Dyadic Deontic Logic of Carmo and Jones

Docent: Prof. Christoph Benzmüller. Speaker: Alexey Gonus.

Standard Deontic Logic: definition

$$\Box \longmapsto \bigcirc \quad \text{"it is obligatory that"}$$

$$\neg\Box\neg \longmapsto \mathcal{P} \quad \text{"it is permitted that"}$$

$$\Box\neg \longmapsto \bigcirc\neg \quad \text{"it is forbidden that"}$$

Standard Deontic Logic: its paradoxes and problems

“Ross paradox” $\vdash \bigcirc A \to \bigcirc(A \vee B)$

“Free Choice Permission paradox” $\nvdash \mathscr{P}(A \vee B) \to (\mathscr{P}A \wedge \mathscr{P}B)$

“Good Samaritan paradox” $\vdash \bigcirc(A \wedge B) \to \bigcirc B$

“Deontic/epistemic paradox” $\bigcirc \mathscr{K}A \to \bigcirc A$

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"Deontic/epistemic paradox" $\bigcirc \mathscr{K} A \rightarrow \bigcirc A$

"Any tautology is obligatory"

"Impossibility of consistent expression of a conflict of obligations"

Standard Deontic Logic: conditional obligations

(option 1) $\bigcirc(B/A) =_{df} A \rightarrow \bigcirc B$

(option 2) $\bigcirc(B/A) =_{df} \bigcirc(A \rightarrow B)$

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(option 2) $\bigcirc(B/A) =_{df} \bigcirc(A \rightarrow B)$

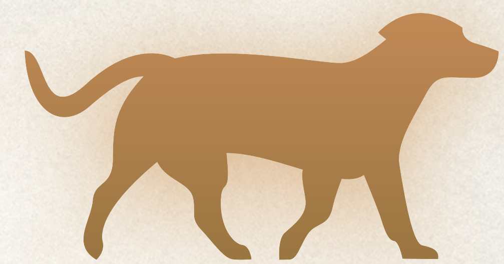
(UN) $\vdash \bigcirc B \leftrightarrow \bigcirc(B/\top)$

(SA) $\vdash \bigcirc(B/A) \rightarrow \bigcirc(B/A \wedge C)$

Desirable Deontic Logic features

- Minimum requirements of consistency
- Logical independence of the sentences
- Applicability to timeless and actionless CTD-scenarios
- The assignment of logical form to every norm in the set should be independent of the other norms in it
- Capacity to derive actual obligations
- Capacity to derive primary obligations
- Capacity to represent the violation of obligations

Contrary-to-Duties Scenario: Dog & Warning Sign



(a) There ought to be no dog

(b) If there is no dog, there ought to be no warning sign

(c) If there is a dog, there ought to be a warning sign

(d) There is a dog

Contrary-to-Duties Scenario: fixity of facts

Temporal

Casual

Agent's decisions

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Agent's decisions

"Considerate assassin"

(a) You should not kill Mr. X

(b) ———

(c) If you kill Mr. X, you should offer him a sigarette

Contrary-to-Duties Scenario: Dyadic Deontic Logic

1) $\Box_a A \longrightarrow \Box_a dog$

2) $\Diamond_p A \longrightarrow \Diamond_p \neg dog$

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1) $\Box_a A \longrightarrow \Box_a \, dog$

2) $\Diamond_p A \longrightarrow \Diamond_p \neg dog$

3) $\bigcirc(B/A) \longrightarrow \bigcirc(\neg dog / \top)$

$\bigcirc(B/A) \longrightarrow \bigcirc(\neg sign / \neg dog)$

$\bigcirc(B/A) \longrightarrow \bigcirc(sign / dog)$

Contrary-to-Duties Scenario: Dyadic Deontic Logic

1)	$\Box_a A$	$\Box_a dog$
2)	$\Diamond_p A$	$\Diamond_p \neg dog$
3)	$\bigcirc(B/A)$	$\bigcirc(\neg dog/\top)$
4)	$\bigcirc_a A$	$\bigcirc(\neg sign/\neg dog)$
5)	$\bigcirc_p A$	$\bigcirc(sign/dog)$

Contrary-to-Duties Scenario: Dyadic Deontic Logic

1) $\Box_a A \longrightarrow \Box_a dog$

2) $\Diamond_p A \longrightarrow \Diamond_p \neg dog$

3) $\bigcirc(B/A) \longrightarrow \bigcirc(\neg dog/\top)$, $\bigcirc(\neg sign/\neg dog)$, $\bigcirc(sign/dog)$

4) $\bigcirc_a A$

5) $\bigcirc_p A$

1)-2) *factual component* **3)** *deontic component*

Contrary-to-Duties Scenario: interpretation

Context — *"set of worlds"*

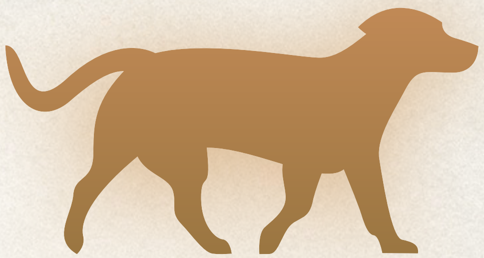
actually open

potentially open

$$\bigcirc(B/A)$$

$$ob(X) : \wp(W) \to \wp(\wp(W))$$

Contrary-to-Duties Scenario: final considerations



$$dog \wedge \Box_a dog \wedge \Diamond_p \neg dog$$

$$\Diamond_p sign$$

Contrary-to-Duties Scenario: final considerations

$$dog \wedge \Box_a dog \wedge \Diamond_p \neg dog$$

$$\Diamond_p sign$$

$$\bigcirc_a sign$$

$$\bigcirc_p \neg dog$$

Violation: $\bigcirc_p \neg dog \wedge dog$

Language of DDL

$q, q_i \in Q$	*countable set of propositional symbols*
$\neg A$	*classical negation*
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$\Box_a A$	*in all actual versions of the current world*
$\Box_p A$	*in all potential versions of the current world*
$\bigcirc(B/A)$	*binary dyadic deontic operator*
$\bigcirc_a A$	*monadic deontic operator for actual obligations*
$\bigcirc_p A$	*monadic deontic operator for primary obligations*

Semantics of DDL

Model $M = \langle W, av, pv, ob, V \rangle$

W – *set of possible worlds* $w, v, \ldots$

V – *function assigning a truth set* $V(q_i) \subseteq W$

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V – *function assigning a truth set* $V(q_i) \subseteq W$

$av : W \to \wp(W)$, $av(w)$ – *set of actual versions of the world* w

$pv : W \to \wp(W)$, $pv(w)$ – *set of potential versions of the world* w

Semantics of DDL

$ob(X) : \wp(W) \rightarrow \wp(\wp(W))$ – *obligatory sentences in the context X*

$$\emptyset \notin ob(X)$$

Semantics of DDL

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$$((Y, Z \in ob(X)) \ \& \ (Y \cap Z \cap X \neq \emptyset)) \implies Y \cap Z \in ob(X)$$

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$$((Y \subseteq X) \;\&\; (Y \in ob(X)) \;\&\; (X \subseteq Z)) \implies ((Z \setminus X) \cup Y) \in ob(Z)$$

$$((Y \subseteq X) \;\&\; (Z \in ob(X)) \;\&\; (Y \cap Z \neq \emptyset)) \implies Z \in ob(Y)$$

Rules of Inference. Axiomatization

$\Box$ $\qquad$ $\Box_p$ $\qquad$ $\Box_a$

normal modal operators of type

S5 $\qquad$ **KT** $\qquad$ **KD**

$$\Box A \rightarrow \Box_p A \qquad\qquad \Box_p A \rightarrow \Box_a A$$

Rules of Inference. Axiomatization

$\bigcirc(B/A) \to \Diamond(B \wedge A)$

$\Diamond(A \wedge B \wedge C) \wedge \bigcirc(B/A) \wedge \bigcirc(C/A) \to \bigcirc(B \wedge C/A)$

$\Box(A \to B) \wedge \Diamond(A \wedge C) \wedge \bigcirc(C/B) \to \bigcirc(C/A)$ **(SA)**

$\Box(A \leftrightarrow B) \to (\bigcirc(C/A) \leftrightarrow \bigcirc(C/B))$

$\Box(C \to (A \leftrightarrow B)) \to (\bigcirc(A/C) \leftrightarrow \bigcirc(B/C))$

$\bigcirc(B/A) \to \Box\bigcirc(B/A)$

$\bigcirc(B/A) \to \bigcirc(A \to B/\top)$

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$\Box(A \rightarrow B) \wedge \Diamond(A \wedge C) \wedge \bigcirc(C/B) \rightarrow \bigcirc(C/A)$ **(SA)**

$\Box(A \leftrightarrow B) \rightarrow (\bigcirc(C/A) \leftrightarrow \bigcirc(C/B))$ **(RE-A)**

$\Box(C \rightarrow (A \leftrightarrow B)) \rightarrow (\bigcirc(A/C) \leftrightarrow \bigcirc(B/C))$

$\bigcirc(B/A) \rightarrow \Box\bigcirc(B/A)$

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$\Box(A \rightarrow B) \wedge \Diamond(A \wedge C) \wedge \bigcirc(C/B) \rightarrow \bigcirc(C/A)$ **(SA)**

$\Box(A \leftrightarrow B) \rightarrow (\bigcirc(C/A) \leftrightarrow \bigcirc(C/B))$ **(RE-A)**

$\Box(C \rightarrow (A \leftrightarrow B)) \rightarrow (\bigcirc(A/C) \leftrightarrow \bigcirc(B/C))$ **(RE-C)**

$\bigcirc(B/A) \rightarrow \Box\bigcirc(B/A)$

$\bigcirc(B/A) \rightarrow \bigcirc(A \rightarrow B/\top)$

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$\bigcirc(B/A) \rightarrow \Diamond(B \wedge A)$

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$\Box(A \leftrightarrow B) \rightarrow (\bigcirc(C/A) \leftrightarrow \bigcirc(C/B))$ **(RE-A)**

$\Box(C \rightarrow (A \leftrightarrow B)) \rightarrow (\bigcirc(A/C) \leftrightarrow \bigcirc(B/C))$ **(RE-C)**

$\bigcirc(B/A) \rightarrow \Box\bigcirc(B/A)$ **(DC)**

$\bigcirc(B/A) \rightarrow \bigcirc(A \rightarrow B/\top)$

Rules of Inference. Axiomatization

$$\Box_{a(p)} A \rightarrow (\neg \bigcirc_{a(p)} A \wedge \neg \bigcirc_{a(p)} \neg A)$$

$$\Box_{a(p)} (A \leftrightarrow B) \rightarrow (\bigcirc_{a(p)} A \leftrightarrow \bigcirc_{a(p)} B)$$

$$\bigcirc(B/A) \wedge \Box_{a(p)} A \wedge \Diamond_{a(p)} B \wedge \Diamond_{a(p)} \neg B \rightarrow \bigcirc_{a(p)} B$$

Rules of Inference. Axiomatization

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$$\bigcirc(B/A) \wedge \Box_{a(p)} A \wedge \Diamond_{a(p)} B \wedge \Diamond_{a(p)} \neg B \rightarrow \bigcirc_{a(p)} B$$

+

2 rules to consistently add conditional obligation sentences

Dog & Warning Sign

$$dog \wedge \Box_a dog$$

$$\Diamond_p \neg dog$$

$$\Diamond_p dog$$

$$\neg \Box_a \neg sign$$

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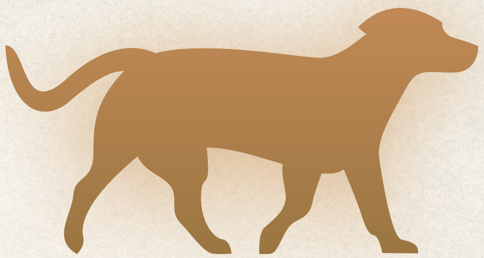
Dog & Warning Sign



$$dog \wedge \Box_a dog \quad \Longrightarrow \Diamond_a dog$$

$$\Diamond_p \neg dog$$

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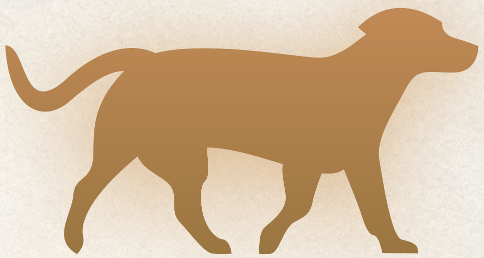
$$\neg \Box_a \neg sign \quad \Longleftrightarrow \Diamond_a \neg sign$$

$$\neg \Box_a sign \quad \Longleftrightarrow \Diamond_a sign$$

Dog & Warning Sign



$dog \wedge \Box_a dog$	$\Longrightarrow \Diamond_a dog$
$\Diamond_p \neg dog$	
$\Diamond_p dog$	
$\neg \Box_a \neg sign$	$\Longleftrightarrow \Diamond_a \neg sign$
$\neg \Box_a sign$	$\Longleftrightarrow \Diamond_a sign$

$$\bigcirc(\neg dog / \top) \wedge \Box_p \top \wedge \Diamond_p \neg dog \wedge \Diamond_p dog \rightarrow \bigcirc_p \neg dog$$

$$\bigcirc(sign/dog) \wedge \Box_a dog \wedge \Diamond_a \neg sign \wedge \Diamond_a sign \rightarrow \bigcirc_a sign$$

$$\bigcirc(B/A) \wedge \Box_{a(p)} A \wedge \Diamond_{a(p)} B \wedge \Diamond_{a(p)} \neg B \rightarrow \bigcirc_{a(p)} B$$

Some Results

Axiomatization

Sound

Complete

Consistent

Logic

Decidable

Thank all for attention.