Normative Reasoning - Dyadic Deontic logic of Carmo & Jones

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Abstract

In this paper we are going to present another particular topic on Normative Reasoning, namely **D**yadic **D**eontic **L**ogic of Carmo & Jones (**DDL** later on). We will start with pointing out the problems, paradoxes and limitation of Standard Deontic Logic to see the need of a new ground. While approaching the **DDL**, we will try to give a feeling and intuition with some examples that motivate the definition of the new logic, the one expressing Contrary-to-Duties scenarios, and introducing some new **DDL** operators and their use on the run with the examples as was done in [1]. Afterwards, the formal language will be defined followed by Semantics section alongside with the explanations of the different parts of the constructed **DDL** model. In continuation, we will axiomatize logic by adding inference rules. At the end we will present some results concerning soundness and consistency of the proposed axiomatization mostly following [1].

1 Introduction

Deontic logic deals with the problems of logical analysis of normative reasoning such as *obligation, permission, right and prohibition.* There is a standard approach to these issues, namely **S**tandard **D**eontic **L**ogic (**SDL** later on) [see reference [8], mostly Chapter 1, for more details and wider introduction to Deontic logic]. It takes the necessity operator $\Box$ to express legal obligation and means "it is obligatory that" and denotes it by $\bigcirc$. There is a dual possibility operator $\Diamond = \neg\Box\neg$ expressing "it is permitted that", which is denoted by $\mathcal{P}$, and an impossibility modal construction $\Box\neg$ meaning "it is forbidden that". Axiomatically, **SDL** is the weakest normal modal system of type $\mathcal{KD}$. It is widely accepted that **SDL** is not adequate as a basic deontic logic since it gives rise to a set of "paradoxes" and there are some deontic concepts and constructions which cannot be expressed in **SDL**.

The first group of paradoxes has its origin in the closure of the $\bigcirc$-operator under logical consequence (**SDL** contains the $(\mathcal{RM})$-rule: "if $\vdash A \rightarrow B$ then $\vdash \bigcirc A \rightarrow \bigcirc B$"):

- Ross paradox: $(\vdash \bigcirc A \rightarrow \bigcirc(A \vee B))$
  "If it is obligatory to mail the letter, then it is obligatory to mail the letter or to burn it"
  Here we have two points of views to the problem. If one supposes that A is obligatory and that A is not the case, how many obligations have been violated then? Also, there is a *fulfilment* perspective: if the obligation that A is fulfilled, then so are all the other obligations which can be derived by application of this Ross theorem.

- Free Choice Permission paradox: (in **SDL**) it is not $\vdash \mathcal{P}(A \vee B) \rightarrow (\mathcal{P}A \wedge \mathcal{P}B)$, while - ordinarily - if it is permitted that A or B then it is usually understood as A is permitted and B is permitted.

- Good Samaritan paradox: $\vdash \bigcirc(A \wedge B) \rightarrow \bigcirc B$
  "If it is obligatory that Mary helps John who has had an accident, then it is obligatory that John has an accident"

- Deontic/epistemic paradox: (a consequence of (T)-schema $\mathcal{K}A \rightarrow A$ for epistemic operator)
  If it is obligatory that Mr. X knows that his wife commits adultery, then it is obligatory that X's wife commits adultery"

There are also other problems in the applications of modal logic arising from the closure of the necessity operator under logical consequence. The second group of problems comes from the $\bigcirc$-necessitation rule itself, when any tautology (more generally, any theorem) becomes obligatory, which is incompatible with the idea that obligations should be possible to fulfil and possible to violate. The third problem of **SDL** is that, because of the (D)-schema ($\bigcirc A \rightarrow \mathcal{P}A$), it is not possible to express consistently the conflict of the obligations.

Finally, the principal reason for seeking an alternative to **SDL** was the problem of how to represent *conditional obligation sentences*. There are two possible ways to do that in **SDL**:

- (option 1) $\bigcirc(B/A) =_{df} A \rightarrow \bigcirc B$
- (option 2) $\bigcirc(B/A) =_{df} \bigcirc(A \rightarrow B)$

Both of them lead to:

- (UN) $\vdash \bigcirc B \leftrightarrow \bigcirc(B/\top)$
- (SA) $\vdash \bigcirc(B/A) \rightarrow \bigcirc(B/A \wedge C)$

As was expressed by Carlos Alchourron in [5] (UN), it was one of the wrong steps which was followed by almost all of the researchers in deontic logic. And (SA) also problematic, since it appears to make the expression of defeasible obligations impossible. Likewise, there is a problem as well that sets deontic logic apart from other branches of modal logic. Those are the issues of representing contrary-to-duty obligation sentences (CTDs). They seem to represent two levels of normative reasoning, namely, *primary* and *actual* obligation. The last point and reason of considering the Dyadic Deontic Logic to mention here is Crisholm's CTD-paradox modelled in **SDL**. With both of options above representing *conditional obligations* the set of four sentences, formulated by Chisholm, have problems either with minimal requirements of consistency or with logical independence while from intuitive point of view it has both.

2 Requirements for Deontic Logic

In this section we would like to collect all the desirable conditions (see [3] for more details) for the *deontic logic*, which is to be constructed, explained and defined a little bit further. Those are here:

1. minimum requirements of consistency,
2. logical independence of the sentences,
3. applicability to timeless and actionless **CTD**-scenarios,
4. the assignment of logical form to every norm in the set should be independent of the other norms in it,
5. capacity to derive *actual* obligations,
6. capacity to derive *primary* obligations,
7. capacity to represent the fact that a violation of an obligation has occured,
8. capacity to avoid the pragmatic oddity.

Now, we briefly explain some of these requirements. The first two of them were encountered already when we were talking about Chisholm set of sentences in previous Section. The forth condition merely says that the sentence in a scenario should always be expressed equally regardless of the presence of the other sentences. So far mentioned demands already require more or less the introduction of the primitive dyadic conditional operator $\bigcirc(/)$. With the fifth and sixth conditions we have a closer look in the next Section. The seventh intuitively says that in real life violations of the norms occur and these violation do not mean the collapse of the normative system and should be expressed as well.

3 Contrary-to-Duties scenario

We are starting our explanation of **D**yadic **D**eontic **L**ogic and its features with the famous Contrary-to-Duties (**CTD** later on) scenario from [1] since it helps one to understand many concepts. It is named the "Dog and warning sign" (**D&WS**) example:

(a) There ought to be no dog.
(b) If there is no dog, there ought to be no warning sign.
(c) If there is a dog, there ought to be a warning sign.
(d) There is a dog.

And now the natural question that arises is so: what is the actual obligation in such circumstances of the agent to whom norms apply? The clue to the answer lies in the fact that the right solution depends on the *status* assigned to the fact described in the line d). In other words, one has to understand what fact is fixed in the given setting in the following sense: whether the dog can be removed or not, i.e., whether the dog owner leaves it at home or he or she stubbornly refuses to remove the dog.

There are various different kinds of reasons why the facts of a given situation might be deemed to be fixed. Previous approaches to **CTD** scenarios considered only one type of reasons, namely the *temporal* approach, when the obligation is violated due to the deadline limitations. But the developing of the **DDL** theory lies upon the the generalization of the previous approach because usually temporal reasons, although very common, are not the only ones, when the facts become fixed. There might be *casual* reasons, when one performs the deed of killing and such act cannot be undone once performed. Sometimes the fixity arises from agent's decisions, as is given in **D&WS** example.

Let us consider complementary example to dig deeper into the problem, "Considerate assassin" as in [1]:

(a) You should not kill Mr X.
(b) $\cdots\cdots\cdots$
(c) If you kill Mr X, you should offer him a cigarette.

Here the question that arises is: when does the assassin have an actual obligation to offer Mr X a cigarette? Obviously, not after he kills him! Thereby, the actual obligation comes into the force when the assassin firmly decides to kill Mr X, i.e. when it becomes a fixed fact that Mr X is going to be killed.

Consideration of these two examples provides us with the idea that two notions of necessity - and their associated notions of possibility - would have to be taken into account. After including them into consideration, the two types of normative conclusions can be drawn from a given **CTD** scenario. One of these normative conclusions is *actual* obligations, the other one is *primary* obligations. Given those notions one can validly reason on the two levels of normativity, which is the property we are seeking for.

Formulae of the form $\Box_a$A, where A represents what is actually fixed, or unavoidable, what the agents concerned have decided to do or not do. So, in **D&WS** example sentence $\Box_a$dog will be true if agent decides not to remove the dog. The dual possibility notion, $\Diamond_a$A, is denoted by $\neg\Box_a\neg$A. $\bigcirc_a$A represents 'it is actually obligatory that A'. In the **D&WS** example if the dog owner had decided differently then the dog might have been removed and to represent it we use the second notion of possibility, $\Diamond_p$A, what is *potentially* possible (and there is a dual necessity notion $\neg\Diamond_p\neg$A). In the **D&WS** scenario, which primary obligations $\bigcirc_p$A are derivable will depend, in particular, on whether or not $\Diamond_p\neg$dog is true. We represent sentences expressing obligation norms by dyadic conditional obligation operator $\bigcirc$(A/B). In our **D&WS** example the first three sentences are written in the following way:

(a) $\bigcirc(\neg\text{dog}/\top)$
(b) $\bigcirc(\neg\text{sign}/\neg\text{dog})$
(c) $\bigcirc(\text{sign}/\text{dog})$

And all the sentences expressed with the help of dyadic obligation operator are of the same *deontic* component while the other ordinary sentences or ones written with $\Box, \Box_a, \Diamond_p$... operators constitute the *factual* component. These parts defines the scenario we wish to consider. And there are also *derived obligational sentences*.

Now comes the interpretation. For example, we think of the sentence (c) as if it was said that in any *context* where there is a dog and such fact is deemed to be fixed then it is obligatory to have a warning sign if it is possible. Thereby, we consider *context* as a set of worlds. And in order to handle the idea of *conditional obligation sentences* in the models of **DDL** the function $ob(X) : \wp(W) \to \wp(\wp(W))$ is introduced, where W is the whole set of worlds. For each context it picks out the propositions, which represent what is obligatory in that context. Hereby, the sentence $\bigcirc$(A/B) is true in our model if and only if, in any context X where B is true and A is possible, it is obligatory that A. As it was pointed out above, we want to represent two different types of obligations, namely *actual* and *primary* obligations. For this purpose we consider *contexts* of what is *actually* open to the agents and *contexts* of what is *potentially* open to them. So, the propositions that are obligatory in the first type of the *contexts* mean *actual* and propositions that are obligatory in the second type mean *primary* obligations.

At this point we can give a logical analysis of the **D&WS** scenario by using those two pairs of *necessity* and *possibility* notions: if it is a fixed fact that (actually necessary that) there is a dog ($\Box_a$dog is true), but it is actually possible that a sign may be erected and potentially possible that there is no dog, then the **DDL** licenses the derivation of the actual obligation to erect a warning sign ($\bigcirc_a$sign) and the primary obligation that there is no dog ($\bigcirc_p\neg$dog). There is no actual obligation that there is no dog precisely because we are supposing that removal of the dog is not an actual possibility due to agents decision. But a violation has nevertheless occurred, as expressed by the conjunction $\bigcirc_p\neg\text{dog} \wedge \text{dog}$.

Connection of *actual* and *primary* obligations may be expressed as the following derivation:

$$\bigcirc_p A \wedge \Diamond_a A \wedge \Diamond_a \neg A \longrightarrow \bigcirc_a A \tag{1}$$

Following, comes the formal part of these considerations.

4 Technical part

4.1 Formal language

For constructing **DDL** formulas we start with the countable set Q of propositional symbols and refer to them as $q, q_1, ...$, and we choose two primitive connectives as $\neg$ and $\vee$. We will use $A, A_1, ..., B, B_1, ...$ to refer to sentences (formulas) and $\Gamma, \Gamma_1, ..., \Delta, \Delta_1, ...$ to refer to set of sentences. Other logical connectives are defined as follows: $A \wedge B := \neg(\neg A \vee \neg B)$, $A \rightarrow B := \neg A \vee B$, $A \leftrightarrow B := (A \rightarrow B) \wedge (A \leftarrow B)$ Now we define the set of **DDL** formulas as the smallest set of formulas obeying the following conditions:

- Each $q, q_i \in Q$ is an atomic formula.
- Given two arbitrary **DDL** formulas A and B, then
  $\neg A$ - classical negation,
  $A \vee B$ - classical disjunction,
  $\Box A$ - in all worlds,
  $\Box_a A$ - in all actual versions of the current world,
  $\Box_p A$ - in all potential versions of the current world),
  $\bigcirc(A/B)$ - dyadic deontic operator: "it ought to be A, given B",
  $\bigcirc_a A$ - monadic deontic operator for actual obligation,
  $\bigcirc_p A$ - monadic deontic operator for primary obligation
  are also the **DDL** formulas.

The duals of $\Box, \Box_a, \Box_p$ are defined as $\Diamond A := \neg\Box\neg A$, $\Diamond_a A := \neg\Box_a\neg A$, $\Diamond_p A := \neg\Box_p\neg A$. The $\top := q \rightarrow q$ for some propositional symbol q, and $\bot := \neg\top$. The violation is described as $\bigcirc_p A \wedge \neg A$.

4.2 Semantics

A **DDL** model is a structure $M = \langle W, av, pv, ob, V\rangle$, where:

1) W is a non-empty set (set of possible worlds to which we refer as $w, v, ...$).

2) V is a function assigning a truth set to each atomic sentence (i.e. $V(q_i) \subseteq W$).

3) av is a function $av : W \rightarrow \wp(W)$ such that:

   (a) $av(w) \neq \varnothing$ ($av(w)$ is the set of actual versions of the world w).

4) pv is a function $pv : W \rightarrow \wp(W)$ such that:

   (a) $av(w) \subseteq pv(w)$

   (b) $w \in pv(w)$ ($pv(w)$ is the set of potential versions of the world w).

5) ob is a function $ob : \wp(W) \rightarrow \wp(\wp(W))$ such that (X, Y, Z denote arbitrary sets of worlds):

(a) $\varnothing \notin ob(X)$

(b) if $Y \cap X = Z \cap X$, then $(Y \in ob(X)$ iff $Z \in ob(X))$

(c) let $\beta \subseteq ob(X)$ and $\beta \neq \varnothing$, if $(\cap\beta) \cap X \neq \varnothing$ (where $\cap\beta = \{w \in W : \forall_{Z\in\beta} w \in Z\}$), then $(\cap\beta) \in ob(X)$.

(d) if $Y \subseteq X$ and $Y \in ob(X)$ and $X \subseteq Z$, then $((Z \backslash X) \cup Y) \in ob(Z)$.

(e) if $Y \subseteq X$ and $Z \in ob(X)$ and $Y \cap Z \neq \varnothing$, then $Z \in ob(Y)$.

Given a model $M = \langle W, av, pv, ob, V \rangle$ a formula A is true in it denoted by $M \models_w A$ and we also define the set $\|A\|^M = \{w \in W : M \models_w A\}$.

$M \models_w p$ iff $w \in V(p)$

$M \models_w \neg A$ iff $M \not\models_w A$

$M \models_w A \vee B$ iff $M \models_w A$ or $M \models_w B$

$M \models_w \Box A$ iff $\|A\| = W$

$M \models_w \Box_a A$ iff $av(w) \subseteq \|A\|$

$M \models_w \Box_p A$ iff $pv(w) \subseteq \|A\|$

$M \models_w \bigcirc(B/A)$ iff $\|A\| \cap \|B\| \neq \varnothing$ and $(\forall X)$(if $X \subseteq \|A\|$ and $X \cap \|B\| \neq \varnothing$, then $\|B\| \in ob(X))$

$M \models_w \bigcirc_a A$ iff $\|A\| \in ob(av(w))$ and $av(w) \cap \|\neg A\| \neq \varnothing$

$M \models_w \bigcirc_p A$ iff $\|A\| \in ob(pv(w))$ and $pv(w) \cap \|\neg A\| \neq \varnothing$

It is said that A is true in a model $M = \langle W, av, pv, ob, V \rangle$ written $M \models A$, iff $\|A\|^M = W$, and that A is valid, written $\models A$, iff $M \models A$ in all models M.

Now we want to point out some observations that help the understanding of the conditions. **1)** One could define $M \models_w \bigcirc(B/A)$ alternatively, in a *strict sense*[1], when any obligation should be possible to fulfil and to violate. **2)** Condition (5a) means that for us a contradiction cannot be obligatory. **3)** Condition (5b) means that if, in any context X, two propositions Y and Z are indistinguishable, then one of them is obligatory (in that context X) iff the other is. **4)** Condition (5c) means (here in infinite case of W) that the conjunction of obligatory propositions (in some fixed context X), unless they are not contradictory, is also obligatory. **5)** Condition (5d) states that if a subset Y of X is an obligatory proposition in a context X, then in a bigger context Z it is obligatory to be either in Y or else in that part of Z which is not in X. **6)** Condition (5e) states that if Z is an obligatory proposition in a context X, then Z is also obligatory in any subcontext Y of Z where it is possible to fulfil Z. **7)** $M \models_w \bigcirc(B/A)$ implies $\|B\| \in ob(\|A\|)$. The inverse is true as well assuming (5e). So we can simplify the definition of truth condition for $M \models_w \bigcirc(B/A)$[2].

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[1] $M \models_w \bigcirc(B/A)$ iff $\|A\| \cap \|B\| \neq \varnothing$ and $\|A\| \cap \|\neg B\| \neq \varnothing$ and $(\forall X)$(if $X \subseteq \|A\|$ and $X \cap \|B\| \neq \varnothing$ and $X \cap \|\neg B\| \neq \varnothing$, then $\|B\| \in ob(X))$. In that case we would require: if $Y \in ob(X)$, then $X \cap (W \backslash Y) \neq \varnothing$ and the truth in a world w of $\bigcirc_a A$ (respectively $\bigcirc_p A$) would be defined simply as follows: $M \models_w \bigcirc_a A$ iff $\|A\| \in ob(av(w))$ (respectively $M \models_w \bigcirc_p A$ iff $\|A\| \in ob(pv(w)))$

[2] $M \models_w \bigcirc(B/A)$ iff $\|B\| \in ob(\|A\|)$

4.3 Axiomatization

In this part the inference rules for modal operators will be given. The system consists likewise of primitive Modus Ponens (MP) rule and tautologies are considered as axioms.

- Characterization of $\Box$:
  – $\Box$ is a normal modal operator of type S5, or more precisely, it has K-[3], T-[4], 5-[5] axiom schemas and Rule of necessitation (RN)[6] in addition to mentioned ones.

- Characterization of $\bigcirc$:
  – $\bigcirc(B/A) \rightarrow \Diamond(B \wedge A)$
  – $\Diamond(A \wedge B \wedge C) \wedge \bigcirc(B/A) \wedge \bigcirc(C/A) \rightarrow \bigcirc(B \wedge C/A)$
  – the principle of strengthening of the antecedent:
  $\Box(A \rightarrow B) \wedge \Diamond(A \wedge C) \wedge \bigcirc(C/B) \rightarrow \bigcirc(C/A)$
  – "RE-axiom"[7] w.r.t the antecedent:
  $\Box(A \leftrightarrow B) \rightarrow (\bigcirc(C/A) \leftrightarrow \bigcirc(C/B))$
  – "RE-axiom" w.r.t the consequent:
  $\Box(C \rightarrow (A \leftrightarrow B)) \rightarrow (\bigcirc(A/C) \leftrightarrow \bigcirc(B/C))$
  – $\bigcirc(B/A) \rightarrow \Box \bigcirc (B/A)$
  – $\bigcirc(B/A) \rightarrow \bigcirc(A \rightarrow B/\top)$

- Characterization of $\Box_p$:
  – $\Box_p$ is a normal modal operator of type KT[8]

- Characterization of $\Box_a$:
  – $\Box_a$ is a normal modal operator of type KD[9]

- Relationships between $\Box, \Box_p$ and $\Box_a$:
  – $\Box A \rightarrow \Box_p A$
  – $\Box_p A \rightarrow \Box_a A$

- Relationships between $\bigcirc_a(\bigcirc_p)$ and $\Box_a(\Box_p)$:
  – $\Box_a A \rightarrow (\neg \bigcirc_a A \wedge \neg \bigcirc_a \neg A)$
  – $\Box_p A \rightarrow (\neg \bigcirc_p A \wedge \neg \bigcirc_p \neg A)$
  – $\Box_a(A \leftrightarrow B) \rightarrow (\bigcirc_a A \leftrightarrow \bigcirc_a B)$
  – $\Box_p(A \leftrightarrow B) \rightarrow (\bigcirc_p A \leftrightarrow \bigcirc_p B)$

- Relationships between $\bigcirc, \bigcirc_a(\bigcirc_p)$ and $\Box_a(\Box_p)$ - those are factual detachment axioms:

[3]K. $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
[4]T. $\Box A \rightarrow A$
[5]5. $\Diamond A \rightarrow \Box\Diamond A$
[6]RN. $\dfrac{A}{\Box A}$
[7]RE. $\dfrac{A \leftrightarrow B}{\Box(A \leftrightarrow B)}$
[8]KT. = T.
[9]KD. = K. + (D. $\Box A \rightarrow \Diamond A$)

– $\bigcirc(B/A) \wedge \Box_a A \wedge \Diamond_a B \wedge \Diamond_a \neg B \to \bigcirc_a B$
– $\bigcirc(B/A) \wedge \Box_p A \wedge \Diamond_p B \wedge \Diamond_p \neg B \to \bigcirc_p B$

- Rules to consistently add $\bigcirc(/)$ formulas:
  – If the propositional symbol q does not occur in any of the formulas $B_1, ..., B_n, A$
  and $\vdash B_1 \wedge ... \wedge B_n \to \neg\Box(\bigcirc_a A \to \Box_a q \wedge \bigcirc(A/q))$
  then $\vdash B_1 \wedge ... \wedge B_n \to \neg\Diamond \bigcirc_a A$
  – If the propositional symbol q does not occur in any of the formulas $B_1, ..., B_n, A$
  and $\vdash B_1 \wedge ... \wedge B_n \to \neg\Box(\bigcirc_p A \to \Box_p q \wedge \bigcirc(A/q))$
  then $\vdash B_1 \wedge ... \wedge B_n \to \neg\Diamond \bigcirc_p A$

The last set of two rules, whose name came from the use given to these rules in the completeness proof, where they are needed to construct the maximal consistent sets, is needed to get a complete axiomatization of the whole logic and.

5 Results

In this part we will mention several important results as well as those that we already have referred to before.

Result 1. The proposed axiomatization is *sound*, i.e. all theorems are valid.

Result 2. The proposed axiomatization is *consistent*, i.e. $\perp$ is not a theorem. (by the **Result 1.** it is sufficient to prove that there exist a model for our logic).

Result 3. As was pointed out before in section 2, inside this logic we can indeed get the formula (with the help of the very last rule of axiomatization): $\vdash \bigcirc_p A \wedge \Diamond_a \neg A \wedge \Diamond_a A \to \bigcirc_a A$.

Result 4. If a formula ψ is consistent (in other words, if in the **DDL** system one has $\dashv \neg\psi$), there is a finite model $M = \langle W, av, pv, ob, V\rangle$ and a world $w \in W$ such that $M \models_w \psi$ (satisfiability of a formula). In this fashion, proposed axiomatization is *complete* and the logic is also *decidable* since it has the finite model property (which in turn considered as a good property for designing *Normative systems* and *Deontic logics* in particular). The way author's proved such result is they built another consistent formula ϕ, from initial (fixed) consistent formula ψ, whose truth in a world implies the truth of ψ in the same world, and built a finite model $M(\phi)$ where ϕ is true in some world.

6 Conclusions

The construction of Dyadic Deontic Logic in this way helps to avoid those mentioned paradoxes of Standard Deontic Logic, its limitations of expressing the relationship between the actual and primary obligations, the Contrary-to-Duties scenarios, the conditional obligations, while also preserving the logical independence of the sentences. It proposes the ground for a valid reasoning inside those fields. The proposed **DDL** system appeared to be complete and decidable.

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