

First-Order Logic: Theory and Practice

Christoph Benzmüller

Freie Universität Berlin

Block Lecture, WS 2012, October 1-12, 2012





Please register in the Campus Management System!

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Course Homepage

http://christoph-benzmueller.de/2012-FOL/

Course Mailinglist & Wiki

https://groups.google.com/d/forum/2012-fol 2012-fol@googlegroups.com



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- ▶ 09:15-11:00 Theory Lectures
- ▶ 11:15-13:00 Exercises: Theory
 - exercises from theory lectures
 - individual work
 - short presentations from students
- ▶ 14:00-16:00 Practice Lectures, Modeling Exercises, Working with Systems
 - modeling of problems, application of theorem provers
 - group work (approx. 5 students per group)
 - short presentations from students
- ▶ 16:15-18:00 Exercises: Implementation
 - Implementation of relevant algorithms, maybe an entire theorem prover
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 Jens Otten, Universität Potsdam
- Guest lecture (First-order Reasoning in Isabelle/HOL) on Friday, October 12, afternoon:
 Nik Sultana, Cambridge University, UK
- Written exam on October 20
 Regular contributions in exercises is mandatory for exam registration and participation

We need to find a time: ...



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Presentations by all students:

- ▶ I will keep a list and mark whenever a student actively contributes (e.g. on blackboard)
- ► Students should ideally contribute in all exercises: theory, modeling, working with systems, implementation
- ▶ If there are no volunteers, I will ask students directly (be smart: volunteer whenever you feel confident!)



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Lecture Organization: Details Monday, October 1



- ► Lecture/Exercises: introduction, propositional logic, syntax, structural induction and recursion, semantics, evaluation, tautology, satisfiability, uniform notation, normal forms, transformation into clause form and dual clause form, propositional semantic tableaux & resolution
- Modeling/Systems: introduction & organization, student team allocation, allocation of prover talks to teams, modeling of simple puzzles in propositional logic, short introduction to TPTP syntax, representation of simple puzzles in TPTP syntax, first use of provers via SystemOnTPTP
- ▶ Implementation: warm up, installation of extended TPTP parser, discussion of some first functions from the lecture

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- ▶ **Implementation:** warm up, installation of extended TPTP parser, discussion of some first functions from the lecture

Lecture Organization: Details Tuesday, October 2



- ► Lecture/Exercises: soundness of semantic tableaux & resolution, Hintikka sets, abstract consistency method, completeness of propositional semantic tableaux & resolution, logical consequence, strong completeness, preview: syntax of first-order logic
- Modeling/Systems: modeling of further propositional puzzles, modeling of first first-order puzzles: Agatha's murderer & others, short introduction to SystemOnTPTP, analyze problems with SystemOnTPTP provers, announcing the modeling challenge: each student team ideally prepares a proposal
- ▶ Implementation: implement selected functions from the lecture including normal form transformations, start implementing semantic tableaux & resolution.

Lecture Organization: Details Tuesday, October 2



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- ► Implementation: implement selected functions from the lecture including normal form transformations, start implementing semantic tableaux & resolution.

Lecture Organization: Details Thursday, October 4



- Lecture/Exercises: propositional hornlogik, syntax first-order logic: terms, formulas, substitutions
- Modeling/Systems: team presentations on provers, SZS ontology, CASC competitions, modeling of city distance problems, modeling of Miami CS courses, discussion of modeling challenge default exercise
- ▶ Implementation: implement propositional semantic tableaux or resolution.

Lecture Organization: Details Thursday, October 4



- ► Lecture/Exercises: propositional hornlogik, syntax first-order logic: terms, formulas, substitutions
- Modeling/Systems: team presentations on provers, SZS ontology, CASC competitions, modeling of city distance problems, modeling of Miami CS courses, discussion of modeling challenge default exercise
- ▶ **Implementation:** implement propositional semantic tableaux or resolution.

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- ▶ **Implementation:** implement propositional semantic tableaux or resolution.

Lecture Organization: Details Friday, October 5



- ▶ Lecture/Exercises: semantics of first-order logic, evaluation of terms and formulas, validity and satisfiability, Herbrand models, uniform notation, first-order structural induction and recursion
- Modeling/Systems: GDV verification, IDV presentation, presentation and discussion of modeling challenge proposals, allocation of modeling tasks
- ▶ Implementation: implement propositional semantic tableaux or resolution

Lecture Organization: Details Friday, October 5



- ► Lecture/Exercises: semantics of first-order logic, evaluation of terms and formulas, validity and satisfiability, Herbrand models, uniform notation, first-order structural induction and recursion
- Modeling/Systems: GDV verification, IDV presentation, presentation and discussion of modeling challenge proposals, allocation of modeling tasks
- ▶ Implementation: implement propositional semantic tableaux or resolution

Lecture Organization: Details Friday, October 5



- ▶ Lecture/Exercises: semantics of first-order logic, evaluation of terms and formulas, validity and satisfiability, Herbrand models, uniform notation, first-order structural induction and recursion
- Modeling/Systems: GDV verification, IDV presentation, presentation and discussion of modeling challenge proposals, allocation of modeling tasks
- ► Implementation: implement propositional semantic tableaux or resolution

Lecture Organization: Details, Monday, October 8



- Lecture/Exercises: first-order Hintikka sets, first-order abstract consistency
- ► Guest Lecture: Jens Otten (Uni Potsdam) DPLL, leanTAB, and leanCoP
- Implementation: implement various functions on first-order terms and formulas

Lecture Organization: Details, Monday, October 8



- Lecture/Exercises: first-order Hintikka sets, first-order abstract consistency
- ► Guest Lecture: Jens Otten (Uni Potsdam) DPLL, leanTAB, and leanCoP
- Implementation: implement various functions on first-order terms and formulas

Lecture Organization: Details, Monday, October 8



- Lecture/Exercises: first-order Hintikka sets, first-order abstract consistency
- Guest Lecture: Jens Otten (Uni Potsdam) DPLL, leanTAB, and leanCoP
- Implementation: implement various functions on first-order terms and formulas

Lecture Organization: Details, Tuesday, October 9



- ► Lecture/Exercises: first-order semantic tableaux, first-order resolution, soundness and completeness, free-variable semantic tableaux & resolution motivation, unification
- ► Modeling/Systems: modeling challenge, close interaction with SystemOnTPTP
- ▶ Implementation: implement first-order unification

Lecture Organization: Details, Tuesday, October 9



- ► Lecture/Exercises: first-order semantic tableaux, first-order resolution, soundness and completeness, free-variable semantic tableaux & resolution motivation, unification
- ► Modeling/Systems: modeling challenge, close interaction with SystemOnTPTP
- ▶ Implementation: implement first-order unification

Lecture Organization: Details, Tuesday, October 9



- ► Lecture/Exercises: first-order semantic tableaux, first-order resolution, soundness and completeness, free-variable semantic tableaux & resolution motivation, unification
- ► Modeling/Systems: modeling challenge, close interaction with SystemOnTPTP
- ▶ Implementation: implement first-order unification

Lecture Organization: Details, Wednesday, October 10



- Lecture/Exercises: free-variable semantic tableaux & resolution without and with restrictions, Skolemization, prenex form, Herbrand's theorem, some history on automated theorem proving,
- ► Modeling/Systems: modeling challenge, close interaction with SystemOnTPTP
- Implementation: implement first-order semantic tableaux or resolution

Lecture Organization: Details, Wednesday, October 10



- Lecture/Exercises: free-variable semantic tableaux & resolution without and with restrictions, Skolemization, prenex form, Herbrand's theorem, some history on automated theorem proving,
- ► Modeling/Systems: modeling challenge, close interaction with SystemOnTPTP
- ▶ Implementation: implement first-order semantic tableaux or resolution

Lecture Organization: Details, Wednesday, October 10



- Lecture/Exercises: free-variable semantic tableaux & resolution without and with restrictions, Skolemization, prenex form, Herbrand's theorem, some history on automated theorem proving,
- ► Modeling/Systems: modeling challenge, close interaction with SystemOnTPTP
- Implementation: implement first-order semantic tableaux or resolution

Lecture Organization: Details, Thursday, October 11



- ► Lecture/Exercises: natural deduction and sequent calculus, some early history in symbolic logic, Gödel's incompleteness theorems
- Modeling/Systems: modeling challenge: present your team results
- Implementation: implement first-order semantic tableaux or resolution

Lecture Organization: Details, Thursday, October 11



- ► Lecture/Exercises: natural deduction and sequent calculus, some early history in symbolic logic, Gödel's incompleteness theorems
- Modeling/Systems: modeling challenge: present your team results
- Implementation: implement first-order semantic tableaux or resolution

Lecture Organization: Details, Thursday, October 11



- ► Lecture/Exercises: natural deduction and sequent calculus, some early history in symbolic logic, Gödel's incompleteness theorems
- Modeling/Systems: modeling challenge: present your team results
- Implementation: implement first-order semantic tableaux or resolution

Lecture Organization: Details, Friday, October 12



- Lecture/Exercises: questions, discussions, exam preparation, etc.
- Guest Lecture: Nik Sultana (Cambridge, UK), First-order Reasoners in Isabelle/HOL
- ► Implementation: present your algorithms and prover(?), run small competition(?)

Lecture Organization: Details, Friday, October 12



- Lecture/Exercises: questions, discussions, exam preparation, etc.
- ► Guest Lecture: Nik Sultana (Cambridge, UK), First-order Reasoners in Isabelle/HOL
- ► Implementation: present your algorithms and prover(?), run small competition(?)

Lecture Organization: Details, Friday, October 12



- Lecture/Exercises: questions, discussions, exam preparation, etc.
- Guest Lecture: Nik Sultana (Cambridge, UK), First-order Reasoners in Isabelle/HOL
- ► Implementation: present your algorithms and prover(?), run small competition(?)

Exercises: Modeling & Systems



Possible Tasks:

- Get Familiar with TPTP syntax and SystemOnTPTP: little exercises, try your own examples
- Short System demonstrations by Student Teams
- Present some successful provers (short talks) (http://www.cs.miami.edu/~tptp/CASC/J6/)
 - Overview on CASC Competitions
 - ► FOF Prover: Vampire, EP, Spass, iProver, Prover9, Z3, Metis (leanCoP)
 - ► FOF Countermodel Finder: iProver-SAT, Vampire-SAT, Paradox
 - TFA Prover: Princess, Spass+T
- aspects to talk about
 - Developers? Since when developed? Where?
 - Calculus, Techniques, Implementation
 - Example, Applications, Awards
 - ▶ Installation, License
 - ▶ References, Documentation

Exercises: Modeling & Systems



Modeling Warm-Up:

- Propositional puzzles
- Degree at U Miami
- Steamroller and Agatha's murderer
- Other first-order puzzles
- Maths examples
- Sudoku
- Cities and distances

Modeling Advanced:

- Suggest you own group project
- Default: Model the conditions of your own masters/bachelor program?

Exercises: Programming



Possible Tasks:

- Get familiar with the TPTP parser and some FOL-2012 related extensions (adapt Makefile, make, ./test.sh FOL2012Test.p)
- test the code with own example formulas
- ► PL:
 - add functionality to replace equivalences and negated equivalences
 - implement propositional normal forms
 - implement tableau and/or resolution calculi
- FOL:
 - ▶ implement functions: free-vars (term), free-vars (formula),
 - implement functions: occurs-free-in(var,term), occurs-free-in(var,formula)
 - implement function: rename-free-vars-in(term), rename-free-vars-in(formula),
 - implement first-order normal forms
 - implement tableau and/or resolution calculi