HOL based Universal Reasoning

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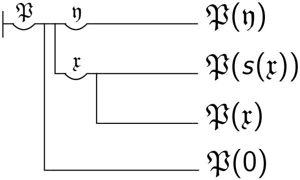
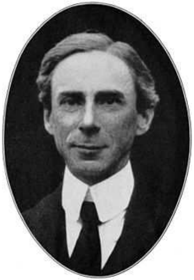
HOL: Church's STT with Henkin Semantics





Automated Reasoners for HOL

TPS ... (Peter Andrews) ?

LEO-I/LEO-II (myself)

Isabelle (Nipkow/Paulson/Blanchette)

Satallax (Brown)

Nitpick (Blanchette)

agsyHOL (Lindblatt)

- all accept TPTP THF Syntax [SutcliffeBenzmüller, J.Form.Reas, 2009]
- can be called remotely via SystemOnTPTP at Miami
- they significantly gained in strength over the last years
- they can be bundled into a combined prover HOL-P

Exploit HOL with Henkin semantics as metalogic
Automate other logics (& combinations) via semantic embeddings
— HOL-P becomes a Universal Reasoner —

Short Demonstration of HOL-P

FO Modal Logic example: $(\Diamond\exists x P f x \wedge \Box\forall y(\Diamond P y \Rightarrow Q y)) \Rightarrow \Diamond\exists z Q z$

encoding in HOL: $\text{valid}\,(\Diamond\exists x P f x \wedge \Box\forall y(\Diamond P y \Rightarrow Q y)) \Rightarrow \Diamond\exists z Q z$

... in THF Syntax: ... not here ...

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```
%> ./HOL-P example.thf -timeout 20 -logic s4 -domain varying
```

Short Demonstration of HOL-P

FO Modal Logic example: $(\Diamond\exists x Pfx \wedge \Box\forall y(\Diamond Py \Rightarrow Qy)) \Rightarrow \Diamond\exists z Qz$
encoding in HOL: valid $(\Diamond\exists x Pfx \wedge \Box\forall y(\Diamond Py \Rightarrow Qy)) \Rightarrow \Diamond\exists z Qz$
... in THF Syntax: ... not here ...

%> ./HOL-P example.thf -timeout 20 -logic s4 -domain varying
Calling HOL Resoners remotely in Miami ... thanks to Geoff Sutcliffe
— LEO-II says Theorem — CPU 0.08s
— Satallax says Theorem — CPU 0.03s
— Isabelle says Unknown — CPU 11.93s
— Nitpick says Unknown — CPU 10.62s
— agsyHOL says Theorem — CPU 0.55s

Short Demonstration of HOL-P

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%> ./HOL-P example.thf -timeout 20 -logic k -domain constant
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Short Demonstration of HOL-P

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Calling HOL Resoners remotely in Miami ... thanks to Geoff Sutcliffe
— LEO-II says Unknown — CPU 11.93s
— Satallax says CounterSatisfiable — CPU 0.04s
— Isabelle says Unknown — CPU 16.19s
— Nitpick says CounterSatisfiable — CPU 8.19s
— agsyHOL says Unknown — CPU 10.82s

HOL — Church's Simple Type Theory [Church, 1940]

Simple Types $\alpha ::= \iota \mid o \mid \alpha_1 \to \alpha_2$

HOL — Church's Simple Type Theory [Church, 1940]

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Simple Types $\alpha ::= \iota \mid o \mid \alpha_1 \to \alpha_2$

Individuals

Booleans (True and False)

Functions/Predicates

HOL — Church's Simple Type Theory [Church, 1940]

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Simple Types $\alpha ::= \iota \mid \mu \mid o \mid \alpha_1 \to \alpha_2$

Possible worlds

Individuals

Booleans (True and False)

Functions/Predicates

HOL — Church's Simple Type Theory [Church, 1940]

HOL $\quad s,t \quad ::= \quad c_\alpha \mid x_\alpha \mid (\lambda x_\alpha\, s_\beta)_{\alpha\to\beta} \mid (s_{\alpha\to\beta}\, t_\alpha)_\beta \mid$
$(\neg_{o\to o}\, s_o)_o \mid (s_o \vee_{o\to o\to o} t_o)_o \mid (\forall x_\alpha\, t_o)_o$

HOL — Church's Simple Type Theory [Church, 1940]

HOL

$$s, t \quad ::= \quad c_\alpha \mid x_\alpha \mid (\lambda x_\alpha\, s_\beta)_{\alpha\to\beta} \mid (s_{\alpha\to\beta}\, t_\alpha)_\beta \mid$$
$$(\neg_{o\to o}\, s_o)_o \mid (s_o \vee_{o\to o\to o} t_o)_o \mid \underbrace{(\forall x_\alpha\, t_o)_o}$$

$$\Pi_{(\alpha\to o)\to o}\lambda x_\alpha t_o$$

HOL — Church's Simple Type Theory [Church, 1940]

HOL $\quad s, t \quad ::= \quad C \mid x \mid (\lambda x\, s) \mid (s\, t) \mid (\neg s) \mid (s \vee t) \mid (\forall x\, t)$

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HOL (with Henkin semantics) is meanwhile very well understood

- Origin [Church, J.Symb.Log., 1940]
- Henkin-Semantics [Henkin, J.Symb.Log., 1950]
  [Andrews, J.Symb.Log., 1971, 1972]
- Extensionality/Intensionality [BenzmüllerBrownKohlhase, J.Symb.Log., 2004]
  [Muskens, J.Symb.Log., 2007]

Embedding of First-order Modal Logic (FML) in HOL

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FML $\quad \varphi, \psi \quad ::= \quad P(t_1, \ldots, t_n) \mid (\neg \varphi) \mid (\varphi \vee \psi) \mid \Box \varphi \mid (\forall x\, \varphi)$

$$
\begin{array}{lll}
M, g, s \models \neg \varphi & \text{iff} & \text{not } M, g, s \models \varphi \\
M, g, s \models \varphi \vee \psi & \text{iff} & M, g, s \models \varphi \text{ or } M, g, s \models \psi \\
M, g, s \models \Box \varphi & \text{iff} & M, g, u \models \varphi \text{ for all } u \text{ with } r(s, u) \\
M, g, s \models \forall x\, \varphi & \text{iff} & M, [d/x]g, s \models \varphi \text{ for all } d \in D
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FML in HOL:

$$
\begin{array}{lll}
\neg & = & \lambda\varphi_{\iota\to o}\, \lambda s_\iota\, \neg\varphi s \\
\vee & = & \lambda\varphi_{\iota\to o}\, \lambda\psi_{\iota\to o}\, \lambda s_\iota\, (\varphi s \vee \psi s) \\
\Box_r & = & \lambda\varphi_{\iota\to o}\, \lambda s_\iota\, \forall u_\iota\, (\neg r s u \vee \varphi u) \\
\Pi & = & \lambda h_{\mu\to(\iota\to o)}\, \lambda s_\iota\, \forall d_\mu\, h d s \\
 & & (\forall x\varphi \text{ stands for } \Pi\lambda x\varphi)
\end{array}
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Embedding of First-order Modal Logic (FML) in HOL

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Idea: Lifting of modal formulas to predicates on worlds

Metalevel notions: valid $= \lambda \varphi_{\iota \to o}\, \forall s_\iota\, \varphi s$

Propositional Quantification

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Propositional Quantification [Fitting, J.Symb.Log., 2002]

...

$M, g, s \models \forall^p p\, \varphi$ iff $M, [v/p]g, s \models \varphi$ for all $v \in P$

(P is a non-empty collection of sets of worlds, it includes atom sets)

Embedding in HOL

...

$\Pi^p = \lambda h_{(\iota\to o)\to(\iota\to o)}\, \lambda s_\iota\, \forall v_\mu\, hvs$ ($\forall \varphi \psi$ stands for $\Pi^p \lambda \varphi \psi$)

Modal logic axioms	Semantical Condition
valid $\forall^p \varphi (\Box \varphi \supset \Diamond \varphi)$	$\forall x \exists y (rxy)$

Bridge rules	Semantical Condition
valid $\forall^p \varphi (\Box_r \varphi \supset \Box_s \varphi)$	$\forall x \forall y (rxy \supset sxy)$

We get a wide range of modal logics and combinations for free!

Propositional Quantification

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We get a wide range of modal logics and combinations for free!

Conditional Logics

Selection Function Semantics [Stalnaker, 1968]

...

$M, g, s \models \varphi \Rightarrow \psi$ iff $M, g, v \models \psi$ for all $v \in f(s, \overbrace{\{u \mid M, g, u \models \varphi\}}^{[\varphi]})$

Embedding in HOL

...

$$\Rightarrow_f \quad = \quad \lambda\varphi_{\iota\to o}\,\lambda\psi_{\iota\to o}\,\lambda s_\iota\,\forall v_\iota\,(\neg f\,s\varphi v \vee \psi v)$$

Selection Function Semantics [Stalnaker, 1968]

...

$$M, g, s \models \varphi \Rightarrow \psi \text{ iff } M, g, v \models \psi \text{ for all } v \in f(s, \overbrace{\{u \mid M, g, u \models \varphi\}}^{[\varphi]})$$

Embedding in HOL

...

$$\Rightarrow \quad = \quad \lambda f_{\iota\to(\iota\to o)\to(\iota\to o)}\, \lambda\varphi_{\iota\to o}\, \lambda\psi_{\iota\to o}\, \lambda s_\iota\, \forall v_\iota\, (\neg f s \varphi v \vee \psi v)$$

Conditional Logics

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Embedding in HOL

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$$\Rightarrow \quad = \quad \lambda f_{\iota\to(\iota\to o)\to(\iota\to o)}\, \lambda\varphi_{\iota\to o}\, \lambda\psi_{\iota\to o}\, \lambda s_\iota\, \forall v_\iota\, (\neg f\, s\varphi v \vee \psi v)$$

Interesting, since selection function semantics is a generalization of Kripke semantics which cannot be naturally translated to FOL.

Soundness and Completeness (and Cut-elimination)

$$\models^{L} \varphi \quad \text{iff} \quad \models^{HOL}_{\text{Henkin}} \text{valid}\ \varphi_{\iota\to o}$$

Logics L studied so far:

- Propositional Multimodal Logics [BenzmüllerPaulson, Log.J.IGPL, 2010]
- Quantified Multimodal Logics [BenzmüllerPaulson, Log.Univ., 2012]
- Intuitionistic Logics [BenzmüllerPaulson, Log.J.IGPL, 2010]
- Access Control Logics [Benzmüller, IFIP SEC, 2009]
- Propositional Conditional Logics [BenzmüllerEtAl., AMAI, 2012]
- Quantified Conditional Logics [Benzmüller, IJCAI, 2013]
- . . . more is on the way . . .

Soundness and Completeness (and Cut-elimination)

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$$\models^{L} \varphi \quad \text{iff} \quad \models^{HOL}_{\text{Henkin}} \text{valid}\, \varphi_{\iota\to o} \quad \text{iff} \quad \vdash^{\text{seq}^{\text{HOL}}}_{\text{cut-free}} \text{valid}\, \varphi_{\iota\to o}$$

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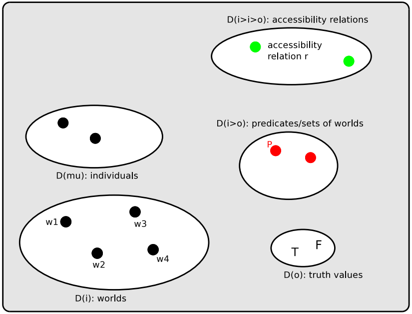
Varying and Cumulative Domain Quantification

Constant Domain



$$\Pi = \lambda h \, \lambda w_\iota \, \forall x_\mu \, hxw$$

Varying and Cumulative Domain Quantification

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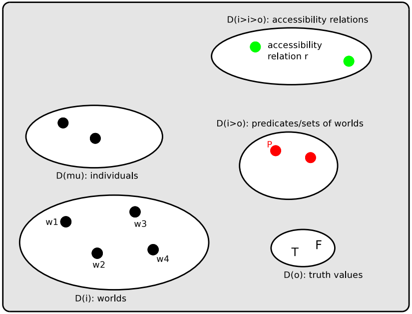
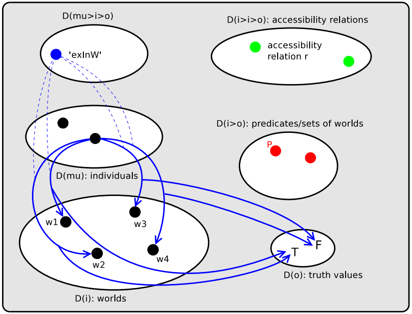


Varying and Cumulative Domain



$$\Pi = \lambda h\, \lambda w_\iota\, \forall x_\mu\; hxw$$

$$\Pi^{va} = \lambda h\, \lambda w_\iota\, \forall x_\mu\; (\neg \mathbf{exInW} xw \vee hxw)$$

Varying and Cumulative Domain Quantification

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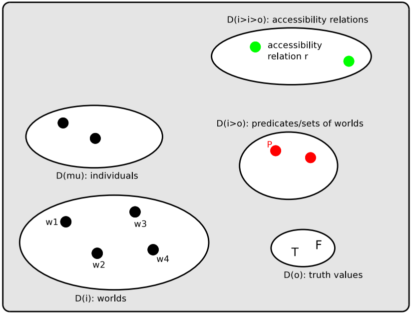
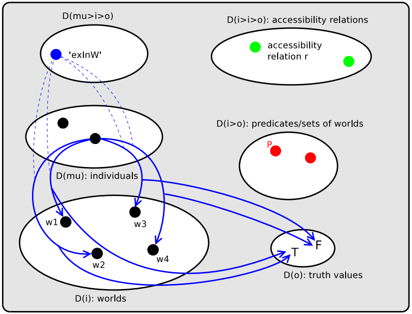


Varying and Cumulative Domain



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domains are non-empty → $\forall w_\iota\, \exists x_\mu\, \mathbf{exInW} xw$

denotation (constants & functions) → $\forall w_\iota\, \mathbf{exInW} cw$

$$\forall w_\iota\, (\mathbf{exInW} t^1 w \wedge \ldots \wedge \mathbf{exInW} t^n w \supset \mathbf{exInW} (f\, t^1 \ldots t^n) w)$$

Varying and Cumulative Domain Quantification

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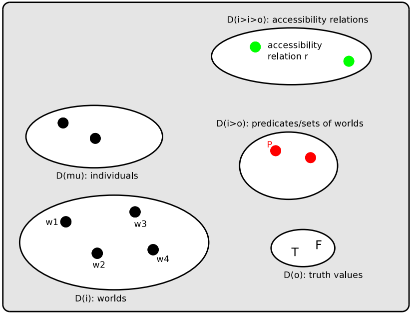

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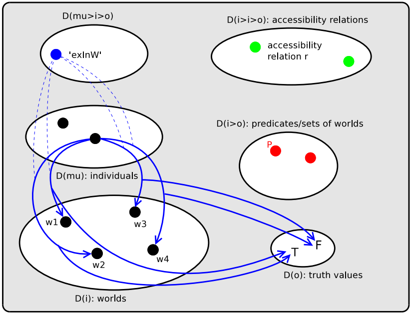

$$\Pi = \lambda h\, \lambda w_\iota\, \forall x_\mu\, hxw \qquad \Pi^{va} = \lambda h\, \lambda w_\iota\, \forall x_\mu\, (\neg \mathbf{exInW} xw \vee hxw)$$

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$$\forall w_\iota\, (\mathbf{exInW} t^1 w \wedge \ldots \wedge \mathbf{exInW} t^n w \supset \mathbf{exInW} (f\, t^1 \ldots t^n) w)$$

cumulative domains → $\forall x, v, w\, (\mathbf{exInW} xv \wedge rvw \supset \mathbf{exInW} xw)$

Instances of (Converse) Barcan Formula:

$$\text{valid } \forall^{*}x(\varphi \Rightarrow \psi(x)) \rightarrow (\varphi \Rightarrow \forall^{*}x\psi(x)) \qquad \text{(BF)}$$

$$\text{valid } (\varphi \Rightarrow \forall^{*}x\psi(x)) \rightarrow \forall^{*}x(\varphi \Rightarrow \psi(x)) \qquad \text{(CBF)}$$

BF:

if $* =$ varying domain then HOL-P: CounterSatisfiable

if $* =$ constant domain then HOL-P: Theorem

CBF:

if $* =$ varying domain then HOL-P: CounterSatisfiable

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Automating Quantified Conditional Logics

Freie Universität Berlin

The following examples are taken from [Delgrande, Artif.Intell., 1998]

$\phi \Rightarrow_x \psi$ stands for $(\exists^{va} x \phi) \Rightarrow \forall^{va} x(\phi \rightarrow \psi)$

"Birds (b) normally fly (f), but Opus (o) is a bird that normally does not fly."

$$b(x) \Rightarrow_x f(x), \quad b(o), \quad b(o) \Rightarrow \neg f(o)$$

HOL-P: Satisfiable (constant domain HOL-P: Unsatisfiable)

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Evaluation: What Systems are there to compare with?

- **Combinations of Quantified Logics** no systems available
- **Quantified Conditional Logics** no systems available
- **Quantified Multimodal Logics** no systems available
- ... ...
- **First-order Monomodal Logics** yes, some systems exist

  There is now even a benchmark library:

  QMLTP-lib (580 Problems): `http://www.iltp.de/qmltp/`

  Earlier experiments (see [BenzmüllerOttenRaths, ECAI, 2012]) already showed that the HOL approach performs quite well.

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Evaluation: FML's (D — constant/varying/cumulative)

No. of solved monomodal problems (out of 580, 600sec timeout)

	MleanSeP labelled sequents	**MleanTAP** labelled tableaux	**f2p-MSPASS** instant. & transform.	**MleanCoP** labelled connections	**HOL-P**
			Logic D, constant domains		
Theorem	135	134	76	**217**	208
Non-Theorem	1	4	107	209	**250**
Solved	136	138	183	426	**458**
			Logic D, cumulative domains		
Theorem	130	120	79	**200**	184
Non-Theorem	4	4	108	224	**269**
Solved	134	124	187	424	**453**
			Logic D, varying domains		
Theorem	-	100	-	**170**	163
Non-Theorem	-	4	-	243	**295**
Solved	-	104	-	413	**458**

Experiments for K, T, S4, S5, ... (const/vary/cumul) still running.

HOL based universal reasoning

- many quantified non-classical logics are fragments of HOL
- logic combinations: bridge rules as axioms
- cut-elimination and automation for free
- applications: expressive ontologies (SUMO, Cyc, Dolce, . . .)

Other (implemented) approaches to compare with?

- Institutions are great — but not helpful for automation

Future work

- more embeddings (eg. multi-valued, paraconsistent)
- other combinations (eg. fibrings)
- range of embeddable logics
- scalability to real world applications

Conclusion

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