Tutorial Dialog with a Mathematical Assistant System

The DIALOG *Project*

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Presentation Overview

- **Role in the** OMEGA **Group at Saarland University & Main Scientific Contributions**
- **Tutorial Dialog with a Mathematical Assistant System**
  - **Project Overview**
  - **Increasing Refinement with *Wizard-of-Oz* (WoZ) Experiments**
  - DIALOG **and the Learning Environment** ACTIVEMATH
  - **Examples from Naive Set Domain: Properties & Requirements**

Research in the OMEGA project

Knowledge-based Proof Planning with multiple Strategies

Interactive Tactical Theorem Proving

Agent-based Reasoning

Ext. Support Systems: FO-ATPs, HO-ATPs, CAS, Constraintssolv., Modelgenerators, . . .

The Mathematical Assistant System
OMEGA
10-13 Researchers

Higher-Order Logic & Calculi, Proof Transformation & Representation

User Interfaces: Graphical, Natural Language

Mathematical Database
MBASE

System Infrastructure: Mathematical Software Bus
MATHWEB

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Organisation and Management of OMEGA research

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Research in the OMEGA project

[AI-02-subm] [IJCAR-WS-01] 2x[CALCULEMUS-00] [CADE-WS-00]	[JSC-02-subm] [LPAR-02-subm]	[CALCULEMUS-02/01/00/99] [AISB-01] [AISB-00] [EPIA-99] [AIMSA-98]
[KI-01] [JUCS-99] [TPHOLS-98]	[CADE-02] [CADE-97]	[JSL-02-subm] [Synthese-02] [TPHOLS- 01] [DISS-99] [CADE-99] 2x[CADE-98]
[CASYS-99] [FAC-99] [UITP-98]	[Festschrift-02] [MKM-01]	[VERIFY-02] [CALCULEMUS-02]

Main Scientific Contributions

Area of PhD thesis: Higher-Order Theorem Proving

- **Extensional Higher-Order Resolution, Paramodulation, RUE-Resolution**
  [Synthese-02] [CADE-99] [CADE-98]
- **Notions of Semantics for Higher-Order Logic (HOL) / Abstract Consistency Proof Principle for HOL**
  [JSL-02-Submitted]
- **Development of my Higher-Order Theorem Prover** LEO **[System-Description-CADE-98]**

The DIALOG Project

The DIALOG project is part of the Collaborative Research Centre SFB 378 *Resource-adaptive Cognitive Processes*

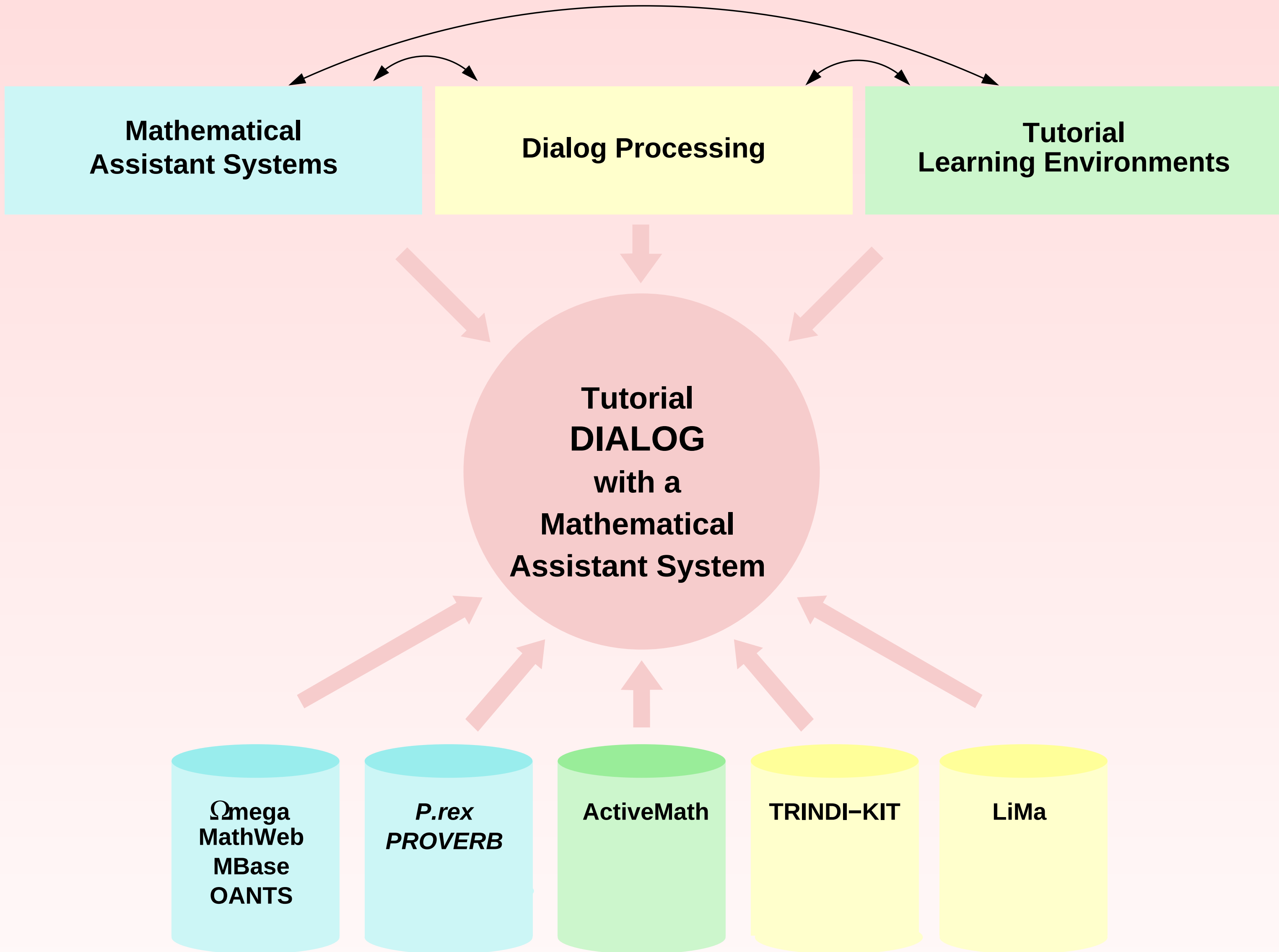
Project Goals:

- **Empirical investigation, modeling, and implementation of natural language (NL) dialog in a tutorial application for a particular mathematical domain**

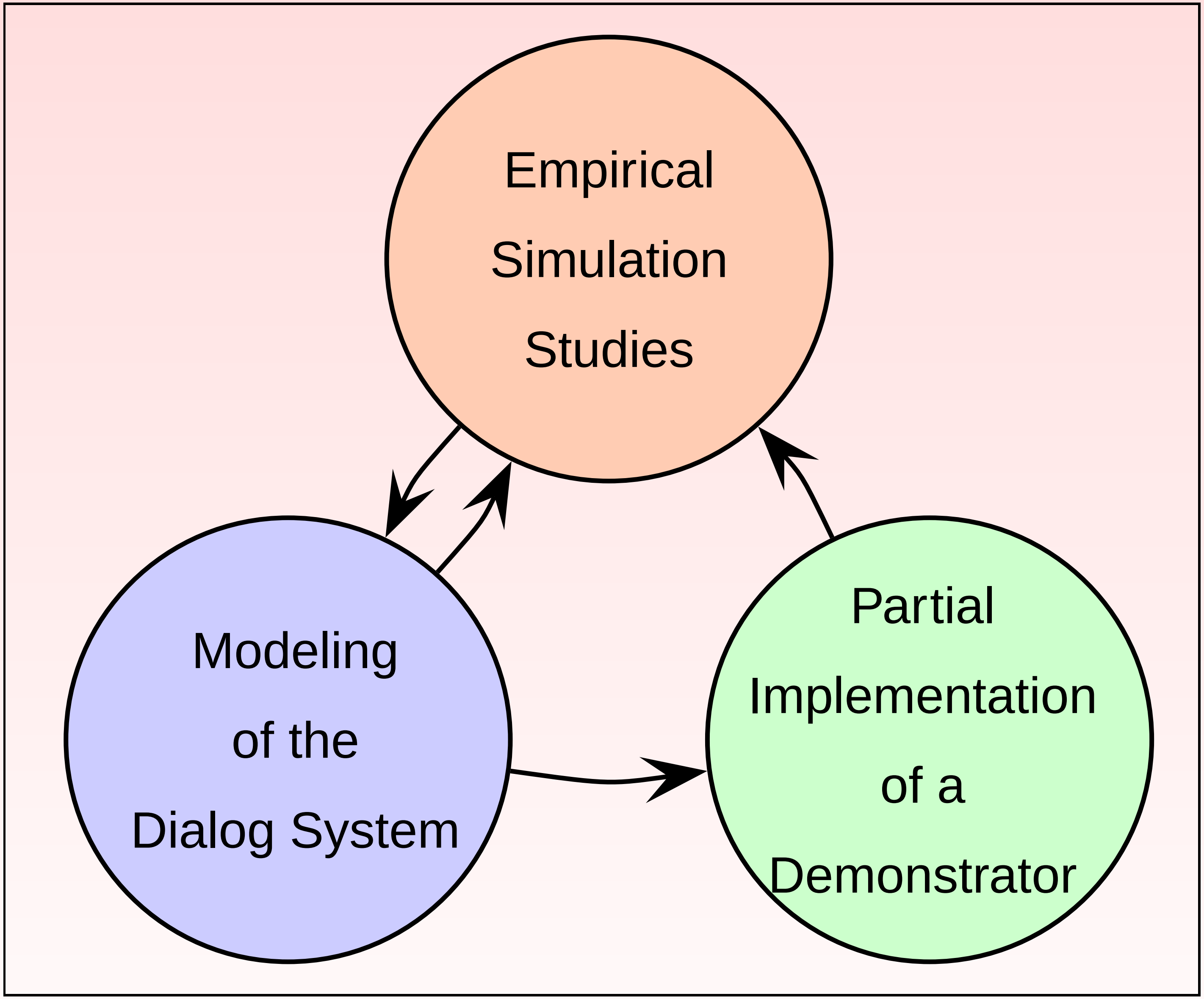
Overall Motivation:

- **Investigation and realisation of dialog techniques in ambitious applications which require deep speech analysis and non-trivial domain reasoning**

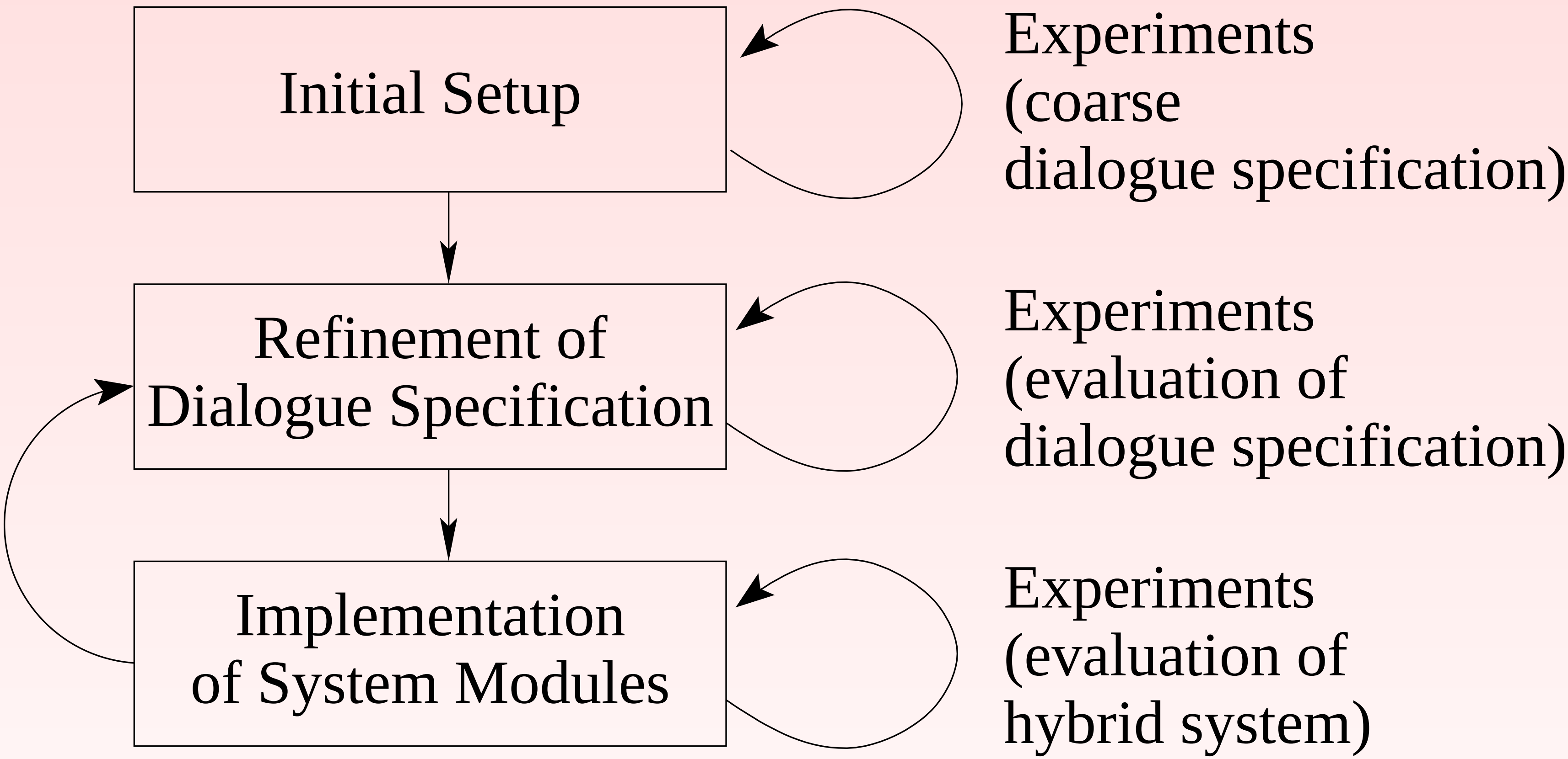
The DIALOG Project



Method: Increasing Refinement



Method: Increasing Refinement

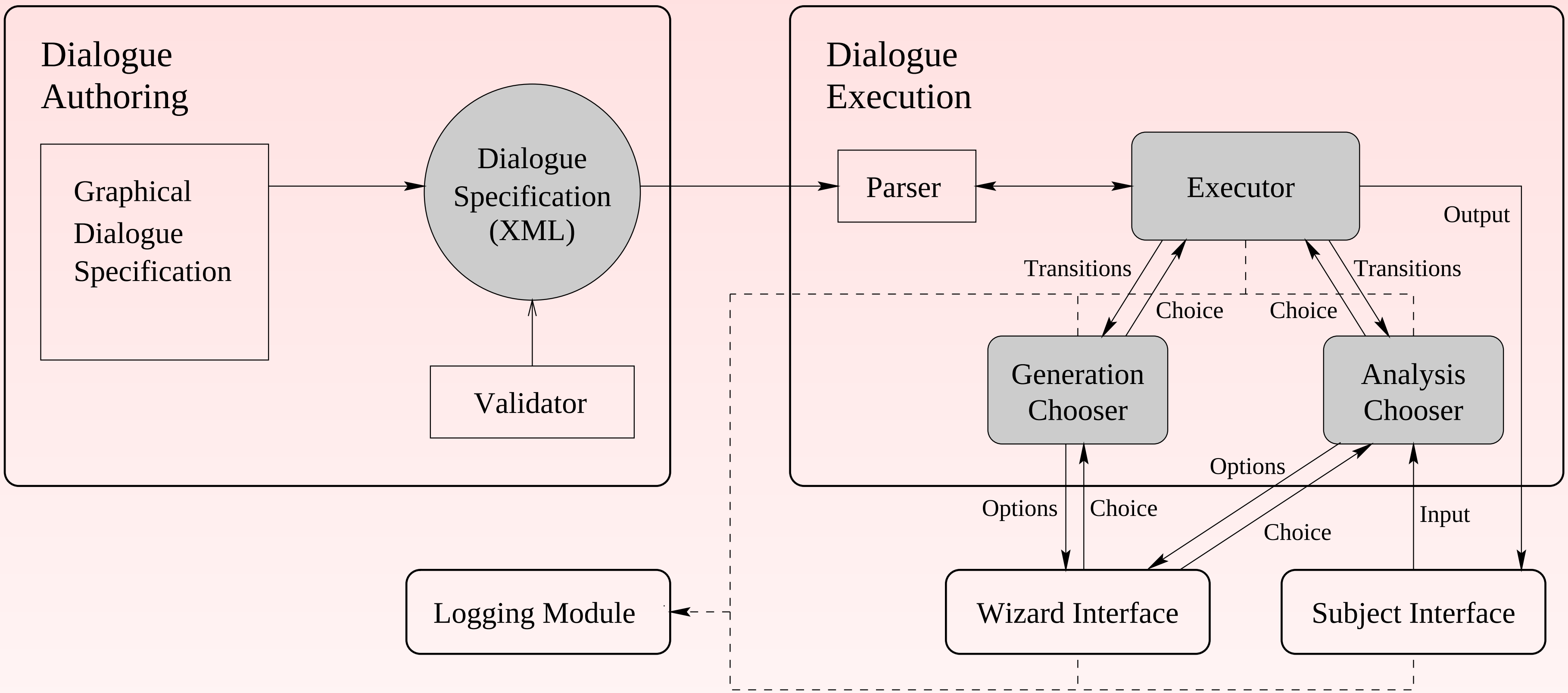


DiaWoZ

System that supports the design and execution of *Wizard-of-Oz* (Bernsen et al.) experiments

- **Combination of finite-state automata and information-state based dialog model (TRINDI)**
- **Global and local variables (for subdialogs)**
- ***Dialog Authoring* and *Dialog Execution* components**

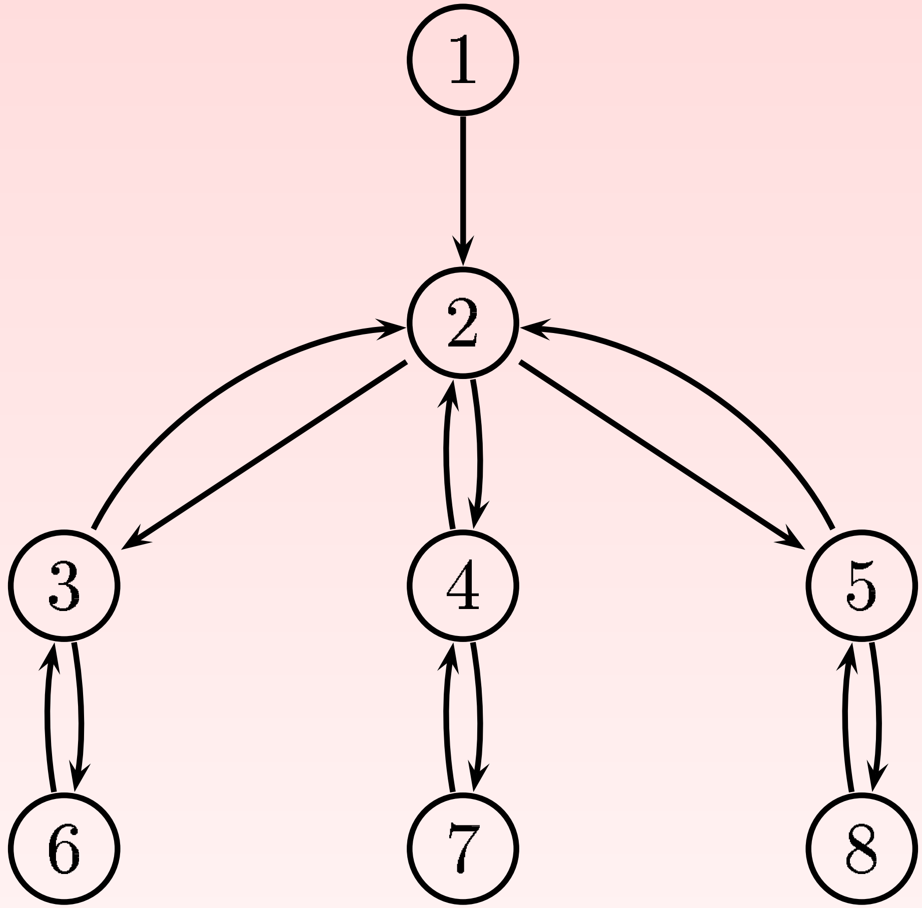
Architecture of DiaWoZ



Dialog Specification

Information State:

NEUTRAL:	**open**
INVERSE:	**open**
ASSOCIATIVE:	**open**



An Example Dialog

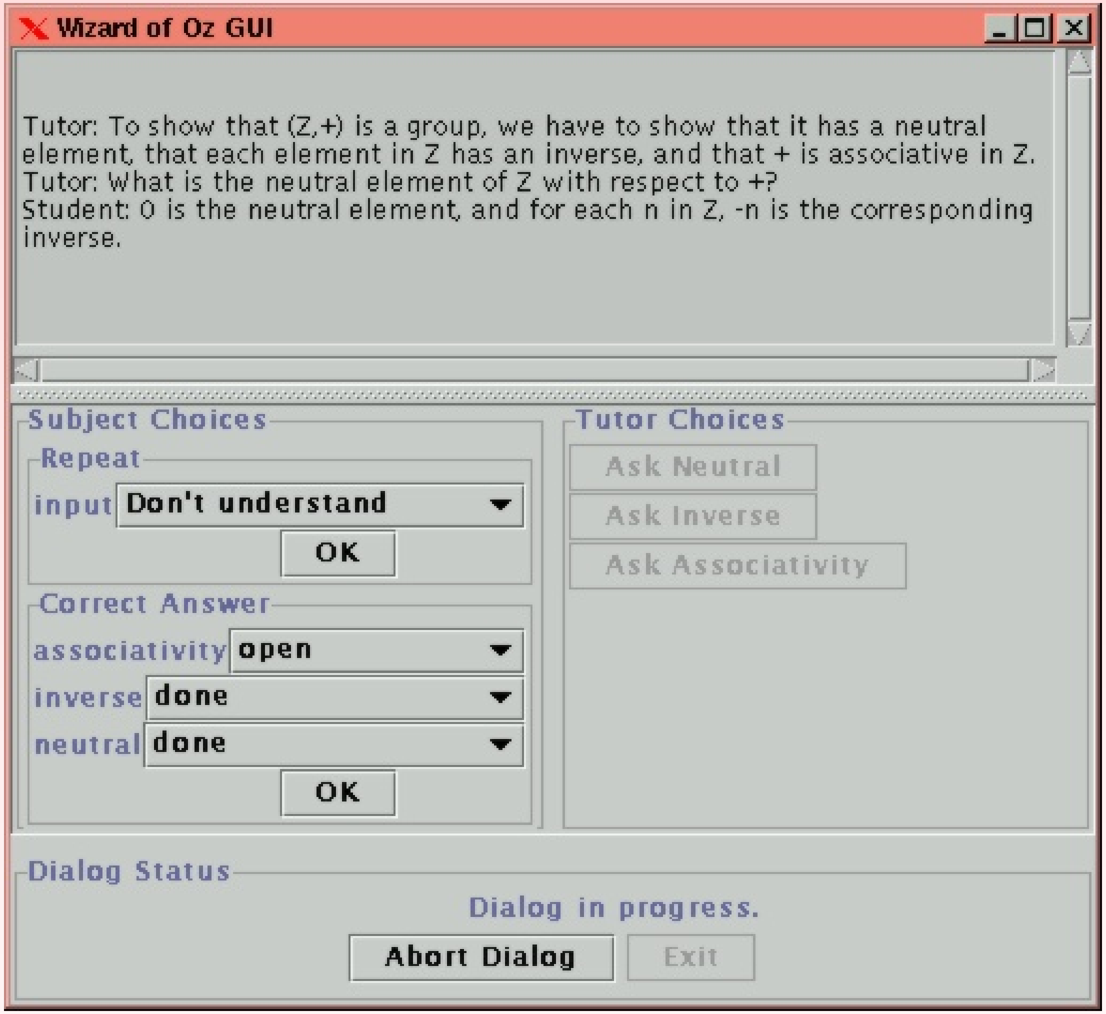
(U1) **Tutor:** To show that $(Z, +)$ is a group, we have to show that it has a neutral element, that each element in Z has an inverse, and that $+$ is associative in Z.

(U2) **Tutor:** What is the neutral element of Z with respect to $+$?

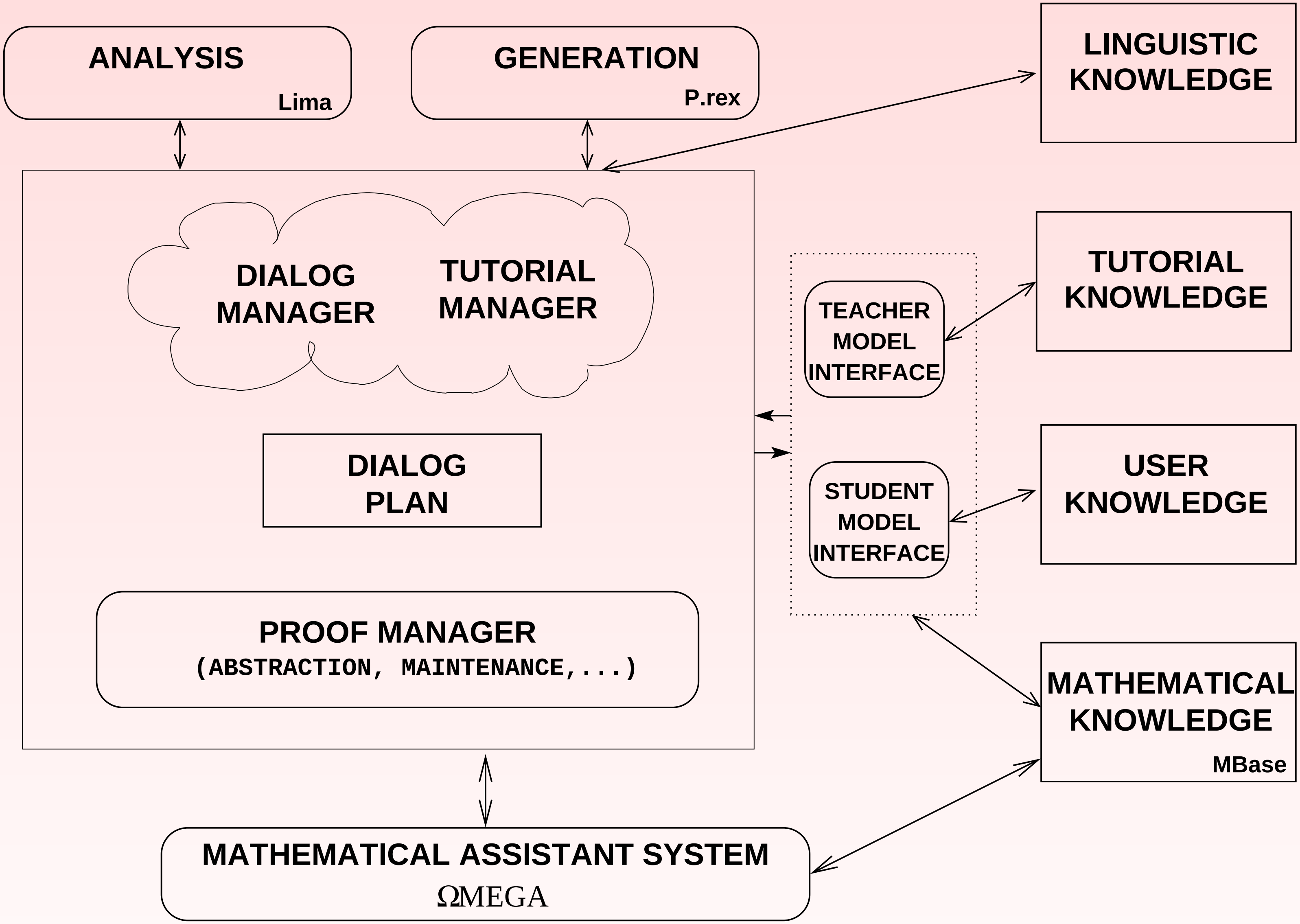
(U3) **Student:** 0 is the neutral element, and for each n in Z, $-n$ is the corresponding inverse.

(U4) **Tutor:** That leaves us to show associativity.

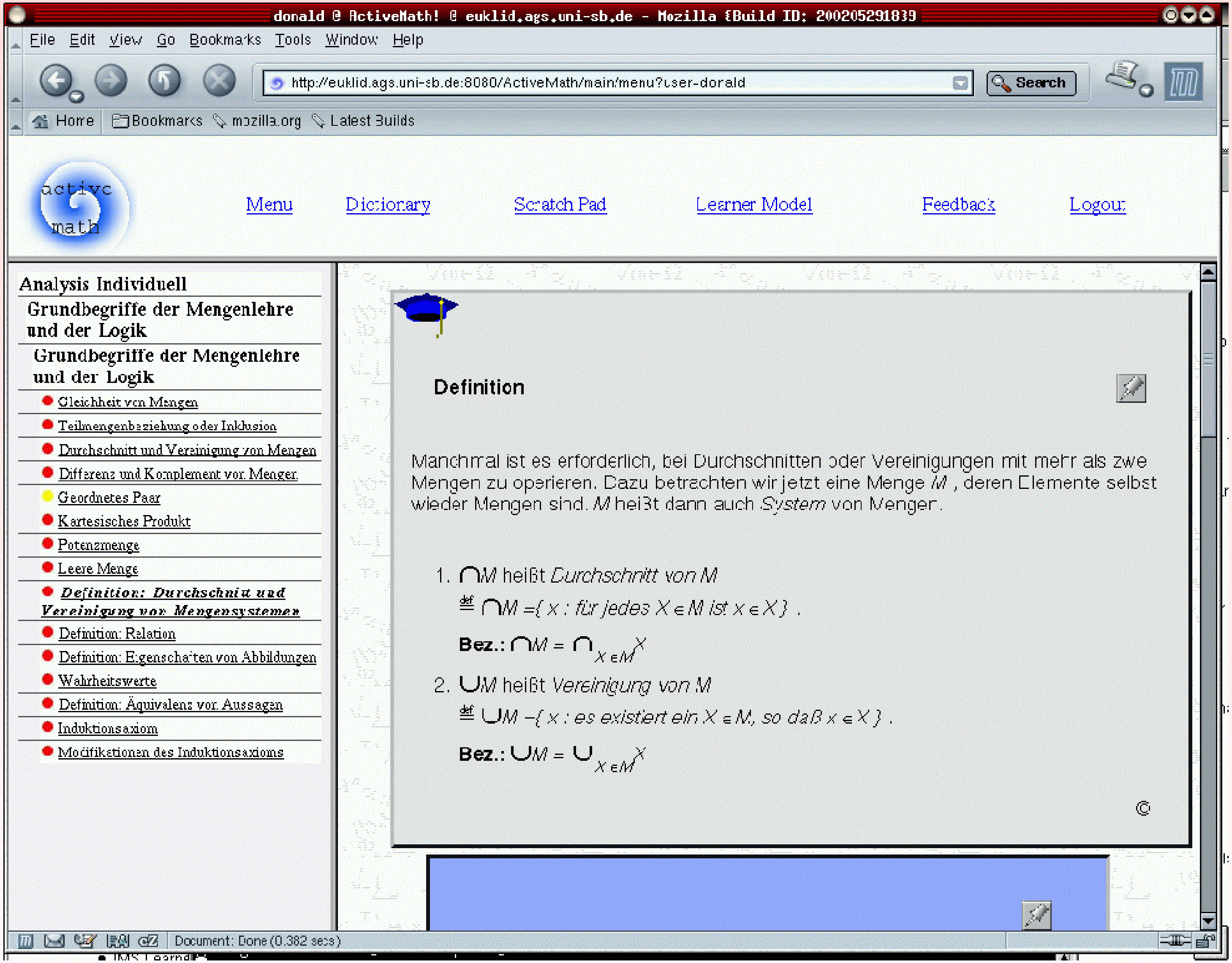
DiaWoZ Interface



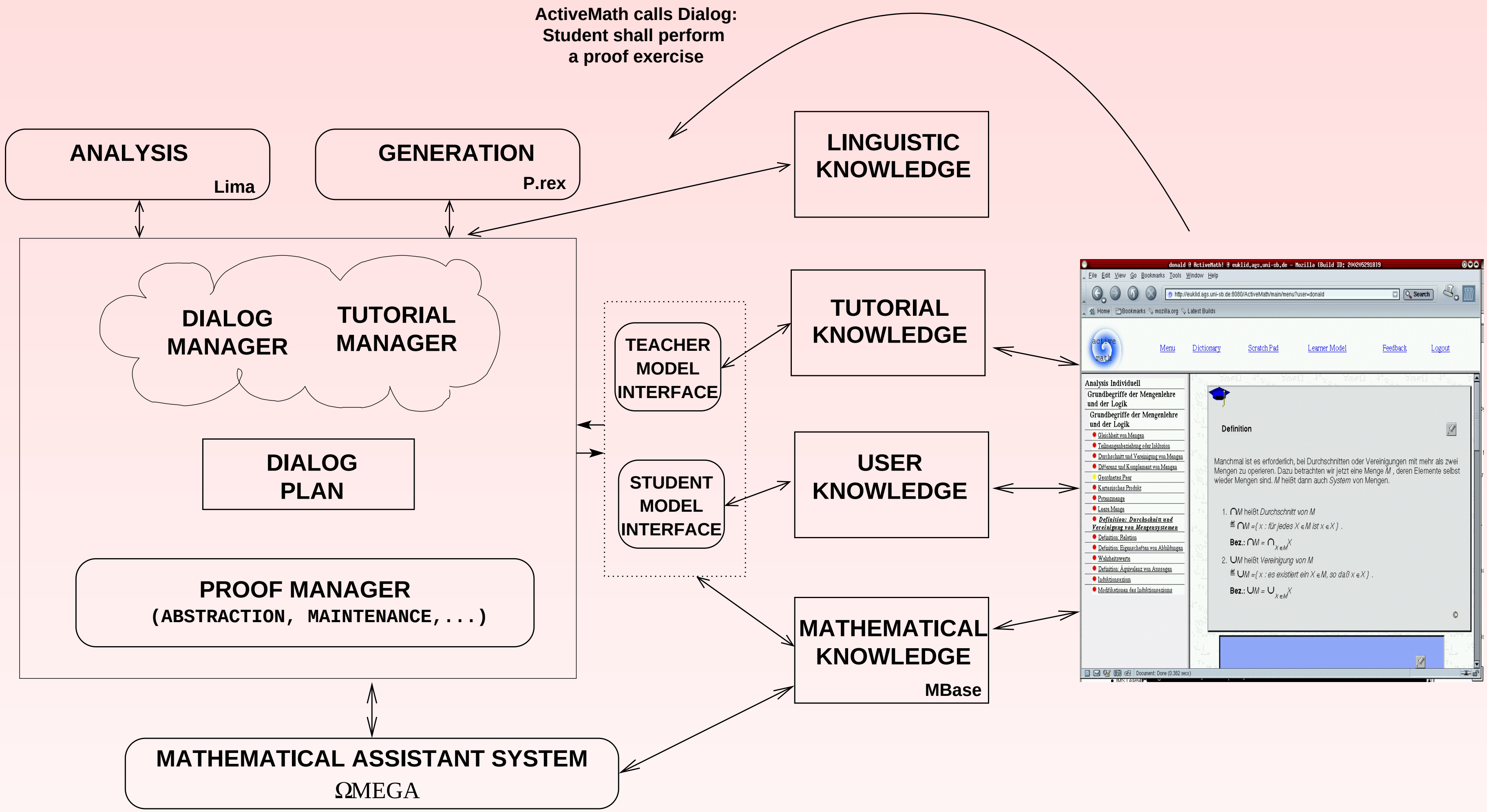
DIALOG Architecture



ACTIVEMATH Learning Environm.



Dialog and ActiveMath



Domain: Naive Set Theory

Is domain well chosen?

- **What advantages has the domain?**
- **Representative also for other domains?**
- **Suitable for empirical studies?**
- **Manageable by** OMEGA**?**
- **Enough interesting structure?**
- **Interesting tutorial aspects?**

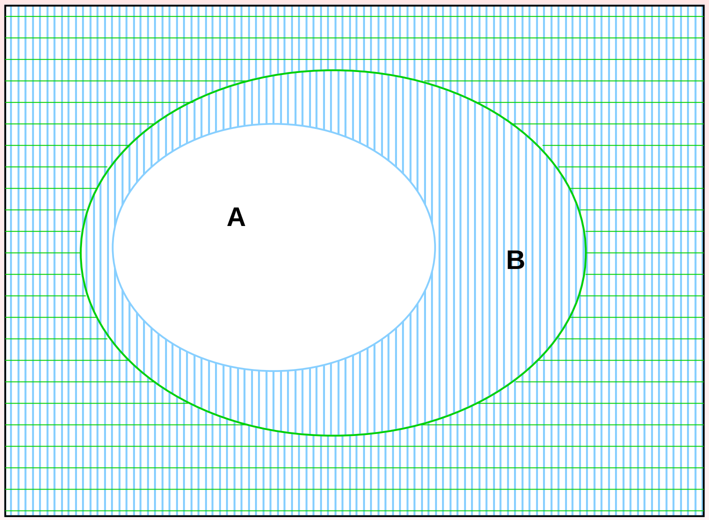
Mathematical Knowledge

Theorem

$$(A \subseteq B) \Rightarrow (B^c \subseteq A^c)$$

Tactic

$$\frac{B \subseteq A}{A^c \subseteq B^c} \subseteq^c\text{-I}$$

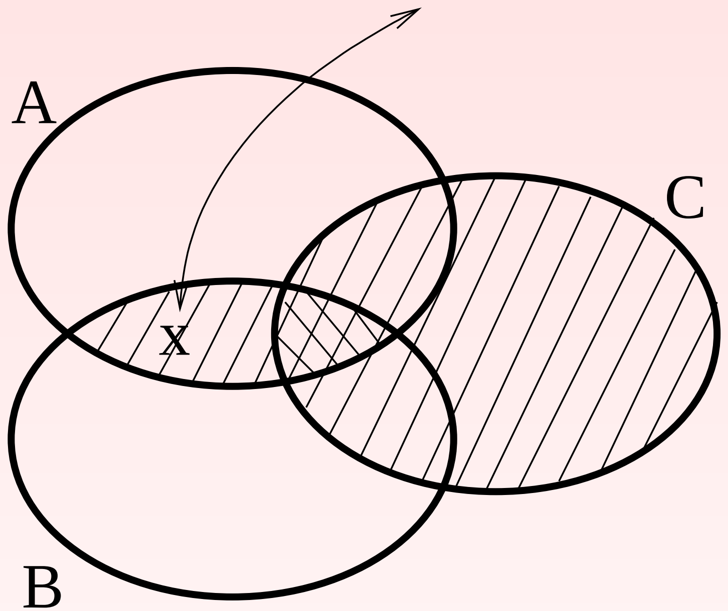
Diagram: 

Mathematical Knowledge

Counterexamples

Non−theorem: $(A \cap B) \cup C = (A \cap B) \cap C$

Counterexample: $x \in A \wedge x \in B \wedge \neg(x \in C)$



Element x is in

$(A \cap B) \cup C$

but not in

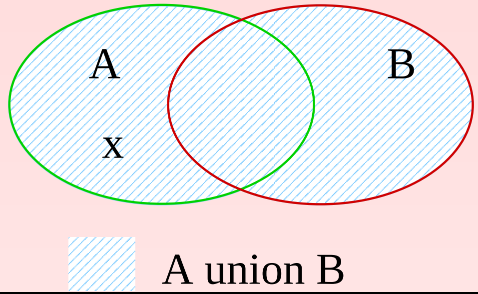
$(A \cap B) \cap C$

VENN Diagram

Further Mathematical Knowledge

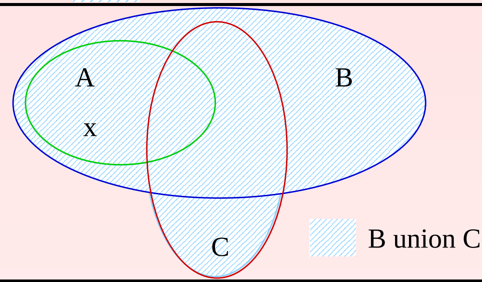
$\in$-$\cup$-IL : $e \in A \Rightarrow (e \in A \cup B)$

$$\frac{e \in A}{e \in A \cup B}\ \in\text{-}\cup\text{-IL}$$



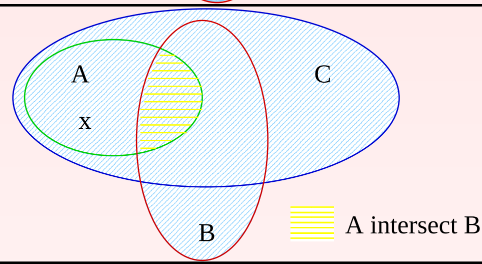
$\subseteq$-$\cup$-IL : $A \subseteq B \Rightarrow (A \subseteq B \cup C)$

$$\frac{A \subseteq B}{A \subseteq B \cup C}\ \subseteq\text{-}\cup\text{-IL}$$



$\subseteq$-$\cap$-IL : $(A \subseteq C) \Rightarrow (A \cap B \subseteq C)$

$$\frac{A \subseteq C}{A \cap B \subseteq C}\ \subseteq\text{-}\cap\text{-IL}$$



$\wp$-I : $(A \subseteq B) \Rightarrow (A \in \wp(B))$

$$\frac{A \subseteq B}{A \in \wp(B)}\ \wp\text{-I}$$

?

$\ldots$

User Knowledge

Student A:

- **Novice in Set Theory**
- **Has studied the following concepts:**
  - **Definitions:** $\in, \cap, \cup, \subseteq, \wp$**, set-complement**
  - **Theorems:** $\subseteq$**-**$\cap$**-IL** : $(A \subseteq C) \Rightarrow (A \cap B \subseteq C)$
  - **etc.**

Student B: $\ldots$

Tutorial Knowledge

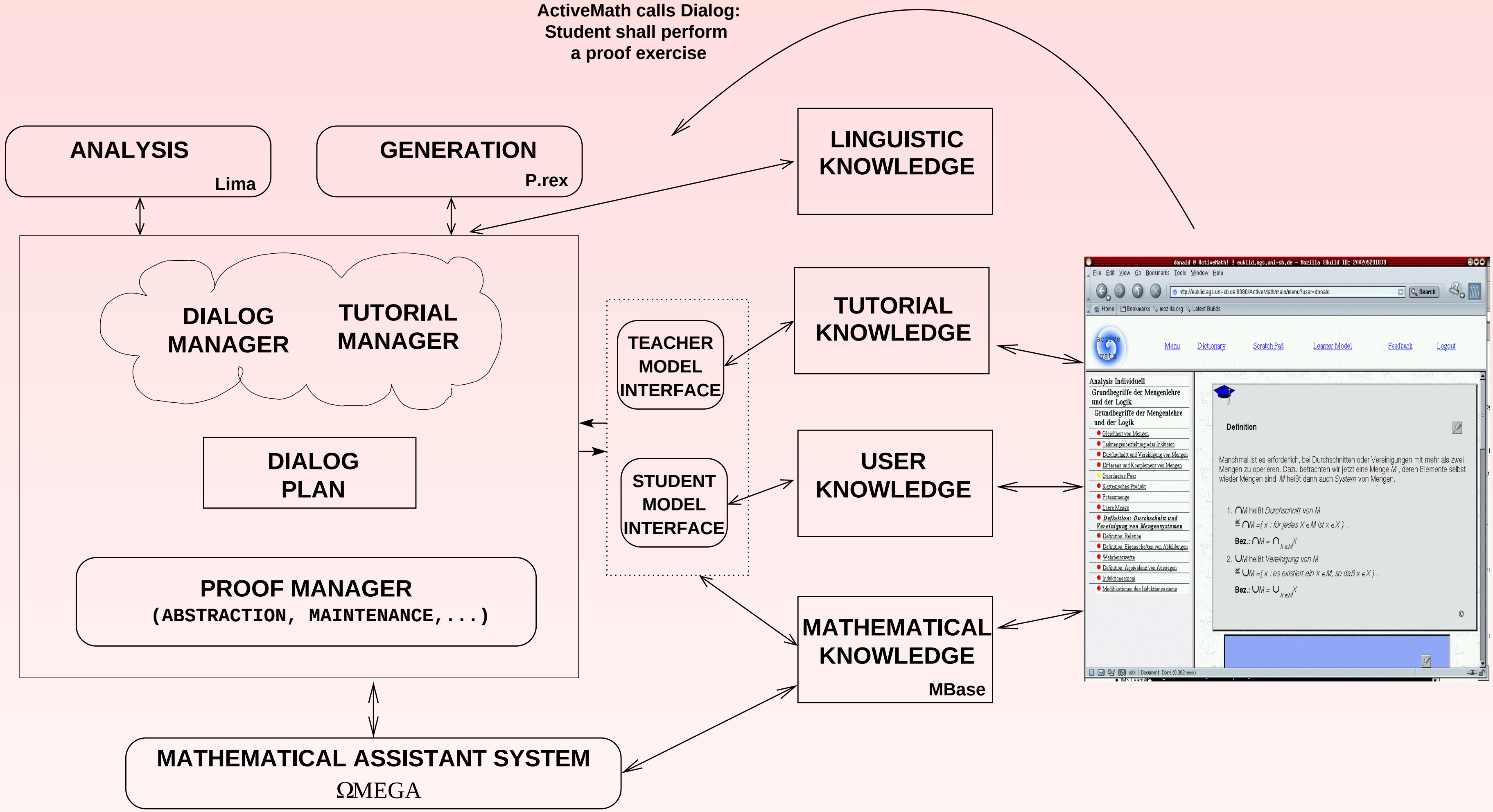
- **Novice student should solve exercises with mathematical knowledge that have been taught in the** ACTIVEMATH **tutorial.**

  Chapter1,Ex1 $\rightarrow R = \{\subseteq$**-REF**$, \in$-$\cup$**-IL**$, \subseteq$-$\cup$**-IL**$, \subseteq$-$\cup$**-IR**$, \subseteq$-$\cap$**-IL**$, \wp$**-I**$\}$

- **If novice student does not understand an application of** $r \in R$ **then employ one of the following**

  1. **show/explain the Venn-Diagram for** r
  2. - **explain instantiation of** r
     - **choose theorem** r **as exercise (recursion!)**
  3. **refer to** ACTIVEMATH**text for** r
  4. **. . .**

Architecture for DIALOG



Christoph Benzmüller, Birmingham, July 2002

Challenge for OMEGA & Ω-Ants

Proof planning in OMEGA & Ω-Ants with resources:

- **inference rules**
- **control knowledge to structure the search space**

Teacher model $\rightarrow$ T = (inf-rules-1, control-1)

User model $\rightarrow$ U = (inf-rules-2, control-2)

CONSTRUCT-PROOF(T) $\rightarrow$ Teacher proof

vs.

CONSTRUCT-PROOF(U) $\rightarrow$ Predictable steps of user

Example Proof

$$A \cap B \in \wp((A \cup C) \cap (B \cup C))$$

Call OMEGA **with**

Rules: ND $\cup$ $\{$**⊆-REF**, **⊆-∩-I**, **⊆-∩-IL**, **⊆-∩-IR**, **⊆-∪-IL**, $\ldots\}$

Strategies: prefer-set-tactics-over-ND

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\top}{A \subseteq A}\ \subseteq\text{-REF}}{A \subseteq A \cup C}\ \subseteq\text{-}\cup\text{-IL}}{A \cap B \subseteq A \cup C}\ \subseteq\text{-}\cap\text{-IL} \qquad \dfrac{\dfrac{\dfrac{\top}{B \subseteq B}\ \subseteq\text{-REF}}{B \subseteq B \cup C}\ \subseteq\text{-}\cup\text{-IL}}{A \cap B \subseteq B \cup C}\ \subseteq\text{-}\cap\text{-IR}}{A \cap B \subseteq (A \cup C) \cap (B \cup C)}\ \subseteq\text{-}\cap\text{-I}}{A \cap B \in \wp((A \cup C) \cap (B \cup C))}\ \wp\text{-I}$$

Example Proof

To show: $A \cap B \in \wp((A \cup C) \cap (B \cup C))$

By $\wp$-I enough to show: $A \cap B \subseteq (A \cup C) \cap (B \cup C)$

By $\subseteq$-$\cap$-I we have to show (1) and (2):

1. $A \cap B \subseteq A \cup C$

   By $\subseteq$-$\cap$-IL enough to show $A \subseteq A \cup C$

   By $\subseteq$-$\cup$-IL enough to show $A \subseteq A$

   Follows by $\subseteq$-REF

2. $A \cap B \subseteq B \cup C$

   By $\subseteq$-$\cup$-IL enough to show $A \cap B \subseteq B$

   By $\subseteq$-$\cap$-IR enough to show $B \subseteq B$

   Follows by $\subseteq$-REF

q.e.d.

$\Rightarrow$ **Note multiplicities in proof: proof abstraction is needed**

Enough structure in Naive Set Theory?

Problem: $(A \cup B)^c \subseteq A^c$

By $\subseteq^c$ enough to show: $A \subseteq A \cup B$

By $\subseteq$-$\cup$-IL enough to show: $A \subseteq A$

Follows by $\subseteq$-REF **q.e.d.**

Assume: Student does not understand application of $\subseteq^c$
According to our tutorial knowledge we would

1. **show/explain Venn diagram for $\subseteq^c$**
2. - **explain instantiation of $\subseteq^c$**
   - **choose theorem $\subseteq^c$ as exercise (recursion!)**
3. **refer to ACTIVEMATH text for $\subseteq^c$**

Enough structure in Naive Set Theory?

(2b) Proof of $\subseteq^c$: $X \subseteq Y \Rightarrow Y^c \subseteq X^c$

By $\Rightarrow$-I we assume $[X \subseteq Y]$ and show $Y^c \subseteq X^c$

By `Defn-I(c)` enough to show $\mathcal{U} \setminus Y \subseteq \mathcal{U} \setminus X$

By $\subseteq -\setminus$ enough to show $\mathcal{U} \subseteq \mathcal{U}$ **and** $X \subseteq Y$

The former follows by $\subseteq$-REF and the latter from the assumption.

q.e.d.

$\Rightarrow$ **Update of User Model**

Assume: Student does not understand $\subseteq -\setminus$ step.

. . .

(2b) Subdialog on proof problem: $A \subseteq B \wedge C \subseteq D \Rightarrow A \setminus D \subseteq \mathbf{B} \setminus C$

. . .

Related Work

Mathematical Assistant Systems: NuPrl, HOL, λ-Clam

Tutor systems: Typically simple dialog capabilities and no clear separation of domain representation, tutorial strategies, and NL dialog

- **Autotutor (Person et al.)**
- **Geometry-tutor (Aleven & Koedinger)**
- **Algebra-tutor Ms. Lindquist (Heferman & Koedinger)**
- **MALIN system**

Related Work

Dialog systems: Recent trend is flexible, domain and content-oriented dialog modeling

- **(Cohen & Levesque): Theory of action and interaction**
- **ARTEMIS (Sadek et al.): Ambitious speech input, reasoning based on cooperation principles**
- **Dialog games (Carlson) (Carletta)**
- **Trindi-Kit developed in EU projects TRINDI and SIRIDUS: Framework for development and evaluation of dialog systems based on dynamical information states**

Summary

Focus of research

- Collect empirical data
- Modeling of dialog system
- Challenge: Interplay with the domain reasoner, the mathematical knowledge base, the tutorial strategies, and the user model

Step by step

- Successive refinement and validation of the model
- Implementation of demonstrator