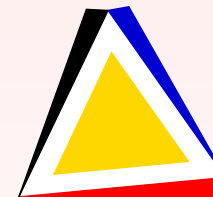

Tutorial Dialog with a Mathematical Assistant System

The DIALOG Project

Christoph Benz Müller
Saarland University, Saarbrücken, Germany



Presentation Overview

- **Role in the OMEGA Group at Saarland University & Main Scientific Contributions**
- **Tutorial Dialog with a Mathematical Assistant System**
 - **Project Overview**
 - **Increasing Refinement with *Wizard-of-Oz* (WoZ) Experiments**
 - **DIALOG and the Learning Environment
ACTIVEMATH**
 - **Examples from Naive Set Domain: Properties & Requirements**

Research in the OMEGA project

**Knowledge-based
Proof Planning with
multiple Strategies**

**Interactive Tactical
Theorem Proving**

**Agent-based
Reasoning**

**Ext. Support Systems:
FO-ATPs, HO-ATPs,
CAS, Constraintssolv.,
Modelgenerators, ...**

**The Mathematical
Assistant System**
OMEGA
10-13 Researchers

**Higher-Order Logic &
Calculi,
Proof Transformation
& Representation**

**User Interfaces:
Graphical,
Natural Language**

**Mathematical
Database**
MBase

**System Infrastructure:
Mathematical
Software Bus**
MATHWEB

Research in the OMEGA project

**Knowledge-based
Proof Planning with
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**Organisation and
Management of
OMEGA research**

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Research in the OMEGA project

[AI-02-subm]
[IJCAR-WS-01]
2x[CALCULEMUS-00]
[CADE-WS-00]

[JSC-02-subm]
[LPAR-02-subm]

[CALCULEMUS-02/01/00/99] [AISB-01]
[AISB-00] [EPIA-99]
[AIMSA-98]

[KI-01] [JUCS-99]
[TPHOLS-98]

[CADE-02] [CADE-97]

[JSL-02-subm]
[Synthese-02]
[TPHOLS-01]
[DISS-99] [CADE-99]
2x[CADE-98]

[CASYS-99] [FAC-99]
[UITP-98]

[Festschrift-02]
[MKM-01]

[VERIFY-02]
[CALCULEMUS-02]

Main Scientific Contributions

Area of PhD thesis: Higher-Order Theorem Proving

- **Extensional Higher-Order Resolution,
Paramodulation, RUE-Resolution**
[Synthese-02] [CADE-99] [CADE-98]
- **Notions of Semantics for Higher-Order Logic (HOL) /
Abstract Consistency Proof Principle for HOL**
[JSL-02-Submitted]
- **Development of my Higher-Order Theorem Prover**
LEO [System-Description-CADE-98]

The DIALOG Project

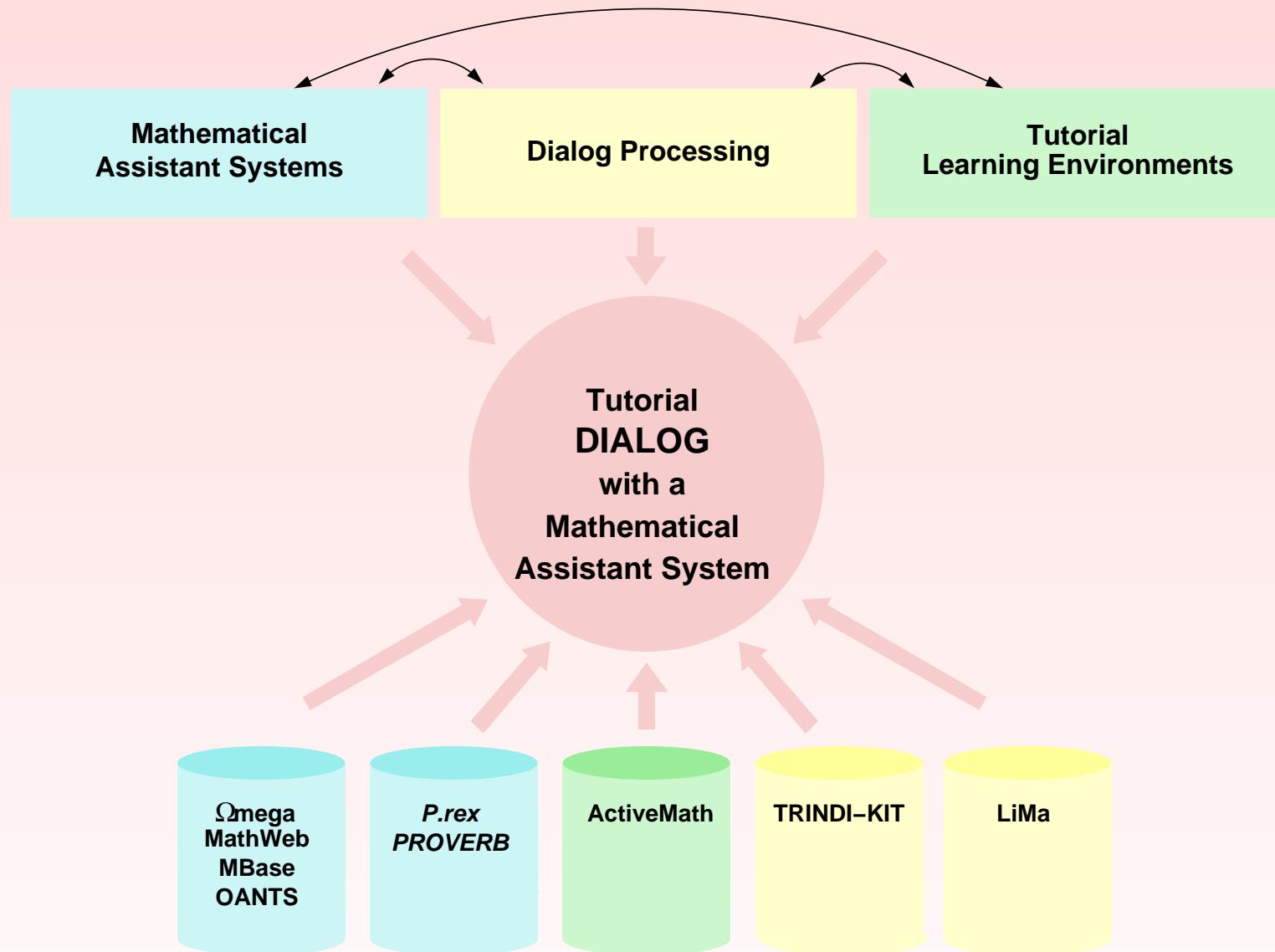
The DIALOG project is part of the Collaborative Research Centre SFB 378 *Resource-adaptive Cognitive Processes*
Project Goals:

- Empirical investigation, modeling, and implementation of natural language (NL) dialog in a tutorial application for a particular mathematical domain

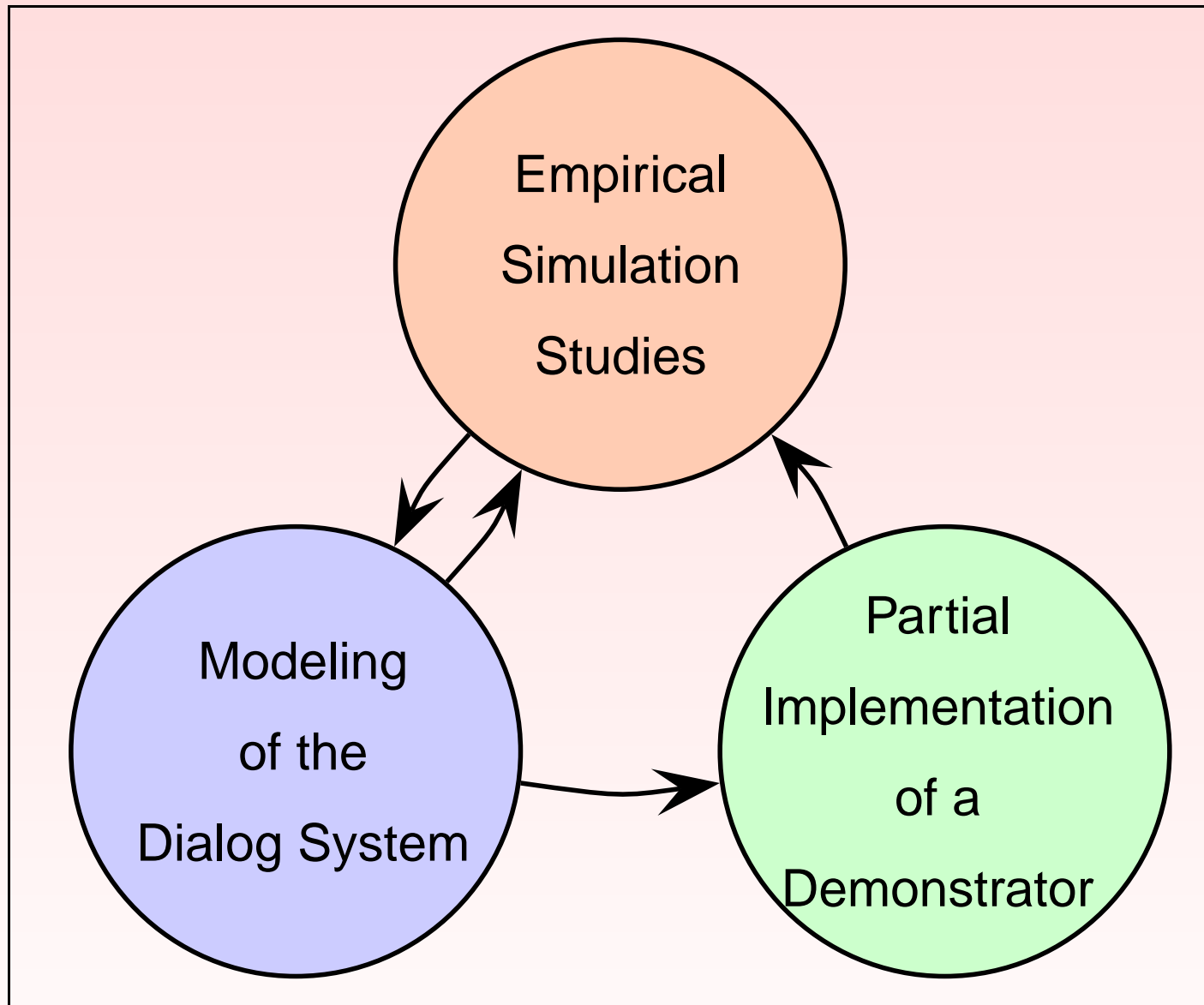
Overall Motivation:

- Investigation and realisation of dialog techniques in ambitious applications which require deep speech analysis and non-trivial domain reasoning

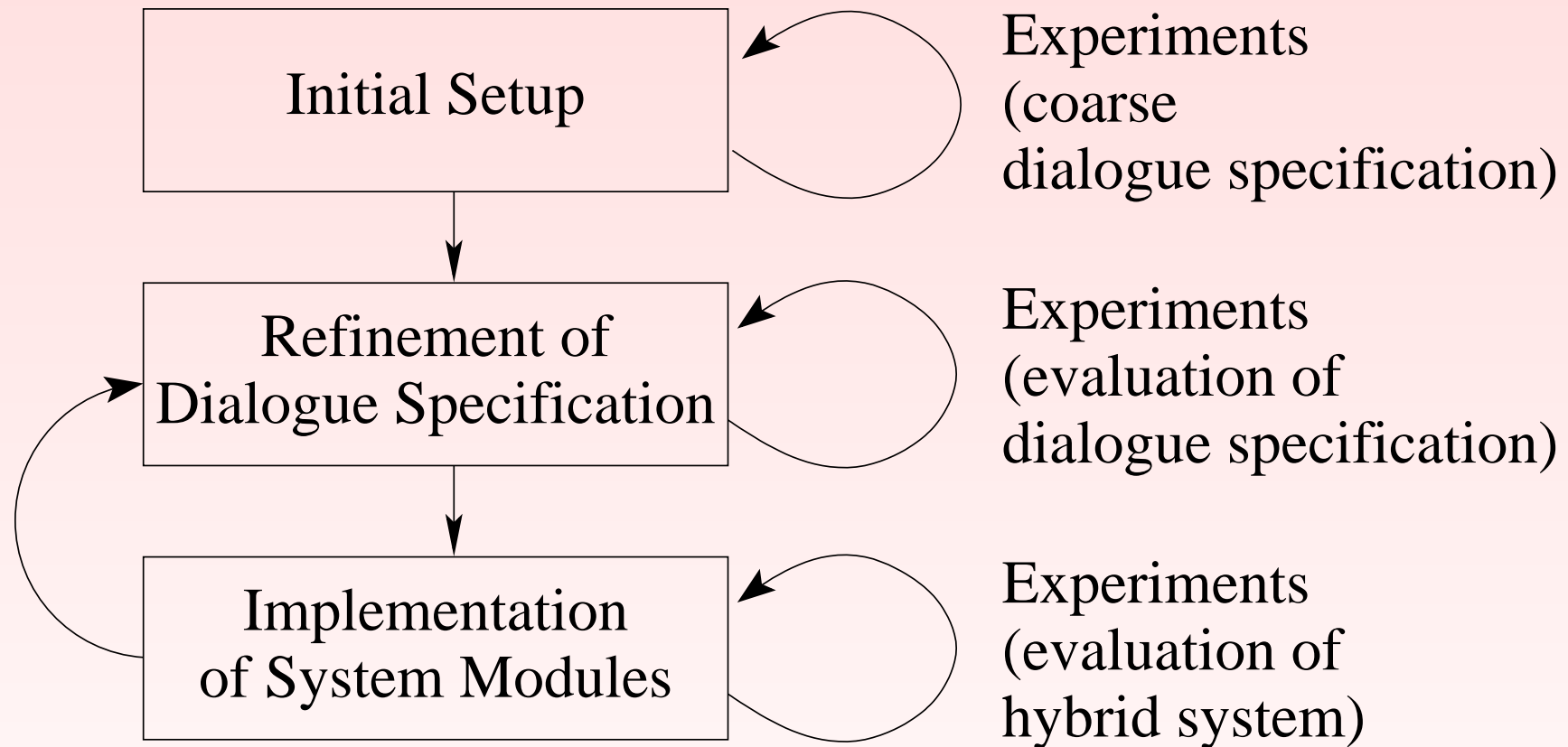
The DIALOG Project



Method: Increasing Refinement



Method: Increasing Refinement

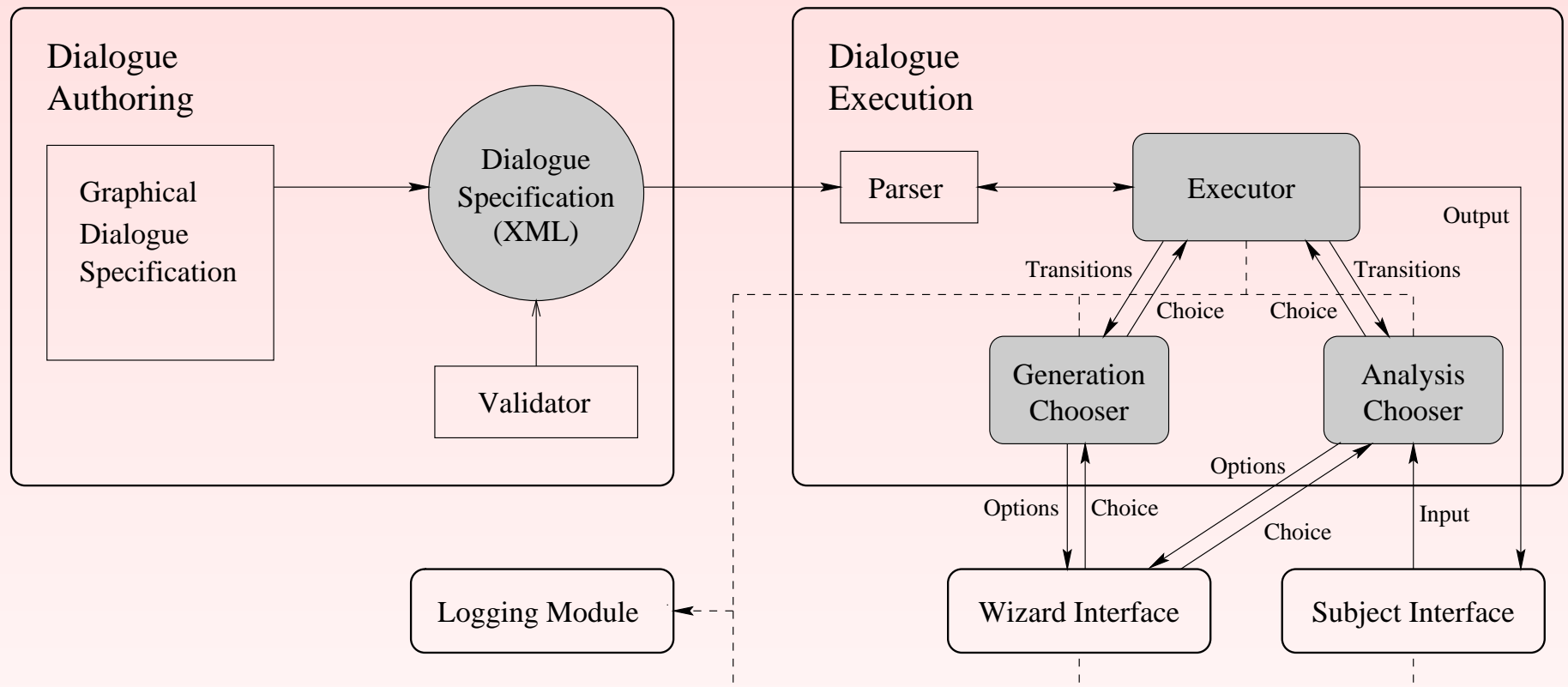


DiaWoZ

System that supports the design and execution of *Wizard-of-Oz* (Bernsen et al.) experiments

- **Combination of finite-state automata and information-state based dialog model (TRINDI)**
- **Global and local variables (for subdialogs)**
- ***Dialog Authoring* and *Dialog Execution* components**

Architecture of DiaWoZ



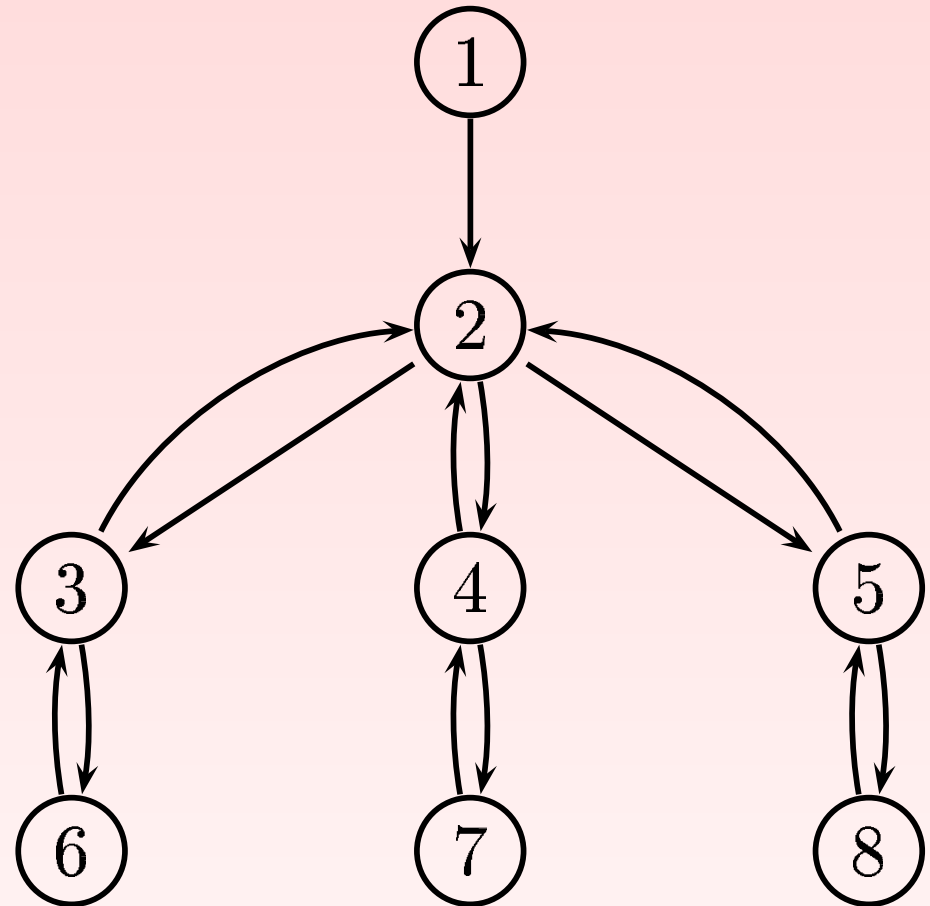
Dialog Specification

Information State:

NEUTRAL: **open**

INVERSE: **open**

ASSOCIATIVE: **open**



An Example Dialog

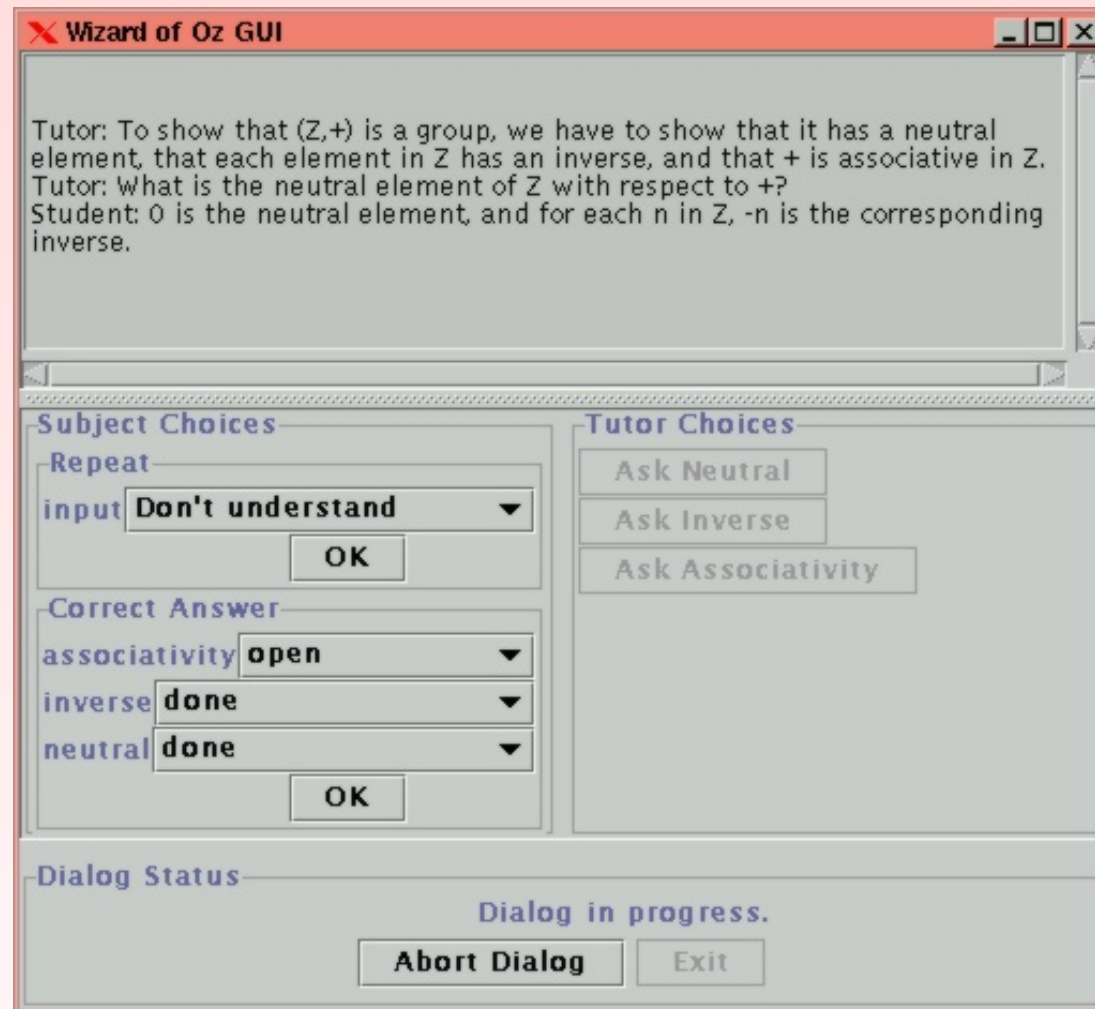
(U1) **Tutor:** To show that $(\mathbb{Z}, +)$ is a group, we have to show that it has a neutral element, that each element in \mathbb{Z} has an inverse, and that $+$ is associative in \mathbb{Z} .

(U2) **Tutor:** What is the neutral element of \mathbb{Z} with respect to $+$?

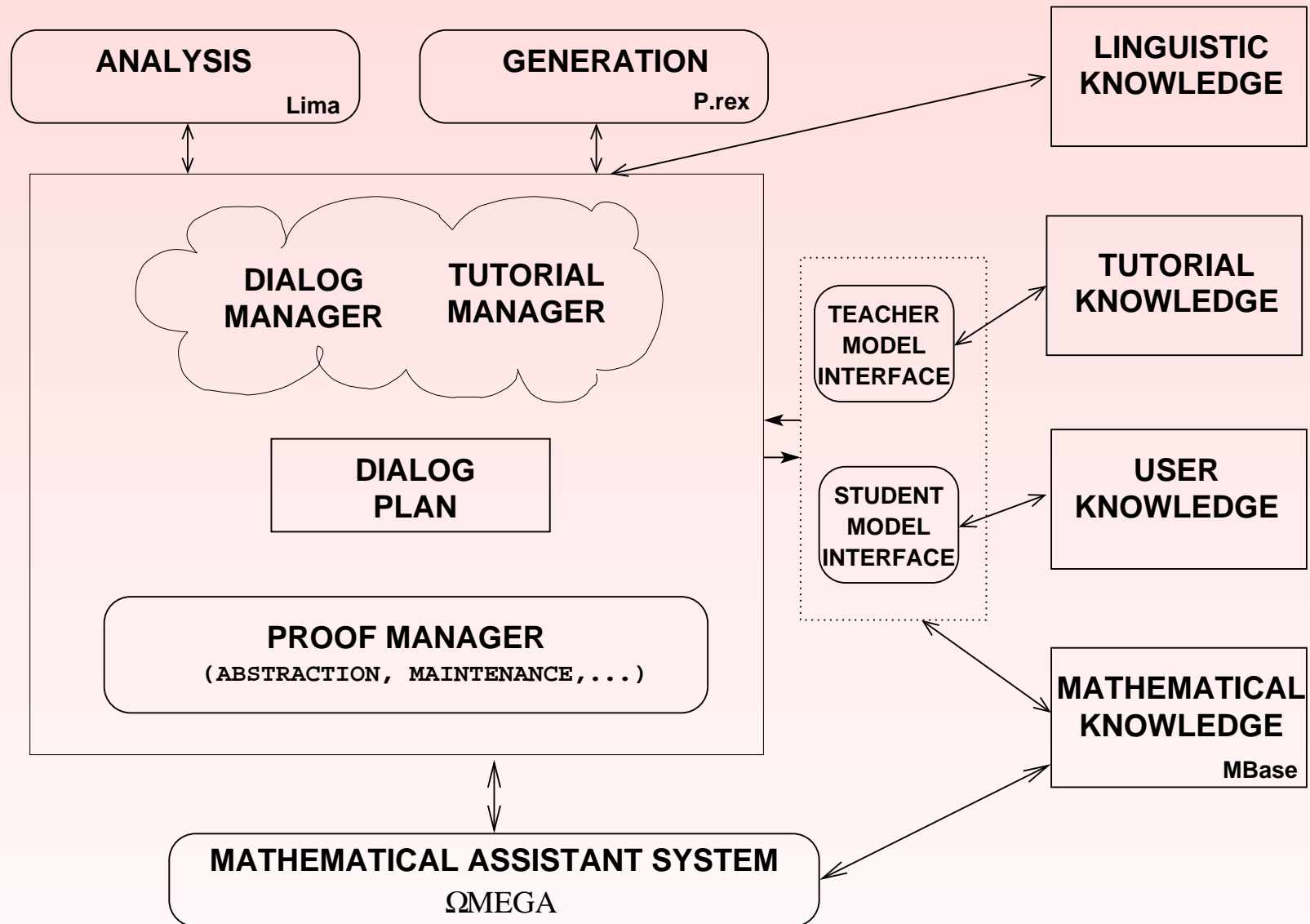
(U3) **Student:** 0 is the neutral element, and for each n in \mathbb{Z} , $-n$ is the corresponding inverse.

(U4) **Tutor:** That leaves us to show associativity.

DiaWoZ Interface



DIALOG Architecture



ACTIVEMATH Learning Environm.

The screenshot shows a Mozilla browser window with the title "donald @ ActiveMath! @ euklid.ags.uni-sb.de - Mozilla {Build ID: 200205291839}". The address bar shows the URL "http://euklid.ags.uni-sb.de:8080/ActiveMath/main/menu?user=donald". The browser's menu bar includes File, Edit, View, Go, Bookmarks, Tools, Window, and Help. The toolbar contains navigation buttons, a search box, and icons for Home, Bookmarks, mozilla.org, and Latest Builds. The main content area features the "active math" logo and a navigation menu with links: Menu, Dictionary, Scratch Pad, Learner Model, Feedback, and Logout. The left sidebar, titled "Analysis Individuell", lists topics under "Grundbegriffe der Mengenlehre und der Logik": Gleichheit von Mengen, Teilmengenbeziehung oder Inklusion, Durchschnitt und Vereinigung von Mengen, Differenz und Komplement von Mengen, Geordnetes Paar, Kartesisches Produkt, Potenzmenge, Leere Menge, Definition: Durchschnitt und Vereinigung von Mengensystemen, Definition: Relation, Definition: Eigenschaften von Abbildungen, Wahrheitswerte, Definition: Äquivalenz von Aussagen, Induktionsaxiom, and Modifikationen des Induktionsaxioms. The main content area displays a "Definition" section with a graduation cap icon. It explains that sometimes it is necessary to operate with more than two sets, and introduces the concept of a "System" of sets M . It defines the intersection $\cap M$ and union $\cup M$ of a system M of sets X . The intersection is defined as $\cap M = \{x : \text{für jedes } X \in M \text{ ist } x \in X\}$ and the union as $\cup M = \{x : \text{es existiert ein } X \in M, \text{ so daß } x \in X\}$. The browser's status bar at the bottom shows "Document: Done (0.382 secs)".

donald @ ActiveMath! @ euklid.ags.uni-sb.de - Mozilla {Build ID: 200205291839}

File Edit View Go Bookmarks Tools Window Help

http://euklid.ags.uni-sb.de:8080/ActiveMath/main/menu?user=donald

active math

Menu Dictionary Scratch Pad Learner Model Feedback Logout

Analysis Individuell

Grundbegriffe der Mengenlehre und der Logik

Grundbegriffe der Mengenlehre und der Logik

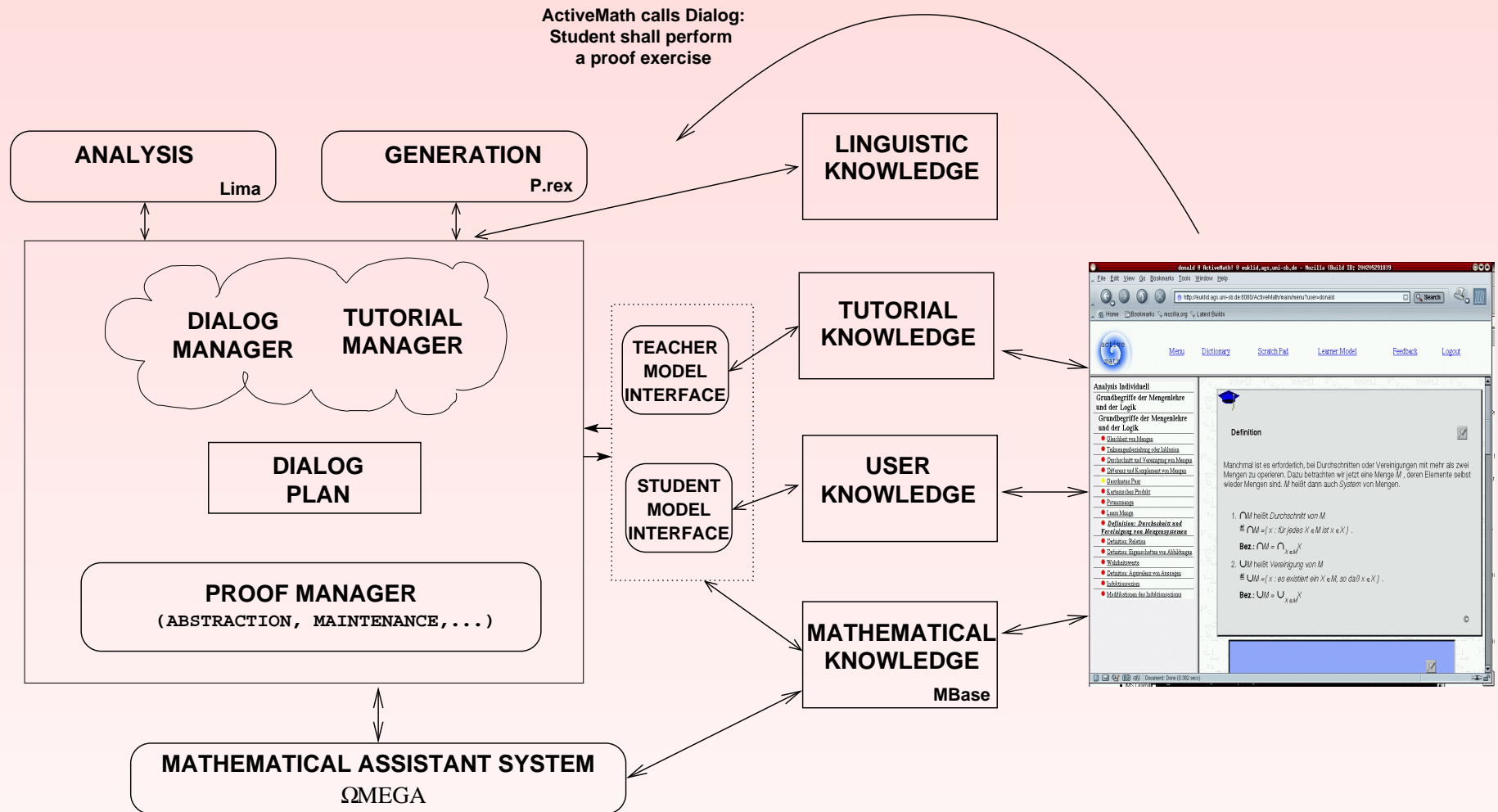
- [Gleichheit von Mengen](#)
- [Teilmengenbeziehung oder Inklusion](#)
- [Durchschnitt und Vereinigung von Mengen](#)
- [Differenz und Komplement von Mengen](#)
- [Geordnetes Paar](#)
- [Kartesisches Produkt](#)
- [Potenzmenge](#)
- [Leere Menge](#)
- [Definition: Durchschnitt und Vereinigung von Mengensystemen](#)
- [Definition: Relation](#)
- [Definition: Eigenschaften von Abbildungen](#)
- [Wahrheitswerte](#)
- [Definition: Äquivalenz von Aussagen](#)
- [Induktionsaxiom](#)
- [Modifikationen des Induktionsaxioms](#)

Definition

Manchmal ist es erforderlich, bei Durchschnitten oder Vereinigungen mit mehr als zwei Mengen zu operieren. Dazu betrachten wir jetzt eine Menge M , deren Elemente selbst wieder Mengen sind. M heißt dann auch *System* von Mengen.

1. $\cap M$ heißt *Durchschnitt* von M
 $\hat{=} \cap M = \{x : \text{für jedes } X \in M \text{ ist } x \in X\}$
Bez.: $\cap M = \bigcap_{X \in M} X$
2. $\cup M$ heißt *Vereinigung* von M
 $\hat{=} \cup M = \{x : \text{es existiert ein } X \in M, \text{ so daß } x \in X\}$
Bez.: $\cup M = \bigcup_{X \in M} X$

DIALOG and ACTIVE MATH



Domain: Naive Set Theory

Is domain well chosen?

- **What advantages has the domain?**
- **Representative also for other domains?**
- **Suitable for empirical studies?**
- **Manageable by OMEGA?**
- **Enough interesting structure?**
- **Interesting tutorial aspects?**

Mathematical Knowledge

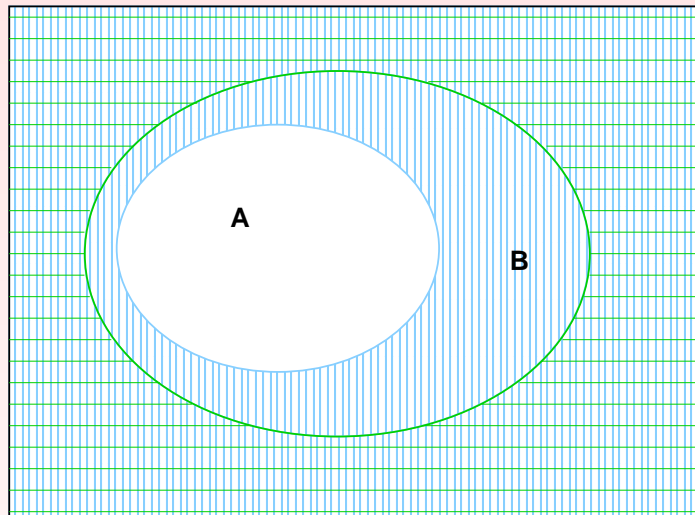
Theorem

$$(A \subseteq B) \Rightarrow (B^c \subseteq A^c)$$

Tactic

$$\frac{B \subseteq A}{A^c \subseteq B^c} \subseteq^{c-1}$$

Diagram:

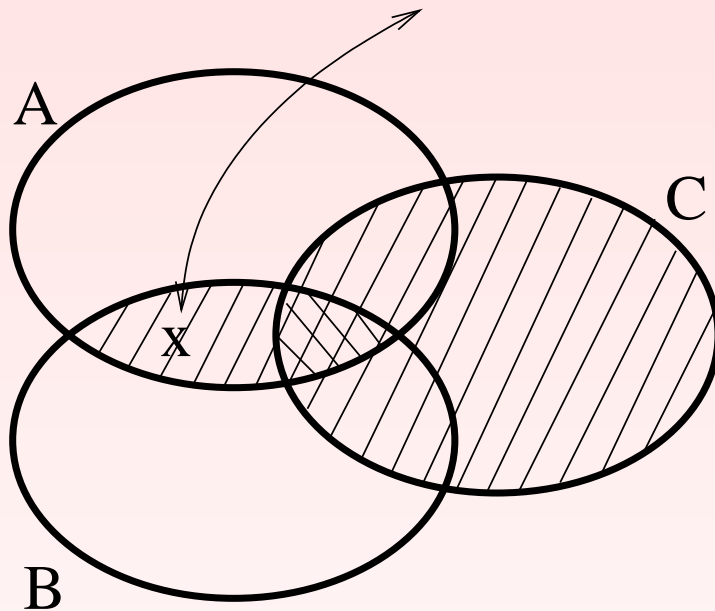


Mathematical Knowledge

Counterexamples

Non-theorem: $(A \cap B) \cup C = (A \cap B) \cap C$

Counterexample: $x \in A \wedge x \in B \wedge \neg(x \in C)$



Element x is in
 $(A \cap B) \cup C$

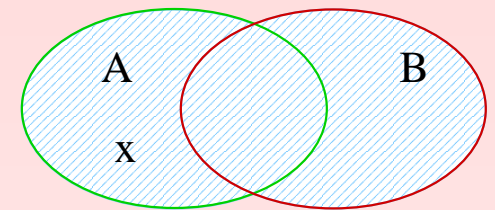
but not in
 $(A \cap B) \cap C$


VENN Diagram

Further Mathematical Knowledge

$$\in\text{-U-IL} : e \in A \Rightarrow (e \in A \cup B)$$

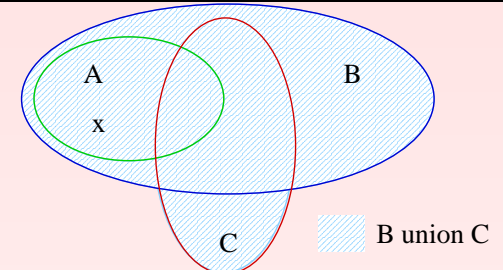
$$\frac{e \in A}{e \in A \cup B} \in\text{-U-IL}$$




 A union B

$$\subseteq\text{-U-IL} : A \subseteq B \Rightarrow (A \subseteq B \cup C)$$

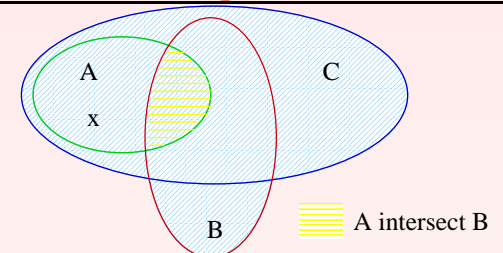
$$\frac{A \subseteq B}{A \subseteq B \cup C} \subseteq\text{-U-IL}$$




 B union C

$$\subseteq\text{-}\cap\text{-IL} : (A \subseteq C) \Rightarrow (A \cap B \subseteq C)$$

$$\frac{A \subseteq C}{A \cap B \subseteq C} \subseteq\text{-}\cap\text{-IL}$$



 A intersect B

$$\wp\text{-I} : (A \subseteq B) \Rightarrow (A \in \wp(B))$$

$$\frac{A \subseteq B}{A \in \wp(B)} \wp\text{-I}$$

?

...

User Knowledge

Student A:

- Novice in Set Theory
- Has studied the following concepts:
 - **Definitions:** $\in, \cap, \cup, \subseteq, \emptyset$, **set-complement**
 - **Theorems:** \subseteq - \cap -IL : $(A \subseteq C) \Rightarrow (A \cap B \subseteq C)$
 - etc.

Student B: ...

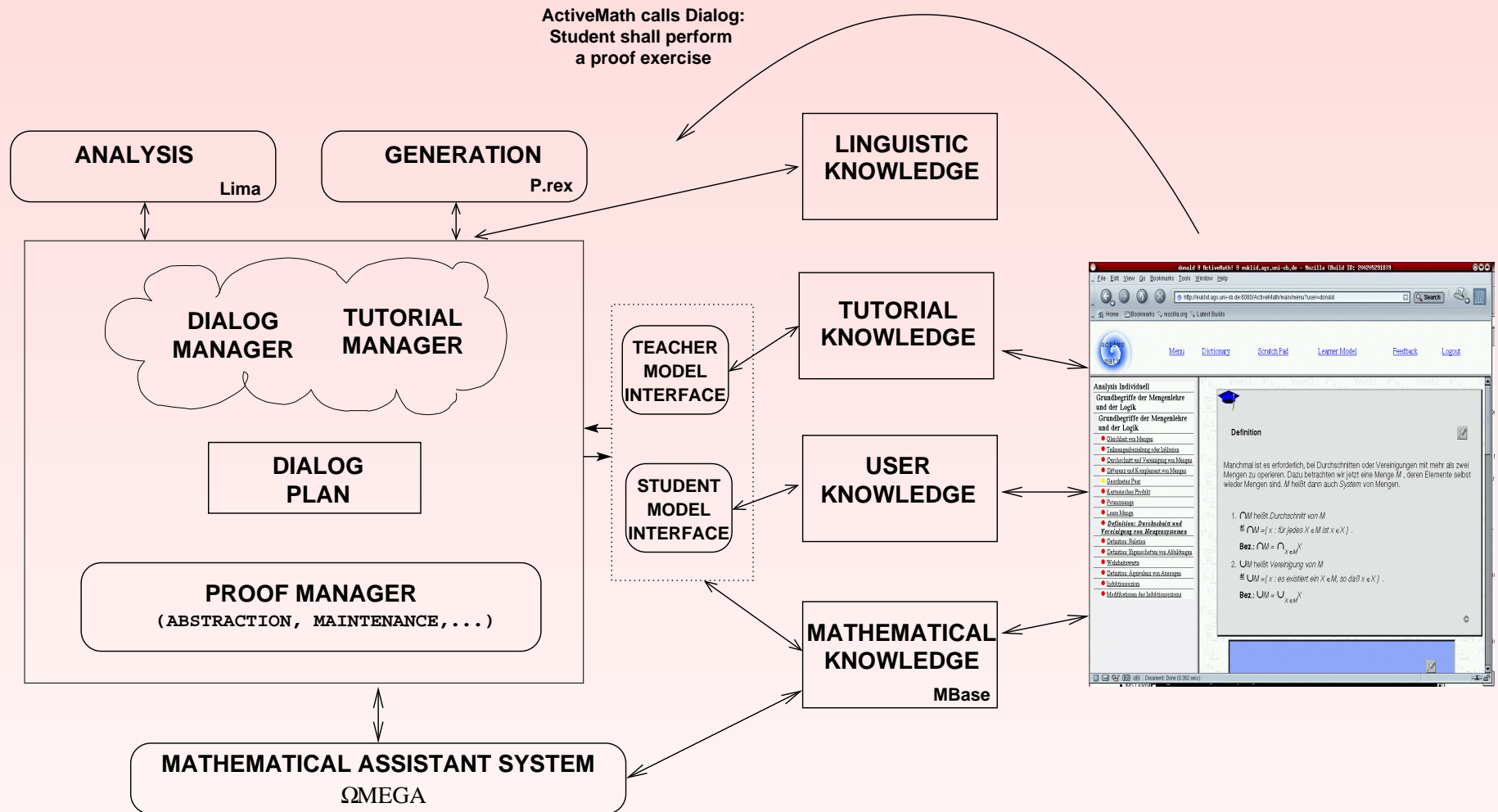
Tutorial Knowledge

- Novice student should solve exercises with mathematical knowledge that have been taught in the ACTIVEMATH tutorial.

Chapter1,Ex1 $\rightarrow R = \{\subseteq\text{-REF}, \in\text{-U-IL}, \subseteq\text{-U-IL}, \subseteq\text{-U-IR}, \subseteq\text{-}\cap\text{-IL}, \emptyset\text{-I}\}$

- If novice student does not understand an application of $r \in R$ then employ one of the following
 1. show/explain the Venn-Diagram for r
 2.
 - explain instantiation of r
 - choose theorem r as exercise (recursion!)
 3. refer to ACTIVEMATHtext for r
 4. ...

Architecture for DIALOG



Challenge for Ω MEGA & Ω -Ants

Proof planning in Ω MEGA & Ω -Ants with resources:

- inference rules
- control knowledge to structure the search space

Teacher model $\rightarrow T = (\text{inf-rules-1}, \text{control-1})$

User model $\rightarrow U = (\text{inf-rules-2}, \text{control-2})$

CONSTRUCT-PROOF(T) \rightarrow Teacher proof

VS.

CONSTRUCT-PROOF(U) \rightarrow Predictable steps of user

Example Proof

$$A \cap B \in \wp((A \cup C) \cap (B \cup C))$$

Call OMEGA with

Rules: **ND** $\cup \{\subseteq\text{-REF}, \subseteq\text{-n-I}, \subseteq\text{-n-IL}, \subseteq\text{-n-IR}, \subseteq\text{-U-IL}, \dots\}$

Strategies: **prefer-set-tactics-over-ND**

$$\begin{array}{c}
 \frac{\frac{\frac{\top}{A \subseteq A} \subseteq\text{-REF}}{A \subseteq A \cup C} \subseteq\text{-U-IL}}{A \cap B \subseteq A \cup C} \subseteq\text{-n-IL} \quad \frac{\frac{\frac{\top}{B \subseteq B} \subseteq\text{-REF}}{B \subseteq B \cup C} \subseteq\text{-U-IL}}{A \cap B \subseteq B \cup C} \subseteq\text{-n-IR} \\
 \hline
 \frac{A \cap B \subseteq (A \cup C) \cap (B \cup C)}{A \cap B \in \wp((A \cup C) \cap (B \cup C))} \wp\text{-I}
 \end{array}$$

Example Proof

To show:

$$A \cap B \in \wp((A \cup C) \cap (B \cup C))$$

By \wp -I enough to show:

$$A \cap B \subseteq (A \cup C) \cap (B \cup C)$$

By \subseteq - \cap -I we have to show (1) and (2):

1.

$$A \cap B \subseteq A \cup C$$

By \subseteq - \cap -IL enough to show

$$A \subseteq A \cup C$$

By \subseteq - \cup -IL enough to show

$$A \subseteq A$$

Follows by \subseteq -REF

2.

$$A \cap B \subseteq B \cup C$$

By \subseteq - \cup -IL enough to show

$$A \cap B \subseteq B$$

By \subseteq - \cap -IR enough to show

$$B \subseteq B$$

Follows by \subseteq -REF

q.e.d.

\Rightarrow Note multiplicities in proof: proof abstraction is needed

Enough structure in Naive Set Theory?

Problem:

$$(A \cup B)^c \subseteq A^c$$

By \subseteq^c enough to show:

$$A \subseteq A \cup B$$

By \subseteq -U-IL enough to show:

$$A \subseteq A$$

Follows by \subseteq -REF

q.e.d.

Assume: Student does not understand application of \subseteq^c

According to our tutorial knowledge we would

- 1. show/explain Venn diagram for \subseteq^c**
- 2. ■ explain instantiation of \subseteq^c**
■ choose theorem \subseteq^c as exercise (recursion!)
- 3. refer to ACTIVEMATHtext for \subseteq^c**

Enough structure in Naive Set Theory?

(2b) Proof of \subseteq^c :

$$X \subseteq Y \Rightarrow Y^c \subseteq X^c$$

By \Rightarrow -I we assume $[X \subseteq Y]$ and show

$$Y^c \subseteq X^c$$

By Defn-I(c) enough to show

$$\mathcal{U} \setminus Y \subseteq \mathcal{U} \setminus X$$

By $\subseteq - \setminus$ enough to show

$$\mathcal{U} \subseteq \mathcal{U} \text{ and } X \subseteq Y$$

The former follows by \subseteq -REF and the latter from the assumption.

q.e.d.

\Rightarrow Update of User Model

Assume: Student does not understand $\subseteq - \setminus$ step.

...

(2b) Subdialog on proof problem: $A \subseteq B \wedge C \subseteq D \Rightarrow A \setminus D \subseteq B \setminus C$

...

Related Work

Mathematical Assistant Systems: NuPrl, HOL, λ -Clam

Tutor systems: Typically simple dialog capabilities and no clear separation of domain representation, tutorial strategies, and NL dialog

- **Autotutor (Person et al.)**
- **Geometry-tutor (Alevan & Koedinger)**
- **Algebra-tutor Ms. Lindquist (Heferman & Koedinger)**
- **MALIN system**

Related Work

Dialog systems: Recent trend is flexible, domain and content-oriented dialog modeling

- **(Cohen & Levesque): Theory of action and interaction**
- **ARTEMIS (Sadek et al.): Ambitious speech input, reasoning based on cooperation principles**
- **Dialog games (Carlson) (Carletta)**
- **Trindi-Kit developed in EU projects TRINDI and SIRIDUS: Framework for development and evaluation of dialog systems based on dynamical information states**

Summary

Focus of research

- **Collect empirical data**
- **Modeling of dialog system**
- **Challenge: Interplay with the domain reasoner, the mathematical knowledge base, the tutorial strategies, and the user model**

Step by step

- **Successive refinement and validation of the model**
- **Implementation of demonstrator**