Problem:	$(A \cup B)^c \subseteq A^c$
By $\subseteq^c$ enough to show:	$A \subseteq A \cup B$
By $\subseteq$-$\cup$-IL enough to show:	$A \subseteq A$
Follows by $\subseteq$-REF	q.e.d.

Assume: Student does not understand application of $\subseteq^c$

According to our tutorial knowledge we would

1. show/explain Venn diagram for $\subseteq^c$
2. - explain instantiation of $\subseteq^c$
   - choose theorem $\subseteq^c$ as exercise (recursion!)
3. refer to ACTIVEMATHtext for $\subseteq^c$

Enough structure in Naive Set Theory

(2b) Proof of $\subseteq^c$: $X \subseteq Y \Rightarrow Y^c \subseteq X^c$

By $\Rightarrow$-I we assume $[X \subseteq Y]$ and show $Y^c \subseteq X^c$

By `Defn-I(c)` enough to show $\mathcal{U} \setminus Y \subseteq \mathcal{U} \setminus X$

By $\subseteq -\setminus$ enough to show $\mathcal{U} \subseteq \mathcal{U}$ and $X \subseteq Y$

The former follows by $\subseteq$-REF and the latter from the assumption.

q.e.d.

$\Rightarrow$ Update of User Model

Assume: Student does not understand $\subseteq -\setminus$ step.

...

(2b) Subdialog on proof problem: $A \subseteq B \wedge C \subseteq D \Rightarrow A \setminus D \subseteq \mathbf{B} \setminus C$

...

Further steps in the project

Empirical Studies

1. Set up a set theory tutorial, fix the set of exercises
2. Analyze the proof information available from OMEGAfor DIALOG
3. First model of system architecture
4. Choose tutorial strategies/pedagogical rules
5. First coarse specification of dialog session with DiaWOZ
6. Perform empirical study and evaluate
7. ...

Further steps in the project

Implementation in ACTIVEMATH

1. Provide section on Naive Set Theory together with examples
2. Analyze and probably adapt features of ACTIVEMATH, e.g. user modeling, according to our needs
3. Perform empirical studies already with ACTIVEMATH?

Further steps in the project

Conception/Realization of Modules

1. Tutor model interface
2. Student model interface
3. Partial realization of single aspects of the proof manager
4. Proof abstraction (e.g. with outline proof search)