

Service Oriented Architectures for Mathematics Assistance Systems

– Motivation, Applications, Technologies –

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Frankfurt a.M., November, 2006



Who am I?
Own Research in AI, HCI, and Distributed Systems



What is a Mathematics Assistance System?



Why Service Oriented Architectures (SOA)
for Mathematics Assistance Systems?



Own SOA Application/Technology:
From MathWeb to MathServe



How to become a World Champion with MathServe?



Who am I?

Who am I? _____

- Grown up in Schoden/Saarburg, near Trier and Luxemburg



- Married/living now in Frankfurt-Nordend (just 500m from here)

Who am I? _____

- Currently working at Cambridge University



- since 2001 head of the OMEGA Group in Saarbrücken:

Dr. Serge Autexier (co-head)

Dr. Chad E. Brown

Dipl.-Inf. Mark Buckley

Dipl.-Inf. Dominik Dietrich

Dr. Armin Fiedler

Dipl.-Inf. Andreas Franke

Dr. Helmut Horacek

Dr. Henri Lesourd

Dipl.-Inf. Marvin Schiller

Dipl.-Math. Ewaryst Schulz

Dipl.-Inf. Frank Theiss

Dipl.-Ling. Dimitra Tsovaltzi

Dipl.-Inf. Marc Wagner

Dipl.-Inf. Jürgen Zimmer

Who am I? _____

- Other stations in career



Carnegie Mellon University,
Pittsburgh, USA



University of Birmingham,
England



University of Edinburgh,
Scotland

Who am I? _____



- An enthusiastic middle- and long-distance-runner



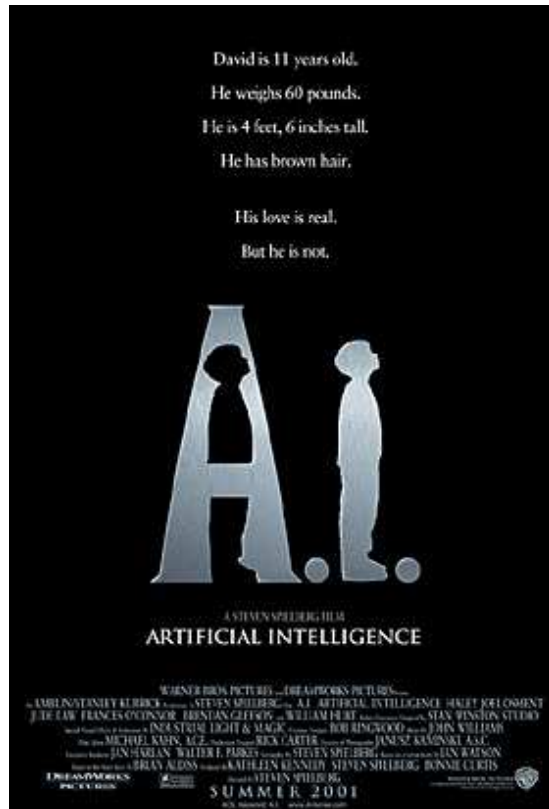
- ▶ German champion 1990 (men's cross-country team)
- ▶ 3rd German championships 1989 over 5000m (Junioren)
- ▶ > 25x Champion of the Rhineland/Rhineland-Palatine
- ▶ Try to beat my personal records:

1000m	2:25min	5000m	14:13min
1500m	3:49min	10000m	30:04min

- So why am I not at the Olympic Games?

Who am I? _____

- By accident I came into contact with AI and Computer Science



Jörg Siekmann cited Marvin Minsky:
“The brain happens to be a meat machine”



Research Interest in AI



Can machines think?

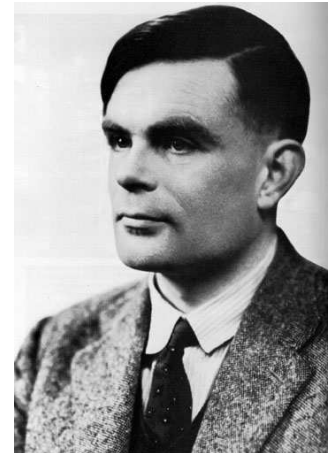
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Can machines think?

*At the end of the century, the use of words and general educated opinion will have changed so much that **one will be able to speak of “machines thinking” without expecting to be contradicted.***

Alan Turing, 1950



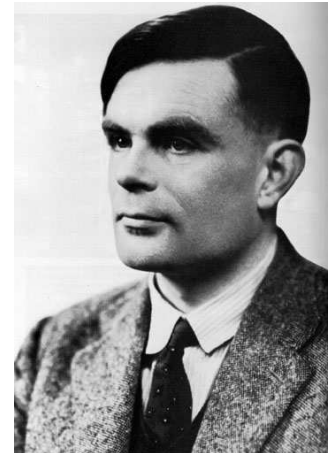
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Can machines play chess? (1997)

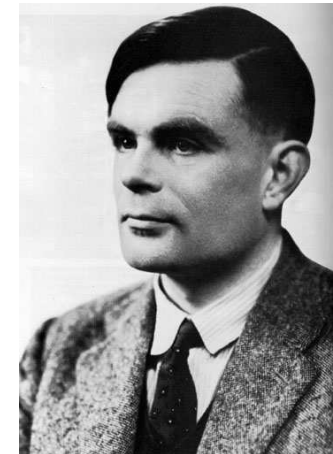
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Alan Turing, 1950



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**And how about mathematics?
(Can we built intelligent
Mathematics Assistant Systems?)**

Distributed System / Distributed Computing: **Own Research**



Development and application of

- Agent oriented Architectures
- Service oriented Architectures

for designing and realizing intelligent, distributed, robust, scalable, and open

Mathematical Assistance Systems
in the emerging

Mathematical Semantic Web

Human Computer Interaction: Own Research



- Multi-modal user interaction for mathematics assistance systems:
 - ▶ formal logic
 - ▶ maths formulas
 - ▶ natural language
 - ▶ graphics
- E-Tutoring of mathematics
- (psychological) experiments



What is a Mathematics Assistance System?

Overall Scientific Goal _____

Mathematics Assistance System(s)



Overall Scientific Goal _____

Mathematics Assistance System(s)

- Computing



Overall Scientific Goal _____

Mathematics Assistance System(s)

- Computing
- Proving



Overall Scientific Goal _____

Mathematics Assistance System(s)



- Computing
- Proving
- Exploring/Inventing

Overall Scientific Goal _____

Mathematics Assistance System(s)



- Computing
- Proving
- Exploring/Inventing
- Illustrating/Publishing

Mathematics Assistance System(s)



- Computing
- Proving
- Exploring/Inventing
- Illustrating/Publishing
- Structuring/Organizing

Mathematics Assistance System(s)



- Computing
- Proving
- Exploring/Inventing
- Illustrating/Publishing
- Structuring/Organizing
- Explaining/Teaching

Mathematics Assistance System(s)



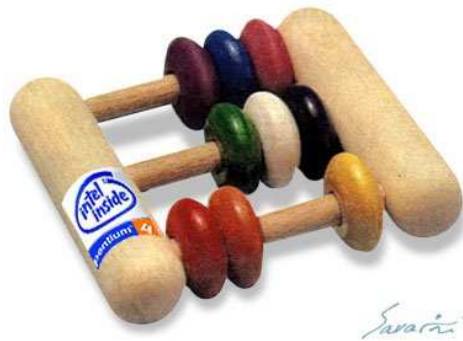
- Computing
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- . . .

Mathematics Assistance System(s)



- Computing
- Proving
- Exploring/Inventing
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- . . .

Computing Environments



Abacus
(~500 a.d.)

Computing Environments

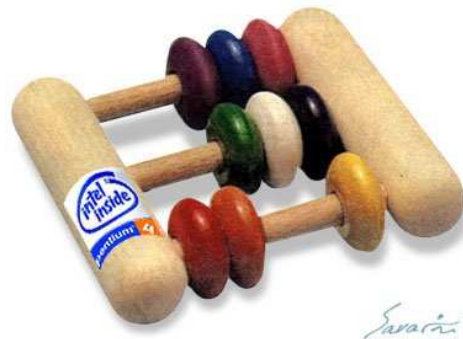


Abacus
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Wilhelm Schickard's Mechanical
Calculator (1592 - 1635)

Computing Environments

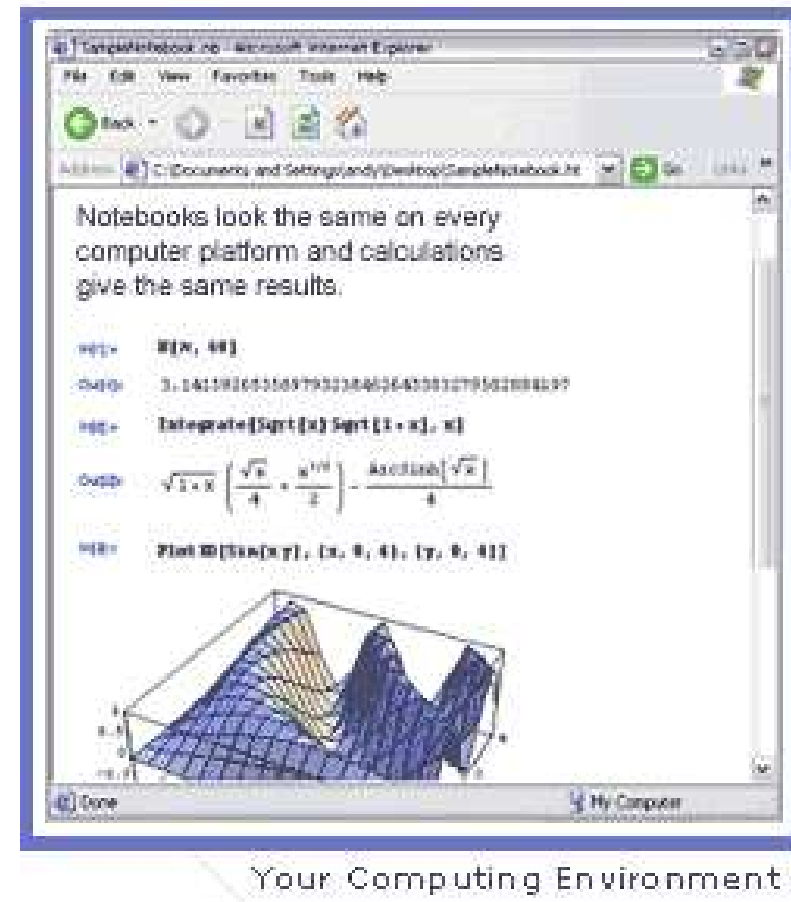


Abacus
(~500 a.d.)



Wilhelm Schickard's Mechanical
Calculator (1592 - 1635)

MATHEMATICA RUNS COMPATIBLY
ACROSS ALL MAJOR COMPUTER SYSTEMS
AND LETS YOU EXCHANGE DATA
IN MANY STANDARD FORMATS.



Modern Computer Algebra System

Frege, Russel, Hilbert Predicate Calculus and Type Theory

$$\forall x, y, z. (x + (y + z)) = ((x + y) + z)$$

$$\neg \exists f_{n \rightarrow (n \rightarrow o)} \forall m_{n \rightarrow o} \exists x_n. f(x) = m$$

Gentzen Natural Deduction Calculus (Gentzen 1935)

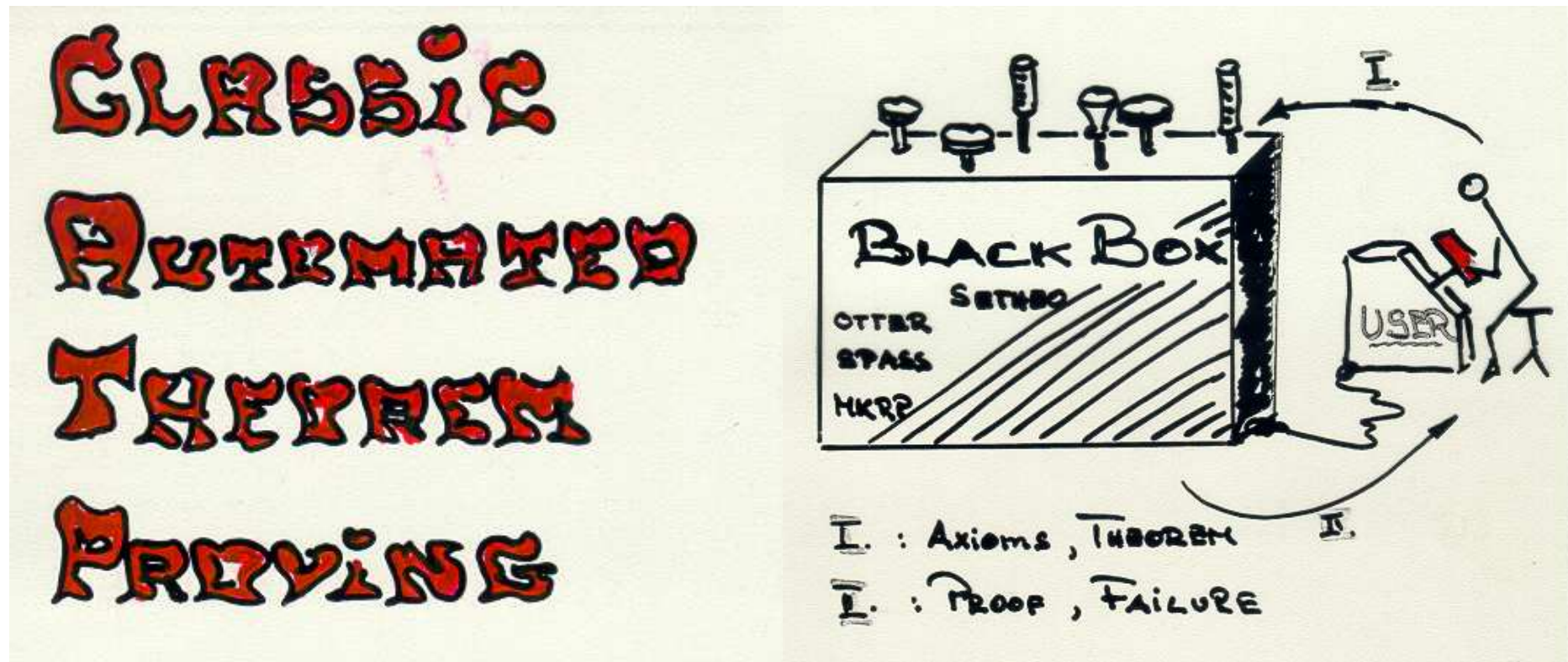
ND Rules (ex.) $\frac{A \Rightarrow B \quad A}{B} \text{ mp}$

$$\begin{array}{l} \frac{A \quad B}{A \wedge B} \wedge I \\ \frac{A \wedge B}{A} \wedge E_l \\ \frac{A \wedge B}{B} \wedge E_r \\ \frac{[A]_1}{\vdots} \\ \frac{\vdots}{B} \\ \frac{B}{A \Rightarrow B} \Rightarrow I^1 \\ \dots \text{ usw. } \dots \end{array}$$

ND Proof for $(A \wedge B) \Rightarrow (B \wedge (C \vee A))$

$$\frac{\frac{\frac{[A \wedge B]_1}{B} \wedge E_r \quad \frac{\frac{[A \wedge B]_1}{A} \wedge E_l}{C \vee A} \vee I_r}{B \wedge (C \vee A)} \wedge I}{(A \wedge B) \Rightarrow (B \wedge (C \vee A))} \Rightarrow I^1$$

Robinson (1965) Resolution Calculus for Black-Box Automation



Graphics: Jörg Siekmann

Theorem Provers: Interaction



Lovely Omega User Interface@brandt (Proof Plan: SQRT2-NOT-RAT-1)

File Presentation Edit View Go Theories Planner Agents Misc Presentation Examples Omega Extern Analogy Omega Basic Tactics Verify 4bas Options Help

Map

Label	Hypothesis	Term	Method	Premises
L1	L1	rat (sqrt 2)	HYP	
L2	L1 RAT-CRITE	\perp	Existse-Sort	L3 L10
RAT-CRITERIO	RAT-CRITERIO	forall-sort ($\lambda x. (\text{exists-sort } (\lambda dc. (\text{common-divisor } dc-248 \text{ dc-251 } dc-255)) \text{ int}))$	THM	
L4	L4	$\langle \text{int } n \rangle \wedge (\text{exists-sort } (\lambda dc. (\text{common-divisor } dc-248 \text{ dc-251 } dc-255)) \text{ int}))$	HYP	
L6	L4	int n	ANDEL	L4
L5	L5	$\langle \text{int } m \rangle \wedge (((\text{sqrt } 2) * n) = m)$	HYP	
L17	L17	$\langle \text{int } k \rangle \wedge (m = (2 * k))$	HYP	
L18	L17	int k	ANDEL	L17
L19	L17	$m = (2 * k)$	ANDER	L17
L20	L17 L4 L5 L1	\perp	ISLAND-TACTI	L12 L24
L8	L5	int m	ANDEL	L5
L9	L5	$((\text{sqrt } 2) * n) = m \wedge \neg (\text{exists-sort } (\lambda dc. (\text{common-divisor } dc-248 \text{ dc-251 } dc-255)) \text{ int}))$	ANDER	L5
L12	L5	$\neg (\text{exists-sort } (\lambda dc. (\text{common-divisor } dc-248 \text{ dc-251 } dc-255)) \text{ int}))$	ANDER	L9
L11	L5	$((\text{sqrt } 2) * n) = m$	ANDE	L9
L13	L4 L5	$(2 * (\text{power } n \text{ } 2)) = (\text{power } m \text{ } 2)$	ISLAND-TACTI	L11 L6 L8
L21	L4 L5 L17	$(\text{power } n \text{ } 2) = (2 * (\text{power } k \text{ } 2))$	ISLAND-TACTI	L19 L13 L6 L8 L18
L22	L5 L4 L17	evenp (power n 2)	ISLAND-TACTI	L21 L6 L18
L23	L17 L5 L4	evenp n	ISLAND-TACTI	L22 L6
L14	L4 L5	evenp (power m 2)	ISLAND-TACTI	L13 L6 L8
L15	L4 L5	evenp m	ISLAND-TACTI	L14 L8
L24	L17 L4 L5	common-divisor n m 2	ISLAND-TACTI	L15 L23 L6 L8
L16	L4 L5	exists-sort ($\lambda dc. (\text{common-divisor } dc-248 \text{ dc-251 } dc-255)) \text{ int}$	DefnE	L15
L10	L4 L5 L1 RAT	\perp	Existse-Sort	L16 L20

Pretty Term

```

( $\lambda dc-251. ( ((\text{sqrt } 2) * dc-248) = dc-251) \wedge \neg (\text{exists-sort } (\lambda dc-255. (\text{common-divisor } dc-248 \text{ dc-251 } dc-255)) \text{ int}))$ )
int
-----
forall-sort
  ( $\lambda x. (\text{exists-sort } (\lambda y. (\text{exists-sort } (\lambda z. (((x * y) = z) \wedge \neg (\text{exists-sort } (\lambda d. (\text{common-divisor } y \text{ z } d)) \text{ int})) \text{ int})) \text{ int}))$ )
  rat
  
```

Output Message Error Warning Trace

0 0 13 0 7 4 1 0 0 Total: 25 Depth: 0 Command: Show-Original-Proof Time: 230ms

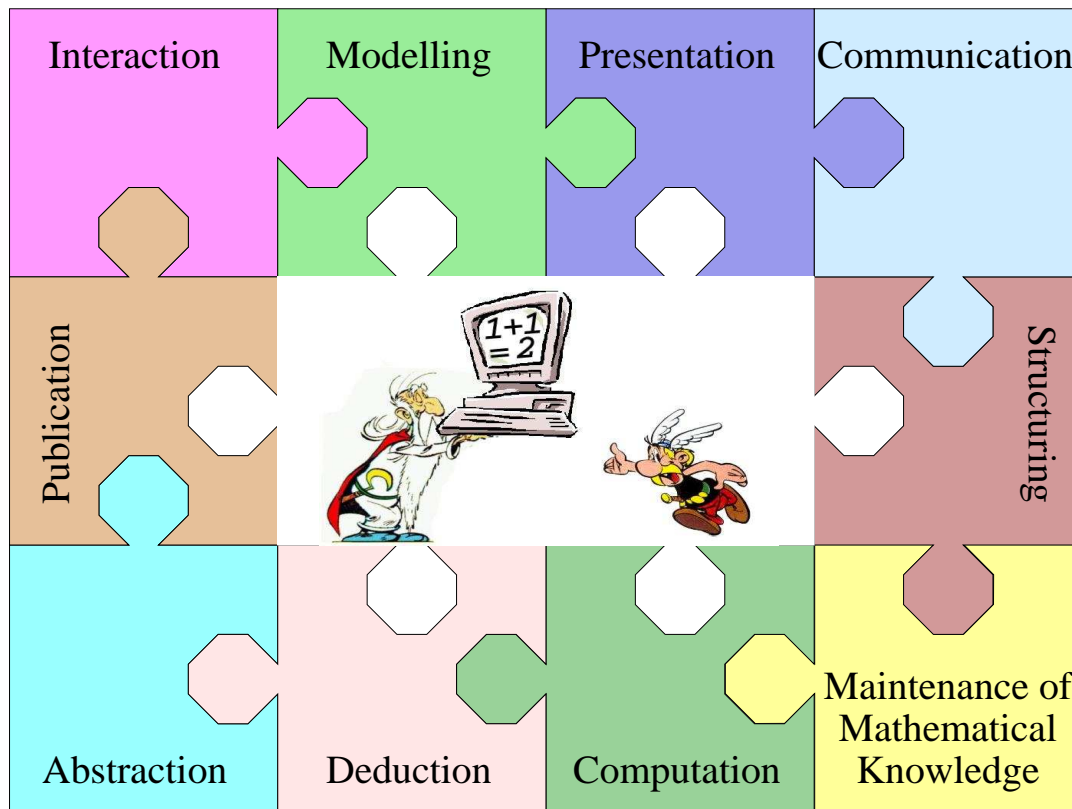
OMEGA Project: Overall Goal



Integrated Mathematical
Assistant Environment

vs.

'Pen-and-Paper'
Mathematics



Which System Architecture?

Challenging Observation I



- Mathematics assistance systems share many characteristics with large AI systems in general
- Subsystems to be integrated are
 - ▶ intelligent systems themselves
 - ▶ developed over years
 - ▶ commercial and non-commercial
 - ▶ use different programming languages, architectures, platforms, etc.
 - ▶ steadily changing and improving

Challenging Observation II



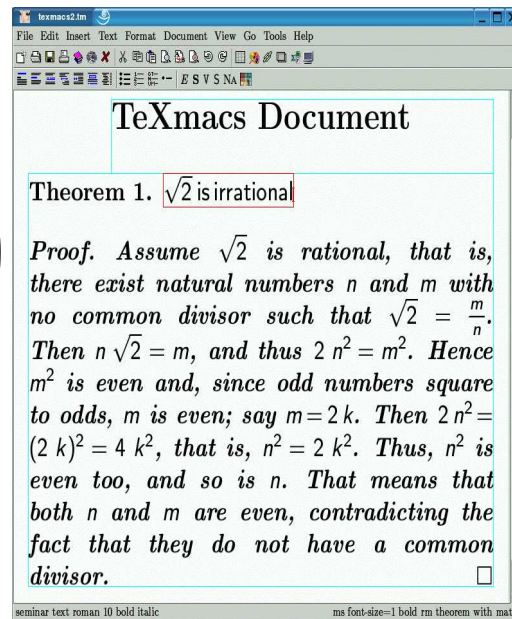
Different application scenarios call for different specialisations and configurations

- Scenario A: Verified mathematical documents
- Scenario B: E-tutoring of proof in mathematics and engineering
- Scenario C: Formal hardware and software verification
- . . .

Scenario A: Verified Maths Documents



SFB 378 Project OMEGA:



Mathematics

Assistance

Environment

Scenario B: E-Tutoring

SFB 378 Project DIALOG:

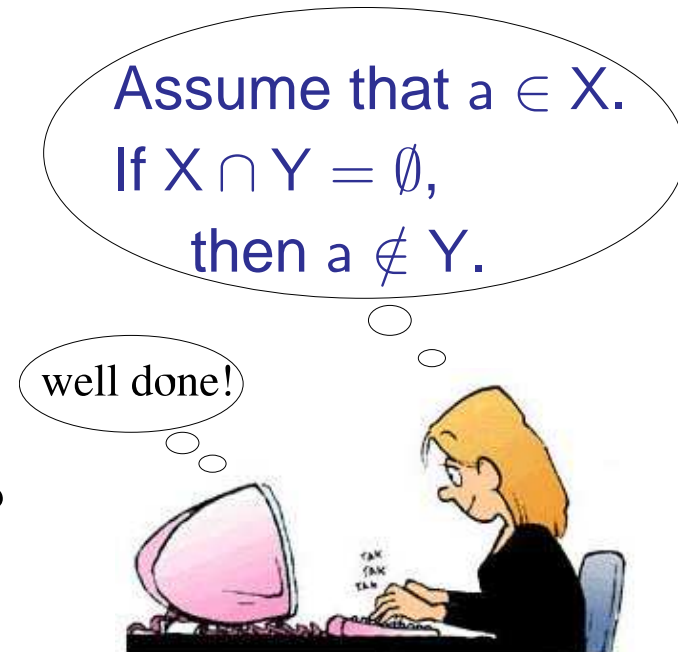
Can we automate NL-based tutoring of mathematical proofs?

- Natural language analysis
- ...
- Theorem Proving with a Mathematical Assistance System
- ...
- Natural language generation

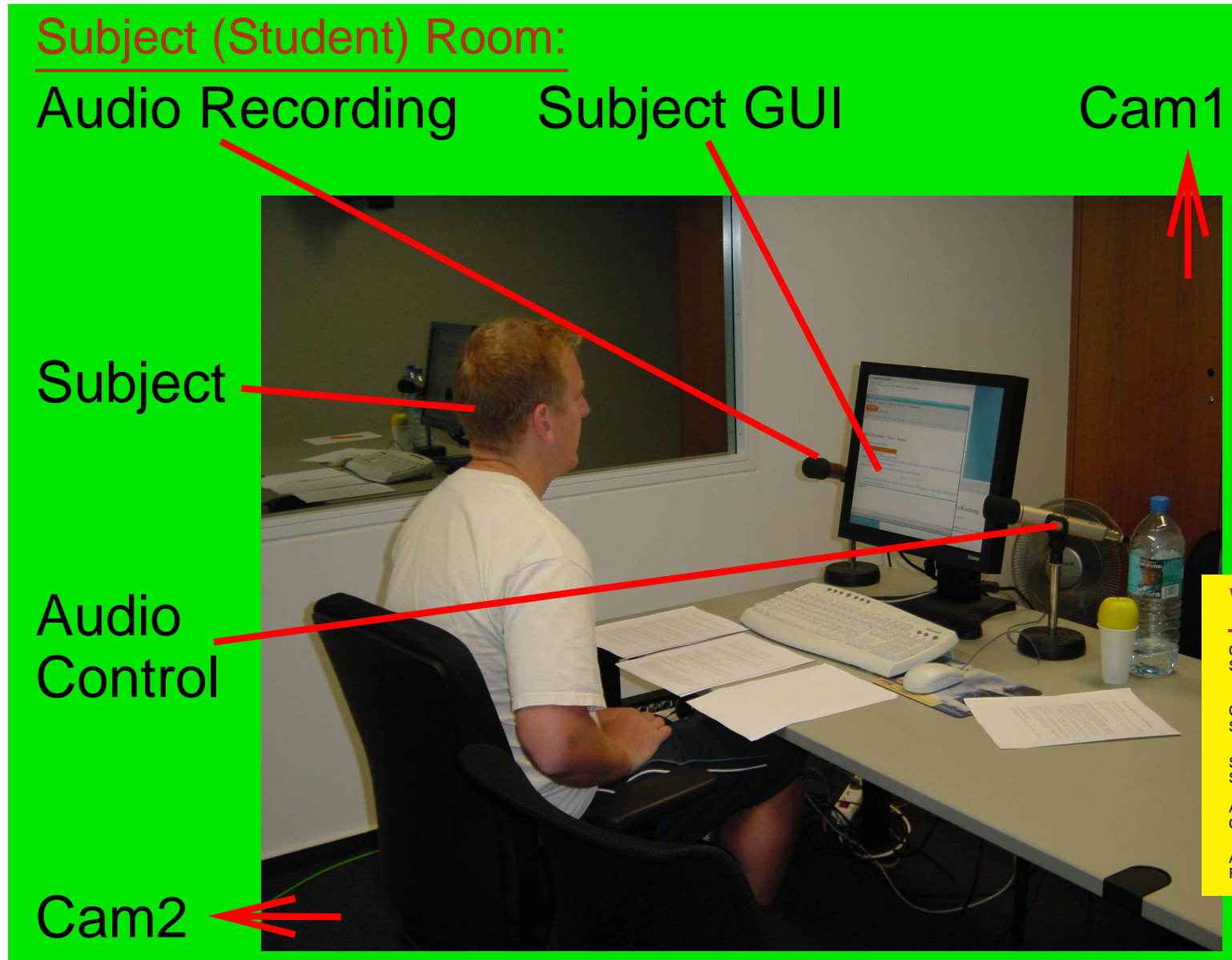
Big challenge:

Where to get real dialog examples from?

→ HCI experiments



Scenario B: Challenge for HCI



Wizard Room:



Scenario B: Challenge for HCI

Wizard (Tutor) Room:

Cam2
Screen

Overall
Control

Experi-
menter

Wizard
GUI

Wizard
(Tutor)

Cam1
Screen

Subject
Screen

Audio
Control

Audio
Recording



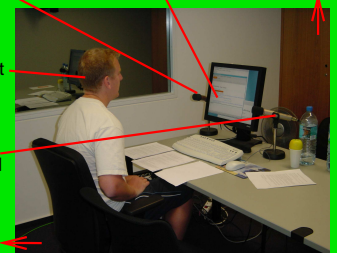
Subject Room:

Audio Recording Subject GUI Cam1

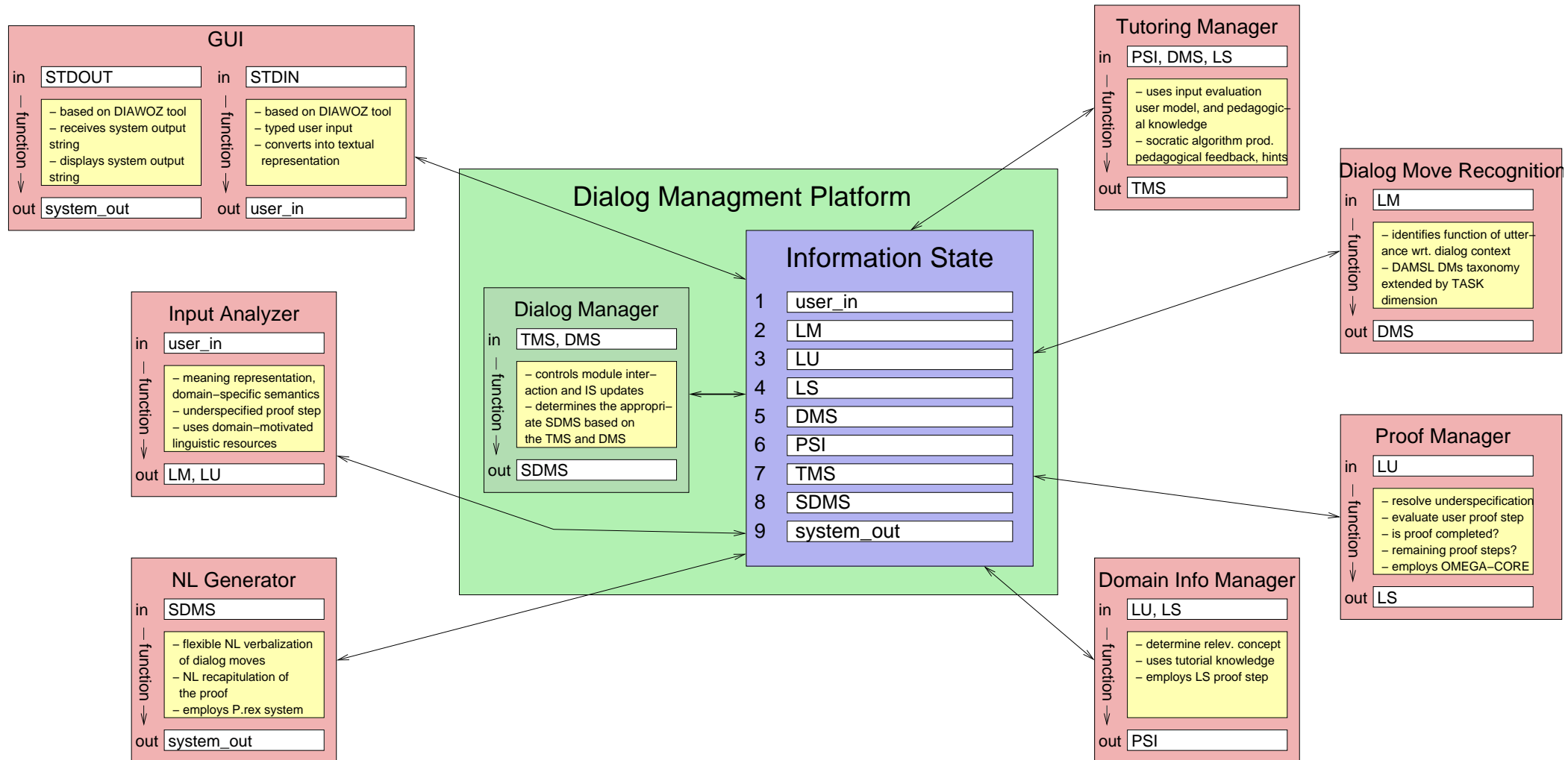
Subject

Audio
Control

Cam2



Scenario B: Need Further Components!





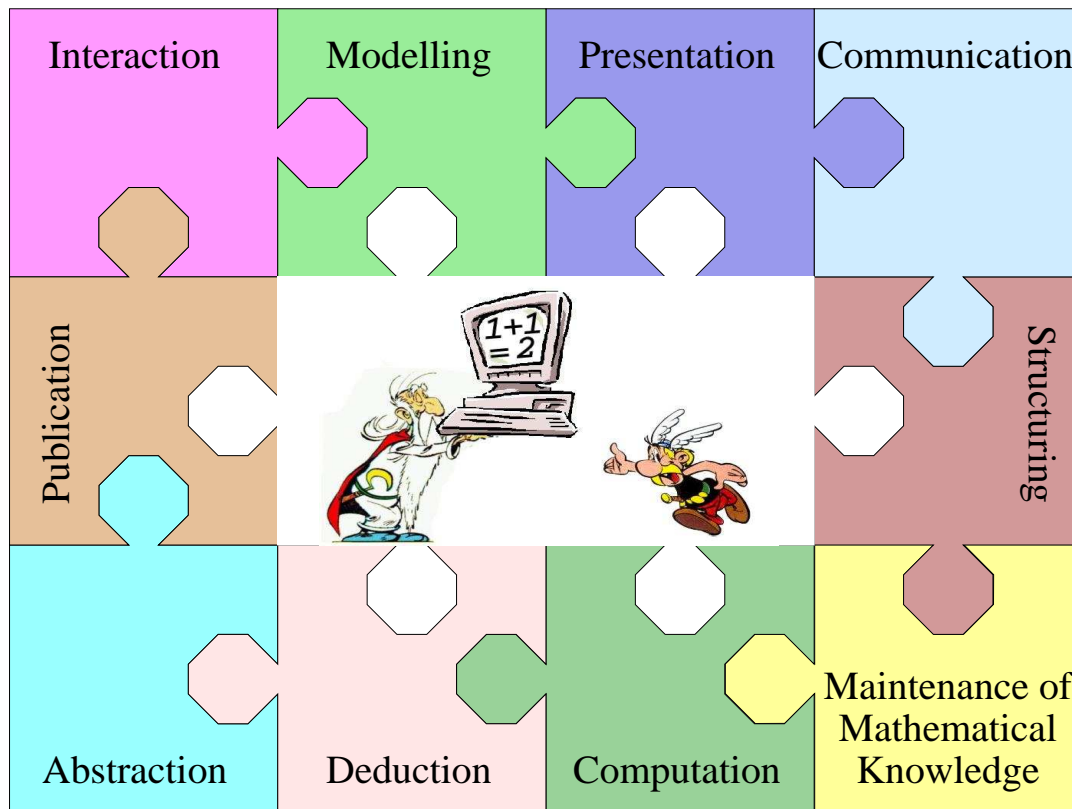
Why Service Oriented Architectures for Mathematics Assistance Systems?

Which System Architecture?

Integrated Mathematical
Assistant Environment

vs.

'Pen-and-Paper'
Mathematics

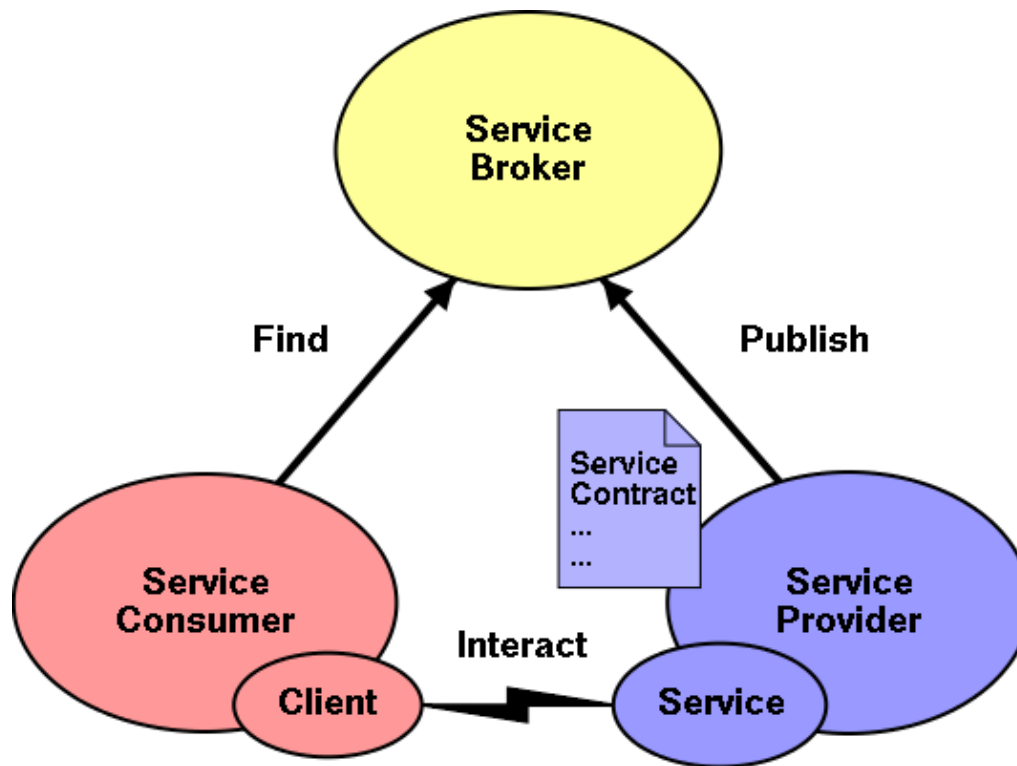


Which System Architecture?

Main Ideas

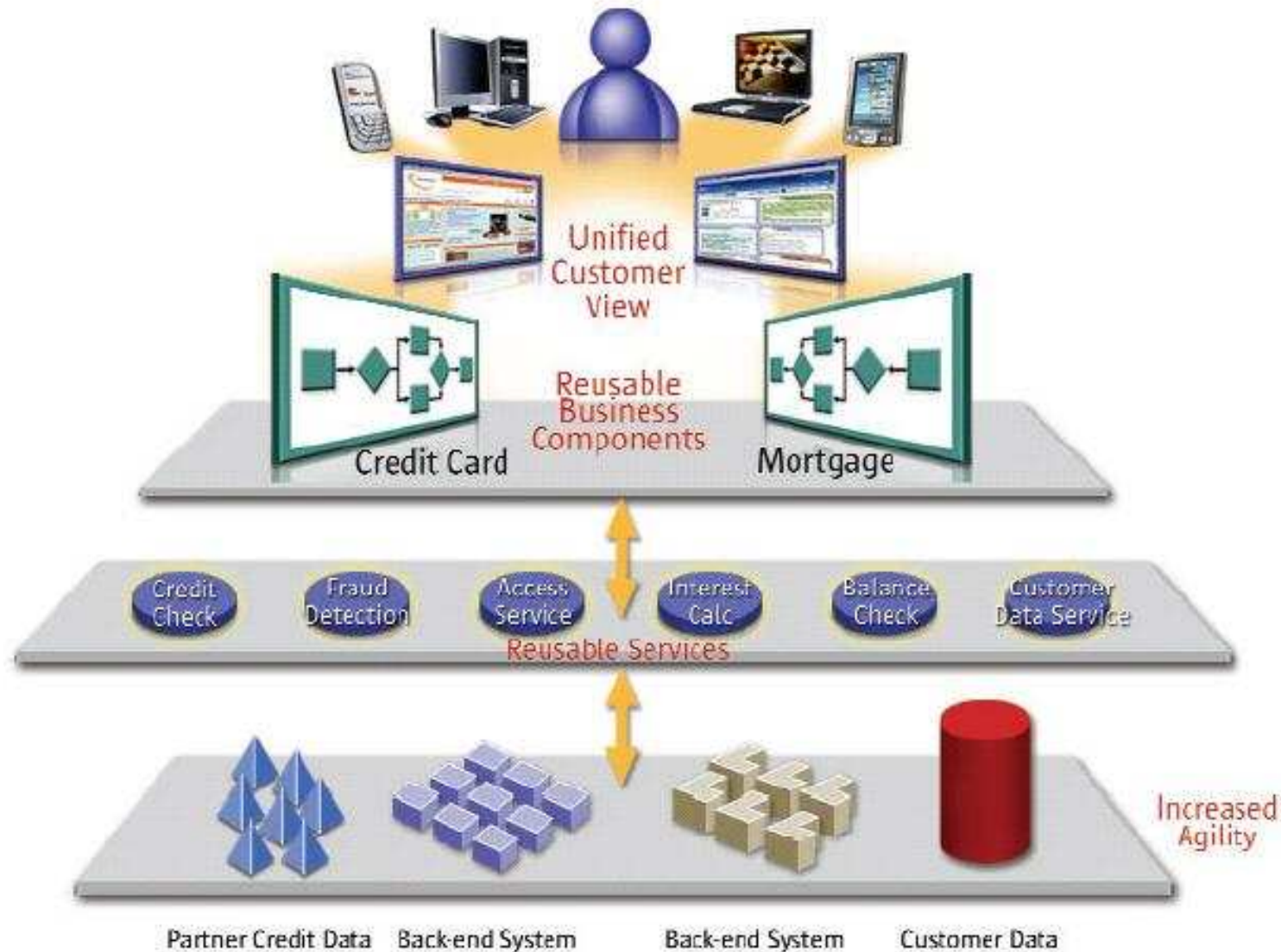
- all software components are modeled as services
- important
 - ▶ services have well-defined interfaces
 - ▶ services are discoverable
 - ▶ services may interoperate via loose coupling
- not important
 - ▶ the 'inner life' of services
- focus of application design: composing services (using their well-defined interfaces) invoked over a network

Service Oriented Architectures (SOA)



1. Service Provider makes service available (with Service Contract) and **advertises** it on Service Broker.
2. Service Consumer **finds** the compatible Service and its Service Contract via the Service Broker.
3. Service Consumer and Service Provider **interact**.

Service Oriented Architectures (SOA)



Benefits of SOA



Using SOA we want to achieve

- standalone and integrated components at the same time
- reusability
- logical and physical distribution
- interchangeability / replaceability
- high configurability and scalability
- availability and accessibility
- loose coupling with minimal interdependencies
- discoverability
- ...

and make first steps towards a mathematical semantic web.



Own SOA Application/Technology:
From MathWeb to MathServe

Difficulties that motivated MathWeb



■ Installation:

- ▶ the (sub-)systems must be available for your OS
- ▶ have to install the individual systems
- ▶ may need to get a license

■ Configuration:

- ▶ must configure the MAS to your local installation

■ Invocation:

- ▶ need to know how the parameters to call each system
- ▶ what are the “services” the systems can provide?
- ▶ how to interpret the systems output (exit status / answer)

Difficulties that motivated MathWeb

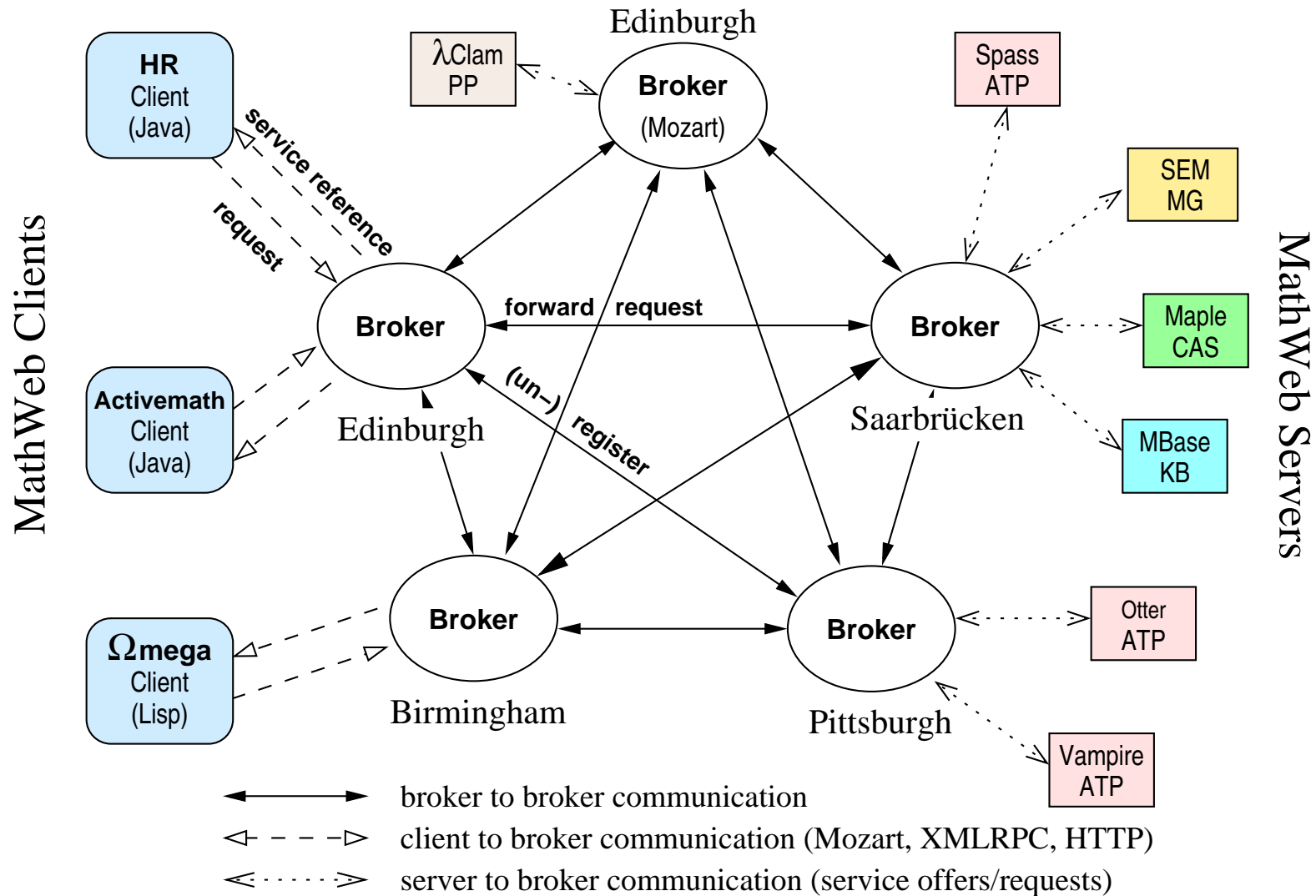


- Language:

- ▶ need to know the interface syntax for each system
- ▶ need to write syntax transformations
- ▶ are the transformations correct?

⇒ SOA provides a means to overcome these difficulties

(Developed our own SOA solution in 1996 without knowing about 'SOA')



Sample XML-RPC to MathWeb



```
<methodCall><methodName> Broker.getService</methodName>
  <params><param><value><string> SPASS</string></value></param></params>
</methodCall>
```

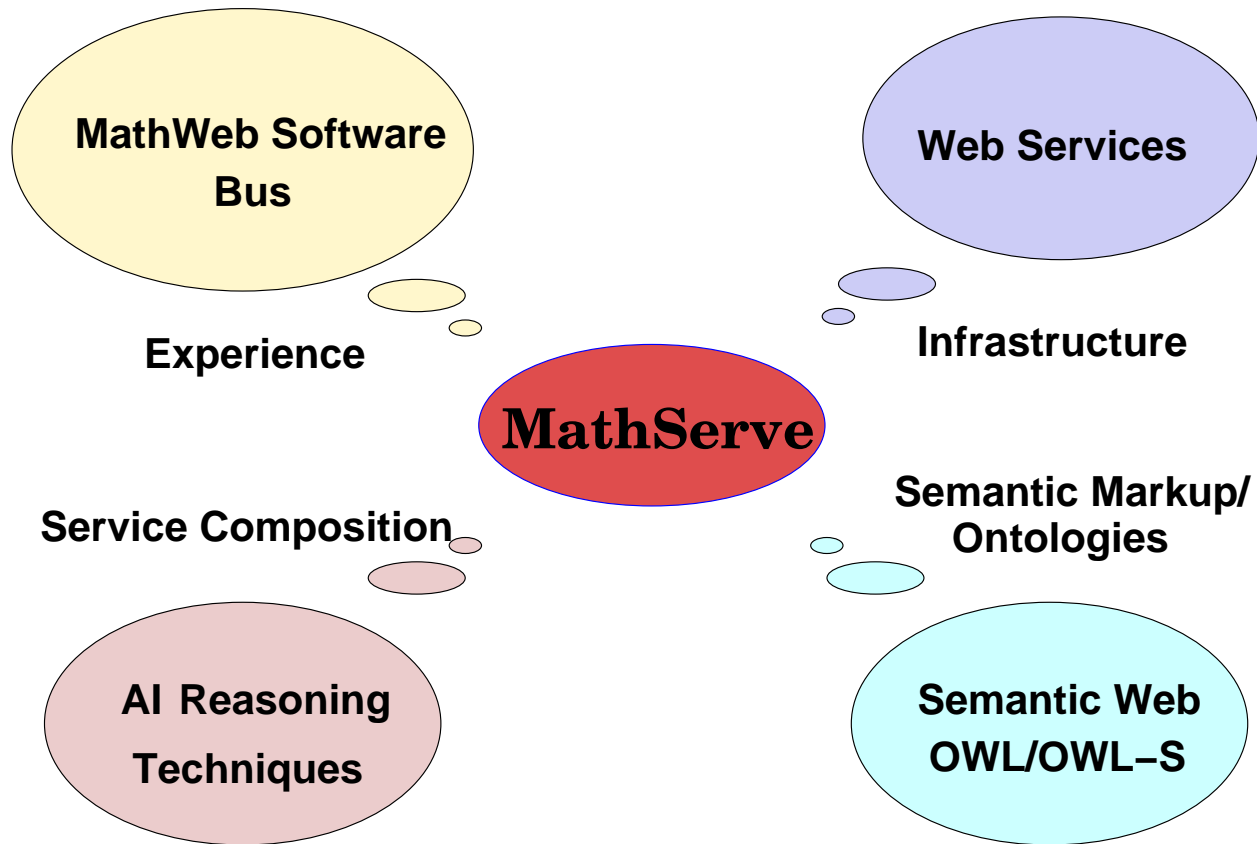
```
<methodCall><methodName> prove</methodName>
  <params><param><struct>
    <member><name>1</name><value><string>
      include('Axioms/EQU001+0.ax').
      include('Axioms/GRP004+0.ax').
      input_formula(conjecture118,conjecture,(! [B,C,D] :
        ((equal(inverse(B),C) & equal(multiply(C,B),D) ) <=>
          (equal(multiply(B,C),D) & equal(multiply(C,B),D) & equal(inverse(C),B))))
    </string></value></member>
    <member><name> syntax</name><value><string> tptp</string>
    <member><name> timeout</name><value><int> 40</int></value></member>
  </struct></param></params>
</methodCall>
```

From MathWeb to MathServe



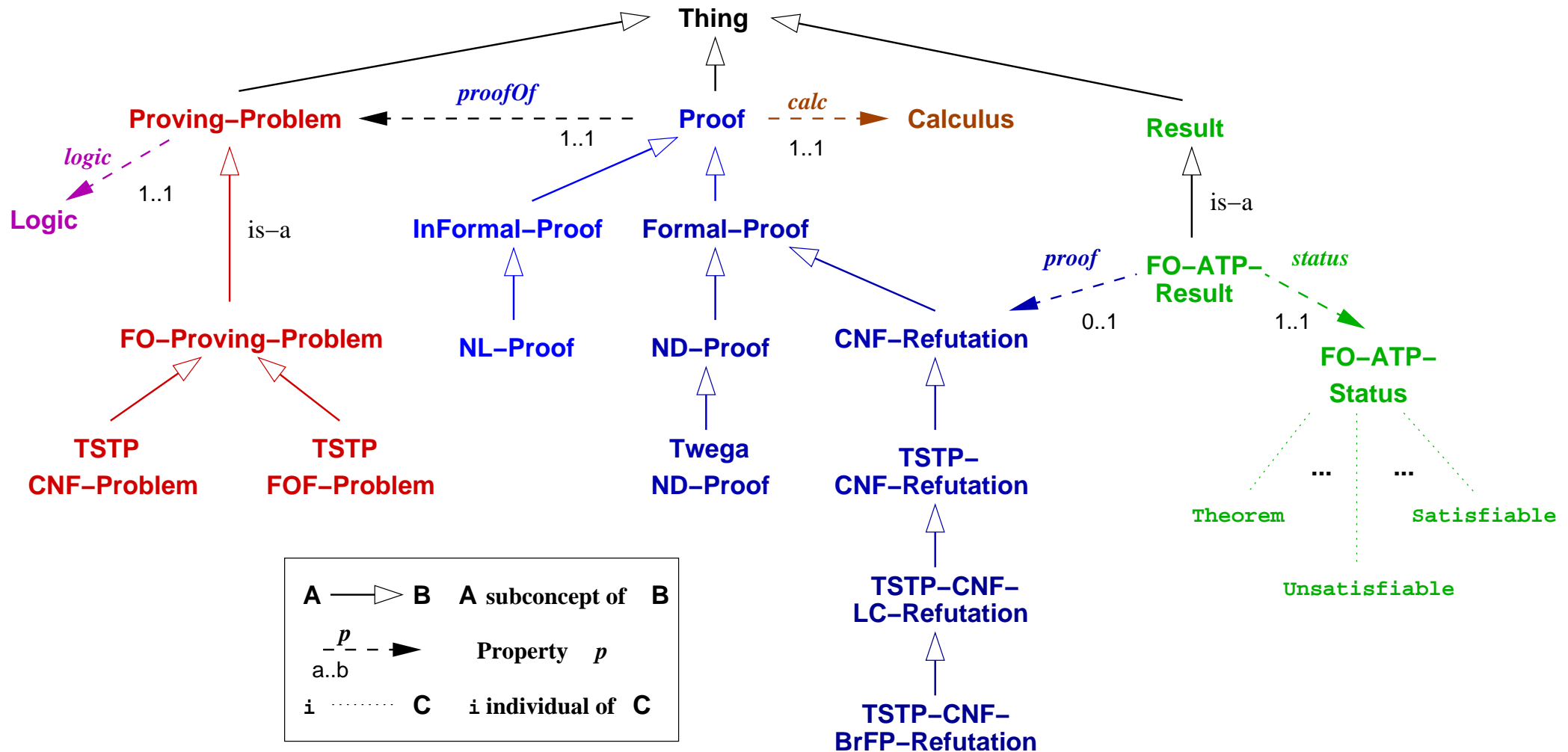
Away from the
system-oriented MathWeb
towards a
problem-oriented MathServe
(supporting also service compositions)

The MathServe System



Jürgen Zimmer (PhD)

OWL Ontology for Service Descriptions



OWL-S Atomic Process for ATP Service



The **central part** of a service description (example)

Service: EpATP	
input parameters:	<i>?problem</i> ::TSTP-CNF-Problem
output parameters:	<i>?result</i> ::FO-ATP-Result
pre-conditions:	\top
post-conditions:	<i>resultFor(?result, ?problem)</i>

- we here completely omit XML details.
- conditions in Semantic Web Rule Language (SWRL)

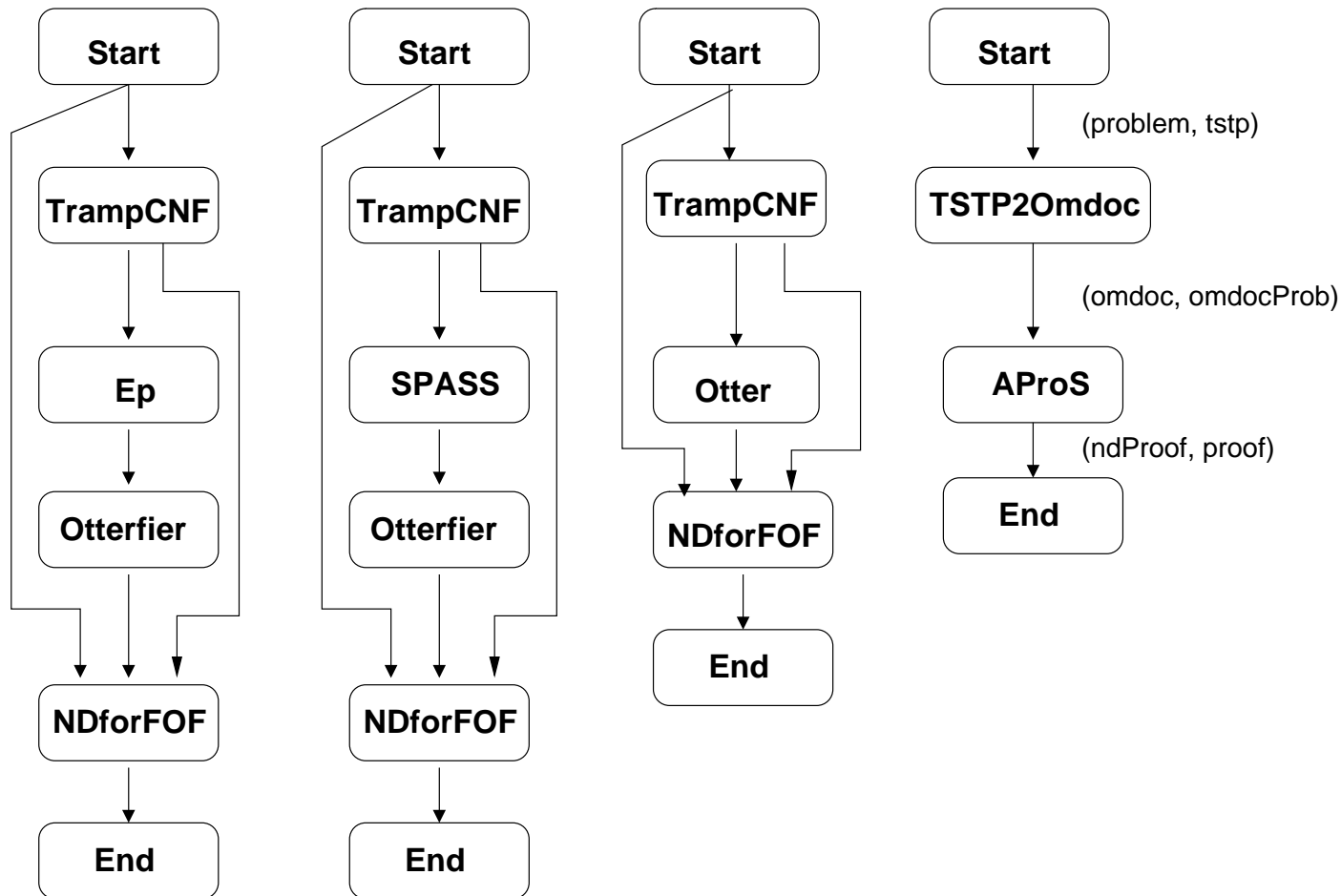
The Service of Tramp



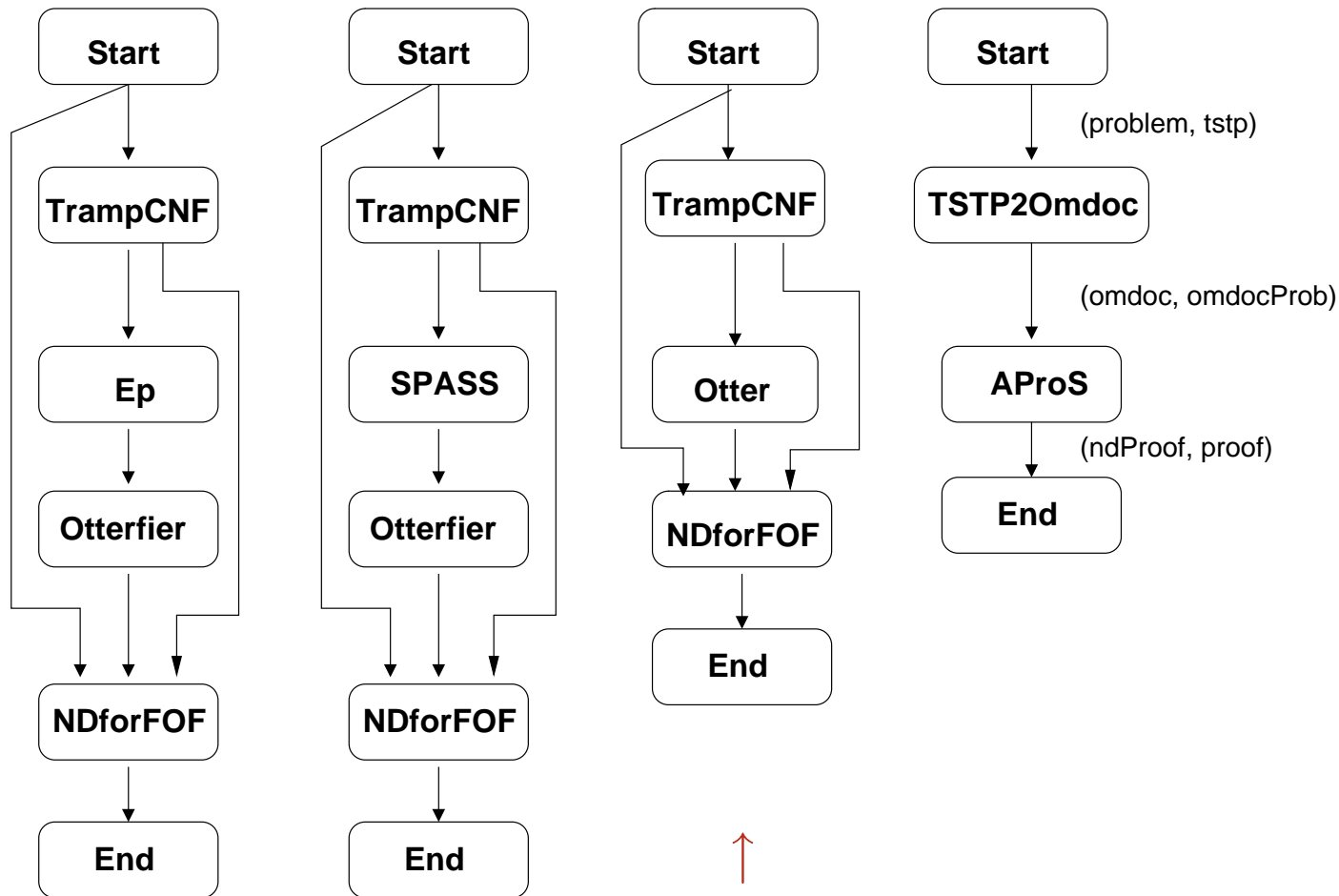
Service: Tramp-NDforFOF	
input parameters:	<i>?fofProblem</i> ::TSTP-FOF-Problem <i>?atpResult</i> ::FO-ATP-Result
output parameters:	<i>?ndProof</i> ::Twega-ND-Proof
pre-conditions:	$\text{resultFor}(\text{?atpResult}, \text{?cnfProblem}) \wedge$ $\text{deltaRelated}(\text{?cnfProblem}, \text{?fofProblem})$
post-conditions:	$\text{proofOf}(\text{?ndProof}, \text{?fofProblem})$

“*deltaRelated*”: \exists mapping $\delta: \text{Literals} \mapsto \text{FOF-Atoms}$

Planning finds many possible plans



Planning finds many possible plans



↑
Best Choice!

(based on probabilistic reasoning)



How to become a World Champion with
MathServe?

Evaluation on CASC-20



System	Problems given	Problems solved	Percentage of given	Percentage complete
MathServe 0.62	660	392	59.4%	59.4%
Ep 0.9pre3	660	409	62.0%	62.0%
Vampire 8.0	540	430	79.6%	65.2%
Vampire 7.0	300	262	87.4%	39.7%
MathServe 0.71	660	440	67.7%	67.7%

- MathServe 0.62 used older versions of EP (0.81) and Vampire (7.0).
- MathServe 0.62 could not handle large problems (> 2MB).
- MathServe 0.71 after CASC-20 with EP 0.9 and Vampire 8.0.

Availability and Usability



- Binary and source distribution available at
<http://www.ags.uni-sb.de/~jzimmer/mathserve.html>
- MathServe is used by
 - ▶ Omega/Core proof assistant (University of Saarbrücken)
 - ▶ Hets toolset (University of Bremen)
 - ▶ Verifun project (Technical University Darmstadt)

See how MathServe 0.8 performs at the next
CASC Championships

SOA and MathServ: Lecture Topics



- More background knowledge

SOA and MathServ: Lecture Topics



- More background knowledge
(for better understanding of the MathServ application scenario)
 - ▶ maths
 - ▶ logic
 - ▶ . . .

SOA and MathServ: Lecture Topics



- More background knowledge
- State-of-the-art web service languages and technologies

SOA and MathServ: Lecture Topics



- More background knowledge
- State-of-the-art web service languages and technologies (as used in MathServ)
 - ▶ **XML** (communication infrastructure)
 - ▶ **RDF** (description of meta-data about web resources)
 - ▶ **Description logics** and **OWL** (express ontological structures)
 - ▶ **WSDL** (web service description language, supports no composition)
 - ▶ **SOAP** (message exchange protocol based on HTTP)
 - ▶ **UDDI** (standard for web service registries)
 - ▶ **BPEL4WS** (business processes execution language)
 - ▶ **OWL-S** (web service markup and description language, supports composition and grounding in WSDL)

SOA and MathServ: Lecture Topics



- More background knowledge
- State-of-the-art web service languages and technologies
- Communication of Maths Content in the Semantic Web

SOA and MathServ: Lecture Topics



- More background knowledge
- State-of-the-art web service languages and technologies
- Communication of Maths Content in the Semantic Web
 - ▶ **MathML** (standard for describing mathematical notation – structure + content)
 - ▶ **OpenMath** (standard for the representation and communication of mathematical objects)
 - ▶ **OMDoc** (semantic markup language for document structure and mathematical content)

SOA and MathServ: Lecture Topics



- More background knowledge
- State-of-the-art web service languages and technologies
- Communication of Maths Content in the Semantic Web
- Composition of Web Services

SOA and MathServ: Lecture Topics



- More background knowledge
- State-of-the-art web service languages and technologies
- Communication of Maths Content in the Semantic Web
- Composition of Web Services
 - ▶ AI planning techniques for service composition
 - ▶ Golog and the situation calculus
 - ▶ Markov decision procedures
 - ▶ Program synthesis

SOA and MathServ: Lecture Topics



- More background knowledge
- State-of-the-art web service languages and technologies
- Communication of Maths Content in the Semantic Web
- Composition of Web Services
- Idea: Become a world champion with a group of students

SOA and MathServ: Lecture Topics



- More background knowledge
- State-of-the-art web service languages and technologies
- Communication of Maths Content in the Semantic Web
- Composition of Web Services
- Idea: Become a world champion with a group of students
 - ▶ Improve implementation of MathWeb with team of students
 - ▶ Participate at the yearly CASC competitions



Agent Oriented Architectures



This is a story for another lecture . . .