— “Gottesbeweis” reloaded —
Analyzing Variants of the Ontological Argument with the Computer

Christoph Benzmüller[1], Freie Universität Berlin

Lange Nacht der Wissenschaften, Berlin, 13. Juni, 2015

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>
>
> Beweise-mit-Leo2 Notwendigerweise-existiert-Gott.p

Leo-II tries to prove
=====================

  Goedel's Theorem T3: "Necessarily, God exists"
  thf(thmT3,conjecture,
      ( v
      @ ( mbox
        @ ( mexists_ind
          @ ^ [X: mu] :
              ( g @ X ) ) ) )).

  Assumptions: D1, C, T2, D3, A5

 . searching for proof ..

 ********************
 *   Proof found    *
 ********************
 % SZS status Theorem for Notwendigerweise-existiert-Gott.p

 . generating proof object
```

[1]Supported by DFG Heisenberg Fellowship BE 2501/9-1/2; jww: Bruno Woltzenlogel-Paleo

Vision of Leibniz (1646–1716): *Calculemus!*

If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other ...: Let us calculate.

(Translation by Russell)

Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo ... dicere: calculemus. (Leibniz, 1684)

Required:
characteristica universalis and **calculus ratiocinator**

Ontological argument for the existence of God

Focus on Gödel's modern version in higher-order modal logic

Automation with provers for higher-order classical logic (HOL)

- verification (or falsification) of known results
- some novel results (by HOL-ATPs)
- HOL as a universal metalogic via logic embeddings (chaRacteristica universalis?)

Different Interests in Ontological Arguments

- Philosophical: Boundaries of Metaphysics & Epistemology
- Theistic: Successful argument could convince atheists?
- Ours: Computational Metaphysics (Leibniz' Vision)

A Long History

pros and cons

… Anselm v. C. Gaunilo … Th. Aquinas …… Descartes Spinoza Leibniz … Hume Kant … Hegel … Frege … Hartshorne Malcolm Lewis Plantinga Gödel …

Anselm’s notion of God (Proslogion, 1078):

“God is that, than which nothing greater can be conceived.”

Gödel’s notion of God:

“A God-like being possesses all ‘positive’ properties.”

To show by logical, deductive reasoning:

“God exists.”

$$\exists x G(x)$$

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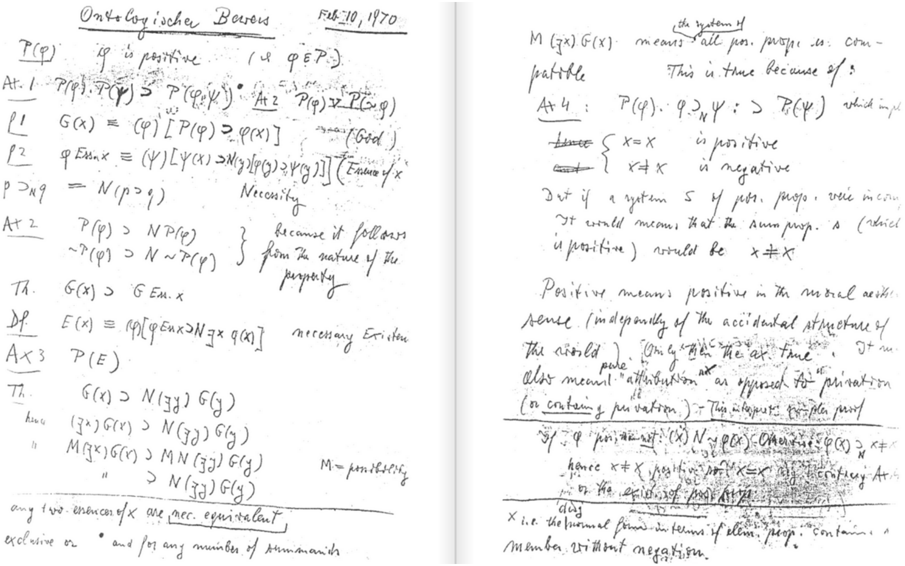
Ontologischer Beweis   Feb 10, 1970

$P(\varphi)$ φ is positive (or $\varphi \epsilon P$)

At. 1 $P(\varphi).P(\psi) \supset P(\varphi.\psi)$ At 2 $P(\varphi) \vee P(\sim\varphi)$

P1 $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ (God)

P2 $\varphi \text{ Ess. } x \equiv (\psi)[\psi(x) \supset N(y)[\varphi(y) \supset \psi(y)]]$ (Essence of x)

$p \supset_N q = N(p \supset q)$ Necessity

At 2 $P(\varphi) \supset N P(\varphi)$, $\sim P(\varphi) \supset N \sim P(\varphi)$ } because it follows from the nature of the property

Th. $G(x) \supset G \text{ Ess. } x$

Df. $E(x) \equiv (\varphi)[\varphi \text{ Ess } x \supset N \exists x\, \varphi(x)]$ necessary Existence

Ax 3 $P(E)$

Th. $G(x) \supset N(\exists y)\, G(y)$

hence $(\exists x)G(x) \supset N(\exists y)\, G(y)$

" $M(\exists x)G(x) \supset MN(\exists y)\, G(y)$

" $\supset N(\exists y)\, G(y)$ M = possibility

any two essences of x are nec. equivalent

exclusive or * and for any number of summands

$M(\exists x)G(x)$ means the system of all pos. props. is compatible. This is true because of:

At 4: $P(\varphi).\varphi \supset_N \psi : \supset P(\psi)$ which impl

{ $x=x$ is positive; $x \neq x$ is negative

But if a system S of pos. props. were incom[patible] It would mean, that the sum prop. s (which is positive) would be $x \neq x$

Positive means positive in the moral aesth[etic] sense (independently of the accidental structure of the world). Only then the ax. true. It m[ay] also mean pure "attribution" as opposed to "privation" (or containing privation). This interpret. simpler proof

If φ pos. not: $(x) N \sim\varphi(x)$. Otherwise: $\varphi(x) \supset_N x \neq x$ hence $x \neq x$ positive not $x=x$ contrary At ... or the existence of pos. prop.

X i.e. the disj normal form in terms of elem. props. contains a member without negation.

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2 A property necessarily implied by a positive property is positive:
$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$

Thm. T1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

Def. D1 A *God-like* being possesses all positive properties: $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

Axiom A3 The property of being God-like is positive: $P(G)$

Cor. C Possibly, God exists: $\Diamond\exists x G(x)$

Axiom A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$

Def. D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess\ x \leftrightarrow \phi(x) \wedge \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Thm. T2 Being God-like is an essence of any God-like being: $\forall x[G(x) \rightarrow G\ ess\ x]$

Def. D3 *Necessary existence* of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall\phi[\phi\ ess\ x \rightarrow \Box\exists y\phi(y)]$

Axiom A5 Necessary existence is a positive property: $P(NE)$

Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

Scott's Version of Gödel's Axioms, Definitions and Theorems

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Difference to Gödel (who omits this conjunct)

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Modal operators are used

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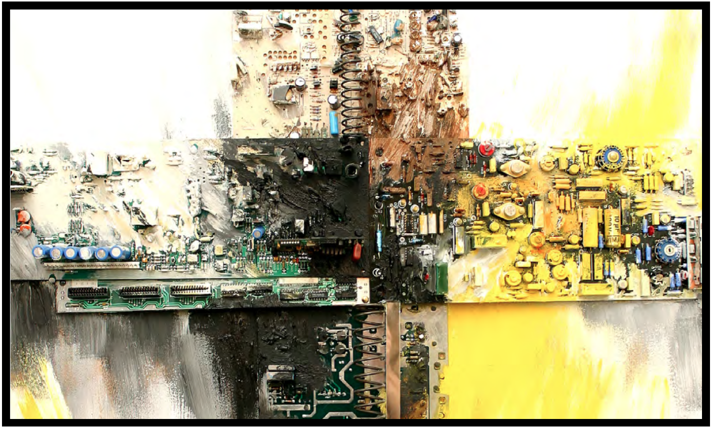
second-order quantifiers



How to automate Higher-Order Modal Logic?

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Challenge: No provers for *Higher-order Modal Logic* (HOML)

Church's Simple Type Theory

Our solution: Embedding in **Higher-order Classical Logic** (HOL)

Then use existing HOL theorem provers for reasoning in HOML

[BenzmüllerPaulson, Logica Universalis, 2013]

Assumption: Henkin semantics for both HOML and HOL

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HOML $\varphi, \psi \quad ::= \quad \ldots \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \rightarrow \psi \mid \Box\varphi \mid \Diamond\varphi \mid \forall x_\gamma\, \varphi \mid \exists x_\gamma\, \varphi$

HOL $s, t \quad ::= \quad C_\alpha \mid x_\alpha \mid (\lambda x_\alpha s_\beta)_{\alpha\rightarrow\beta} \mid (s_{\alpha\rightarrow\beta}\, t_\alpha)_\beta \mid \neg s_o \mid s_o \vee t_o \mid \forall x_\alpha\, t_o$

HOML in HOL: HOML formulas φ are mapped to HOL predicates $\varphi_{\mu\rightarrow o}$ (explicit representation of labelled formulas)

$$
\begin{array}{lcl}
\neg & = & \lambda\varphi_{\mu\rightarrow o}\lambda w_\mu \neg\varphi w \\
\wedge & = & \lambda\varphi_{\mu\rightarrow o}\lambda\psi_{\mu\rightarrow o}\lambda w_\mu(\varphi w \wedge \psi w) \\
\rightarrow & = & \lambda\varphi_{\mu\rightarrow o}\lambda\psi_{\mu\rightarrow o}\lambda w_\mu(\neg\varphi w \vee \psi w) \\
\forall & = & \lambda h_{\gamma\rightarrow(\mu\rightarrow o)}\lambda w_\mu \forall d_\gamma\, h d w \\
\exists & = & \lambda h_{\gamma\rightarrow(\mu\rightarrow o)}\lambda w_\mu \exists d_\gamma\, h d w \\
\Box & = & \lambda\varphi_{\mu\rightarrow o}\lambda w_\mu \forall u_\mu\, (\neg r w u \vee \varphi u) \\
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\text{valid} & = & \lambda\varphi_{\mu\rightarrow o}\forall w_\mu \varphi w
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Ax (polymorphic over γ)

The equations in Ax are given as axioms to the HOL provers!

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Embedding HOML in HOL

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Ax (polymorphic over γ)

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Example

HOML formula	$\Diamond \exists x G(x)$
HOML formula in HOL	valid $(\Diamond \exists x G(x))_{\mu \to o}$
expansion, $\beta\eta$-conversion	$\forall w_\mu (\Diamond \exists x G(x))_{\mu \to o}\, w$
expansion, $\beta\eta$-conversion	$\forall w_\mu \exists u_\mu (rwu \wedge (\exists x G(x))_{\mu \to o} u)$
expansion, $\beta\eta$-conversion	$\forall w_\mu \exists u_\mu (rwu \wedge \exists x Gxu)$

Expansion: user or prover may flexibly choose expansion depth

What are we doing?

In order to prove that φ is valid in HOML,
–> we instead prove that valid $\varphi_{\mu \to o}$ can be derived from Ax in HOL.

This can be done with interactive or automated HOL theorem provers.

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Modal logic axioms	Semantical conditions
M: valid $\forall\varphi(\Box\varphi \rightarrow \varphi)$	$\forall x(rxy)$
B: valid $\forall\varphi(\varphi \rightarrow \Box\Diamond\varphi)$	$\forall x\forall y(rxy \rightarrow ryx)$
D: valid $\forall\varphi(\Box\varphi \rightarrow \Diamond\varphi)$	$\forall x\exists y(rxy)$
4: valid $\forall\varphi(\Box\varphi \rightarrow \Box\Box\varphi)$	$\forall x\forall y\forall z(rxy \wedge ryz \rightarrow rxz)$
5: valid $\forall\varphi(\Diamond\varphi \rightarrow \Box\Diamond\varphi)$	$\forall x\forall y\forall z(rxy \wedge rxz \rightarrow ryz)$

Possibilist vs. Actualist Quantification

Modified Quantifiers:

$\forall \;\; = \lambda h_{\gamma\to(\mu\to o)} \lambda w_\mu \forall d_\gamma \, h d w$ (possibilist/constant domain)

becomes

$\forall^{va} = \lambda h_{\gamma\to(\mu\to o)} \lambda w_\mu \forall d_\gamma \, (\mathbf{ExInW} d w \to h d w)$ (actualist/varying domain)

where **ExInW** is an existence predicate.

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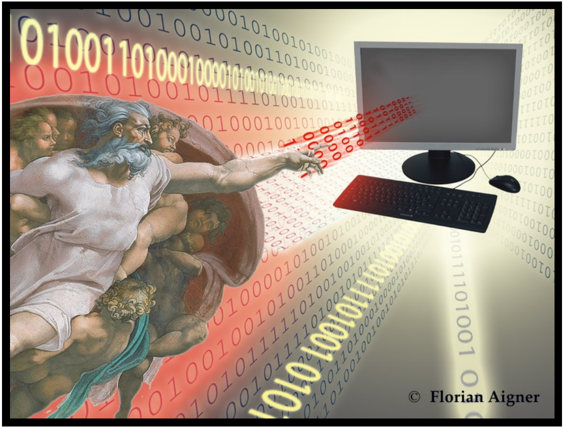
Soundness and Completeness (and Cut-elimination)

$$\models^{L} \varphi \quad \text{iff} \quad \mathsf{Ax} \models^{HOL}_{\mathsf{Henkin}} \mathit{valid}\, \varphi_{\mu\to o} \quad (\text{iff} \quad \mathsf{Ax} \vdash^{\mathsf{HOL}}_{\mathsf{cut\text{-}free}} \mathit{valid}\, \varphi_{\mu\to o})$$

Logic L:

- Higher-order Modal Logics [BenzmüllerWoltezenlogelPaleo, ECAI, 2014]
- First-order Multimodal Logics [BenzmüllerPaulson, LogicaUniversalis, 2013]
- Propositional Multimodal Logics [BenzmüllerPaulson, Log.J.IGPL, 2010]
- Quantified Conditional Logics [Benzmüller, IJCAI, 2013]
- Propositional Conditional Logics [BenzmüllerEtAl., AMAI, 2012]
- Intuitionistic Logics [BenzmüllerPaulson, Log.J.IGPL, 2010]
- Access Control Logics [Benzmüller, IFIP SEC, 2009]
- Logic Combinations [Benzmüller, AMAI, 2011]
- ...more is on the way ... including:
  - Description Logics
  - Nominal Logics
  - Multivalued Logics (SIXTEEN)
  - Logics based on Neighborhood Semantics
  - (Mathematical) Fuzzy Logics
  - Paraconsistent Logics



Automated and Interactive Theorem Provers for HOL

Automated Theorem Provers and Model Finders for HOL

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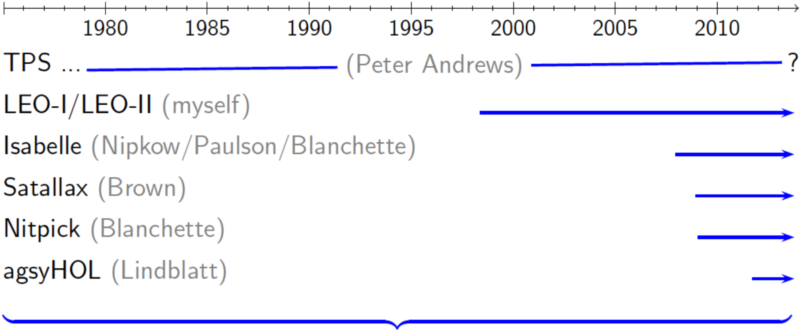

- all accept TPTP THF Syntax [SutcliffeBenzmüller, J.Form.Reas, 2009]
- can be called remotely via SystemOnTPTP at Miami
- they significantly gained in strength over the last years
- they can be bundled into a combined prover HOL-P

Exploit HOL with Henkin semantics as metalogic
Automate other logics (& combinations) via semantic embeddings
— HOL-P becomes a Universal Reasoner —

Proof Automation with LEO-II


```
>
>
> Beweise-mit-Leo2 Notwendigerweise-existiert-Gott.p

Leo-II tries to prove
=====================

  Goedel's Theorem T3: "Necessarily, God exists"
  thf(thmT3,conjecture,
      ( v
      @ ( mbox
        @ ( mexists_ind
          @ ^ [X: mu] :
              ( g @ X ) ) ) )).

  Assumptions: D1, C, T2, D3, A5

 . searching for proof ..

 ********************
 *   Proof found    *
 ********************
 % SZS status Theorem for Notwendigerweise-existiert-Gott.p

 . generating proof object
```

Provers can be called remotely in Miami — no local installation needed!

Download our experiments from

`https://github.com/FormalTheology/GoedelGod/tree/master/Formalizations/THF`

Gödel's God in Isabelle/HOL

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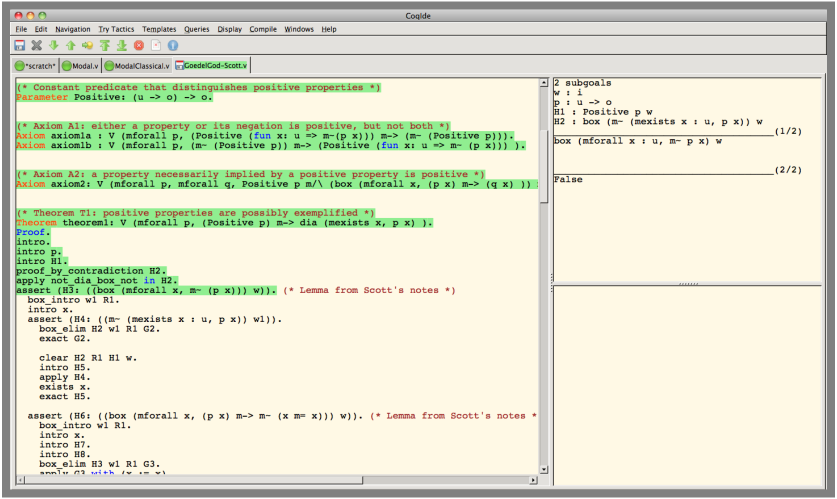
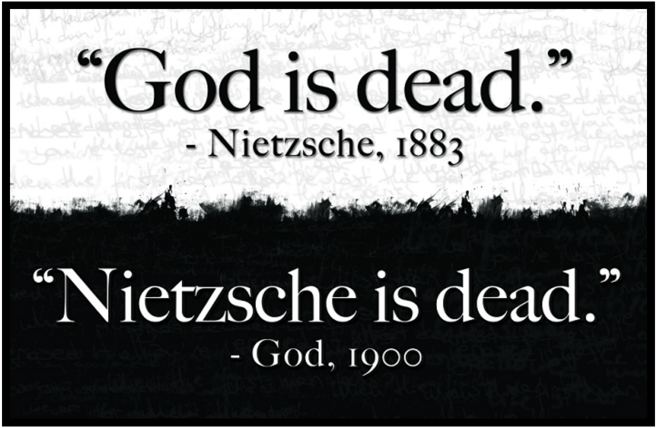
See verifiable Isabelle/HOL document (Archive of Formal Proofs) at:

`http://afp.sourceforge.net/entries/GoedelGod.shtml`



See verifiable Coq document at:

`https://github.com/FormalTheology/GoedelGod/tree/master/Formalizations/Coq`



Findings from our study

[BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

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	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg(\phi X)) \,\dot\equiv\, \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\wedge\, \dot\Box\dot\forall X_\mu.\, (\phi X \,\dot\supset\, \psi X)) \,\dot\supset\, p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\supset\, \dot\Diamond\dot\exists X_\mu.\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\supset\, \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma} X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\supset\, \dot\Box p\phi]$						
D2	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \phi X \,\dot\wedge\, \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \,\dot\supset\, \dot\Box\dot\forall Y_\mu.\, (\phi Y \,\dot\supset\, \psi Y))$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma} X \,\dot\supset\, (\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathbf{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, (\mathbf{ess}\, \phi X \,\dot\supset\, \dot\Box\dot\exists Y_\mu.\, \phi Y)$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\, \mathbf{NE}_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma} X]$	D1, C, T2, D3, A5	K	CSA	—/—	—/—	3.8/6.2
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \,\dot\supset\, \dot\Box s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall X_\mu.\, (g_{\mu\to\sigma} X \,\dot\supset\, (\dot\neg(p_{(\mu\to\sigma)\to\sigma}\phi) \,\dot\supset\, \dot\neg(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot\forall X_\mu.\, \dot\forall Y_\mu.\, (g_{\mu\to\sigma} X \,\dot\supset\, (g_{\mu\to\sigma} Y \,\dot\supset\, X \doteq Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \,\dot\supset\, \dot\Box\dot\forall Y_\mu.\, (\phi Y \,\dot\supset\, \psi Y))$						
CO'	$\emptyset$ (no goal, check for consistency)	A1(⊃), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

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	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg(\phi X)) \,\dot\equiv\, \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\wedge\, \dot\Box\dot\forall X_\mu.\, (\phi X \,\dot\supset\, \psi X)) \,\dot\supset\, p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\supset\, \dot\Diamond\dot\exists X_\mu.\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\supset\, \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\supset\, \dot\Box p\phi]$						
D2	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \phi X \,\dot\wedge\, \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \,\dot\supset\, \dot\Box\dot\forall Y_\mu.\, (\phi Y \,\dot\supset\, \psi Y))$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma}X \,\dot\supset\, (\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, gX)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, (\mathrm{ess}\,\phi X \,\dot\supset\, \dot\Box\dot\exists Y_\mu.\, \phi Y)$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$	D1, C, T2, D3, A5	K	CSA	—/—	—/—	3.8/6.2
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \,\dot\supset\, \dot\Box s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall X_\mu.\, (g_{\mu\to\sigma}X \,\dot\supset\, (\dot\neg(p_{(\mu\to\sigma)\to\sigma}\phi) \,\dot\supset\, \dot\neg(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot\forall X_\mu.\, \dot\forall Y_\mu.\, (g_{\mu\to\sigma}X \,\dot\supset\, (g_{\mu\to\sigma}Y \,\dot\supset\, X \,\dot=\, Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \,\dot\supset\, \dot\Box\dot\forall Y_\mu.\, (\phi Y \,\dot\supset\, \psi Y))$						
CO'	$\emptyset$ (no goal, check for consistency)	A1(⊃), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

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	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg(\phi X)) \dot\equiv \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \dot\wedge \dot\Box\dot\forall X_\mu.\, (\phi X \dot\supset \psi X)) \dot\supset p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Diamond\dot\exists X_\mu.\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Box p\phi]$						
D2	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \phi X \dot\wedge \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \dot\supset \dot\Box\dot\forall Y_\mu.\, (\phi Y \dot\supset \psi Y))$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma}X \dot\supset (\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, (e$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \dot\supset \dot\Box s_\sigma]$						
FG	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall X_\mu.\, (g_{\mu\to\sigma}X \dot\supset ($						
MT	$[\dot\forall X_\mu.\, \dot\forall Y_\mu.\, (g_{\mu\to\sigma}X \dot\supset (g_\mu$						
CO	∅ (no goal, check for cons						
D2'	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda$						
CO'	∅ (no goal, check for cons						

Automating Scott's proof script

T1: "Positive properties are possibly exemplified" proved by LEO-II and Satallax

- **in logic: K**
- **from assumptions:**
  - **A1 and A2**
  - **A1(⊃) and A2**
- **notion of quantification**
  - **possibilist quantifiers (constant dom.)**
  - **actualist quantifiers for individuals (varying dom.)**

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg(\phi X)) \,\dot\equiv\, \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\wedge\, \dot\Box\dot\forall X_\mu.\, (\phi X \,\dot\supset\, \psi X)) \,\dot\supset\, p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\supset\, \dot\Diamond\dot\exists X_\mu.\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\supset\, \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\supset\, \dot\Box p\phi]$						
D2	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \phi X \,\dot\wedge\, \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \,\dot\supset\, \dot\Box\dot\forall Y_\mu.\, (\phi Y \,\dot\supset\, \psi Y))$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma}X \,\dot\supset\, (\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, (\mathrm{e}$ …						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \,\dot\supset\, \dot\Box s_\sigma]$						
FG	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall X_\mu.\, (g_{\mu\to\sigma}X \,\dot\supset\, ($ …						
MT	$[\dot\forall X_\mu.\, \dot\forall Y_\mu.\, (g_{\mu\to\sigma}X \,\dot\supset\, (g_\mu$ …						
CO	$\emptyset$ (no goal, check for cons…						
D2'	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda$ …						
CO'	$\emptyset$ (no goal, check for cons…						

Automating Scott's proof script

C: "Possibly, God exists"
proved by LEO-II and Satallax

- **in logic: K**
- **from assumptions:**
  - **T1, D1, A3**
- **for domain conditions:**
  - **possibilist quantifiers (constant dom.)**
  - **actualist quantifiers for individuals (varying dom.)**

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

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	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\dot\neg(\phi X)) \dot\equiv \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\dot\forall\psi_{\mu\to\sigma}.(p_{(\mu\to\sigma)\to\sigma}\phi \dot\wedge \dot\Box\dot\forall X_\mu.(\phi X \dot\supset \psi X)) \dot\supset p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Diamond\dot\exists X_\mu.\phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\dot\forall\phi_{\mu\to\sigma}.p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma} g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot\forall\phi_{\mu\to\sigma}.p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Box p\phi]$						
D2	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\lambda X_\mu.\phi X \dot\wedge \dot\forall\psi_{\mu\to\sigma}.(\psi X \dot\supset \dot\Box\dot\forall Y_\mu.(\phi Y \dot\supset \psi Y))$						
T2	$[\dot\forall X_\mu.g_{\mu\to\sigma}X \dot\supset (ess_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$NE_{\mu\to\sigma} = \lambda X_\mu.\dot\forall\phi_{\mu\to\sigma}.(e$						
A5	$[p_{(\mu\to\sigma)\to\sigma}NE_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \dot\supset \dot\Box s_\sigma]$						
FG	$[\dot\forall\phi_{\mu\to\sigma}.\dot\forall X_\mu.(g_{\mu\to\sigma}X \dot\supset ($						
MT	$[\dot\forall X_\mu.\dot\forall Y_\mu.(g_{\mu\to\sigma}X \dot\supset (g_\mu$						
CO	$\emptyset$ (no goal, check for cons						
D2'	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\lambda$						
CO'	$\emptyset$ (no goal, check for cons						

Automating Scott's proof script

T2: "Being God-like is an ess. of any God-like being" proved by LEO-II and Satallax

- **in logic: K**
- **from assumptions:**
  - **A1, D1, A4, D2**
- **for domain conditions:**
  - **possibilist quantifiers (constant dom.)**
  - **actualist quantifiers for individuals (varying dom.)**

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg(\phi X)) \dot\equiv \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\wedge\, \dot\Box\dot\forall X_\mu.\, (\phi X \dot\supset \psi X)) \dot\supset p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Diamond\dot\exists X_\mu.\, \phi X]$	A1($\supset$), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Box p\phi]$						
D2	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \phi X \,\dot\wedge\, \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \dot\supset \dot\Box\dot\forall Y_\mu.\, (\phi Y \dot\supset \psi Y))$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma}X \dot\supset (\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, (\mathrm{e}$						
A5	[p ... NE ...]						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \dot\supset \dot\Box s_\sigma]$						
FG	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall X_\mu.\, (g_{\mu\to\sigma}X \dot\supset ($						
MT	$[\dot\forall X_\mu.\, \dot\forall Y_\mu.\, (g_{\mu\to\sigma}X \dot\supset (g_\mu$						
CO	$\emptyset$ (no goal, check for cons						
D2’	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda$						
CO’	$\emptyset$ (no goal, check for cons						

Automating Scott’s proof script

T3: "Necessarily, God exists” proved by LEO-II and Satallax

- **in logic: KB**
- **from assumptions:**
  - **D1, C, T2, D3, A5**
- **for domain conditions:**
  - **possibilist quantifiers (constant dom.)**
  - **actualist quantifiers for individuals (varying dom.)**

For logic K we got a countermodel by Nitpick

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

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	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg(\phi X)) \dot\equiv \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\,\dot\forall\psi_{\mu\to\sigma}.\,(p_{(\mu\to\sigma)\to\sigma}\phi \dot\wedge \dot\Box\dot\forall X_\mu.\,(\phi X \dot\supset \psi X)) \dot\supset p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Diamond\dot\exists X_\mu.\,\phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\,\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Box p\phi]$						
D2	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\,\lambda X_\mu.\,\phi X \dot\wedge \dot\forall\psi_{\mu\to\sigma}.\,(\psi X \dot\supset \dot\Box\dot\forall Y_\mu.\,(\phi Y \dot\supset \psi Y))$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma}X \dot\supset (\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\,\dot\forall\phi_{\mu\to\sigma}.\,($e						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \dot\supset \dot\Box s_\sigma]$						
FG	$[\dot\forall\phi_{\mu\to\sigma}.\,\dot\forall X_\mu.\,(g_{\mu\to\sigma}X \dot\supset ($						
MT	$[\dot\forall X_\mu.\,\dot\forall Y_\mu.\,(g_{\mu\to\sigma}X \dot\supset (g_\mu$						
CO	∅ (no goal, check for cons						
D2'	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\,\lambda$						
CO'	∅ (no goal, check for cons						

Automating Scott's proof script

Summary

- proof verified and automated
- KB is sufficient (critisized logic S5 not needed!)
- possibilist and actualist quantifiers (individuals)
- exact dependencies determined experimentally
- ATPs have found alternative proofs
  e.g. self-identity $\lambda x(x = x)$ is not needed

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg(\phi X)) \dot\equiv \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \dot\wedge \dot\Box\dot\forall X_\mu.\, (\phi X \dot\supset \psi X)) \dot\supset p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Diamond\dot\exists X_\mu.\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu-}$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
A4	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Box p$						
D2	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma}X \dot\supset (\mathrm{ess}_{(\mu\to\sigma)}$						
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, (e$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \dot\supset \dot\Box s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall X_\mu.\, (g_{\mu\to\sigma}X \dot\supset (\dot\neg(p_{(\mu\to\sigma)\to\sigma}\phi) \dot\supset \dot\neg(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot\forall X_\mu.\, \dot\forall Y_\mu.\, (g_{\mu\to\sigma}X \dot\supset (g_{\mu\to\sigma}Y \dot\supset X \dot= Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	∅ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \dot\supset \dot\Box\dot\forall Y_\mu.\, (\phi Y \dot\supset \psi Y))$						
CO'	∅ (no goal, check for consistency)	A1(⊃), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

Consistency check: Gödel vs. Scott

- **Scott's assumptions are consistent; shown by Nitpick**
- **Gödel's assumptions are inconsistent; shown by LEO-II (new philosophical result?)**

Main Findings

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	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall\phi_{\mu\to\sigma}\centerdot p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu\centerdot\dot\neg(\phi X))\,\dot\equiv\,\dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}\centerdot\dot\forall\psi_{\mu\to\sigma}\centerdot(p_{(\mu\to\sigma)\to\sigma}\phi\,\dot\wedge\,\dot\Box\dot\forall X_\mu\centerdot(\phi X\,\dot\supset\,\psi X))\,\dot\supset\,p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}\centerdot p_{(\mu\to\sigma)\to\sigma}\phi\,\dot\supset\,\dot\Diamond\dot\exists X_\mu\centerdot\phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu\centerdot\dot\forall\phi_{\mu\to\sigma}\centerdot p_{(\mu-}$ … $\phi\,\dot\supset\,\phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu\centerdot g_{\mu\to\sigma}X]$						
A4	$[\dot\forall\phi_{\mu\to\sigma}\centerdot p_{(\mu\to\sigma)\to\sigma}\phi\,\dot\supset\,\dot\Box p$ …						
D2	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to}$ … $\lambda\phi_{\mu\to\sigma}\centerdot\lambda$ …						
T2	$[\dot\forall X_\mu\centerdot g_{\mu\to\sigma}X\,\dot\supset\,(\mathbf{ess}_{(\mu\to\sigma)-}$ …						
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu\centerdot\dot\forall\phi_{\mu\to\sigma}\centerdot(\mathrm{e}$ …						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu\centerdot g_{\mu\to\sigma}X]$	D1, C, T2, D3, A5	K	CSA	—/—	—/—	3.8/6.2
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma\,\dot\supset\,\dot\Box s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot\forall\phi_{\mu\to\sigma}\centerdot\dot\forall X_\mu\centerdot(g_{\mu\to\sigma}X\,\dot\supset\,(\dot\neg(p_{(\mu\to\sigma)\to\sigma}\phi)\,\dot\supset\,\dot\neg(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot\forall X_\mu\centerdot\dot\forall Y_\mu\centerdot(g_{\mu\to\sigma}X\,\dot\supset\,(g_{\mu\to\sigma}Y\,\dot\supset\,X\,\dot=\,Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}\centerdot\lambda X_\mu\centerdot\dot\forall\psi_{\mu\to\sigma}\centerdot(\psi X\,\dot\supset\,\dot\Box\dot\forall Y_\mu\centerdot(\phi Y\,\dot\supset\,\psi Y))$						
CO'	$\emptyset$ (no goal, check for consistency)	A1(⊃), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

Further Results

- **Monotheism holds**
- **God is flawless**

Main Findings

Modal Collapse (Sobel)

$$\forall\varphi(\varphi \supset \Box\varphi)$$

- proved by LEO-II and Satallax
- for possibilist and actualist quantification (ind.)

Main critique on Gödel's ontological proof:

- there are no contingent truths
- everything is determined / no free will

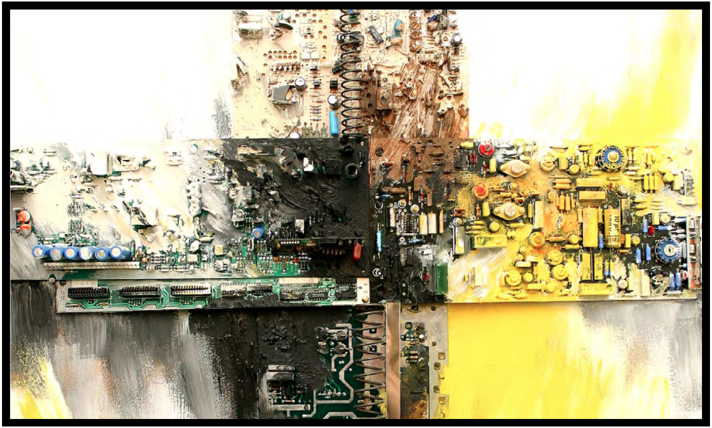
	HOL encoding						
A1	$[\dot\forall\phi_{\mu\to\sigma}.\ p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\ \dot\neg$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\ \dot\forall\psi_{\mu\to\sigma}.\ (p_{(\mu\to\sigma)\to}$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\ p_{(\mu\to\sigma)\to\sigma}\phi \ \dot\supset\ \dot\Diamond\dot\exists$						
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\ \dot\forall\phi_{\mu\to\sigma}.\ p_{(\mu-}$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\ g_{\mu\to\sigma}X]$						
A4	$[\dot\forall\phi_{\mu\to\sigma}.\ p_{(\mu\to\sigma)\to\sigma}\phi \ \dot\supset\ \dot\Box p$						
D2	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\ \lambda$						
T2	$[\dot\forall X_\mu.\ g_{\mu\to\sigma}X \ \dot\supset\ (\mathrm{ess}_{(\mu\to\sigma)}$						
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\ \dot\forall\phi_{\mu\to\sigma}.\ (e$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\ g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \ \dot\supset\ \dot\Box s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot\forall\phi_{\mu\to\sigma}.\ \dot\forall X_\mu.\ (g_{\mu\to\sigma}X \supset (\neg(p_{(\mu\to\sigma)\to\sigma}\phi) \supset \neg(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot\forall X_\mu.\ \dot\forall Y_\mu.\ (g_{\mu\to\sigma}X \ \dot\supset\ (g_{\mu\to\sigma}Y \ \dot\supset\ X \doteq Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\ \lambda X_\mu.\ \dot\forall\psi_{\mu\to\sigma}.\ (\psi X \ \dot\supset\ \dot\Box\dot\forall Y_\mu.\ (\phi Y \ \dot\supset\ \psi Y))$						
CO'	$\emptyset$ (no goal, check for consistency)	A1($\supset$), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

Main Findings

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot{\neg}(\phi X)) \,\dot{\equiv}\, \dot{\neg}(p\phi)]$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma}.\, \dot{\forall}\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\wedge}\, \dot{\Box}\dot{\forall}X_\mu.\, (\phi X \,\dot{\supset}\, \psi X)) \,\dot{\supset}\, p\psi]$						
T1	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\supset}\, \dot{\Diamond}\dot{\exists}X_\mu.\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\supset}\, \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot{\Diamond}\dot{\exists}X_\mu.\, g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\supset}\, \dot{\Box}p\phi]$						
D2	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \phi X \,\dot{\wedge}\, \dot{\forall}\psi_{\mu\to\sigma}.\, (\psi X \,\dot{\supset}\, \dot{\Box}\dot{\forall}Y_\mu.\, (\phi Y \,\dot{\supset}\, \psi Y))$						
T2	$[\dot{\forall}X_\mu.\, g_{\mu\to\sigma}X \,\dot{\supset}\, (\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathbf{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot{\forall}\phi_{\mu\to\sigma}.\, (\mathbf{ess}\,\phi X \,\dot{\supset}\, \dot{\Box}\dot{\exists}Y_\mu.\, \phi Y)$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathbf{NE}_{\mu\to\sigma}]$						
T3	$[\dot{\Box}\dot{\exists}X_\mu.\, g_{\mu\to\sigma}X]$	D1, C, T2, D3, A5	K	CSA	—/—	—/—	3.8/6.2
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
			KB	THM	17.9/—	3.3/3.2	—/—
		3, A5	KB	THM	—/—	—/—	—/—
			KB	THM	16.5/—	0.0/0.0	—/—
		3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
			KB	THM	—/—	0.0/3.3	—/—
		3, A5	KB	THM	—/—	—/—	—/—
		3, A5	KB	SAT	—/—	—/—	7.3/7.4
			KB	UNS	7.5/7.8	—/—	—/—
		D3, A5	KB	UNS	—/—	—/—	—/—

Observation

- **good performance of ATPs**
- **excellent match between argumentation granularity in papers and the reasoning strength of the ATPs**



Reconstruction of the Inconsistency of Gödel's Axioms

Inconsistency (Gödel): Proof by LEO-II in KB

DemoMaterial — bash — 166×52

```
@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: $false))@SV3)=$true))),inference(prim_subst,[status(thm)],[66:[bind(SV11,$thf(^[SV23:mu,SV24:$i]: $false))]])).
 thf(84,plain,(![SV22:(mu>($i>$o)),SV3:$i,SV8:(mu>($i>$o))]: ((((SV8@(((sK2_SY33@SV3)@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV8))@(((sK1_SY31@(^[SX0:mu,SX1:$i]: (
~ ((SV22@SX0)@SX1))))@SV8)@SV3))=$true) | (((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV3)=$true))),inference(prim_subst,[status(thm)],[66
:[bind(SV11,$thf(^[SV20:mu,SV21:$i]: (~ ((SV22@SV20)@SV21))))]])).
 thf(85,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false) | ((((p@SV9)@SV4) = ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)
@SY28))))@SV4))=$false))),inference(fac_restr,[status(thm)],[56])).
 thf(86,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true) | ((((p@SV9)@SV4) = ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@
SY30))))@SV4))=$false))),inference(fac_restr,[status(thm)],[57])).
 thf(87,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((~ (((p@SV9)@SV4) | ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))) | (~ ((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY27:mu,
SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)))))=$false) | (((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_equal_neg,[status(thm)],[85])).
 thf(89,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((~ (((p@SV9)@SV4) | ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4))) | (~ ((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY29:mu,
SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)))))=$false) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true))),inference(extcnf_equal_neg,[status(thm)],[86])).
 thf(92,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((~ ((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))))=$false) | (((p@(^[SY27:mu,SY28:$i]: (
~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_or_neg,[status(thm)],[87])).
 thf(93,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((~ (((p@SV9)@SV4) | ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)))=$false) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@
SY29)@SY30))))@SV4)=$true))),inference(extcnf_or_neg,[status(thm)],[89])).
 thf(96,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)))=$true) | (((p@(^[SY27:mu,SY28:$i]: (~ ((S
V9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_not_neg,[status(thm)],[92])).
 thf(97,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((((p@SV9)@SV4) | ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4))=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)
@SY30))))@SV4)=$true))),inference(extcnf_not_neg,[status(thm)],[93])).
 thf(100,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((~ ((p@SV9)@SV4))=$true) | ((~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))=$true) | (((p@(^[SY27:mu,SY28:$i]:
(~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_or_pos,[status(thm)],[96])).
 thf(101,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@SV9)@SV4)=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV
9@SY29)@SY30))))@SV4)=$true))),inference(extcnf_or_pos,[status(thm)],[97])).
 thf(103,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@SV9)@SV4)=$false) | ((~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))=$true) | (((p@(^[SY27:mu,SY28:$i]: (~
((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_not_pos,[status(thm)],[100])).
 thf(105,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false) | (((p@SV9)@SV4)=$false) | (((p@(^[SY27:mu,SY28:$i]: (~ ((
SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_not_pos,[status(thm)],[103])).
 thf(107,plain,(![SV8:(mu>($i>$o)),SV3:$i,SV22:(mu>($i>$o))]: ((((SV22@(((sK2_SY33@SV3)@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV8))@(((sK1_SY31@(^[SX0:mu,SX1:$i]:
(~ ((SV22@SX0)@SX1))))@SV8)@SV3))=$true) | (((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV3)=$true))),inference(extcnf_not_neg,[status(thm
)],[78])).
 thf(108,plain,(![SV11:(mu>($i>$o)),SV3:$i,SV15:(mu>($i>$o))]: ((((SV15@(((sK2_SY33@SV3)@SV11)@(^[SX0:mu,SX1:$i]: (~ ((SV15@SX0)@SX1)))))@(((sK1_SY31@SV11)@(^[SX0:mu,
SX1:$i]: (~ ((SV15@SX0)@SX1))))@SV3))=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV15@SX0)@SX1))))@SV3)=$false) | (((p@SV11)@SV3)=$true))),inference(extcnf_not_pos,[statu
s(thm)],[81])).
 thf(109,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false) | (((p@SV9)@SV4)=$false))),inference(sim,[status(thm)],[10
5])).
 thf(110,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@SV9)@SV4)=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true))),inference(sim,[status(thm)],[101]
)).
 thf(111,plain,(![SV3:$i,SV8:(mu>($i>$o))]: ((((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: $true))@SV3)=$true))),inference(sim,[status(thm)],[76])).
 thf(112,plain,(![SV11:(mu>($i>$o)),SV3:$i]: ((((p@(^[SX0:mu,SX1:$i]: $false))@SV3)=$false) | (((p@SV11)@SV3)=$true))),inference(sim,[status(thm)],[80])).
 thf(113,plain,((($false)=$true)),inference(fo_atp_e,[status(thm)],[25,112,111,110,109,108,107,84,83,82,75,74,73,72,71,70,69,68,67,66,65,62,57,56,51,42,29])).
 thf(114,plain,($false),inference(solved_all_splits,[solved_all_splits(join,[])],[113])).
% SZS output end CNFRefutation

%***** End of derivation protocol ****
%***** no. of clauses in derivation: 97 ****
%***** clause counter: 113 ****

% SZS status Unsatisfiable for ConsistencyWithoutFirstConjunctinD2.p : (rf:0,axioms:6,ps:3,u:6,ude:false,rLeibEQ:true,rAndEQ:true,use_choice:true,use_extuni:true,use_
extcnf_combined:true,expand_extuni:false,foatp:e,atp_timeout:25,atp_calls_frequency:10,ordering:none,proof_output:1,clause_count:113,loop_count:0,foatp_calls:2,transl
ation:fof_full)
ontoleo:DemoMaterial cbenzmueller$ 
```

Inconsistency (Gödel): Verification in Isabelle/HOL (KB)

Freie Universität Berlin

GoedelGodWithoutConjunctInEss_KB.thy

GoedelGodWithoutConjunctInEss_KB.thy (~/GoedelGod/Talks/2015/LogicAndReligion/DemoMaterial/)

```
theory GoedelGodWithoutConjunctInEss_KB imports QML
begin
  consts P :: "(μ ⇒ σ) ⇒ σ"
  axiomatization where A1a: "[∀(λΦ. P (λx. m¬ (Φ x)) m→ m¬ (P Φ))]"
                   and A2:  "[∀(λΦ. ∀(λΨ. (P Φ m∧ □ (∀(λx. Φ x m→ Ψ x))) m→ P Ψ))]"

-- {* Positive properties are possibly exemplified. *}
  theorem T1: "[∀(λΦ. P Φ m→ ◇ (∃ Φ))]"                      by (metis A1a A2)

  definition ess (infixr "ess" 85) where "Φ ess x = ∀(λΨ. Ψ x m→ □ (∀(λy. Φ y m→ Ψ y)))"

-- {* The empty property is an essence of every individual. *}
  lemma Lemma1: "[(∀(λx.( (λy.λw. False) ess x)))]"          by (metis ess_def)

  definition NE  where "NE x = ∀(λΦ. Φ ess x m→ □ (∃ Φ))"
  axiomatization where sym: "x r y ⟶ y r x"

-- {* Exemplification of necessary existence is not possible. *}
  lemma Lemma2: "[m¬ (◇ (∃ NE))]"                  by (metis sym Lemma1 NE_def)

  axiomatization where A5:  "[P NE]"

-- {* Now the inconsistency follows from A5, T1 and Lemma2 *}
  lemma False                                       by (metis A5 T1 Lemma2)
end
```

Documentation Sidekick Theories

Output Query Sledgehammer Symbols

11,1 (477/1095) (isabelle,sidekick,UTF-8-Isabelle)Nmro UG 263/347MB 17:18

Gödel's Manuscript

Ontologischer Beweis — Feb 10, 1970

$P(\varphi)$ — φ is positive (or $\varphi \in P$)

At. 1 — $P(\varphi) . P(\psi) \supset P(\varphi . \psi)$ — At 2 — $P(\varphi) \vee P(\sim\varphi)$

P1 — $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ — (God)

P2 — $\varphi \text{ Ess. } x \equiv (\psi)[\psi(x) \supset N(y)[\varphi(y) \supset \psi(y)]]$ — (Essence of x)

$p \supset_N q = N(p \supset q)$ — Necessity

At 2 — $P(\varphi) \supset N P(\varphi)$, $\sim P(\varphi) \supset N \sim P(\varphi)$ — because it follows from the nature of the property

Th. — $G(x) \supset G \text{ Ess. } x$

Df. — $E(x) \equiv (\varphi)[\varphi \text{ Ess } x \supset N \exists x\, \varphi(x)]$ — necessary Existence

Ax 3 — $P(E)$

Th. — $G(x) \supset N(\exists y)\, G(y)$

hence $(\exists x) G(x) \supset N(\exists y)\, G(y)$

" $M(\exists x) G(x) \supset M N(\exists y)\, G(y)$ — M = possibility

" $\supset N(\exists y)\, G(y)$

any two essences of x are nec. equivalent

exclusive or • and for any number of summands

$M(\exists x) G(x)$ means the system of all pos. prop. is compatible. This is true because of:

At 4 : $P(\varphi) . \varphi \supset_N \psi : \supset P(\psi)$ which impl

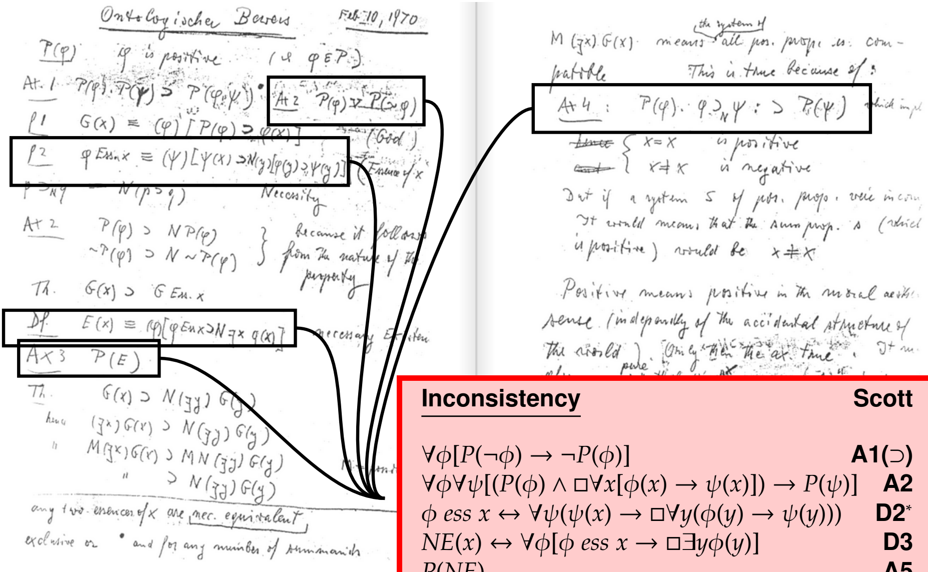
$x = x$ is positive
$x \neq x$ is negative

But if a system S of pos. prop. were incom it would mean that the sum prop. s (which is positive) would be $x \neq x$

Positive means positive in the moral aesthetic sense (independently of the accidental structure of the world). Only then the ax. true. It may also mean pure "attribution"* as opposed to "privation" (or containing privation). This interpret. simpler proof

If φ pos. not: $(x) N \sim \varphi(x)$. Otherwise: $\varphi(x) \supset_N x \neq x$ hence $x \neq x$ positive not $x = x$ therefore contrary At 5 or the exist. of pos. prop.

* i.e. the disj. normal form in terms of elem. prop. contains a member without negation.



Inconsistency **Scott**

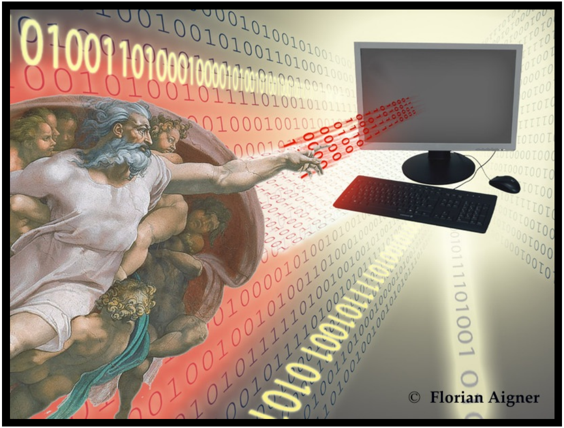
$\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$ **A1(⊃)**

$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$ **A2**

$\phi\ ess\ x \leftrightarrow \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$ **D2***

$NE(x) \leftrightarrow \forall\phi[\phi\ ess\ x \rightarrow \Box\exists y\phi(y)]$ **D3**

$P(NE)$ **A5**



Avoiding the Modal Collapse

Avoiding the Modal Collapse: Recent Variants

SOME EMENDATIONS OF GÖDEL'S ONTOLOGICAL PROOF

C. Anthony Anderson

Kurt Gödel's version of the ontological argument was shown by J. Howard Sobel to be defective, but some plausible modifications in the argument result in a version which is immune to Sobel's objection. A definition is suggested which permits the proof of some of Gödel's axioms.

Gödel's Ontological Proof Revisited *

C. Anthony Anderson and Michael Gettings

University of California, Santa Barbara
Department of Philosophy

Gödel's version of the modal ontological argument for the existence of God has been criticized by J. Howard Sobel [5] and modified by C. Anthony Anderson [1]. In the present paper we consider the extent to which Anderson's emendation is defeated by the type of objection first offered by the Monk Gaunilo to St. Anselm's original Ontological Argument. And we try to push the analysis of this Gödelian argument a bit further to bring it into closer agreement with the details of Gödel's own formulation. Finally, we indicate what seems to be the main weakness of this emendation of Gödel's attempted proof.

Magari and others on Gödel's ontological proof

Petr Hájek
Institute of Computer Science, Academy of Sciences
182 07 Prague, Czech Republic
e-mail: hajek@uivt.cas.cz

1 Introduction

This paper is a continuation of my paper [H] and concentrates almost exclusively to mathematical properties of logical systems underlying Gödel's ontological proof [G] and its variant by Anderson [A], with special care paid to Magari's criticism [M]. Since [H] is written in German, we shall try to summarize its content in such a way that knowledge of [H] will be not obligatory for reading the present paper (even it remains advantageous). Here we describe

Der Mathematiker und die Frage der Existenz Gottes

(betreffend Gödels ontologischen Beweis)

Es ist gut, daß wir nicht wissen, sondern glauben, daß ein Gott sei. (Kant, Nachlaß)

1. Einführung

Gödels zu Lebzeiten unveröffentlichter Beweis für die notwendige Existenz eines Gott-ähnlichen Wesens hat sowohl philosophisches als auch mathematisches Interesse geweckt. Zweck der vorliegenden Arbeit ist es, zu einer Deutung des Gödelschen Textes beizutragen, 1. durch Kommentierung der einschlägigen Literatur und 2. durch Bereitstellung von etwas Modelltheorie. Die Arbeit enthält keinen philosophischen Beitrag. Während der letzten Jahre habe ich etliche Male über Gödels Gottesbeweis vorgetragen, insbesondere auf dem Symposium zur Feier von Professor Gert Müller (Heidelberg, Jänner 1991), doch habe ich niemals beabsichtigt, eine Veröffentlichung über das Thema zu machen. Da ich wiederholt um eine schriftliche Version gebeten wurde, entschloß ich mich, schnell eine „erweiterte Kurzfassung"[1] zu schreiben, ohne aus ihr einen

PETR HÁJEK

A New Small Emendation of Gödel's Ontological Proof

Keywords: Ontological proof, Gödel, modal logic, comprehension, positive properties.

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Gödel's ontological proof of necessary existence of a godlike being was finally published in the third volume of Gödel's collected works [7]; but it became known in 1970 when Gödel showed the proof to Dana Scott and Scott presented it (in fact a variant of it) at a seminar at Princeton. Detailed history is found in Adams' introductory remarks to the ontological proof in [7]. The proof uses modal logic and its analysis is an exciting exercise in systems of formal modal logic. Needles to say, formal modal logic has found several

Understanding Gödel's Ontological Argument

FRODE BJØRDAL

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Avoiding the Modal Collapse: Some Emendations

Freie Universität Berlin

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Computer-supported Clarification of Controversy
First World Congress on Logic and Religion

Conclusion

Overall Achievements

- significant contribution towards a **Computational Metaphysics**
- **novel results** contributed by **HOL-ATPs**
- infrastructure can be adapted for **other logics and logic combinations**
- **our technology is sufficiently mature** for use by philosophers

Relevance (wrt foundations and applications)

- Philosophy, AI, Computer Science, Computational Linguistics, Maths

Little related work: only for Anselm's simpler argument

- first-order ATP PROVER9 [OppenheimerZalta, 2011]
- interactive proof assistant PVS [Rushby, 2013]

Ongoing/Future work

- Landscape of verified/falsified ontological arguments
- You may consider to contribute:
  `https://github.com/FormalTheology/GoedelGod.git`

Personal Statement

(Interim) Culmination of two decades of related own research

- Theory of classical higher-order logic (HOL) (since 1995)
- Automation of HOL / own LEO provers (since 1998)
- Integration of interactive and automated theorem proving (since 1999)
- International TPTP infrastructure for HOL (since 2006)
- HOL as a universal logic via semantic embeddings (since 2008)
- jww Bruno Woltzenlogel-Paleo:
  Application in Metaphysics: Ontological Argument (since 2013)

... success story (despite strong criticism/opposition on the way!) ...
... huge media attention ...

(Interim) Own standpoint

- I am not fully convinced (yet) by the ontological argument.
- However, it seems to me that **the belief in a (God-like) supreme being is at least not necessarily irrational/inconsistent**.

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Formel von Kurt Gödel: Mathematiker bestätigen Gottesbeweis

Von Tobias Hürter

picture-alliance/ Imagno/ Wiener Stadt- und Landesbibliothek

Kurt Gödel (um das Jahr 1935): Der Mathematiker hielt seinen Gottesbeweis jahrzehntelang geheim

Ein Wesen existiert, das alle positiven Eigenschaften in sich vereint. Das bewies der legendäre Mathematiker Kurt Gödel mit einem komplizierten Formelgebilde. Zwei Wissenschaftler haben diesen Gottesbeweis nun überprüft - und für gültig befunden.

Montag, 09.09.2013 – 12:03 Uhr

Drucken | Versenden | Merken

Jetzt sind die letzten Zweifel ausgeräumt: Gott existiert tatsächlich. Ein Computer hat es mit kalter Logik bewiesen - das MacBook des Computerwissenschaftlers Christoph Benzmüller von der Freien Universität Berlin.

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English Site > Germany > Science > Scientists Use Computer to Mathematically Prove Gödel God Theorem

Holy Logic: Computer Scientists 'Prove' God Exists

By David Knight

picture-alliance/ Imagno/ Wiener Stadt- und Landesbibliothek

Austrian mathematician Kurt Gödel kept his proof of God's existence a secret for decades. Now two scientists say they have proven it mathematically using a computer.

Two scientists have formalized a theorem regarding the existence of God penned by mathematician Kurt Gödel. But the God angle is somewhat of a red herring -- the real step forward is the example it sets of how computers can make scientific progress simpler.

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DIE WELT

FREITAG, 18. OKTOBER 2013

Zippert zappt

Teuer erkaufte Harmonie

Mindestlohn lockt SPD in dritte Ehe mit der Union

Mittwoch soll über eine große Koalition verhandelt werden. Votum des SPD-Konvents steht aus

THEMEN

Sport – Bundestrainer Löw unterschreibt beim DFB bis 2016

Wirtschaft – Der Jumbojet wird bei Boeing zum Ladenhüter

Karriere – Stress im Job: So legen Sie sich einen dicken Panzer zu

Aus aller Welt – Mietprozess in Paris: Dürfen 15 Quadratmeter 330 Euro kosten?

Dax – Im Minus

Göttliche Mathematik

Haben Forscher mit einem Computerprogramm die Existenz Gottes bewiesen?

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SCIENCE NEWS

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Researchers say they used MacBook to prove Goedel's God theorem

Oct. 23, 2013 | 8:14 PM | 1 comments

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`https://github.com/FormalTheology/GoedelGod/tree/master/Press`

CADE-25 website: `http://cade-25.info`

Inconsistency (Gödel): Reconstruction of Informal Argument (KB)

(special thanks to Chad Brown for a fruitful discussion)

Axiom A1(⊃) $\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$

Axiom A2 $\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$

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Inconsistency $\bot$

by A5, T1, Lemma2