DIALOG: Natural Language-based Interaction with a Mathemtics Assistance System

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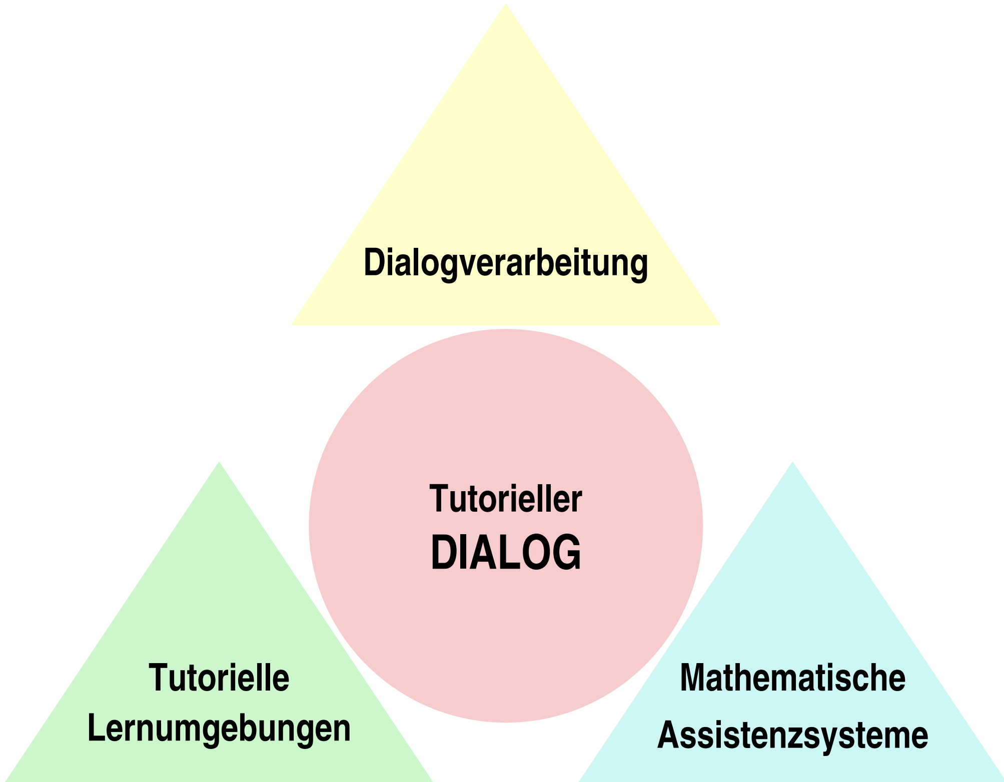
The DIALOG Project

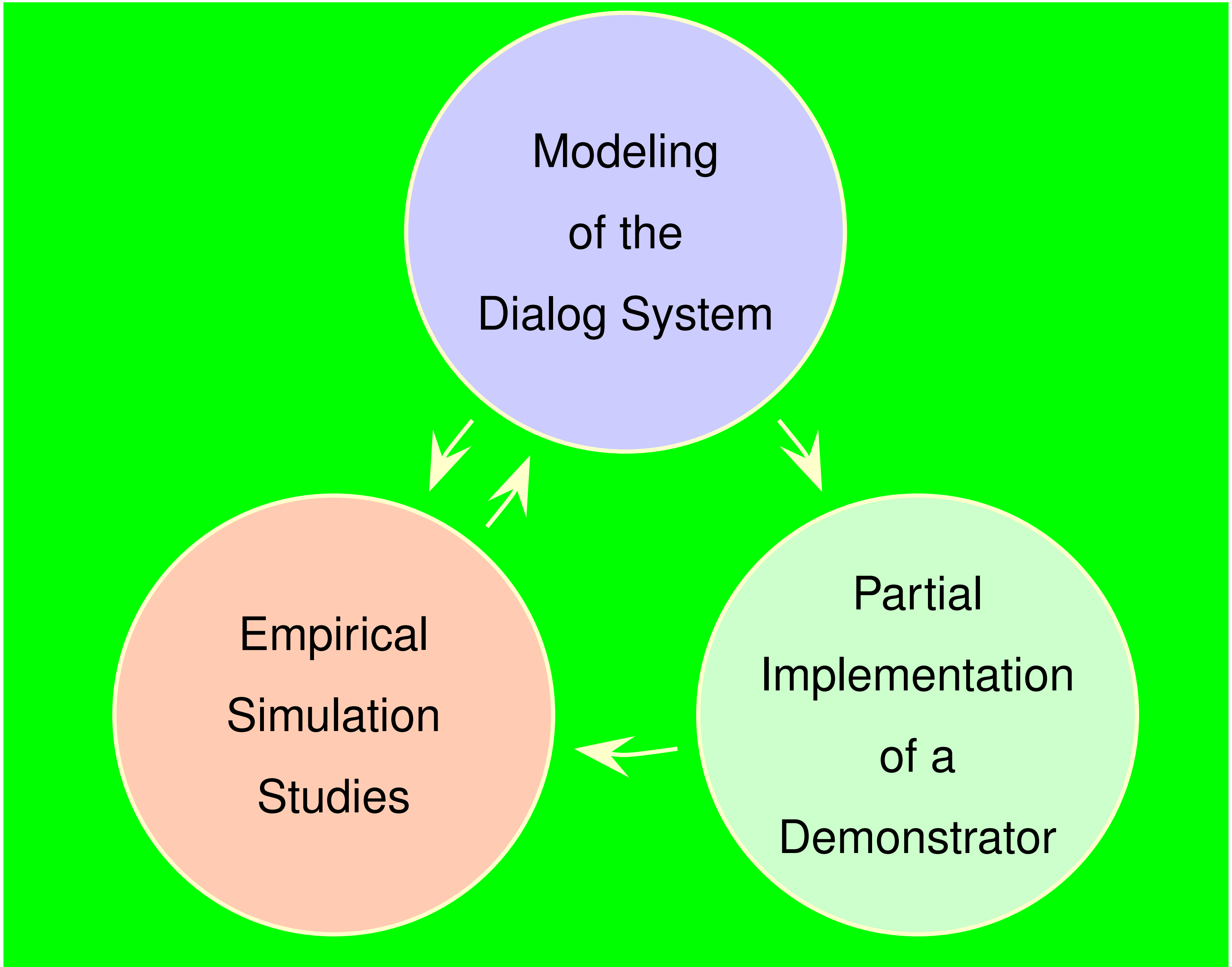


- Joint project (between Coli and CS) as part of the SFB378 on *Resource-adaptive cognitive processes*
- Selected mathematical domain: naive set theory

Team

- Directly Involved Researchers: Manfred Pinkal (Coli), Jörg Siekmann (CS), Christoph Benzmüller (CS), Ivana Kruijff-Korbayova (Coli), Magdalena Wolska (Coli), Quoc Bao Vo (CS), Armin Fiedler (CS), Dimitra Tsovaltzis (CS)
- Collaborators: Serge Autexier (CS), Malte Gabsdil (Coli), Helmut Horacek (CS), Alexander Koller (Coli), Erica Melis (CS)
- Hiwis and Students: Mark Buckley (CS), Hussain Syed Sajjad (CS), Marvin Schiller, Jochen Setz (CS), Michael Wirth (Coli), Sreedhar Ellisetty (CS), Andrea Schuch (Coli), Beata Biehl (Coli), Oliver Culo (Coli)

Method: Progressive Refinement

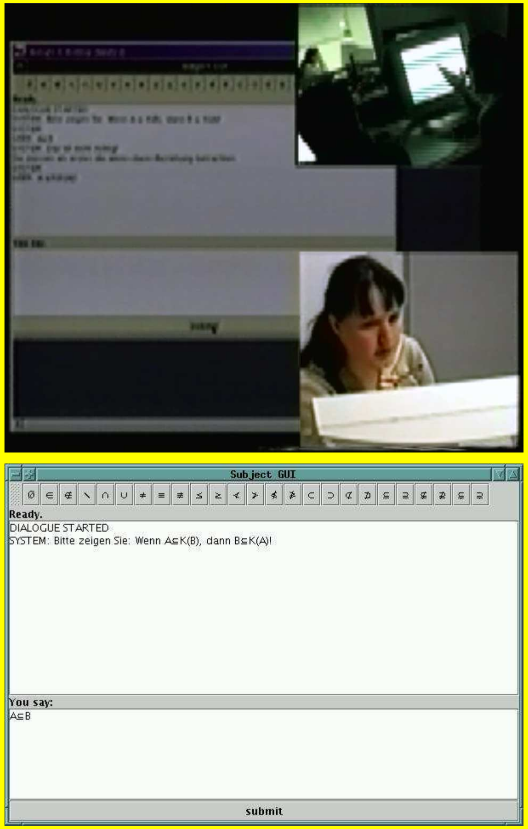


WOZ-Experiment $\longrightarrow$ Own Corpus

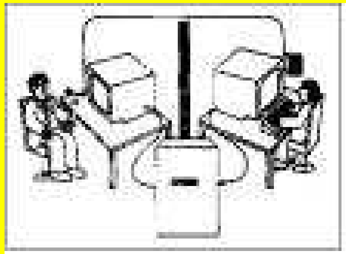
- collect data (for analysis of phenomena)
- test coarse grained model of system
- test hinting algorithm

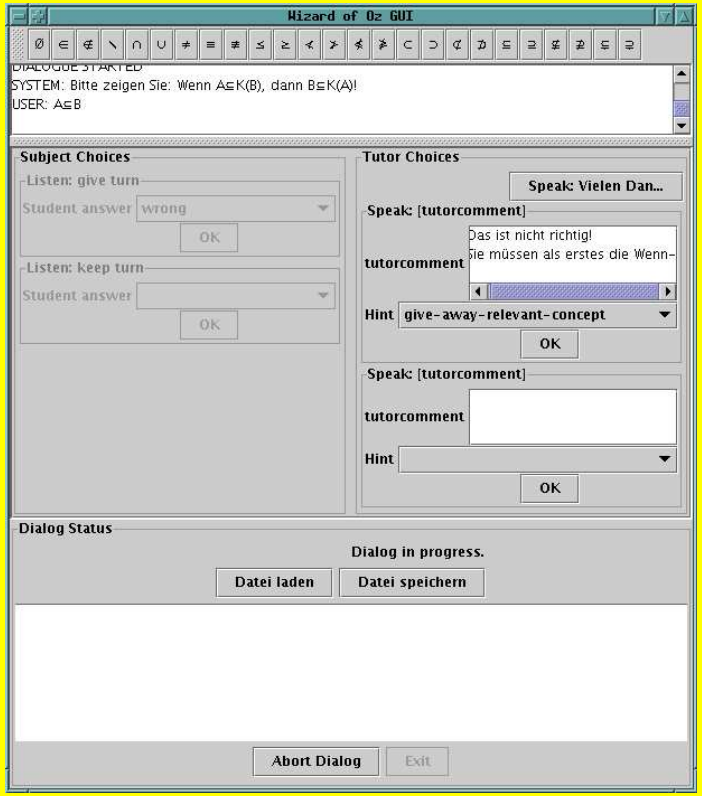
- 24 Subjects:
  - university students
  - varying background
  - varying math knowledge
- Wizard:
  - mathematician with tutoring experience
  - assisted by developers of hinting algorithm
- Experimenter

WOZ-Experiment ⟶ Own Corpus



The subject window





The wizard window

WOZ-Experiment

1. Preparation and pre-test:
   - fill in background questionnaire
   - study written lesson material (basic concepts and a collection of lemmata)
   - (try to) prove (on paper) the theorem $\mathrm{K}(\mathrm{A}) \in \mathrm{P}(\mathrm{K}(\mathrm{A} \cap \mathrm{B}))$
2. Tutoring session: evaluate a tutoring system with NL dialog capabilities (enter partial steps of a proof and having an interaction with the system)
3. Post-test and evaluation questionnaire:
   - (try to) prove (on paper) another theorem
   - fill in a questionnaire addressing various aspects of the system and its usability

Corpus Example

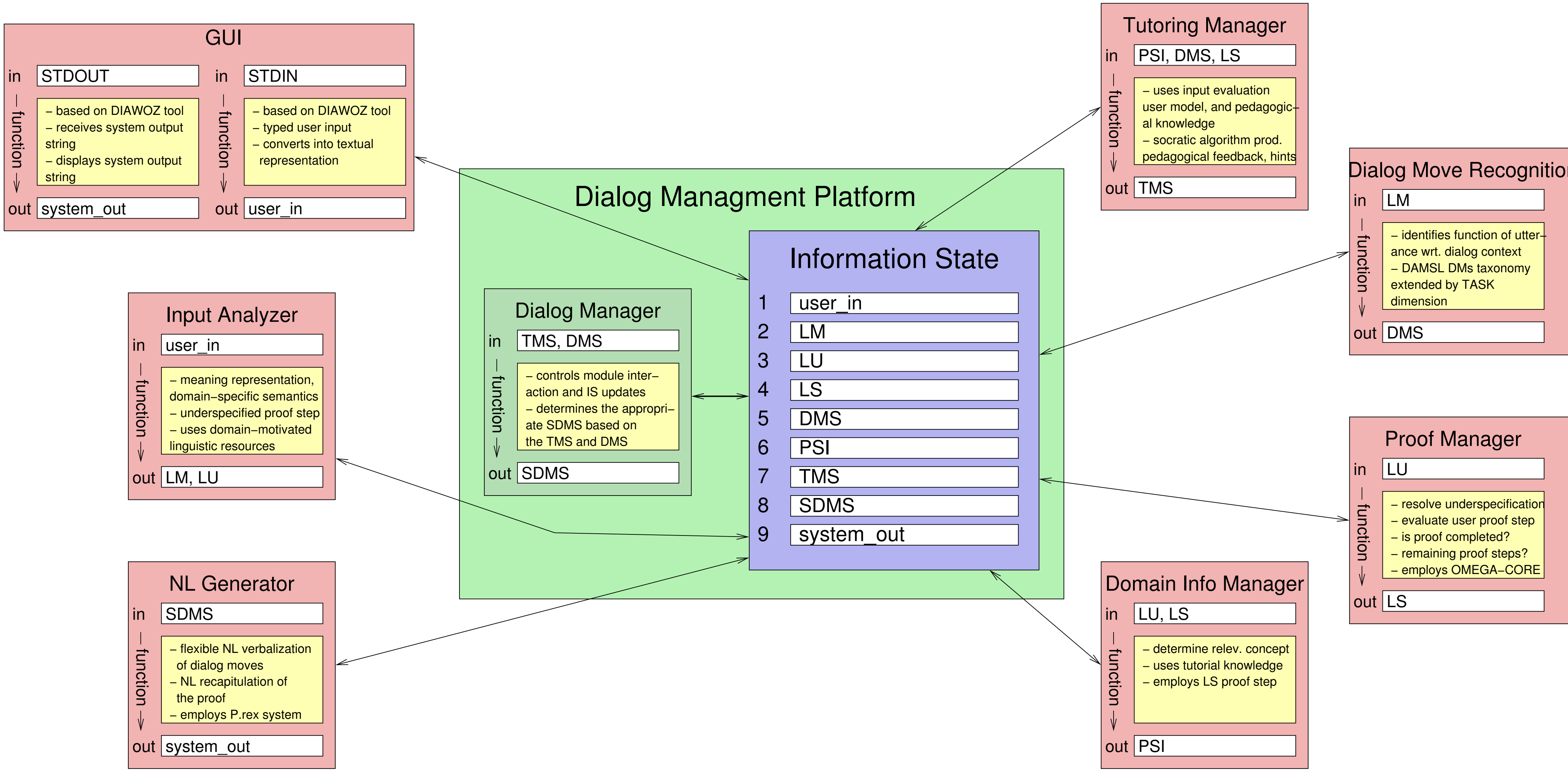
Example: didactic, vp16, dryrun

T1: Bitte zeigen Sie : K ((A ∪ B) ∩ (C ∪ D)) = (K (A) ∩ K (B)) ∪ (K (C) ∩ K (D)) !</s>
S1: (`correct`) nach deMorgan-Regel-2 ist K ((A ∪ B) ∩ (C ∪ D)) = (K (A ∪ B) ∪ K (C ∪ D))</s>
T2: Das ist richtig !</s>
S2: (`correct`) K (A ∪ B) ist laut DeMorgan-1 K (A) ∩ K (B)</s>
T3: Das stimmt auch .</s>
S3: (`correct`) und K (C ∪ D) ist ebenfalls laut DeMorgan-1 K (C) ∩ K (D)</s>
T4: Auch das stimmt .</s>
S4: (`correct`) also folgt letztendlich : K ((A ∪ B) ∩ (C ∪ D)) = (K (A) ∩ K (B)) ∪ (K (C) ∩ K (D)) .</s>
T5: Das stimmt genau .</s> Ich wiederhole noch einmal : Auf die linke Seite der Gleichung kann ich zuerst die zweite und danach die erste de-Morgan-Regel anwenden , so daß sich folgende Argumentationskette ergibt : K ((A ∪ B) ∩ (C ∪ D)) = K (A ∪ B) ∪ K (C ∪ D) = (K (A) ∩ K (B)) ∪ (K (C) ∩ K (D)) .</s>

Project Progress: First Phase

- opened and entered new field of NL based mathematical tutoring dialogs
- foremost aim: obtain a general view of the interplay between advanced NL processing in a flexible tutoring dialog, and dynamic, abstract-level mathematical domain reasoning.
- moved from collecting empirical data through modeling of the different components and their interfaces to a demonstrator implementation.

Demonstrator Architecture



Example Utterance

T1: Bitte zeigen Sie : K ((A ∪ B) ∩ (C ∪ D)) = (K (A) ∩ K (B)) ∪ (K (C) ∩ K (D)) !</s>
S1: (`correct`) nach deMorgan-Regel-2 ist K ((A ∪ B) ∩ (C ∪ D)) = (K (A ∪ B) ∪ K (C ∪ D))</s>
T2: Das ist richtig !</s>

Abbrev.	Meaning	Example
STDIN	standard input	*"nach deMorgan-Regel-2 ist* $\mathsf{K}((\mathsf{A} \cup \mathsf{B}) \cap (\mathsf{C} \cup \mathsf{D})) = (\mathsf{K}(\mathsf{A} \cup \mathsf{B}) \cup \mathsf{K}(\mathsf{C} \cup \mathsf{D}))$*"*
LM	linguistic meaning	s : @h1(holds ∧ <Norm>(d1 ∧ deMorgan–Regel–2)∧ <Patient>(f1 ∧ FORMULA1))
LU	proof language with underspecification	(input (label 1_1) (formula (= (complement (intersection (union a b) (union c d))) (union (complement (union a b)) (complement (union c d)))))) (type ?) (direction ?) (justifications (just (reference DeMorgan-2) (formula ?) (substitution ?) (role:from))))
LS	system-oriented proof language (+ evaluation)	((KEY 1_1) ⟶ ((Evaluation (expStepRepr (label 1_1) (formula (=(complement(intersection(union(A B) union(C D))) union(complement(union(A B)) complement(union(C D)))))) (type inference) (direction forward) (justification ((reference demorgan-2) (formula nil) (substitution ((X union(A B) Y union(C D)))) (role nil)))) (StepCat correct))) (ProofCompleted false) (completeProofs . . . see next poster . . .))
DMS	dialog move specification	{ fwd = "Assert", bwd = "Address_statement", commm = "", taskm = "", comms = "", task = "Domain_contribution" }
PSI	proof step information	{domConCat: "correct", proofCompleted: false, proofstepCompleted: true, proofStep: "", relConU: true, hypConU: true, domRelU: false, iRU: true, relCon: "" "+(char)8745", hypCon: "" "+(char)8746", domRel: "", iR: "deMorgan-Regel-2"}
TMS	tutorial move specification	{mode= "min"; task= (signalAccept; {proofStep= ""; relCon= ""; hypCon= ""; domRel= ""; iR= ""; taskSet= ""; completeProof= ""})}
SDMS	system dialog move specification	{ mode = "min"; fwd = "Assert"; bwd = "Address_statement"; task = ("signalCorrect", {proofStep= "", relCon= "", hypCon= "", domRel= "", iR= "", taskSet= "", completeProof= ""}); comms = ""; commm = ""; taskm = "" }
STD_out	textual representation of NL output	"Das ist richtig".

Achievements

- Experiment design, empirical test environment, and Wizard of Oz tool.
- First experiment in naive set theory domain, with written dialog input and output.
- Preliminary corpus investigation and subsequent formal annotation at several levels of interpretation: deep semantic structure, dialog moves, and tutorial task aspects.
- Coarse grained architecture and specification of refined modules for input analysis, proof management, and tutorial dialog moves (especially hinting).

Achievements (contd.)

- Realization of input analysis using a deep dependency-based grammar, focusing on uniform interpretation of informal interleaved natural language and mathematical formulae.
- Proposal of a user-oriented proof language with underspecification.
- Realization of the proof manager: proof representation languages for the proof manager, interfacing to the underlying domain reasoner (the Omega theorem prover); agent-based assertion reasoning that can enumerate proof step suggestions.
- Design and development of a demonstrator.

Project Progress: First Phase

- Experiment & WOZ Tool & Corpus
- NL Analysis
- Tutoring Aspects
- Dialog Management
- Proof Manager
- Proof Step Evaluation
- Modeling of System & Demonstrator

Overview, Papers, Corpus, etc.: see

`http://www.ags.uni-sb.de/~chris/dialog/`

Proof Manager: Tasks

- Resolution of
  - Ambiguities
  - Underspecification
- Proof Step Evaluation
  - Soundness: Can the proof step be verified by a formal inference system?
  - Granularity: Is the granularity (i.e., ’logical size’ or ’argumentative complexity’) of the proof step acceptable?
  - Relevance: Is the proof step needed or useful in achieving the goal?

Ambiguities and Underspecification

Discourse:

(1) From previous observations we know that A or B.

(2) The former implies D by Lemma X.

(3) Similarly, from the latter follows C.

Alternative user utterances with underspecification:

(a) From this follows D since C implies D by Lemma Y.

— *this may refer to (1)+(2)+(3), to (3), or even (1) or (2) with wrong justification*

(b) It holds D since C implies D by Lemma Y.

— *no underspecified anaphoric reference but ambiguity at domain reasoning level*

Ambiguities and Underspecification

Example	*Where does ambiguity arise?*	*Ambiguity resolution means*
(1) $x \in B$ und somit $x \subseteq K(B)$ und $x \subseteq K(A)$ wegen Vorraussetzung (2) A enthaelt B	linguistic meaning level; attachment, coordination	linguistic means; type checking in (2)
(3) P((A∪C)∩(B∪C))=PC ∪(A∩B) (4) K((A∪C)∩(B∪C))=KC ∪(A∩B)	linguistic meaning level; informal character of discourse	type checking for (3); domain reasoning for (4)
(5) T1: Bitte zeigen Sie: $K((A \cup B) \cap (C \cup D)) = (K(A) \cap K(B)) \cup (K(C) \cap K(D))$! S1: nach deMorgan-Regel-2 ist $K((A \cup B) \cap (C \cup D)) = (K(A \cup B) \cup K(C \cup D))$	underspecified proof step	domain reasoning

Example: Proof Step Evaluation

Assertions already introduced

(A1) $A \land B$.

(A2) $A \Rightarrow C$.

(A3) $C \Rightarrow D$.

(A4) $F \Rightarrow B$.

(G) $D \lor E$.

Alternative proof step directives.

(a) Aus den Annahmen folgt D.

(b) B gilt.

(c) Es genügt D zu zeigen.

(d) Wir zeigen E.

Proof Step Evaluation

Criterion	*Task (first approach)*	*Requirements for theorem prover*
Soundness	$E \vdash^{?}_{C} D \vee E$	'Yes' or 'No' answer; any theorem prover resp. calculus C
Granularity	proof-steps($E \vdash^{?}_{C} D \vee E$)	adequate abstract-level theorem prover resp. calculus C; measure 'shortest' proof; take tutorial constraints into account; proof planning or assertion level reasoning?
Relevance	$\begin{array}{l} A \wedge B \\ A \Rightarrow C \\ C \Rightarrow D \\ F \Rightarrow B \end{array} \vdash^{?}_{C} E$	recognize detours; compare with other 'shorter' proofs; take tutorial constraints into account; forward case more challenging

What are the right provers?

???

Conclusion

To support flexible, tutorial NL dialog on mathematical proofs we need to

- adapt our deduction systems to support specific tasks
- e.g., to reason about user uttered proof steps
- some of these tasks require abstract-level reasoning systems

From a perspective of cognitive science:

Very fascinating application domain for deduction systems.

Example: Proof Step Evaluation

Assertions already introduced

(A1) $A \wedge B$.

(A2) $A \Rightarrow C$.

(A3) $C \Rightarrow D$.

(A4) $F \Rightarrow B$.

(G) $D \vee E$.

Alternative proof step directives.

(a) Aus den Annahmen folgt D.

(b) B gilt.

(c) Es genügt D zu zeigen.

(d) Wir zeigen E.

Soundness Evaluation

Soundness verification of utterance (a) boils down to proving the theorem:

$$\mathsf{P1:} \quad (A \wedge B), (A \Rightarrow C), (C \Rightarrow D), (F \Rightarrow B) \vdash D$$

Analogously, for the backward reasoning step given in (d) we get:

$$\mathsf{P2:} \quad E \vdash (D \vee E)$$

No specific requirements imposed on the proof system $\vdash$ (any kind of first-order theorem prover)

Granularity Evaluation

Requires analyzing the ‘complexity’ or ‘size’ of proofs: For utterances (a) and (d) above, it thus boils down to judging about the complexity of the proof tasks (P1) and (P2).

For illustration consider Gentzen’s ND as proof system $\vdash$.

Define *argumentative complexity*: number of $\vdash$-steps in the smallest $\vdash$-proof.

Judgements:

- argumentative complexity of (a) is bigger than that of (b)
- argumentative complexity of (a) is above a (tutorially) motivated complexity threshold

Granularity Evaluation (contd.)

Natural deduction calculus propably not appropriate: two intuitively very similar user proof steps may actually expand into natural deduction proofs of completely different size.

Question: What is an appropriate choice of a proof system $\vdash$?
It should reflect the argumentative level of human reasoning.

Empirical studies are possible and planned.

Relevance Evaluation

Alternative approaches listed according to increasing difficulty:

- Statically choose one or a few “golden proofs” and match the uttered partial proofs against them.
- Generate from the initially chosen golden proofs larger sets modulo, for instance, (allowed) re-orderings of proof steps and match against this extended set.
- Dynamically support relevance analysis with domain reasoning. For this, we test whether a proof can still be obtained from the new proof situation (using an abstract-level proof system). Resource-bound enumeration of possible proofs and proof step matching is additionally required.
- Stimulate research in proof theory: compact and tractable representation of the proofs in the proof space.

Relevance Evaluation (contd.)

Backward reasoning case (c): $D \vee E$ is refined to goal D.

Relevance question: can a proof still be generated? The task is thus identical to proof task (P1) as before.
A backward proof step that is not relevant according to this criterion is (d) since it reduces to:

$$P3: \quad (A \wedge B), (A \Rightarrow C), (C \Rightarrow D), (F \Rightarrow B) \vdash E$$

for which no proof can be generated. Thus, (d) is a sound refinement step that is not relevant, in contrast to utterance (c).

Approach needs to be refined: exclude detours and take tutorial aspects into account (teaching particular styles of proofs, particular proof methods, etc.).

Relevance Evaluation (contd.)

More challenging forward reasoning case is discussed next.

Example (a):

$$\mathsf{P4}: \quad (A \wedge B), (A \Rightarrow C), (C \Rightarrow D), (F \Rightarrow B), D \vdash (D \vee E)$$

The question whether D is relevant reduces to the question whether there exists a proof for the given task that employs D and which is shorter than the best proof that can be obtained when deleting D from the available knowledge. According to this approach, utterance (b) describes an non-relevant proof step.

Note that we do not just ask about the existence of an arbitrary proof but about the existence of a proof with particular properties. This requires techniques such as (resource-bound and heuristic guided) enumeration of proofs.