New Directions in the ΩMEGA Project

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joint work with: The OMEGA Group

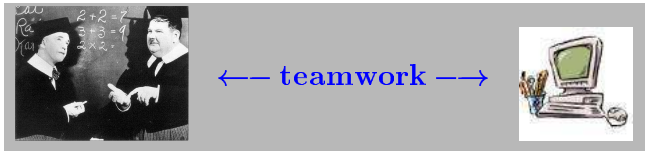
Issues of this Talk

- History and Aims of the OMEGA Project

- A New Logic Layer: CORE
- A New Interface to the Logic Layer: TASKS
- An Improved Proof Datastructure: PDS
- Adding Management of Change: MAYA
- Adaptation of Proof Planning, Agent-based Reasoning, User-Interaction
- A New User Interface: TEX_{MACS}
- Bridging between (multimodal) NL Proofs and Logic-level Proofs

History and Aims of the OMEGA Project

- THE GOAL:



  → integrated mathematical assistant environment

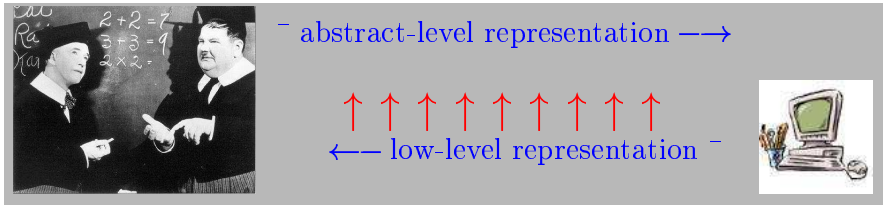
  → support for typical tasks of a mathematician

- THE PROBLEM:

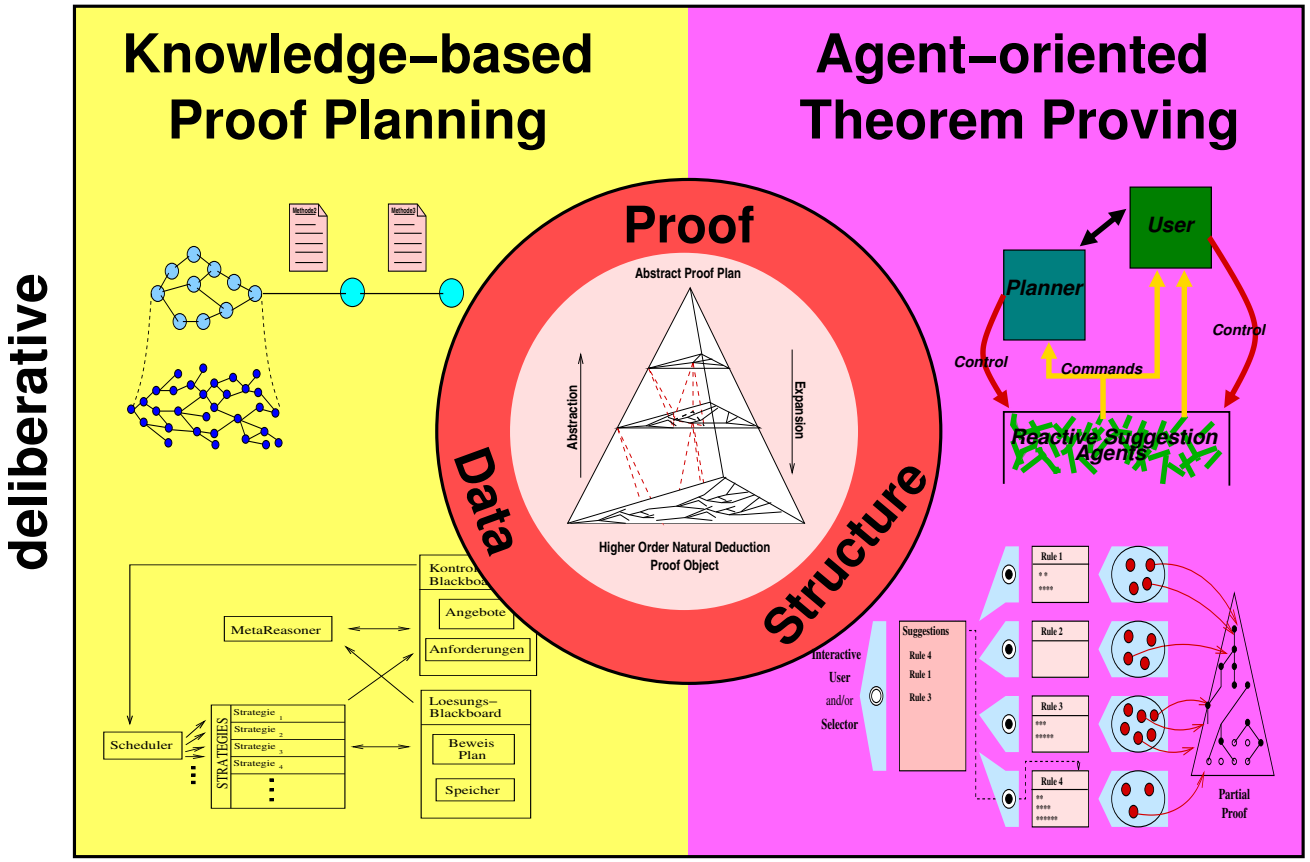
  → resource "mathematical knowledge" represented and employed differently

  → majority in community: "human should adapt to computer"

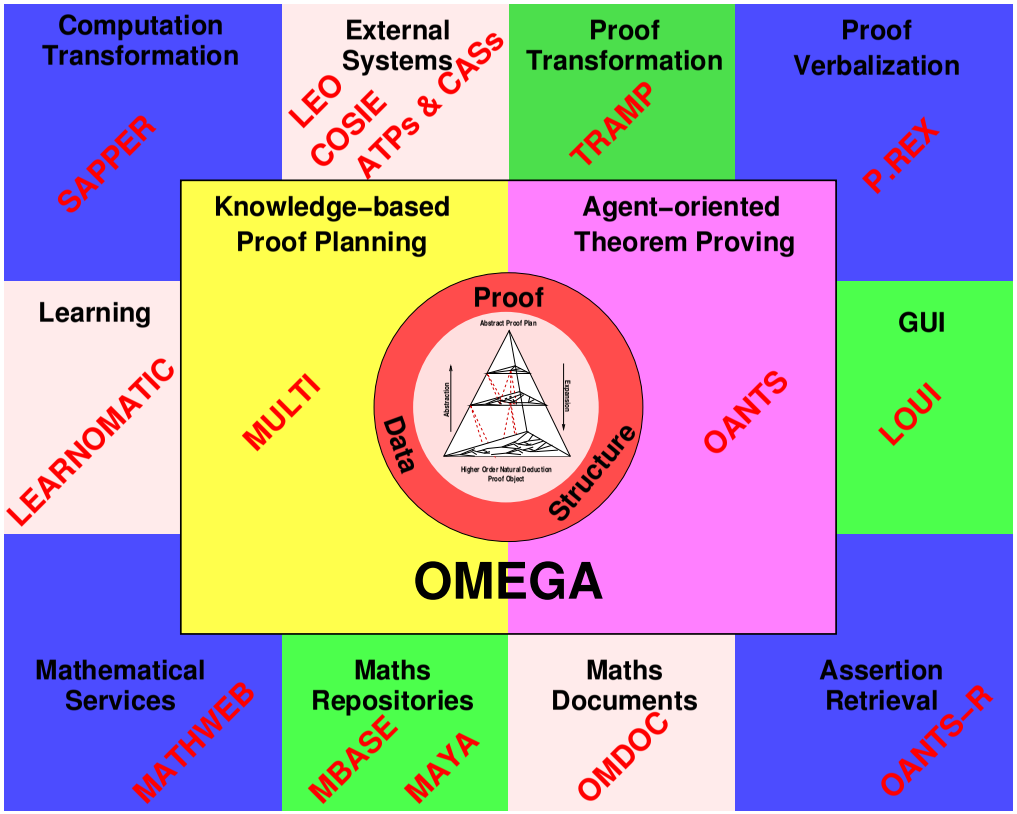
- OUR APPROACH:



History and Aims of the OMEGA Project



History and Aims of the OMEGA Project



COLLABORATIONS
(with joint publication)

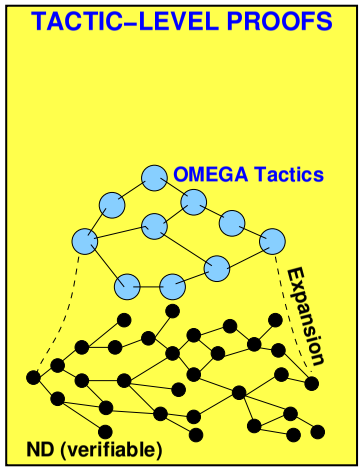
Yale University, USA
CMU, Pittsburgh, USA
University of Miami, USA
The Univ. of Edinburgh, UK
The Univ. of Birmingham, UK
Imperial College London, UK
Eindhoven TU, Netherlands
Cambridge University, UK
International Univ. of Bremen, D
University of Kaiserslautern, D
TU Munich, D
EU RTN CALCULEMUS

OMEGA Base Calculus: ND vs CORE

Theorem 1. $\sqrt{2}$ is irrational.

Proof. (by contradiction)
Assume $\sqrt{2}$ is rational, that is, there exist natural numbers m, n with no common divisor such that $\sqrt{2} = \frac{m}{n}$. Then $n\sqrt{2} = m$, and thus $2n^2 = m^2$. Hence m^2 is even and, since odd numbers square to odds, m is even; say $m = 2k$. ...

□

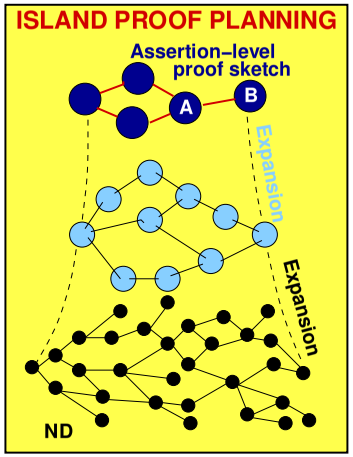


Result:
Tactic-based theorem provers like PVS, ISABELLE, COQ, etc. (and OMEGA) can construct a proof ——
but not at an adequate level

A New Logic Layer: CORE

SOLUTION A

[35years-of-AUTOMATH]



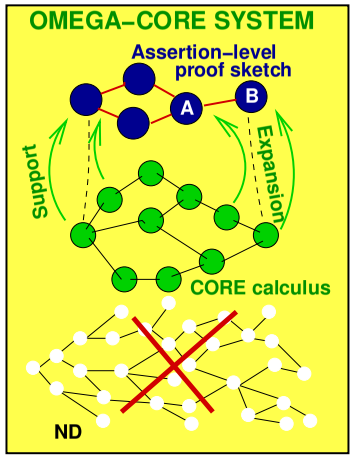
- new explicit layer for proof sketches
- easy to add to proof data structure
- proof sketches may be unsound
- verification by expansion to ND

Problem:
- expansion distance to base-layer

SOLUTION B

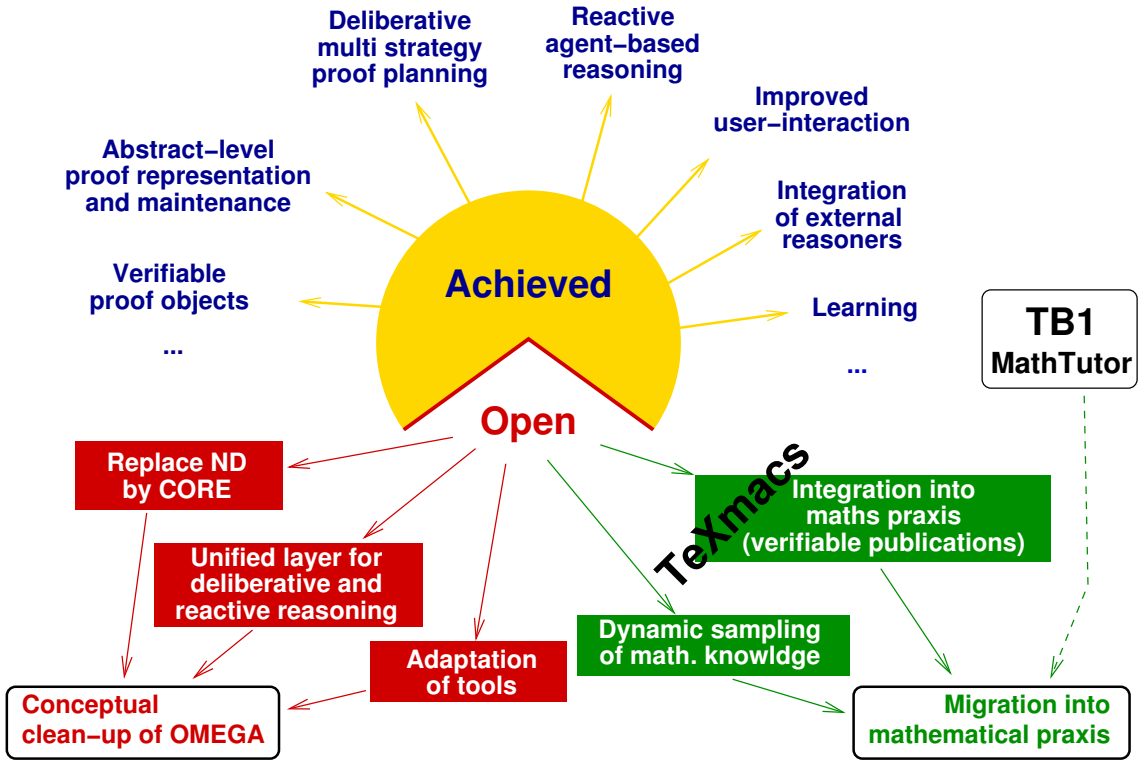
[PhD-Autexier-2003]

(realization started)



- CORE is more natural/abstract-level base calculus for proof assistants
- subsumes old ND base and (parts of) the old tactic layer
- provides more direct, constructive support for upper layer(s)

OMEGA Project: Open Tasks



A New Interface to the Logic Layer: TASKS

- Problem in the current OMEGA system:
  - → interactive theorem proving
  - → deliberative knowledge-based proof planning
  - → reactive agent-based proof planning

  do not employ a common interface to the OMEGA base calculus

- **OMEGA's current proof planner Multi based on "proof (plan) tasks"**
  - → **maintained by Multi on an agenda**
  - → **employed to structure a proof into subtasks**
  - → **basically only a proof goal together with its supporting assertions**

A New Interface to the Logic Layer: TASKS

Deliberative Knowledge-based Proof-Planning
Interactive Theorem Proving
Agent-based Proof Planning

Tasks

...	...
(rat-criterion)	...
(even)	$\lambda x.\exists y.x = 2\,y$
(square-even)	$\text{even}(x^2) \Rightarrow \text{even}(x)$
(goal)	$\sqrt{2}$ is not rational

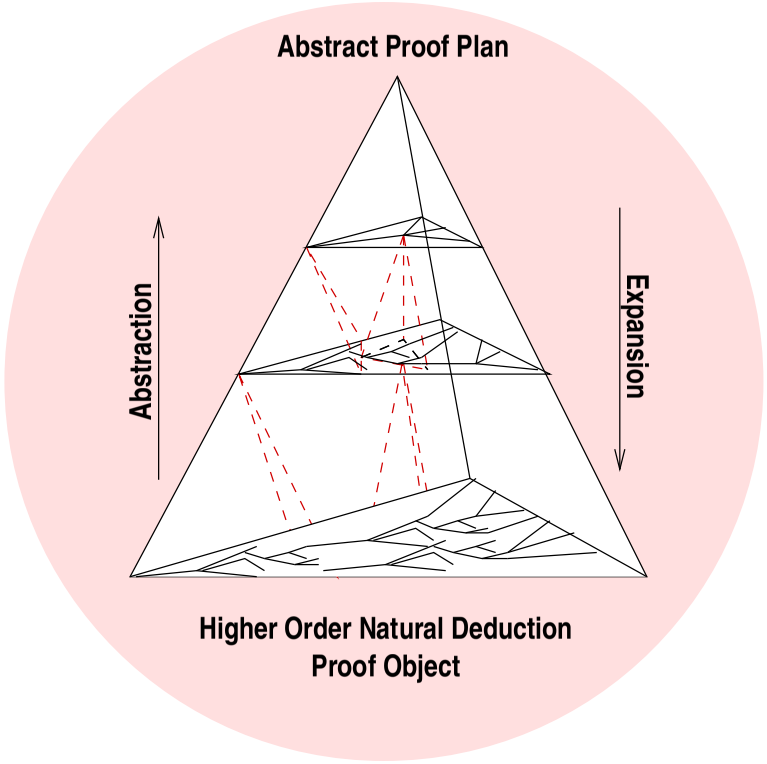
CORE calculus

Technically a task is a

→ multi-conclusion sequent: $\text{ass}_1^-, \ldots, \text{ass}_n^-, \text{goal}_1^+, \ldots, \text{goal}_m^+$
(premise-conclusion-status defined via polarities
which are maintained by CORE)

→ with focus of attention: $\text{ass}_1^-, \ldots, \text{ass}_n^-, \text{goal}_1^+, \ldots, \text{goal}_m^+$

An Improved Proof Datastructure: PDS



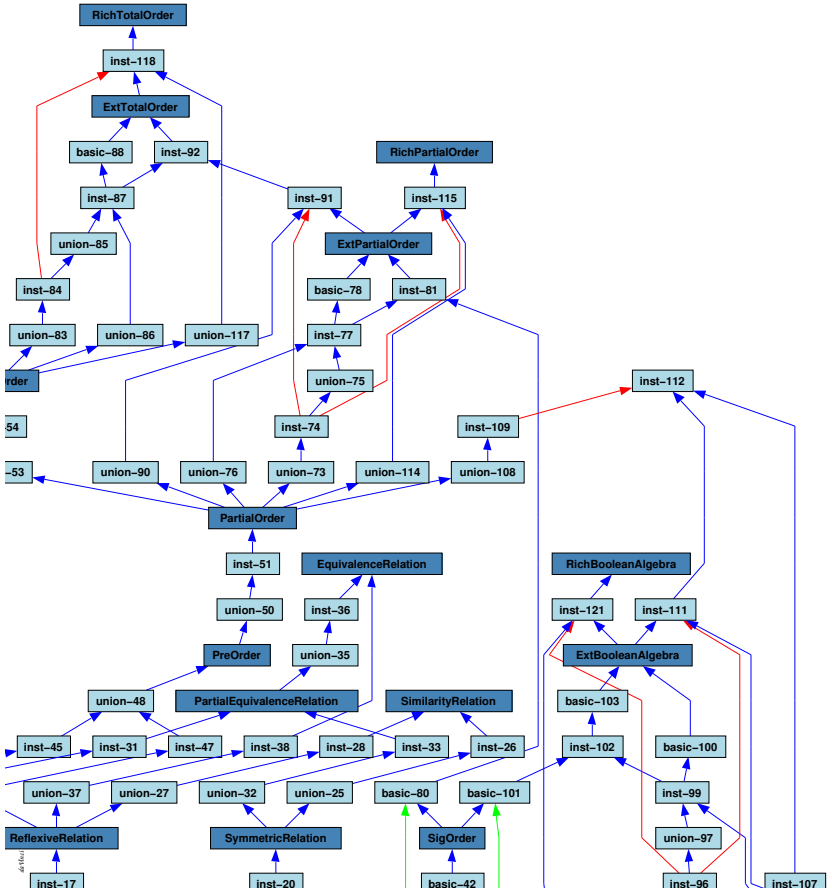
Problems and Missing Features

- (loosely) dependent on ND calculus
- no proof forests (lemmata)
- no OR-branching

An Improved Proof Datastructure: PDS

- Separation of
  - → PDS framework
  - → from its instantiation by an inference system
- Instantion of PDS framework for the new OMEGA with Proof Tasks
- Support for Proof Forests:
  - → one proof tree for each lemma
  - → taken from Quodlibet framework
- Addition of OR-branches: AND-OR proof trees

Management of Change: MAYA



Management of Change: MAYA

- Integration of OMEGA and MAYA
  - → obtain support for management of change in theory exploration in mathematics
  - → MAYA as tool between user and OMEGA
  - → MAYA as tool for joint document development (multiple users)
- Extension of MAYA
  - → support for other theory specific mathematical knowledge:
    proof tactics, proof methods, mathematical documents
- Important Sidegoal:
  - → reunification/integration of the systems developed in the AGS via MAYA (OMEGA, INKA, VSE)

Adaptation of Proof Planning & Co

→ Knowledge-based Proof-Planning (PP) ←
→ Interactive Theorem Proving (ITP) ←
→ Agent-based Proof Planning (APP) ←

Tasks

...	...
(rat-criterion)	...
(even)	$\lambda x.\exists y.x = 2\,y$
(square-even)	$\text{even}(x^2) \Rightarrow \text{even}(x)$
(goal)	$\sqrt{2}$ is not rational

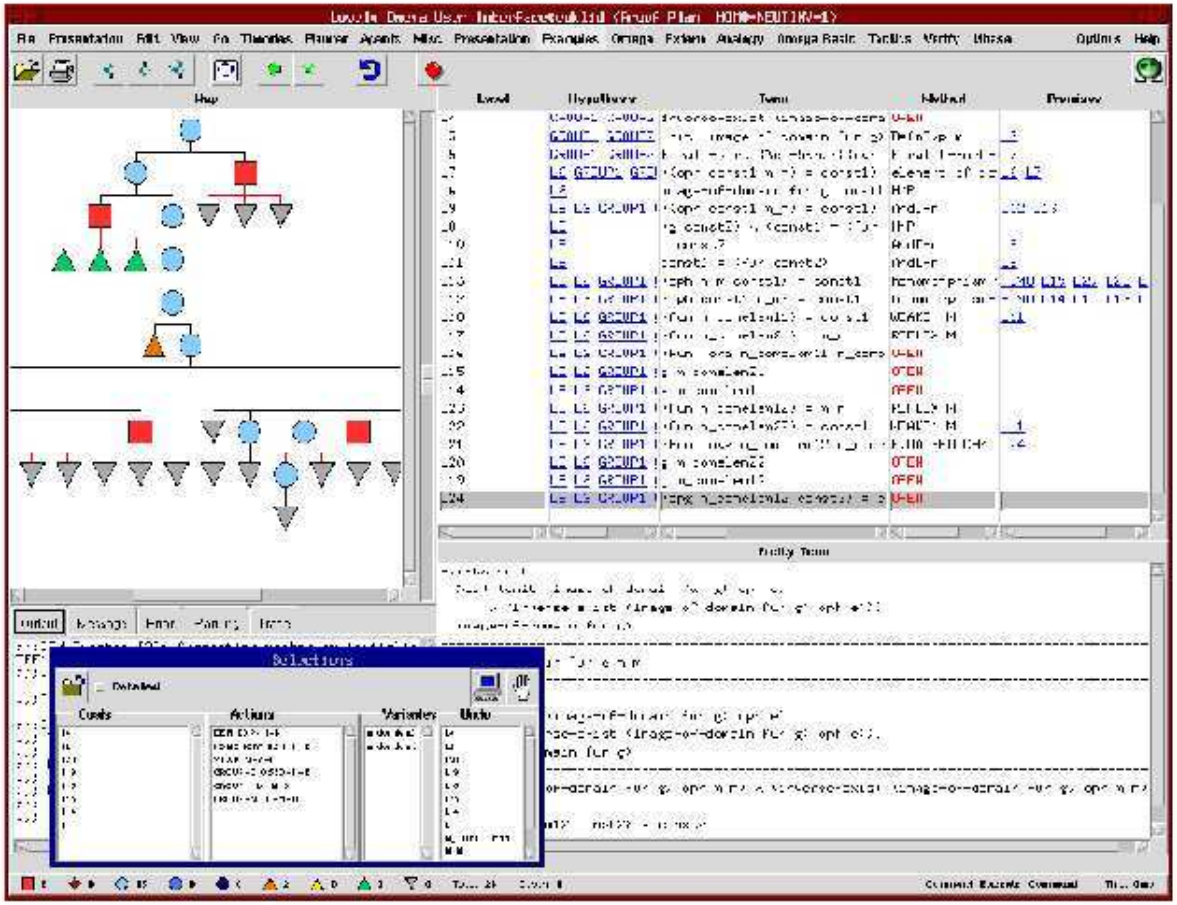
CORE calculus

What Benefits?

- Conceptual clean-up of OMEGA
  → proof planning with proof methods (and not wit
- Better intergration of PP, ITP, APP

Adaptation of Proof Planning & Co

A New OMEGA Interface



OMEGA Interface: Why TEX_{MACS} ?

What is TEX_{MACS}? :

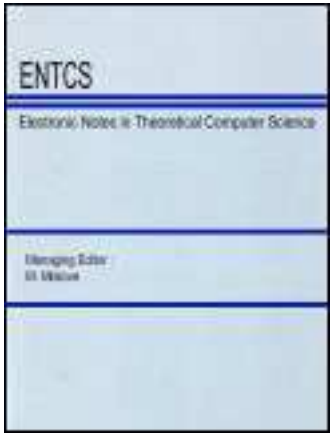
- **symbiosis of LaTeX and Emacs; supports professional mathematical editing**
- **integrates many support tools: CASs (Maple ...), Gnuplot,..., Scheme, Lisp, ...**
- **Springer official document format for maths journals (in test)**
- **missing: integrated proof assistant (OMEGA**

A small Demo: **see here!**

$\sqrt[j]{k}\quad\sqrt{4+8}$

This is an example for an action

OMEGA-TEX_{MACS} Interface: Our Vision



- Verifiable Scientific Documents (also Marking of Exams)
- Active Scientific Documents
- Overcome Need for Multiple Encodings of Mathematical Knowledge

A Short Demo

Human-Oriented Problem Representation	Formal Representation
Chris invites Jörg, Claus-Peter, and Erica to his Party. He receives the following replies: Jörg: "Claus-Peter or Erica will come" Claus-Peter: "Either Jörg or Erica will come" Erica: "Either Jörg or Claus-Peter will come"	Chris v. Erica (Joerg & ~Erica) v. (~Joerg & Erica) (Joerg & ~Chris) v. (~Joerg & Chris)
Theorem: Erica will be at the Party.	"above axioms" |= Erica

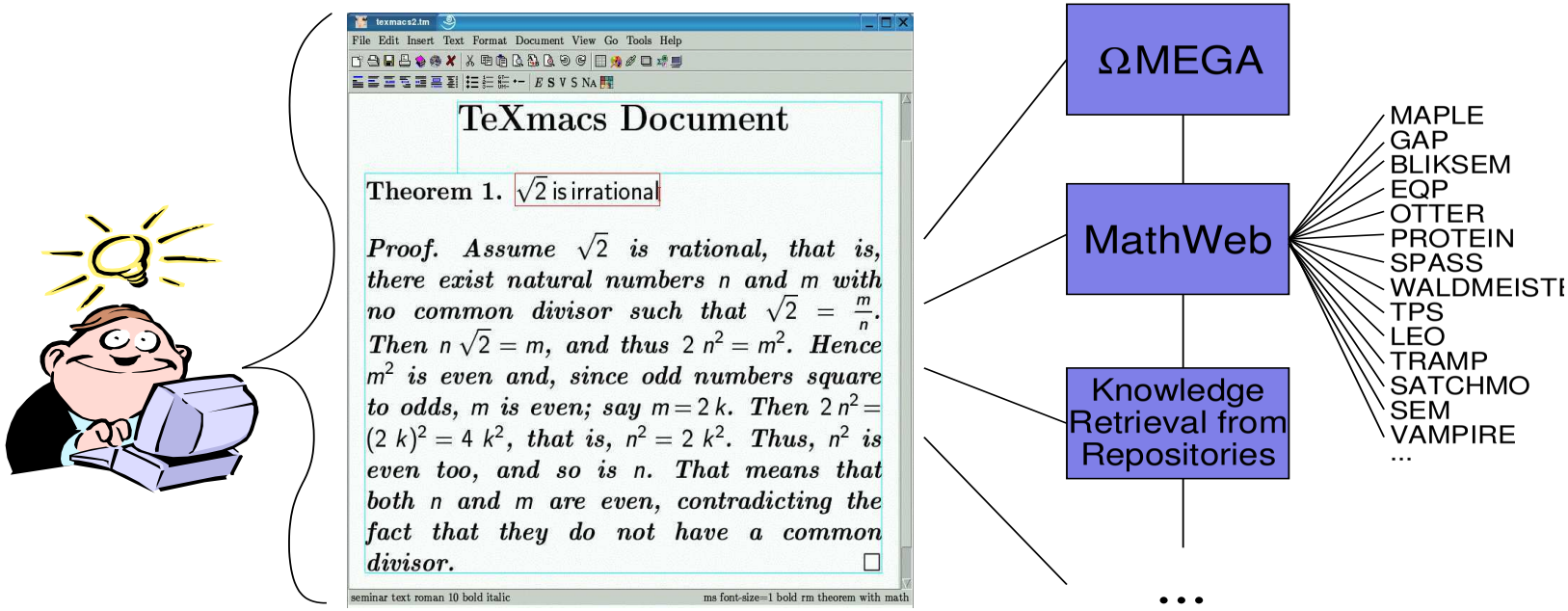
Theorem: {Chris v. Erica, (Joerg & ~Erica) v. (~Joerg & Erica), (Joerg & ~Chris) v. (~Joerg & Chris)}|= Erica

Proof:

\"translatedBuf{{}{C}{h}{r}{i}{s}{ }{v}{.}{ }{E}{r}{i}{c}{a}{,}{ }{(}{J}{o}{e}{r}{g}{ }{&}{ }{~}{E}{r}{i}{c}{a}{)}{ }{v}{.}{ }{(}{~}{J}{o}{e}{r}{g}{ }{&}{ }{E}{r}{i}{c}{a}{)}{,}{ }{(}{J}{o}{e}{r}{g}{ }{&}{ }{~}{C}{h}{r}{i}{s}{)}{ }{v}{.}{ }{(}{~}{J}{o}{e}{r}{g}{ }{&}{ }{C}{h}{r}{i}{s}{)}{}}{|}{=}{ }{E}{r}{i}{c}{a}\"

OMEGA-TEX_{MACS} Interface: Our Vision

- TEX_{MACS} (www.texmacs.org) as scientific wysiwyg Texteditor



- **Bridging between NL proofs and Logic-level proofs: VeriMath & DIALOG**

There is no conclusion yet ....