On a (Quite) Universal Theorem Proving Approach and its Application to Metaphycis

Christoph Benzmüller[1], FU Berlin

jww: **B. Woltzenlogel Paleo**, L. Paulson , C. Brown, G. Sutcliffe and many others!

Tableaux 2015

```
>
>
> Beweise-mit-Leo2 Notwendigerweise-existiert-Gott.p

Leo-II tries to prove
=====================

  Goedel's Theorem T3: "Necessarily, God exists"
  thf(thmT3,conjecture,
      ( v
      @ ( mbox
        @ ( mexists_ind
          @ ^ [X: mu] :
              ( g @ X ) ) ) )).

  Assumptions: D1, C, T2, D3, A5

 . searching for proof ..

 *******************
 *   Proof found    *
 *******************
 % SZS status Theorem for Notwendigerweise-existiert-Gott.p

 . generating proof object
```

[1] Supported by DFG Heisenberg Fellowship BE 2501/9-1/2

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A: HOL as a Universal (Meta-)Logic via Semantic Embeddings

B: HOL ATPs contributed New Knowledge in Metaphysics

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Nachrichten > Wissenschaft > Mensch > Mathematik > Formel von Kurt Gödel: Mathematiker bestätigen Gottesbeweis

Formel von Kurt Gödel: Mathematiker bestätigen Gottesbeweis

Von Tobias Hürter

picture-alliance/ Imagno/ Wiener Stadt- und Landesbibliothek

Kurt Gödel (um das Jahr 1935): Der Mathematiker hielt seinen Gottesbeweis jahrzehntelang geheim

Ein Wesen existiert, das alle positiven Eigenschaften in sich vereint. Das bewies der legendäre Mathematiker Kurt Gödel mit einem komplizierten Formelgebilde. Zwei Wissenschaftler haben diesen Gottesbeweis nun überprüft - und für gültig befunden.

Montag, 09.09.2013 – 12:03 Uhr

Drucken | Versenden | Merken

Jetzt sind die letzten Zweifel ausgeräumt: Gott existiert tatsächlich. Ein Computer hat es mit kalter Logik bewiesen - das MacBook des Computerwissenschaftlers Christoph Benzmüller von der Freien Universität Berlin.

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English Site > Germany > Science > Scientists Use Computer to Mathematically Prove Gödel God Theorem

Holy Logic: Computer Scientists 'Prove' God Exists

By David Knight

picture-alliance/ Imagno/ Wiener Stadt- und Landesbibliothek

Austrian mathematician Kurt Gödel kept his proof of God's existence a secret for decades. Now two scientists say they have proven it mathematically using a computer.

Two scientists have formalized a theorem regarding the existence of God penned by mathematician Kurt Gödel. But the God angle is somewhat of a red herring -- the real step forward is the example it sets of how computers can make scientific progress simpler.

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DIE WELT

FREITAG, 18. OKTOBER 2013

Zippert zappt

Teuer erkaufte Harmonie

Mindestlohn lockt SPD in dritte Ehe mit der Union

Mittwoch soll über eine große Koalition verhandelt werden. Votum des SPD-Konvents steht aus

THEMEN

Göttliche Mathematik

Haben Forscher mit einem Computerprogramm die Existenz Gottes bewiesen?

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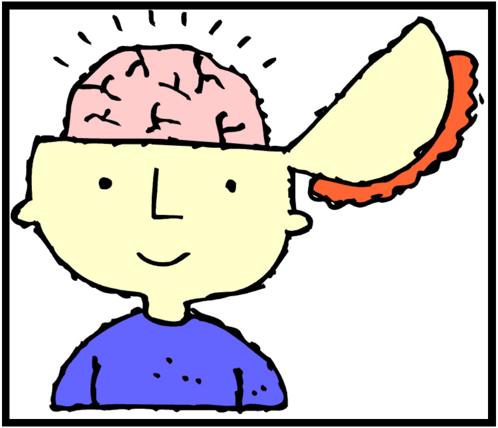
SCIENCE NEWS

HOME / SCIENCE NEWS / RESEARCHERS SAY THEY USED MACBOOK TO PROVE GOEDEL'S GOD THEOREM

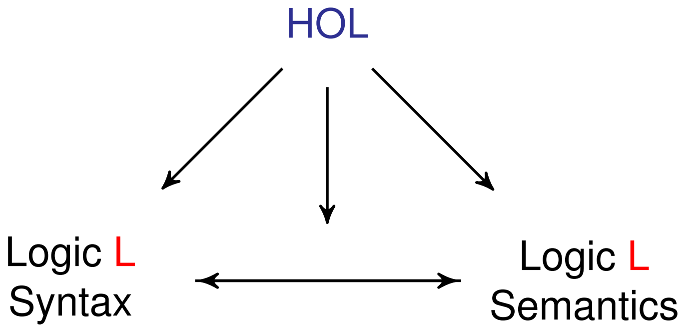
Researchers say they used MacBook to prove Goedel's God theorem

Oct. 23, 2013 | 8:14 PM | 1 comments

See more serious and funny news links at
`https://github.com/FormalTheology/GoedelGod/tree/master/Press`

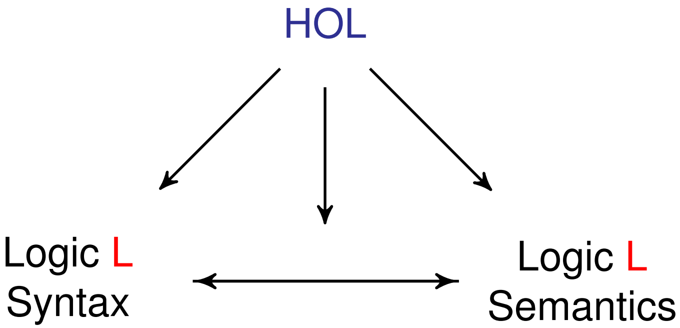


Part A:
HOL as a Universal (Meta-)Logic via Semantic Embeddings



Examples for L we have already studied:
Modal Logics, Conditional Logics, Intuitionistic Logics, Access Control Logics, Nominal Logics, Multivalued Logics (SIXTEEN), Logics based on Neighborhood Semantics, (Mathematical) Fuzzy Logics, Paraconsistent Logics, . . .

Works also for (first-order & higher-order) quantifiers



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Works also for (first-order & higher-order) quantifiers

Embedding Approach — Idea

HOL (meta-logic) $\varphi ::=$

Your-logic (object-logic) $\psi ::=$

Embedding of in

=

=

=

=

Embedding of meta-logical notions on in

valid =

satisfiable =

... =

Pass this set of equations to a higher-order automated theorem prover

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HOL $\quad s,t \quad ::= \quad C_\alpha \mid x_\alpha \mid (\lambda x_\alpha s_\beta)_{\alpha\to\beta} \mid (s_{\alpha\to\beta}\, t_\alpha)_\beta \mid \neg s_o \mid s_o \vee t_o \mid \forall x_\alpha\, t_o$

HOML $\quad \varphi,\psi \quad ::= \quad \ldots \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \rightarrow \psi \mid \Box\varphi \mid \Diamond\varphi \mid \forall x_\gamma\, \varphi \mid \exists x_\gamma\, \varphi$

HOML in HOL: HOML formulas φ are mapped to HOL predicates $\varphi_{\mu\to o}$ (explicit representation of labelled formulas)

$$
\begin{array}{lcl}
\neg & = & \lambda\varphi_{\mu\to o}\lambda w_\mu \neg\varphi w \\
\wedge & = & \lambda\varphi_{\mu\to o}\lambda\psi_{\mu\to o}\lambda w_\mu(\varphi w \wedge \psi w) \\
\rightarrow & = & \lambda\varphi_{\mu\to o}\lambda\psi_{\mu\to o}\lambda w_\mu(\neg\varphi w \vee \psi w) \\
\forall & = & \lambda h_{\gamma\to(\mu\to o)}\lambda w_\mu \forall d_\gamma\, hdw \\
\exists & = & \lambda h_{\gamma\to(\mu\to o)}\lambda w_\mu \exists d_\gamma\, hdw \\
\Box & = & \lambda\varphi_{\mu\to o}\lambda w_\mu \forall u_\mu\, (\neg rwu \vee \varphi u) \\
\Diamond & = & \lambda\varphi_{\mu\to o}\lambda w_\mu \exists u_\mu\, (rwu \wedge \varphi u) \\
\text{valid} & = & \lambda\varphi_{\mu\to o}\forall w_\mu \varphi w
\end{array}
$$

Ax (polymorphic over γ)

The equations in Ax are given as axioms to the HOL provers!

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Example

HOML formula	$\Diamond \exists x G(x)$
HOML formula in HOL	valid $(\Diamond \exists x G(x))_{\mu\to o}$
expansion	$(\lambda\varphi \forall w_\mu \varphi\, w)(\Diamond \exists x G(x))_{\mu\to o}$
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expansion	$\forall w_\mu \exists u_\mu (rwu \wedge ((\lambda h_{\gamma\to(\mu\to o)} \lambda w_\mu \exists d_\gamma\, hdw)(\lambda x G(x)))_{\mu\to o} u)$
$\beta\eta$-normalisation	$\forall w_\mu \exists u_\mu (rwu \wedge \exists x Gxu)$

Expansion: user or prover may flexibly choose expansion depth

What are we doing?

In order to prove that φ is valid in HOML,
–> we instead prove that valid $\varphi_{\mu\to o}$ can be derived from Ax in HOL.

This can be done with interactive or automated HOL theorem provers.

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Advantages of the Embedding Approach

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1. Pragmatics and convenience:
   - implementing new provers made simple (even for not yet automated logics)
2. Availability:
   - simply reuse and adapt our existing encodings (THF, Isabelle/HOL, Coq)
3. Flexibility:
   - rapid experimentation with logic variations and logic combinations
4. Relation to labelled deductive systems:
   - extra-logical labels vs. intra-logical labels (here)
5. Relation to standard translation:
   - extra-logical translation vs. extended intra-logical translation (here)
6. Meta-logical reasoning:
   - various examples already exist, e.g. verification of modal logic cube
7. Direct calculi and user intuition:
   - possible: tactics on top of embedding, hiding of embedding
8. Soundness and completeness:
   - already proven for many non-classical logics (wrt Henkin semantics)
9. Cut-elimination:
   - generic indirect result, since HOL enjoys cut-elimination (Henkin semantics)

Advantage: 1. Pragmatics and convenience

implementing new provers made simple (even for not yet automated logics)

A very “Lean” Prover for HOML KB

```
%----The base type $i (already built-in) stands here for worlds and
%----mu for individuals; $o (also built-in) is the type of Booleans
thf(mu_type,type,(mu:$tType)).
%----Reserved constant r for accessibility relation
thf(r,type,(r:$i>$i>$o)).
%----Modal logic operators not, or, and, implies, box, diamond
thf(mnot_type,type,(mnot:($i>$o)>$i>$o)).
thf(mnot,definition,(mnot = (^[A:$i>$o,W:$i]:~(A@W)))).
thf(mor_type,type,(mor:($i>$o)>($i>$o)>$i>$o)).
thf(mor,definition,(mor = (^[A:$i>$o,Psi:$i>$o,W:$i]:((A@W)|(Psi@W))))).
thf(mand_type,type,(mand:($i>$o)>($i>$o)>$i>$o)).
thf(mand,definition,(mand = (^[A:$i>$o,Psi:$i>$o,W:$i]:((A@W)&(Psi@W))))).
thf(mimplies_type,type,(mimplies:($i>$o)>($i>$o)>$i>$o)).
thf(mimplies,definition,(mimplies = (^[A:$i>$o,Psi:$i>$o,W:$i]:((A@W)&(Psi@W))))).
thf(mbox_type,type,(mbox:($i>$o)>$i>$o)).
thf(mbox,definition,(mbox = (^[A:$i>$o,W:$i]:![V:$i]:(~(r@W@V)|(A@V))))).
thf(mdia_type,type,(mdia:($i>$o)>$i>$o)).
thf(mdia,definition,(mdia = (^[A:$i>$o,W:$i]:?[V:$i]:((r@W@V)&(A@V))))).
%----Quantifiers (constant domains) for individuals and propositions
thf(mforall_ind_type,type,(mforall_ind:(mu>$i>$o)>$i>$o)).
thf(mforall_ind,definition,(mforall_ind = (^[A:mu>$i>$o,W:$i]:![X:mu]:(A@X@W)))).
thf(mforall_indset_type,type,(mforall_indset:((mu>$i>$o)>$i>$o)>$i>$o)).
thf(mforall_indset,definition,(mforall_indset = (^[A:(mu>$i>$o)>$i>$o,W:$i]:![X:mu>$i>$o]:(A@X@W)))).
thf(mexists_ind_type,type,(mexists_ind:(mu>$i>$o)>$i>$o)).
thf(mexists_ind,definition,(mexists_ind = (^[A:mu>$i>$o,W:$i]:?[X:mu]:(A@X@W)))).
thf(mexists_indset_type,type,(mexists_indset:((mu>$i>$o)>$i>$o)>$i>$o)).
thf(mexists_indset,definition,(mexists_indset = (^[A:(mu>$i>$o)>$i>$o,W:$i]:?[X:mu>$i>$o]:(A@X@W)))).
%----Definition of validity (grounding of lifted modal formulas)
thf(v_type,type,(v:($i>$o)>$o)).
thf(mvalid,definition,(v = (^[A:$i>$o]:![W:$i]:(A@W)))).
%----Properties of accessibility relations: symmetry
thf(msymmetric_type,type,(msymmetric:($i>$i>$o)>$o)).
thf(msymmetric,definition,(msymmetric = (^[R:$i>$i>$o]:![S:$i,T:$i]:((R@S@T)=>(R@T@S))))).
%----Here we work with logic KB, i.e., we postulate symmetry for r
thf(sym,axiom,(msymmetric@r)).
```

TPTP THF0 syntax: [SutcliffeBenzmüller, J.Formalized Reasoning, 2010]

Advantage: 1. Pragmatics and convenience

implementing new provers made simple (even for not yet automated logics)

Approach is competitive

- First-order modal logic: see experiments in

  [BenzmüllerOttenRaths, ECAI, 2012]
  [BenzmüllerRaths, LPAR, 2013]
  [Benzmüller, ARQNL, 2014]

- Higher-order modal logics:

  There are no other systems yet!

Advantage: 2. Availability

simply reuse and adapt our existing encodings (THF, Isabelle/HOL, Coq)

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HOML in Isabelle/HOL

HOML.thy

```
abbreviation mnot :: "σ ⇒ σ" (*<*)("¬m")(*>*)
  where "¬m φ ≡ (λw. ¬ φ w)"
abbreviation mand :: "σ ⇒ σ ⇒ σ" (*<*)(infixr "∧m" 52)(*>*)
  where "φ ∧m ψ ≡ (λw. φ w ∧ ψ w)"
abbreviation mor :: "σ ⇒ σ ⇒ σ" (*<*)(infixr "∨m" 51)(*>*)
  where "φ ∨m ψ ≡ (λw. φ w ∨ ψ w)"
abbreviation mimplies :: "σ ⇒ σ ⇒ σ" (*<*)(infixr "⟶m" 49)(*>*)
    where "φ ⟶m ψ ≡ (λw. φ w ⟶ ψ w)"
abbreviation mequiv:: "σ ⇒ σ ⇒ σ" (*<*)(infixr "≡m" 48)(*>*)
  where "φ ≡m ψ ≡ (λw. φ w ⟷ ψ w)"
abbreviation mforall :: "('a ⇒ σ) ⇒ σ" (*<*)("∀")(*>*)
  where "∀ Φ ≡ (λw. ∀x. Φ x w)"
abbreviation mexists :: "('a ⇒ σ) ⇒ σ" (*<*)("∃")(*>*)
    where "∃ Φ ≡ (λw. ∃x. Φ x w)"
abbreviation mbox :: "(i ⇒ i ⇒ bool) ⇒ σ ⇒ σ" (*<*)("□- _")(*>*)
  where "□R φ ≡ (λw. ∀v. (R w v) ⟶ φ v)"
abbreviation mdia :: "(i ⇒ i ⇒ bool) ⇒ σ ⇒ σ" (*<*)("◇- _")(*>*)
    where "◇R φ ≡ (λw. ∃v. R w v ∧ φ v)"
```

Auto update | Update | Search: | 100%

Output | Query | Sledgehammer | Symbols

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Documentation | Sidekick | Theories

Postulating modal axioms or semantical constraints

HOL

Sahlqvist axioms | **Semantical constraints**

M:	valid $\forall\varphi(\Box^r\varphi \rightarrow \varphi)$	$\leftrightarrow$	$\forall x(rxx)$	(reflexivity)
B:	valid $\forall\varphi(\varphi \rightarrow \Box^r\Diamond^r\varphi)$	$\leftrightarrow$	$\forall x\forall y(rxy \rightarrow ryx)$	(symmetry)
D:	valid $\forall\varphi(\Box^r\varphi \rightarrow \Diamond^r\varphi)$	$\leftrightarrow$	$\forall x\exists y(rxy)$	(serial)
4:	valid $\forall\varphi(\Box^r\varphi \rightarrow \Box^r\Box^r\varphi)$	$\leftrightarrow$	$\forall x\forall y\forall z(rxy \wedge ryz \rightarrow rxz)$	(transitivity)
5:	valid $\forall\varphi(\Diamond^r\varphi \rightarrow \Box^r\Diamond^r\varphi)$	$\leftrightarrow$	$\forall x\forall y\forall z(rxy \wedge rxz \rightarrow ryz)$	(euclidean)

Advantage: 3. Flexibility

rapid experimentation with logic variations and logic combinations

Possibilist vs. Actualist Quantification

$\forall \;\; = \lambda h_{\gamma\rightarrow(\mu\rightarrow o)} \lambda w_\mu \forall d_\gamma \, h d w$ (constant domains)

becomes

$\forall^{va} = \lambda h_{\gamma\rightarrow(\mu\rightarrow o)} \lambda w_\mu \forall d_\gamma \, (\mathbf{ExInW} \, d w \rightarrow h d w)$ (varying domains)

where **ExInW** is an existence predicate

(additional axioms: non-empty domains, denotation of constants & functions)

Advantage: 4. Relation to labelled deductive systems

extra-logical labels vs. intra-logical labels (here)

$$\Diamond\exists x G(x)^{\text{worldlabel}} \longrightarrow ((\Diamond\exists x G(x))_{\mu\to o}\ \text{worldlabel}_{\mu})$$

Advantage: 4. Relation to labelled deductive systems

extra-logical labels vs. intra-logical labels (here)

$$\Diamond\exists x G(x)^{\text{worldlabel}} \longrightarrow ((\Diamond\exists x G(x))_{\mu\to o}\ \text{worldlabel}_{\mu})$$

Advantage: 5. Relation to standard translation

extra-logical translation vs. extended intra-logical translation (here)

[BenzmüllerPaulson, LogicaUniversalis, 2013]
[BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

Intra-logical realisation of the standard translation

$$(\Box\phi)^{a}$$
$$\longrightarrow \quad ((\Box\phi)_{\mu\to o}\, a)$$
$$\longrightarrow \quad (((\lambda\varphi_{\mu\to o}\lambda w_\mu \forall u_\mu\, (\neg rwu \vee \varphi u))\, \phi)_{\mu\to o}\, a)$$
$$\longrightarrow \quad (\forall u_\mu\, (\neg rau \vee \phi_{\mu\to o}\, u))$$

We have extended this also for first-order and higher-order quantifiers!

$$(\forall x\, \phi(x))^{a}$$
$$\longrightarrow \quad ((\forall x\, \phi(x))_{\mu\to o}\, a)$$
$$\longrightarrow \quad ((\forall(\lambda x\, \phi(x)))_{\mu\to o}\, a)$$
$$\longrightarrow \quad (((\lambda h_{\gamma\to(\mu\to o)}\lambda w_\mu \forall d_\gamma\, hdw)(\lambda x\, \phi(x)))_{\mu\to o}\, a)$$
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Intra-logical realisation of the standard translation

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\begin{array}{ll}
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\longrightarrow & (((\lambda\varphi_{\mu\to o}\lambda w_\mu \forall u_\mu\, (\neg rwu \vee \varphi u))\, \phi)_{\mu\to o}\, a) \\
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\end{array}
$$

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$$
\begin{array}{ll}
 & (\forall x\, \phi(x))\boxed{a} \\
\longrightarrow & ((\forall x\, \phi(x))_{\mu\to o}\, a) \\
\longrightarrow & ((\forall(\lambda x\, \phi(x)))_{\mu\to o}\, a) \\
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Intra-logical realisation of the standard translation

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\begin{array}{ll}
 & (\Box\phi)\ a \\
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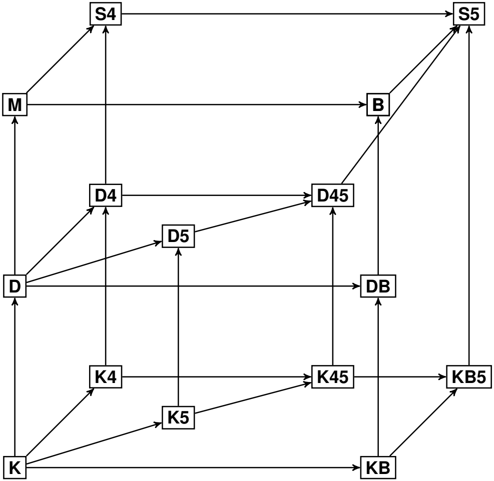
\longrightarrow & \forall d\, (\phi(d)_{\mu\to o}\, a)
\end{array}
$$

Advantage: 6. Meta-logical reasoning

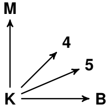
various examples already exist, e.g. verification of modal logic cube

[Benzmüller, FestschriftWalther, 2010]
[BenzmüllerClausSultana, PxTP, 2015]



≡ M5 ≡ MB5 ≡ M4B5
≡ M45 ≡ M4B ≡ D4B
≡ D4B5 ≡ DB5

M: $\Box P \rightarrow P$
B: $P \rightarrow \Box\Diamond P$
D: $\Box P \rightarrow \Diamond P$
4: $\Box P \rightarrow \Box\Box P$
5: $\Diamond P \rightarrow \Box\Diamond P$



≡ K4B5 ≡ K4B

Verification of cube in less than 1 minute in Isabelle/HOL

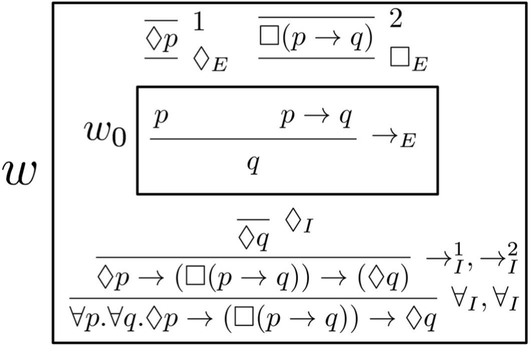
Advantage: 7. Direct calculi and user intuition

abstract level tactics (here in Coq) on top of embedding, hiding of embedding

[BenzmüllerWoltzenlogelPaleo, CSR'2015]

```
Lemma mp_dia:
  [mforall p, mforall q, (dia p) m-> (box (p m-> q)) m-> (dia q)].
```

```
Proof. mv.
intros p q H1 H2. dia_e H1. dia_i w0. box_e H2 H3. apply H3. exact H1.
Qed.
```



Advantage: 8. Soundness and completeness

already proven for many non-classical logics (wrt Henkin semantics)

Soundness and Completeness

$$\models^{L} \varphi \quad \text{iff} \quad Ax \models^{HOL}_{\text{Henkin}} valid\, \varphi_{\mu\to o}$$

Logic L:

- Higher-order Modal Logics [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]
- First-order Multimodal Logics [BenzmüllerPaulson, LogicaUniversalis, 2013]
- Propositional Multimodal Logics [BenzmüllerPaulson, Log.J.IGPL, 2010]
- Quantified Conditional Logics [Benzmüller, IJCAI, 2013]
- Propositional Conditional Logics [BenzmüllerEtAl., AMAI, 2012]
- Intuitionistic Logics [BenzmüllerPaulson, Log.J.IGPL, 2010]
- Access Control Logics [Benzmüller, IFIP SEC, 2009]
- Logic Combinations [Benzmüller, AMAI, 2011]
- . . . more is on the way . . . including:
  - Description Logics
  - Nominal Logics
  - Multivalued Logics (SIXTEEN)
  - Logics based on Neighborhood Semantics
  - (Mathematical) Fuzzy Logics
  - Paraconsistent Logics

Advantage: 9. Cut-elimination

generic indirect result, since HOL enjoys cut-elimination (Henkin semantics)

Soundness and Completeness

$\models^{L} \varphi$ iff $Ax \models^{HOL}_{\text{Henkin}} valid\, \varphi_{\mu\to o}$

Logic L:

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Soundness and Completeness and Cut-elimination

$$\models^{L} \varphi \quad \text{iff} \quad Ax \models^{HOL}_{\text{Henkin}} \mathit{valid}\,\varphi_{\mu\to o} \quad \text{iff} \quad Ax \vdash^{\text{HOL}}_{\text{cut-free}} \mathit{valid}\,\varphi_{\mu\to o}$$

Logic L:

- Higher-order Modal Logics [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]
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Advantage: 9. Cut-elimination

generic indirect result, since HOL enjoys cut-elimination (Henkin semantics)

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Soundness and Completeness and Cut-elimination

$$\models^{L} \varphi \quad \text{iff} \quad \mathrm{Ax} \models^{HOL}_{\text{Henkin}} \mathit{valid}\, \varphi_{\mu\to o} \quad \text{iff} \quad \mathrm{Ax} \vdash^{\text{HOL}}_{\text{cut-free}} \mathit{valid}\, \varphi_{\mu\to o}$$

One-sided sequent calculus $\mathcal{G}_{\beta\mathfrak{fb}}$ [BenzmüllerBrownKohlhase, LMCS, 2009]
(Δ: finite sets of β-normal closed formulas, $\Delta * \mathbf{A}$ stands for $\Delta \cup \{\mathbf{A}\}$,
$cwff_\alpha$: set of closed terms of type α, $\doteq$ abbreviates Leibniz equality):

Base Rules

$$\frac{\mathbf{A} \text{ atomic (\& } \beta\text{-normal)}}{\Delta * \mathbf{A} * \neg\mathbf{A}}\,\mathcal{G}(init) \qquad \frac{\Delta * \mathbf{A}}{\Delta * \neg\neg\mathbf{A}}\,\mathcal{G}(\neg) \qquad \frac{\Delta * \neg\mathbf{A} \quad \Delta * \neg\mathbf{B}}{\Delta * \neg(\mathbf{A} \vee \mathbf{B})}\,\mathcal{G}(\vee_{-})$$

$$\frac{\Delta * \mathbf{A} * \mathbf{B}}{\Delta * (\mathbf{A} \vee \mathbf{B})}\,\mathcal{G}(\vee_{+}) \qquad \frac{\Delta * \neg\, (\mathbf{AC}){\downarrow_\beta} \quad \mathbf{C} \in cwff_\alpha}{\Delta * \neg\Pi^\alpha \mathbf{A}}\,\mathcal{G}(\Pi^{\mathbf{C}}_{-}) \qquad \frac{\Delta * (\mathbf{A}c){\downarrow_\beta} \quad c_\alpha\, new}{\Delta * \Pi^\alpha \mathbf{A}}\,\mathcal{G}(\Pi^{c}_{+})$$

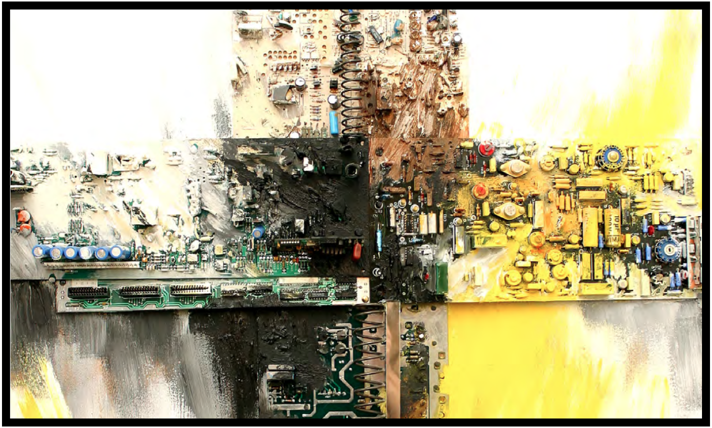
Full Extensionality

$$\frac{\Delta * (\forall X_\alpha . \mathbf{A}X \doteq^\beta \mathbf{B}X){\downarrow_\beta}}{\Delta * (\mathbf{A} \doteq^{\alpha\to\beta} \mathbf{B})}\,\mathcal{G}(\mathfrak{f}) \qquad \frac{\Delta * \neg\mathbf{A} * \mathbf{B} \quad \Delta * \neg\mathbf{B} * \mathbf{A}}{\Delta * (\mathbf{A} \doteq^o \mathbf{B})}\,\mathcal{G}(\mathfrak{b})$$

Initial. and Decomp. of Leibniz Equality

$$\frac{\Delta * (\mathbf{A} \doteq^o \mathbf{B}) \quad \mathbf{A},\mathbf{B} \text{ atomic}}{\Delta * \neg\mathbf{A} * \mathbf{B}}\,\mathcal{G}(Init^{\doteq})$$

$$\frac{\Delta * (\mathbf{A}^1 \doteq^{\alpha_1} \mathbf{B}^1) \ \cdots\ \Delta * (\mathbf{A}^n \doteq^{\alpha_n} \mathbf{B}^n) \quad n \geq 1, \beta \in \{o, \iota\}, h_{\overline{\alpha^n}\to\beta} \in \Sigma}{\Delta * (h\overline{\mathbf{A}^n} \doteq^\beta h\overline{\mathbf{B}^n})}\,\mathcal{G}(d)$$



Part B:
HOL ATP's (in particular LEO-II) contributed New Knowledge in Metaphysics

Vision of Leibniz (1646–1716): *Calculemus!*

If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other ...: Let us calculate.

(Translation by Russell)

Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo ... dicere: calculemus. (Leibniz, 1684)

Required:
characteristica universalis and **calculus ratiocinator**

Ontological argument for the existence of God

- Focus on Gödel's modern version in higher-order modal logic
- Experiments with HO provers and embedding approach

Different interests in ontological arguments

- Philosophical: Boundaries of metaphysics & epistemology
- Theistic: Successful argument could convince atheists?
- Ours: Computational metaphysics (Leibniz' vision)

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A Long History

pros and cons

… Anselm v. C. Gaunilo … Th. Aquinas …… Descartes Spinoza Leibniz … Hume Kant … Hegel … Frege … Hartshorne Malcolm Lewis Plantinga Gödel …

Anselm's notion of God (Proslogion, 1078):

"God is that, than which nothing greater can be conceived."

Gödel's notion of God:

"A God-like being possesses all 'positive' properties."

To show by logical, deductive reasoning:

"God exists."

$$\exists x G(x)$$

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To show by logical, deductive reasoning:

"Necessarily, God exists."

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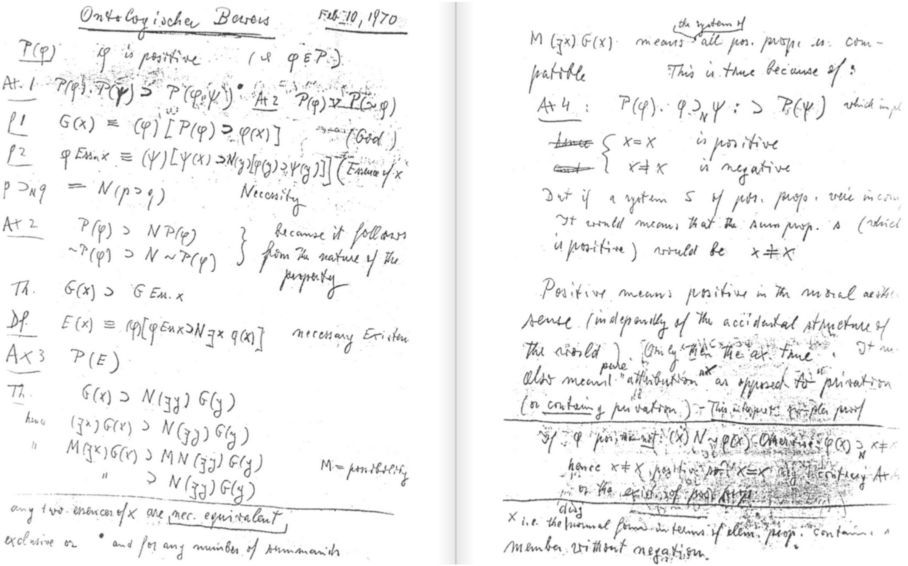
Gödel's Manuscript: 1930's, 1941, 1946-1955, 1970



Ontologischer Beweis — Feb 10, 1970

$P(\varphi)$ — φ is positive (or $\varphi \in P$)

At. 1 $P(\varphi).P(\psi) \supset P(\varphi.\psi)$* — At 2 $P(\varphi) \vee P(\sim\varphi)$

P1 $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ (God)

P2 $\varphi \, Ess.x \equiv (\psi)[\psi(x) \supset N(y)[\varphi(y) \supset \psi(y)]]$ (Essence of x)

$p \supset_N q = N(p \supset q)$ Necessity

At 2 $P(\varphi) \supset NP(\varphi)$, $\sim P(\varphi) \supset N\sim P(\varphi)$ } because it follows from the nature of the property

Th. $G(x) \supset G \, Ess.x$

Df. $E(x) \equiv (\varphi)[\varphi \, Ess\, x \supset N \exists x \, \varphi(x)]$ necessary Existence

Ax 3 $P(E)$

Th. $G(x) \supset N(\exists y) G(y)$

hence $(\exists x) G(x) \supset N(\exists y) G(y)$

" $M(\exists x) G(x) \supset MN(\exists y) G(y)$ — M = possibility

" $\supset N(\exists y) G(y)$

any two essences of x are nec. equivalent

exclusive or — * and for any number of summands

$M(\exists x) G(x)$ means the system of all pos. prop. is compatible. This is true because of:

At 4: $P(\varphi).\varphi \supset_N \psi : \supset P(\psi)$ which impl[ies]

{ $x = x$ is positive; $x \neq x$ is negative

But if a system S of pos. props. were incom[patible] it would mean that the sum prop. s (which is positive) would be $x \neq x$

Positive means positive in the moral aesth[etic] sense (independently of the accidental structure of the world). Only then the ax. true. It m[ay] also mean pure "attribution"* as opposed to "privation" (or containing privation). This interpret[ation] simpler proof

~~If φ pos., not: (x) N~φ(x). Otherwise: φ(x) ⊃N x≠x hence x≠x positive not x=x contrary Ax ... or the existence of pos. prop.~~

* i.e. the disj. normal form in terms of elem. prop. contains a member without negation.

Scott's Version of Gödel's Axioms, Definitions and Theorems

Freie Universität Berlin

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2 A property necessarily implied by a positive property is positive: $\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$

Thm. T1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

Def. D1 A *God-like* being possesses all positive properties: $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

Axiom A3 The property of being God-like is positive: $P(G)$

Cor. C Possibly, God exists: $\Diamond\exists x G(x)$

Axiom A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$

Def. D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \leftrightarrow \phi(x) \wedge \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Thm. T2 Being God-like is an essence of any God-like being: $\forall x[G(x) \rightarrow G\ ess.\ x]$

Def. D3 *Necessary existence* of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$

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Difference to Gödel (who omits this conjunct)

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Modal operators are used

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second-order quantifiers

Gödel's God in TPTP THF


```
>
>
> Beweise-mit-Leo2 Notwendigerweise-existiert-Gott.p

Leo-II tries to prove
=====================

  Goedel's Theorem T3: "Necessarily, God exists"
  thf(thmT3,conjecture,
      ( v
      @ ( mbox
        @ ( mexists_ind
          @ ^ [X: mu] :
              ( g @ X ) ) ) )).

  Assumptions: D1, C, T2, D3, A5

 . searching for proof ..

 ********************
 *   Proof found    *
 ********************
 % SZS status Theorem for Notwendigerweise-existiert-Gott.p

 . generating proof object
```

GoedelGod.thy (~/Varia/Talks/2015-ReasoningWeb/DemoMaterial/DEMO-Isabelle/)

```
text {* QML formulas are translated as HOL terms of type @{typ "i ⇒ bool"}.
This type is abbreviated as @{text "σ"}. *}

  type_synonym σ = "(i ⇒ bool)"

text {* The classical connectives $\neg, \wedge, \rightarrow$, and $\forall$
(over individuals and over sets of individuals) and $\exists$ (over individuals) are
lifted to type $\sigma$. The lifted connectives are @{text "m¬"}, @{text "m∧"}, @{text "m→"},
@{text "∀"}, and @{text "∃"} (the latter two are modeled as constant symbols).
Other connectives can be introduced analogously. We exemplarily do this for @{text "m∨"} ,
@{text "m≡"}, and @{text "mL="} (Leibniz equality on individuals). Moreover, the modal
operators @{text "□"} and @{text "◇"}  are introduced. Definitions could be used instead of
abbreviations. *}

  abbreviation mnot :: "σ ⇒ σ" ("m¬") where "m¬ φ ≡ (λw. ¬ φ w)"
  abbreviation mand :: "σ ⇒ σ ⇒ σ" (infixr "m∧" 51) where "φ m∧ ψ ≡ (λw. φ w ∧ ψ w)"
  abbreviation mor :: "σ ⇒ σ ⇒ σ" (infixr "m∨" 50) where "φ m∨ ψ ≡ (λw. φ w ∨ ψ w)"
  abbreviation mimplies :: "σ ⇒ σ ⇒ σ" (infixr "m→" 49) where "φ m→ ψ ≡ (λw. φ w ⟶ ψ w)"
  abbreviation mequiv:: "σ ⇒ σ ⇒ σ" (infixr "m≡" 48) where "φ m≡ ψ ≡ (λw. φ w ⟷ ψ w)"
  abbreviation mforall :: "('a ⇒ σ) ⇒ σ" ("∀") where "∀ Φ ≡ (λw. ∀x. Φ x w)"
  abbreviation mexists :: "('a ⇒ σ) ⇒ σ" ("∃") where "∃ Φ ≡ (λw. ∃x. Φ x w)"
  abbreviation mLeibeq :: "μ ⇒ μ ⇒ σ" (infixr "mL=" 52) where "x mL= y ≡ ∀(λφ. (φ x m→ φ y))"
  abbreviation mbox :: "σ ⇒ σ" ("□") where "□ φ ≡ (λw. ∀v.  w r v ⟶ φ v)"
  abbreviation mdia :: "σ ⇒ σ" ("◇") where "◇ φ ≡ (λw. ∃v. w r v ∧ φ v)"

text {* For grounding lifted formulas, the meta-predicate @{text "valid"} is introduced. *}

  (*<*) no_syntax "_list" :: "args ⇒ 'a list" ("[(_)]") (*>*)
  abbreviation valid :: "σ ⇒ bool" ("[_]") where "[p] ≡ ∀w. p w"
```

100% Auto update Update Detach

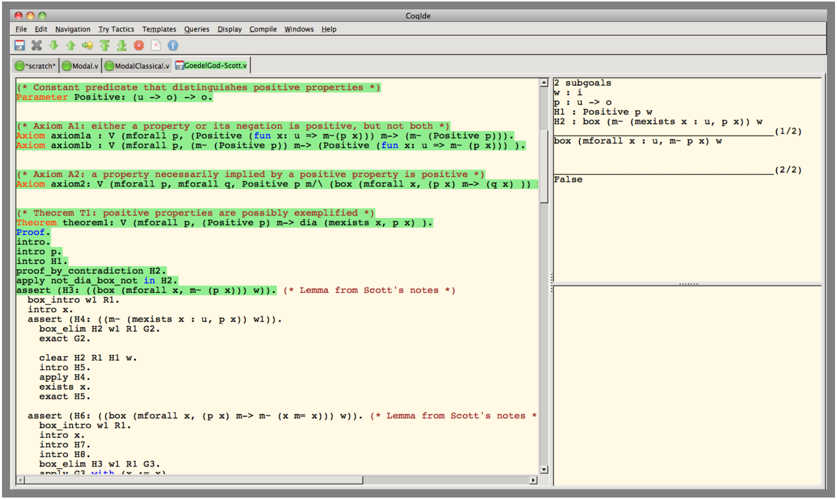
Output README Symbols

41,7 (1708/9125) (isabelle,sidekick,UTF-8-Isabelle) N m r o UG 68/124MB 6:28 PM

See verifiable Isabelle/HOL document (Archive of Formal Proofs) at:

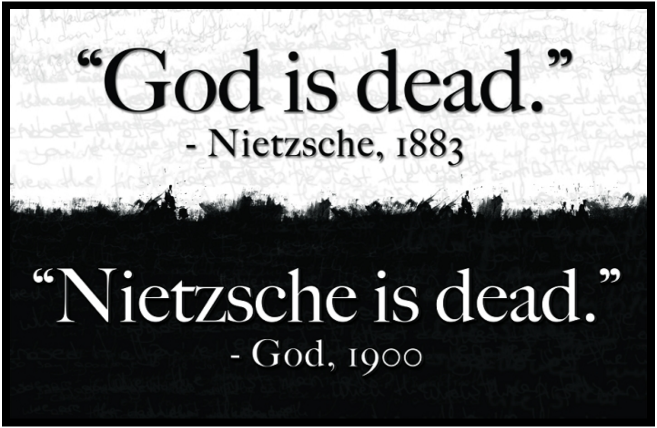
`http://afp.sourceforge.net/entries/GoedelGod.shtml`

Gödel's God in Coq



See verifiable Coq document at:

`https://github.com/FormalTheology/GoedelGod/tree/master/Formalizations/Coq`



Findings from our study

[BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot{\neg}(\phi X)) \dot{\equiv} \dot{\neg}(p\phi)]$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma}.\, \dot{\forall}\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\wedge}\, \dot{\Box}\dot{\forall}X_\mu.\, (\phi X \dot{\supset} \psi X)) \dot{\supset} p\psi]$						
T1	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Diamond}\dot{\exists}X_\mu.\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot{\Diamond}\dot{\exists}X_\mu.\, g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Box}p\phi]$						
D2	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \phi X \,\dot{\wedge}\, \dot{\forall}\psi_{\mu\to\sigma}.\, (\psi X \dot{\supset} \dot{\Box}\dot{\forall}Y_\mu.\, (\phi Y \dot{\supset} \psi Y))$						
T2	$[\dot{\forall}X_\mu.\, g_{\mu\to\sigma}X \dot{\supset} (\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathbf{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot{\forall}\phi_{\mu\to\sigma}.\, (\mathbf{ess}\, \phi X \dot{\supset} \dot{\Box}\dot{\exists}Y_\mu.\, \phi Y)$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathbf{NE}_{\mu\to\sigma}]$						
T3	$[\dot{\Box}\dot{\exists}X_\mu.\, g_{\mu\to\sigma}X]$	D1, C, T2, D3, A5	K	CSA	—/—	—/—	3.8/6.2
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \dot{\supset} \dot{\Box}s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot{\forall}\phi_{\mu\to\sigma}.\, \dot{\forall}X_\mu.\, (g_{\mu\to\sigma}X \dot{\supset} (\dot{\neg}(p_{(\mu\to\sigma)\to\sigma}\phi) \dot{\supset} \dot{\neg}(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot{\forall}X_\mu.\, \dot{\forall}Y_\mu.\, (g_{\mu\to\sigma}X \dot{\supset} (g_{\mu\to\sigma}Y \dot{\supset} X \dot{=} Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2’	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \dot{\forall}\psi_{\mu\to\sigma}.\, (\psi X \dot{\supset} \dot{\Box}\dot{\forall}Y_\mu.\, (\phi Y \dot{\supset} \psi Y))$						
CO’	$\emptyset$ (no goal, check for consistency)	A1(⊃), A2, D2’, D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2’, D3, A5	KB	UNS	—/—	—/—	—/—

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Freie Universität Berlin

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T1	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Diamond}\dot{\exists}X_\mu.\,\phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
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		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\,\dot{\forall}\phi_{\mu\to\sigma}.(\mathrm{ess}\,\phi X \dot{\supset} \dot{\Box}\dot{\exists}Y_\mu.\,\phi Y)$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot{\Box}\dot{\exists}X_\mu.\, g_{\mu\to\sigma}X]$	D1, C, T2, D3, A5	K	CSA	—/—	—/—	3.8/6.2
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \dot{\supset} \dot{\Box}s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot{\forall}\phi_{\mu\to\sigma}.\dot{\forall}X_\mu.(g_{\mu\to\sigma}X \dot{\supset} (\dot{\neg}(p_{(\mu\to\sigma)\to\sigma}\phi) \dot{\supset} \dot{\neg}(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot{\forall}X_\mu.\dot{\forall}Y_\mu.(g_{\mu\to\sigma}X \dot{\supset}(g_{\mu\to\sigma}Y \dot{\supset} X \dot{=} Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	∅ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\lambda X_\mu.\dot{\forall}\psi_{\mu\to\sigma}.(\psi X \dot{\supset} \dot{\Box}\dot{\forall}Y_\mu.(\phi Y \dot{\supset}\psi Y))$						
CO'	∅ (no goal, check for consistency)	A1(⊃), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu. \dot{\neg}(\phi X)) \doteq \dot{\neg}(p\phi)]$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma}. \dot{\forall}\psi_{\mu\to\sigma}. (p_{(\mu\to\sigma)\to\sigma}\phi \dot{\wedge} \dot{\Box}\dot{\forall}X_\mu. (\phi X \dot{\supset} \psi X)) \dot{\supset} p\psi]$						
T1	$[\dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Diamond}\dot{\exists}X_\mu. \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu. \dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma} g_{\mu\to\sigma}]$						
C	$[\dot{\Diamond}\dot{\exists}X_\mu. g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Box}p\phi]$						
D2	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}. \lambda X_\mu. \phi X \dot{\wedge} \dot{\forall}\psi_{\mu\to\sigma}. (\psi X \dot{\supset} \dot{\Box}\dot{\forall}Y_\mu. (\phi Y \dot{\supset} \psi Y))$						
T2	$[\dot{\forall}X_\mu. g_{\mu\to\sigma}X \dot{\supset} (\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} gX)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu. \dot{\forall}\phi_{\mu\to\sigma}. (e$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot{\Box}\dot{\exists}X_\mu. g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \dot{\supset} \dot{\Box}s_\sigma]$						
FG	$[\dot{\forall}\phi_{\mu\to\sigma}. \dot{\forall}X_\mu. (g_{\mu\to\sigma}X \dot{\supset} ($						
MT	$[\dot{\forall}X_\mu. \dot{\forall}Y_\mu. (g_{\mu\to\sigma}X \dot{\supset} (g_\mu$						
CO	∅ (no goal, check for cons						
D2'	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}. \lambda$						
CO'	∅ (no goal, check for cons						

Automating Scott's proof script

T1: "Positive properties are possibly exemplified" proved by LEO-II and Satallax

- **in logic: K**
- **from assumptions:**
  - **A1 and A2**
  - **A1(⊃) and A2**
- **notion of quantification**
  - **possibilist quantifiers (constant dom.)**
  - **actualist quantifiers for individuals (varying dom.)**

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma}\cdot p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu\cdot \dot{\neg}(\phi X)) \,\dot{\equiv}\, \dot{\neg}(p\phi)]$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma}\cdot\dot{\forall}\psi_{\mu\to\sigma}\cdot(p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\wedge}\, \dot{\Box}\dot{\forall}X_\mu\cdot(\phi X \,\dot{\supset}\, \psi X)) \,\dot{\supset}\, p\psi]$						
T1	$[\dot{\forall}\phi_{\mu\to\sigma}\cdot p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\supset}\, \dot{\Diamond}\dot{\exists}X_\mu\cdot\phi X]$	A1($\supset$), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu\cdot\dot{\forall}\phi_{\mu\to\sigma}\cdot p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\supset}\, \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot{\Diamond}\dot{\exists}X_\mu\cdot g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot{\forall}\phi_{\mu\to\sigma}\cdot p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\supset}\, \dot{\Box}p\phi]$						
D2	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}\cdot\lambda X_\mu\cdot\phi X \,\dot{\wedge}\, \dot{\forall}\psi_{\mu\to\sigma}\cdot(\psi X \,\dot{\supset}\, \dot{\Box}\dot{\forall}Y_\mu\cdot(\phi Y \,\dot{\supset}\, \psi Y))$						
T2	$[\dot{\forall}X_\mu\cdot g_{\mu\to\sigma}X \,\dot{\supset}\, (ess_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$NE_{\mu\to\sigma} = \lambda X_\mu\cdot\dot{\forall}\phi_{\mu\to\sigma}\cdot$(e…						
A5	$[p_{(\mu\to\sigma)\to\sigma}NE_{\mu\to\sigma}]$						
T3	$[\dot{\Box}\dot{\exists}X_\mu\cdot g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \,\dot{\supset}\, \dot{\Box}s_\sigma]$						
FG	$[\dot{\forall}\phi_{\mu\to\sigma}\cdot\dot{\forall}X_\mu\cdot(g_{\mu\to\sigma}X \,\dot{\supset}\,$ (…						
MT	$[\dot{\forall}X_\mu\cdot\dot{\forall}Y_\mu\cdot(g_{\mu\to\sigma}X \,\dot{\supset}\, (g_\mu$…						
CO	$\emptyset$ (no goal, check for cons…						
D2'	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}\cdot\lambda$…						
CO'	$\emptyset$ (no goal, check for cons…						

Automating Scott's proof script

C: "Possibly, God exists"
proved by LEO-II and Satallax

- in logic: K
- from assumptions:
  - T1, D1, A3
- for domain conditions:
  - possibilist quantifiers (constant dom.)
  - actualist quantifiers for individuals (varying dom.)

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

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	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall \phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu. \dot\neg(\phi X)) \dot\equiv \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\dot\forall\psi_{\mu\to\sigma}.(p_{(\mu\to\sigma)\to\sigma}\phi \dot\wedge \dot\Box\dot\forall X_\mu.(\phi X \dot\supset \psi X)) \dot\supset p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Diamond\dot\exists X_\mu.\phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\dot\forall\phi_{\mu\to\sigma}.p_{(\mu\to\sigma)\to\sigma}\phi\dot\supset\phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma} g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu. g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot\forall\phi_{\mu\to\sigma}.p_{(\mu\to\sigma)\to\sigma}\phi\dot\supset\dot\Box p\phi]$						
D2	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\lambda X_\mu.\phi X \dot\wedge \dot\forall\psi_{\mu\to\sigma}.(\psi X \dot\supset \dot\Box\dot\forall Y_\mu.(\phi Y\dot\supset\psi Y))$						
T2	$[\dot\forall X_\mu. g_{\mu\to\sigma}X \dot\supset (ess_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$NE_{\mu\to\sigma} = \lambda X_\mu.\dot\forall\phi_{\mu\to\sigma}.(e$						
A5	$[p_{(\mu\to\sigma)\to\sigma}NE_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu. g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \dot\supset \dot\Box s_\sigma]$						
FG	$[\dot\forall\phi_{\mu\to\sigma}.\dot\forall X_\mu.(g_{\mu\to\sigma}X\dot\supset ($						
MT	$[\dot\forall X_\mu.\dot\forall Y_\mu.(g_{\mu\to\sigma}X\dot\supset(g_\mu$						
CO	$\emptyset$ (no goal, check for cons						
D2'	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\lambda$						
CO'	$\emptyset$ (no goal, check for cons						

Automating Scott's proof script

T2: "Being God-like is an ess. of any God-like being" proved by LEO-II and Satallax

- **in logic: K**
- **from assumptions:**
  - **A1, D1, A4, D2**
- **for domain conditions:**
  - **possibilist quantifiers (constant dom.)**
  - **actualist quantifiers for individuals (varying dom.)**

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

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	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall \phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg(\phi X)) \dot\equiv \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\,\dot\forall\psi_{\mu\to\sigma}.\,(p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\wedge\, \dot\Box\dot\forall X_\mu.\,(\phi X \dot\supset \psi X)) \dot\supset p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Diamond\dot\exists X_\mu.\,\phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\,\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Box p\phi]$						
D2	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\,\lambda X_\mu.\,\phi X \,\dot\wedge\, \dot\forall\psi_{\mu\to\sigma}.\,(\psi X \dot\supset \dot\Box\dot\forall Y_\mu.\,(\phi Y \dot\supset \psi Y))$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma}X \dot\supset (\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/
D3	$NE_{\mu\to\sigma} = \lambda X_\mu.\,\dot\forall\phi_{\mu\to\sigma}.\,(e$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\, NE_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \dot\supset \dot\Box s_\sigma]$						
FG	$[\dot\forall\phi_{\mu\to\sigma}.\,\dot\forall X_\mu.\,(g_{\mu\to\sigma}X \dot\supset ($						
MT	$[\dot\forall X_\mu.\,\dot\forall Y_\mu.\,(g_{\mu\to\sigma}X \dot\supset (g_\mu$						
CO	$\emptyset$ (no goal, check for cons						
D2'	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\,\lambda$						
CO'	$\emptyset$ (no goal, check for cons						

Automating Scott's proof script

T3: "Necessarily, God exists"
proved by LEO-II and Satallax

- **in logic: KB**
- **from assumptions:**
  - **D1, C, T2, D3, A5**
- **for domain conditions:**
  - **possibilist quantifiers (constant dom.)**
  - **actualist quantifiers for individuals (varying dom.)**

For logic K we got a countermodel by Nitpick

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

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	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\dot\neg(\phi X)) \dot\equiv \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\dot\forall\psi_{\mu\to\sigma}.(p_{(\mu\to\sigma)\to\sigma}\phi \dot\wedge \dot\Box\dot\forall X_\mu.(\phi X \dot\supset \psi X)) \dot\supset p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Diamond\dot\exists X_\mu.\phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\dot\forall\phi_{\mu\to\sigma}.p_{(\mu\to\sigma)\to\sigma}\phi\dot\supset\phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma} g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu. g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot\forall\phi_{\mu\to\sigma}.p_{(\mu\to\sigma)\to\sigma}\phi\dot\supset\dot\Box p\phi]$						
D2	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\lambda X_\mu.\phi X\dot\wedge\dot\forall\psi_{\mu\to\sigma}.(\psi X\dot\supset\dot\Box\dot\forall Y_\mu.(\phi Y\dot\supset\psi Y))$						
T2	$[\dot\forall X_\mu. g_{\mu\to\sigma}X\dot\supset(ess_{(\mu\to\sigma)\to\mu\to\sigma}gX)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$NE_{\mu\to\sigma} = \lambda X_\mu.\dot\forall\phi_{\mu\to\sigma}.($						
A5	$[p_{(\mu\to\sigma)\to\sigma}NE_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu. g_{\mu\to\sigma}X]$						
MC	$[s_\sigma\dot\supset\dot\Box s_\sigma]$						
FG	$[\dot\forall\phi_{\mu\to\sigma}.\dot\forall X_\mu.(g_{\mu\to\sigma}X\dot\supset($						
MT	$[\dot\forall X_\mu.\dot\forall Y_\mu.(g_{\mu\to\sigma}X\dot\supset(g_\mu$						
CO	∅ (no goal, check for cons						
D2'	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\lambda$						
CO'	∅ (no goal, check for cons						

Automating Scott's proof script

Summary

- proof verified and automated
- KB is sufficient (critisized logic S5 not needed!)
- possibilist and actualist quantifiers (individuals)
- exact dependencies determined experimentally
- ATPs have found alternative proofs
  e.g. self-identity $\lambda x(x = x)$ is not needed

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

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	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu. \dot{\neg}(\phi X)) \dot{\equiv} \dot{\neg}(p\phi)]$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma}. \dot{\forall}\psi_{\mu\to\sigma}. (p_{(\mu\to\sigma)\to\sigma}\phi \dot{\wedge} \dot{\Box}\dot{\forall}X_\mu. (\phi X \dot{\supset} \psi X)) \dot{\supset} p\psi]$						
T1	$[\dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Diamond}\dot{\exists}X_\mu. \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu. \dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu-}$						
A3	$[p_{(\mu\to\sigma)\to\sigma} g_{\mu\to\sigma}]$						
C	$[\dot{\Diamond}\dot{\exists}X_\mu. g_{\mu\to\sigma} X]$						
A4	$[\dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Box}p$						
D2	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}. \lambda$						
T2	$[\dot{\forall}X_\mu. g_{\mu\to\sigma} X \dot{\supset} (ess_{(\mu\to\sigma)-}$						
D3	$NE_{\mu\to\sigma} = \lambda X_\mu. \dot{\forall}\phi_{\mu\to\sigma}. (e$						
A5	$[p_{(\mu\to\sigma)\to\sigma} NE_{\mu\to\sigma}]$						
T3	$[\dot{\Box}\dot{\exists}X_\mu. g_{\mu\to\sigma} X]$						
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \dot{\supset} \dot{\Box} s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot{\forall}\phi_{\mu\to\sigma}. \dot{\forall}X_\mu. (g_{\mu\to\sigma} X \dot{\supset} (\dot{\neg}(p_{(\mu\to\sigma)\to\sigma}\phi) \dot{\supset} \dot{\neg}(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot{\forall}X_\mu. \dot{\forall}Y_\mu. (g_{\mu\to\sigma} X \dot{\supset} (g_{\mu\to\sigma} Y \dot{\supset} X \dot{=} Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}. \lambda X_\mu. \dot{\forall}\psi_{\mu\to\sigma}. (\psi X \dot{\supset} \dot{\Box}\dot{\forall}Y_\mu. (\phi Y \dot{\supset} \psi Y))$						
CO'	$\emptyset$ (no goal, check for consistency)	A1(⊃), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

Consistency check: Gödel vs. Scott

- Scott's assumptions are consistent; shown by Nitpick
- Gödel's assumptions are **inconsistent**; shown by LEO-II (**new philosophical result!**)

Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot{\neg}(\phi X)) \dot{\equiv} \dot{\neg}(p\phi)]$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma}.\, \dot{\forall}\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\wedge}\, \dot{\Box}\dot{\forall}X_\mu.\, (\phi X \dot{\supset} \psi X)) \dot{\supset} p\psi]$						
T1	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Diamond}\dot{\exists}X_\mu.\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to \dots}$ … $\phi \dot{\supset} \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma} g_{\mu\to\sigma}]$						
C	$[\dot{\Diamond}\dot{\exists}X_\mu.\, g_{\mu\to\sigma}X]$						
A4	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Box}p \dots$						
D2	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda \dots$						
T2	$[\dot{\forall}X_\mu.\, g_{\mu\to\sigma}X \dot{\supset} (\mathrm{ess}_{(\mu\to\sigma)\dots}$						
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot{\forall}\phi_{\mu\to\sigma}.\, (\mathrm{e}\dots$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot{\Box}\dot{\exists}X_\mu.\, g_{\mu\to\sigma}X]$	D1, C, T2, D3, A5	K	CSA	—/—	—/—	3.8/6.2
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \dot{\supset} \dot{\Box}s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot{\forall}\phi_{\mu\to\sigma}.\, \dot{\forall}X_\mu.\, (g_{\mu\to\sigma}X \dot{\supset} (\dot{\neg}(p_{(\mu\to\sigma)\to\sigma}\phi) \dot{\supset} \dot{\neg}(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot{\forall}X_\mu.\, \dot{\forall}Y_\mu.\, (g_{\mu\to\sigma}X \dot{\supset} (g_{\mu\to\sigma}Y \dot{\supset} X \dot{=} Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \dot{\forall}\psi_{\mu\to\sigma}.\, (\psi X \dot{\supset} \dot{\Box}\dot{\forall}Y_\mu.\, (\phi Y \dot{\supset} \psi Y))$						
CO'	$\emptyset$ (no goal, check for consistency)	A1(⊃), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

Further Results

- Monotheism holds
- God is flawless

Modal Collapse (Sobel)

$$\forall\varphi(\varphi \supset \Box\varphi)$$

- proved by LEO-II and Satallax
- for possibilist and actualist quantification (ind.)

Main critique on Gödel's ontological proof:

- there are no contingent truths
- everything is determined / no free will

	HOL encoding						
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to}$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \mathbin{\dot\supset} \dot\Diamond\dot\exists$						
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu-}$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
A4	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \mathbin{\dot\supset} \dot\Box p$						
D2	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma}X \mathbin{\dot\supset} (\mathrm{ess}_{(\mu\to\sigma)}$						
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, (e$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \mathbin{\dot\supset} \dot\Box s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall X_\mu.\, (g_{\mu\to\sigma}X \mathbin{\dot\supset} (\dot\neg(p_{(\mu\to\sigma)\to\sigma}\phi) \mathbin{\dot\supset} \dot\neg(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot\forall X_\mu.\, \dot\forall Y_\mu.\, (g_{\mu\to\sigma}X \mathbin{\dot\supset} (g_{\mu\to\sigma}Y \mathbin{\dot\supset} X \doteq Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \mathbin{\dot\supset} \dot\Box\dot\forall Y_\mu.\, (\phi Y \mathbin{\dot\supset} \psi Y))$						
CO'	$\emptyset$ (no goal, check for consistency)	A1($\supset$), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot{\neg}(\phi X)) \,\dot{\equiv}\, \dot{\neg}(p\phi)]$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma}.\, \dot{\forall}\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\wedge}\, \dot{\Box}\dot{\forall}X_\mu.\, (\phi X \,\dot{\supset}\, \psi X)) \,\dot{\supset}\, p\psi]$						
T1	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\supset}\, \dot{\Diamond}\dot{\exists}X_\mu.\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\supset}\, \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot{\Diamond}\dot{\exists}X_\mu.\, g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot{\forall}\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\supset}\, \dot{\Box}p\phi]$						
D2	$\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \phi X \,\dot{\wedge}\, \dot{\forall}\psi_{\mu\to\sigma}.\, (\psi X \,\dot{\supset}\, \dot{\Box}\dot{\forall}Y_\mu.\, (\phi Y \,\dot{\supset}\, \psi Y))$						
T2	$[\dot{\forall}X_\mu.\, g_{\mu\to\sigma}X \,\dot{\supset}\, (\mathrm{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, gX)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot{\forall}\phi_{\mu\to\sigma}.\, (\mathrm{ess}\,\phi X \,\dot{\supset}\, \dot{\Box}\dot{\exists}Y_\mu.\, \phi Y)$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot{\Box}\dot{\exists}X_\mu.\, g_{\mu\to\sigma}X]$	D1, C, T2, D3, A5	K	CSA	—/—	—/—	3.8/6.2
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
			KB	THM	17.9/—	3.3/3.2	—/—
		3, A5	KB	THM	—/—	—/—	—/—
			KB	THM	16.5/—	0.0/0.0	—/—
		3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
			KB	THM	—/—	0.0/3.3	—/—
		3, A5	KB	THM	—/—	—/—	—/—
		3, A5	KB	SAT	—/—	—/—	7.3/7.4
			KB	UNS	7.5/7.8	—/—	—/—
		D3, A5	KB	UNS	—/—	—/—	—/—

Observation

- good performance of ATPs
- excellent match between argumentation granularity in papers and the reasoning strength of the ATPs

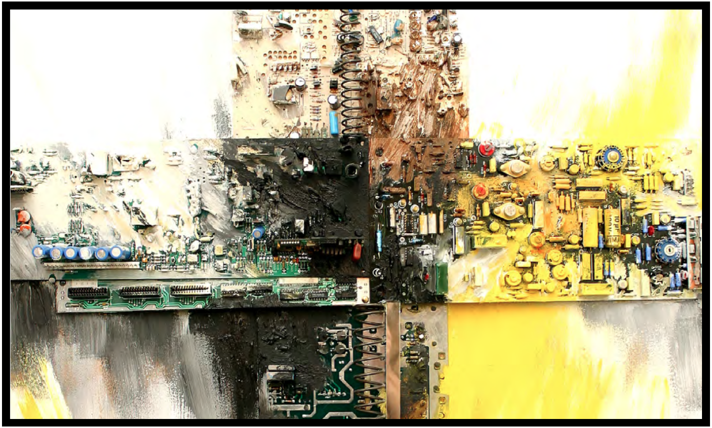
Main Findings [BenzmüllerWoltzenlogelPaleo, ECAI, 2014]

Freie Universität Berlin

Consistency check: Gödel vs. Scott

- **Scott's assumptions are consistent; shown by Nitpick**
- **Gödel's assumptions are inconsistent; shown by LEO-II (new philosophical result!)**

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall \phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg(\phi X)) \dot\equiv \dot\neg(p\phi)]$						
A2	$[\dot\forall \phi_{\mu\to\sigma}.\,\dot\forall\psi_{\mu\to\sigma}.\,(p_{(\mu\to\sigma)\to\sigma}\phi \dot\wedge \dot\Box\dot\forall X_\mu.(\phi X \dot\supset \psi X)) \dot\supset p\psi]$						
T1	$[\dot\forall \phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Diamond\dot\exists X_\mu.\,\phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\,\dot\forall\phi_{\mu\to\sigma}.\,p_{(\mu}$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
A4	$[\dot\forall \phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Box p$						
D2	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\,\lambda$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma}X \dot\supset (ess_{(\mu\to\sigma)}$						
D3	$NE_{\mu\to\sigma} = \lambda X_\mu.\,\dot\forall\phi_{\mu\to\sigma}.(e$						
A5	$[p_{(\mu\to\sigma)\to\sigma} NE_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \dot\supset \dot\Box s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot\forall\phi_{\mu\to\sigma}.\,\dot\forall X_\mu.(g_{\mu\to\sigma}X \dot\supset (\dot\neg(p_{(\mu\to\sigma)\to\sigma}\phi) \dot\supset \dot\neg(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot\forall X_\mu.\,\dot\forall Y_\mu.(g_{\mu\to\sigma}X \dot\supset (g_{\mu\to\sigma}Y \dot\supset X \dot= Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\,\lambda X_\mu.\,\dot\forall\psi_{\mu\to\sigma}.(\psi X \dot\supset \dot\Box\dot\forall Y_\mu.(\phi Y \dot\supset \psi Y))$						
CO'	$\emptyset$ (no goal, check for consistency)	A1(⊃), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—



Reconstruction of the Inconsistency of Gödel’s Axioms

Inconsistency (Gödel): Proof by LEO-II in KB

DemoMaterial — bash — 166×52

```
@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: $false))@SV3)=$true))),inference(prim_subst,[status(thm)],[66:[bind(SV11,$thf(^[SV23:mu,SV24:$i]: $false))]])).
 thf(84,plain,(![SV22:(mu>($i>$o)),SV3:$i,SV8:(mu>($i>$o))]: ((((SV8@(((sK2_SY33@SV3)@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV8))@(((sK1_SY31@(^[SX0:mu,SX1:$i]: (
~ ((SV22@SX0)@SX1))))@SV8)@SV3))=$true) | (((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV3)=$true))),inference(prim_subst,[status(thm)],[66
:[bind(SV11,$thf(^[SV20:mu,SV21:$i]: (~ ((SV22@SV20)@SV21))))]])).
 thf(85,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false) | ((((p@SV9)@SV4) = ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)
@SY28))))@SV4))=$false))),inference(fac_restr,[status(thm)],[56])).
 thf(86,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true) | ((((p@SV9)@SV4) = ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@
SY30))))@SV4))=$false))),inference(fac_restr,[status(thm)],[57])).
 thf(87,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((~ (((p@SV9)@SV4) | ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))) | (~ ((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY27:mu,
SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)))))=$false) | (((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_equal_neg,[status(thm)],[85])).
 thf(89,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((~ (((p@SV9)@SV4) | ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4))) | (~ ((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY29:mu,
SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)))))=$false) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true))),inference(extcnf_equal_neg,[status(thm)],[86])).
 thf(92,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((~ ((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))))=$false) | (((p@(^[SY27:mu,SY28:$i]: (
~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_or_neg,[status(thm)],[87])).
 thf(93,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((~ (((p@SV9)@SV4) | ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)))=$false) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@
SY29)@SY30))))@SV4)=$true))),inference(extcnf_or_neg,[status(thm)],[89])).
 thf(96,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)))=$true) | (((p@(^[SY27:mu,SY28:$i]: (~ ((S
V9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_not_neg,[status(thm)],[92])).
 thf(97,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((((p@SV9)@SV4) | ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4))=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)
@SY30))))@SV4)=$true))),inference(extcnf_not_neg,[status(thm)],[93])).
 thf(100,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((~ ((p@SV9)@SV4))=$true) | ((~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))=$true) | (((p@(^[SY27:mu,SY28:$i]:
(~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_or_pos,[status(thm)],[96])).
 thf(101,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@SV9)@SV4)=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV
9@SY29)@SY30))))@SV4)=$true))),inference(extcnf_or_pos,[status(thm)],[97])).
 thf(103,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@SV9)@SV4)=$false) | ((~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))=$true) | (((p@(^[SY27:mu,SY28:$i]: (~
((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_not_pos,[status(thm)],[100])).
 thf(105,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false) | (((p@SV9)@SV4)=$false) | (((p@(^[SY27:mu,SY28:$i]: (~ ((
SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_not_pos,[status(thm)],[103])).
 thf(107,plain,(![SV8:(mu>($i>$o)),SV3:$i,SV22:(mu>($i>$o))]: ((((SV22@(((sK2_SY33@SV3)@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV8))@(((sK1_SY31@(^[SX0:mu,SX1:$i]:
(~ ((SV22@SX0)@SX1))))@SV8)@SV3))=$true) | (((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV3)=$true))),inference(extcnf_not_neg,[status(thm
)],[78])).
 thf(108,plain,(![SV11:(mu>($i>$o)),SV3:$i,SV15:(mu>($i>$o))]: ((((SV15@(((sK2_SY33@SV3)@SV11)@(^[SX0:mu,SX1:$i]: (~ ((SV15@SX0)@SX1)))))@(((sK1_SY31@SV11)@(^[SX0:mu,
SX1:$i]: (~ ((SV15@SX0)@SX1))))@SV3))=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV15@SX0)@SX1))))@SV3)=$false) | (((p@SV11)@SV3)=$true))),inference(extcnf_not_pos,[statu
s(thm)],[81])).
 thf(109,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false) | (((p@SV9)@SV4)=$false))),inference(sim,[status(thm)],[10
5])).
 thf(110,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@SV9)@SV4)=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true))),inference(sim,[status(thm)],[101]
)).
 thf(111,plain,(![SV3:$i,SV8:(mu>($i>$o))]: ((((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: $true))@SV3)=$true))),inference(sim,[status(thm)],[76])).
 thf(112,plain,(![SV11:(mu>($i>$o)),SV3:$i]: ((((p@(^[SX0:mu,SX1:$i]: $false))@SV3)=$false) | (((p@SV11)@SV3)=$true))),inference(sim,[status(thm)],[80])).
 thf(113,plain,((($false)=$true)),inference(fo_atp_e,[status(thm)],[25,112,111,110,109,108,107,84,83,82,75,74,73,72,71,70,69,68,67,66,65,62,57,56,51,42,29])).
 thf(114,plain,($false),inference(solved_all_splits,[solved_all_splits(join,[])],[113])).
% SZS output end CNFRefutation

%***** End of derivation protocol ****
%***** no. of clauses in derivation: 97 ****
%***** clause counter: 113 ****

% SZS status Unsatisfiable for ConsistencyWithoutFirstConjunctinD2.p : (rf:0,axioms:6,ps:3,u:6,ude:false,rLeibEQ:true,rAndEQ:true,use_choice:true,use_extuni:true,use_
extcnf_combined:true,expand_extuni:false,foatp:e,atp_timeout:25,atp_calls_frequency:10,ordering:none,proof_output:1,clause_count:113,loop_count:0,foatp_calls:2,transl
ation:fof_full)
ontoleo:DemoMaterial cbenzmueller$
```

Inconsistency (Gödel): Verification in Isabelle/HOL (KB)

Freie Universität Berlin

GoedelGodWithoutConjunctInEss_KB.thy

GoedelGodWithoutConjunctInEss_KB.thy (~/GoedelGod/Talks/2015/LogicAndReligion/DemoMaterial/)

```
theory GoedelGodWithoutConjunctInEss_KB imports QML
begin
  consts P :: "(μ ⇒ σ) ⇒ σ"
  axiomatization where A1a: "[∀(λΦ. P (λx. m¬ (Φ x)) m→ m¬ (P Φ))]"
                   and A2:  "[∀(λΦ. ∀(λΨ. (P Φ m∧ □ (∀(λx. Φ x m→ Ψ x))) m→ P Ψ))]"

-- {* Positive properties are possibly exemplified. *}
  theorem T1: "[∀(λΦ. P Φ m→ ◇ (∃ Φ))]"                          by (metis A1a A2)

  definition ess (infixr "ess" 85) where "Φ ess x = ∀(λΨ. Ψ x m→ □ (∀(λy. Φ y m→ Ψ y)))"

-- {* The empty property is an essence of every individual. *}
  lemma Lemma1: "[(∀(λx.( (λy.λw. False) ess x)))]"               by (metis ess_def)

  definition NE  where "NE x = ∀(λΦ. Φ ess x m→ □ (∃ Φ))"
  axiomatization where sym: "x r y ⟶ y r x"

-- {* Exemplification of necessary existence is not possible. *}
  lemma Lemma2: "[m¬ (◇ (∃ NE))]"                       by (metis sym Lemma1 NE_def)

  axiomatization where A5:  "[P NE]"

-- {* Now the inconsistency follows from A5, T1 and Lemma2 *}
  lemma False                                              by (metis A5 T1 Lemma2)
end
```

Output Query Sledgehammer Symbols

Documentation Sidekick Theories

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Inconsistency (Gödel): Reconstruction of Informal Argument (KB)

(special thanks to Chad Brown for a fruitful discussion)

Axiom A1(⊃) $\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$

Axiom A2 $\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$

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Theorem 1 Positive Properties are possibly exemplified. $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

by A1(⊃), A2

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Freie Universität Berlin

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Def. D2* $\phi\ ess.\ x \leftrightarrow \cancel{\phi(x) \wedge}\ \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

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Lemma 1 The empty property is an essence of every entity. $\forall x\,(\emptyset\ ess.\ x)$

by D2*

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Def. D3 $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$

Axiom B $\forall\varphi(\varphi \rightarrow \Box\Diamond\varphi)$ (resp. $\forall x\forall y(rxy \rightarrow ryx)$)

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Lemma 2 Exemplification of necessary existence is not possible. $\neg\Diamond\exists x\, NE(x)$

by B, D3, Lemma1

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(special thanks to Chad Brown for a fruitful discussion)

Axiom A1($\supset$) $\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$

Axiom A2 $\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$

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by A1($\supset$), A2

Def. D2* $\phi\ ess.\ x \leftrightarrow \cancel{\phi(x) \wedge}\ \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Lemma 1 The empty property is an essence of every entity. $\forall x\,(\emptyset\ ess.\ x)$

by D2*

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Axiom A5 $P(NE)$

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(special thanks to Chad Brown for a fruitful discussion)

Axiom A1(⊃) $\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$

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Theorem 1 Positive Properties are possibly exemplified. $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

by A1(⊃), A2

Def. D2* $\phi\ ess.\ x \leftrightarrow \cancel{\phi(x) \wedge}\ \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

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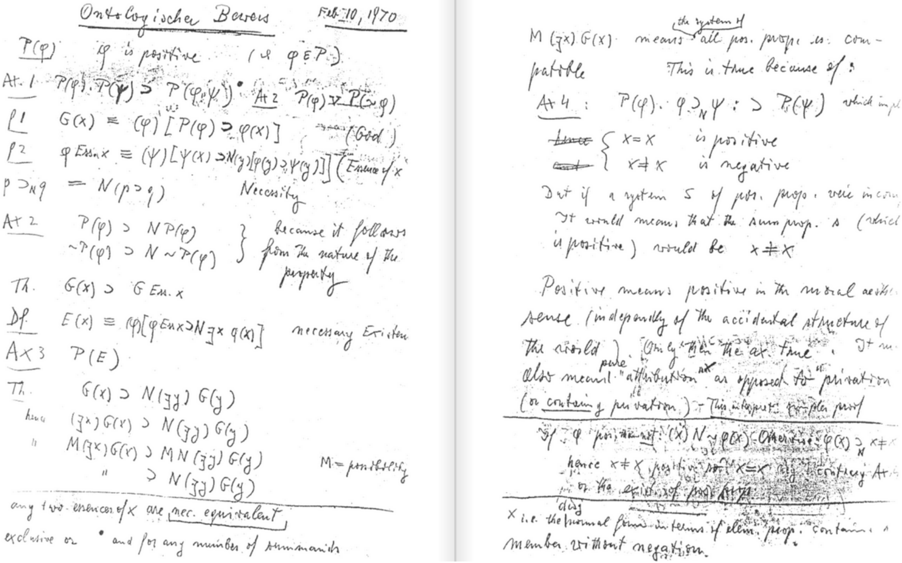
by B, D3, Lemma1

Axiom A5 $P(NE)$

Inconsistency $\bot$

by A5, T1, Lemma2

Gödel's Manuscript: Identifying the Inconsistent Axioms



Ontologischer Beweis Feb 10, 1970

$P(\varphi)$ φ is positive (or $\varphi \in P$)

At. 1 $P(\varphi).P(\psi) \supset P(\varphi.\psi)$* At 2 $P(\varphi) \vee P(\sim\varphi)$

P1 $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ (God)

P2 $\varphi\, Ess.x \equiv (\psi)[\psi(x) \supset N(y)[\varphi(y) \supset \psi(y)]]$ (Essence of x)

$p \supset_N q = N(p \supset q)$ Necessity

At 2 $P(\varphi) \supset NP(\varphi)$, $\sim P(\varphi) \supset N\sim P(\varphi)$ } because it follows from the nature of the property

Th. $G(x) \supset G\, Ess.\, x$

Df. $E(x) \equiv (\varphi)[\varphi\, Ess\, x \supset N \exists x\, \varphi(x)]$ necessary Existence

Ax 3 $P(E)$

Th. $G(x) \supset N(\exists y)\, G(y)$

hence $(\exists x)\, G(x) \supset N(\exists y)\, G(y)$

" $M(\exists x)G(x) \supset MN(\exists y)\, G(y)$ M = possibility

" $\supset N(\exists y)\, G(y)$

any two essences of x are nec. equivalent

exclusive or * and for any number of summands

$M(\exists x)\, G(x)$ means the system of all pos. prop. is compatible. This is true because of:

At 4: $P(\varphi).\ \varphi \supset_N \psi : \supset P(\psi)$ which impl

$\left\{ \begin{array}{ll} x = x & \text{is positive} \\ x \neq x & \text{is negative} \end{array} \right.$

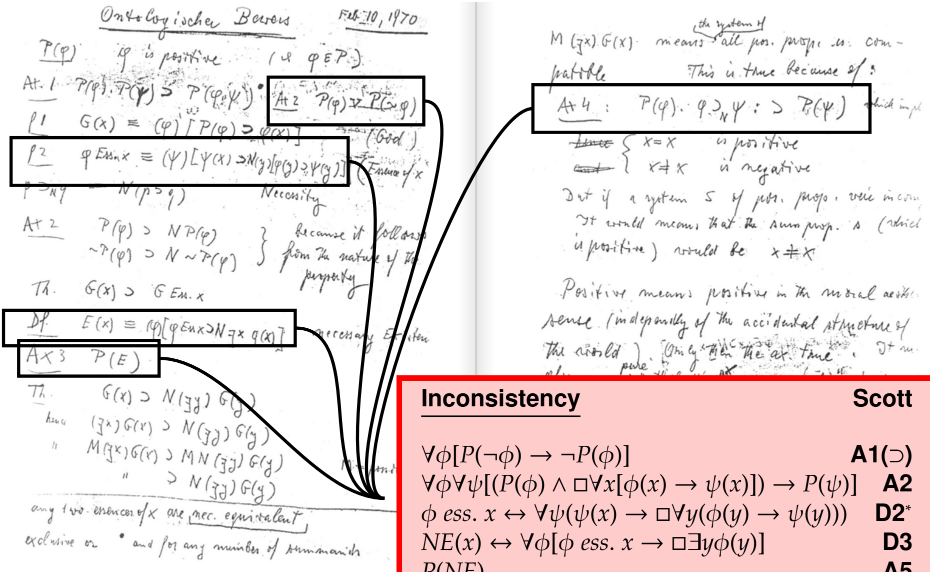
But if a system S of pos. prop. were incom it would mean, that the sum prop. s (which is positive) would be $x \neq x$

Positive means positive in the moral aesth sense (independently of the accidental structure of the world). Only then the ax. true. It m also mean pure "attribution" as opposed to "privation" (or containing privation). This interpret simpler proof

If φ pos. not: $(x)\, N \sim\varphi(x)$. Otherwise: $\varphi(x) \supset_N x \neq x$ hence $x \neq x$ positive not $x = x$ contrary At or the existence of pos. prop.

X i.e. the disj. normal form in terms of elem. prop. contains a member without negation.

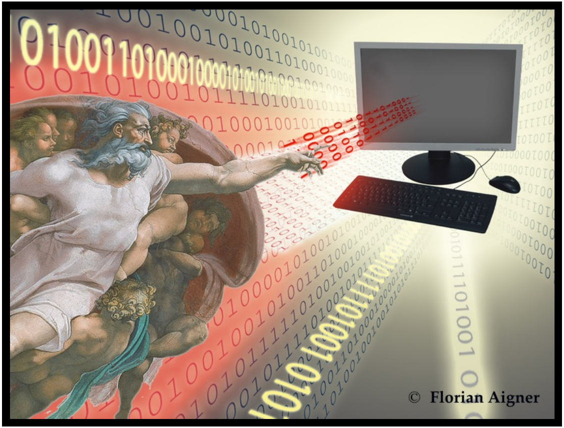
Gödel's Manuscript: Identifying the Inconsistent Axioms



Inconsistency

Scott

$\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$	**A1(⊃)**
$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$	**A2**
$\phi\ ess.\ x \leftrightarrow \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$	**D2***
$NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$	**D3**
$P(NE)$	**A5**



Avoiding the Modal Collapse — More Recent Papers

Avoiding the Modal Collapse: Recent Variants

SOME EMENDATIONS OF GÖDEL'S ONTOLOGICAL PROOF

C. Anthony Anderson

Kurt Gödel's version of the ontological argument was shown by J. Howard Sobel to be defective, but some plausible modifications in the argument result in a version which is immune to Sobel's objection. A definition is suggested which permits the proof of some of Gödel's axioms.

Gödel's Ontological Proof Revisited *

C. Anthony Anderson and Michael Gettings

University of California, Santa Barbara
Department of Philosophy

Gödel's version of the modal ontological argument for the existence of God has been criticized by J. Howard Sobel [5] and modified by C. Anthony Anderson [1]. In the present paper we consider the extent to which Anderson's emendation is defeated by the type of objection first offered by the Monk Gaunilo to St. Anselm's original Ontological Argument. And we try to push the analysis of this Gödelian argument a bit further to bring it into closer agreement with the details of Gödel's own formulation. Finally, we indicate what seems to be the main weakness of this emendation of Gödel's attempted proof.

Magari and others on Gödel's ontological proof

Petr Hájek
Institute of Computer Science, Academy of Sciences
182 07 Prague, Czech Republic
e-mail: hajek@uivt.cas.cz

1 Introduction

This paper is a continuation of my paper [H] and concentrates almost exclusively to mathematical properties of logical systems underlying Gödel's ontological proof [G] and its variant by Anderson [A], with special care paid to Magari's criticism [M]. Since [H] is written in German, we shall try to summarize its content in such a way that knowledge of [H] will be not obligatory for reading the present paper (even it remains advantageous). Here we describe

Der Mathematiker und die Frage der Existenz Gottes
(betreffend Gödels ontologischen Beweis)

Es ist gut, daß wir nicht wissen,
sondern glauben, daß ein Gott sei.
(Kant, Nachlaß)

1. Einführung

Gödels zu Lebzeiten unveröffentlichter Beweis für die notwendige Existenz eines Gott-ähnlichen Wesens hat sowohl philosophisches als auch mathematisches Interesse geweckt. Zweck der vorliegenden Arbeit ist es, zu einer Deutung des Gödelschen Textes beizutragen, 1. durch Kommentierung der einschlägigen Literatur und 2. durch Bereitstellung von etwas Modelltheorie. Die Arbeit enthält keinen philosophischen Beitrag. Während der letzten Jahre habe ich etliche Male über Gödels Gottesbeweis vorgetragen, insbesondere auf dem Symposium zur Feier von Professor Gert Müller (Heidelberg, Jänner 1991), doch habe ich niemals beabsichtigt, eine Veröffentlichung über das Thema zu machen. Da ich wiederholt um eine schriftliche Version gebeten wurde, entschloß ich mich, schnell eine „erweiterte Kurzfassung"[1] zu schreiben, ohne aus ihr einen

PETR HÁJEK

A New Small Emendation of Gödel's Ontological Proof

Keywords: Ontological proof, Gödel, modal logic, comprehension, positive properties.

1. Introduction

Gödel's ontological proof of necessary existence of a godlike being was finally published in the third volume of Gödel's collected works [7]; but it became known in 1970 when Gödel showed the proof to Dana Scott and Scott presented it (in fact a variant of it) at a seminar at Princeton. Detailed history is found in Adams' introductory remarks to the ontological proof in [7]. The proof uses modal logic and its analysis is an exciting exercise in systems of formal modal logic. Needles to say, formal modal logic has found several

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Computer-supported Clarification of Controversy
1st World Congress on Logic and Religion, 2015

Results Obtained with Fully Automated Reasoners

A controversy between Magari, Hájek and Anderson regarding the redundancy of some axioms

Proof	D1'	A:A1'	A2'	A3'	A4	A4'	H:A4	A5	A5'	H:A5	T3	T3'	MC
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (K4B)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (K4B)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Anderson (const)	-	-	-	-	R (K4B)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (K4B)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

Leibniz (1646–1716)

characteristica universalis *and* ***calculus ratiocinator***

If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other . . . : Let us calculate.

But: **Intuitive proofs/models are needed to convince philosophers**

Discussion

- LEO-II detected relevant new knowledge:
  Inconsistency in Gödel's original ontological argument
  Key step: 'non-analytic' instantiation of a second-order variable!
- LEO-II's proof object actually contains the proof idea
- first: failed to identify the relevant puzzle pieces
- only later: extracted easily accessible abstract-level proof
- Once a beautiful structure has been revealed it can't be missed anymore
- **Unmated low-level formal proofs, in contrast, are lacking persuasive power**
  Cut-introduction instead of cut-elimination!

We need (better) tools and means to bridge between machine-oriented and human-intuitive proofs and (counter-)models

Philosophers nevertheless seem interested

Conclusion

Overall Achievements

- significant contribution towards a **Computational Metaphysics**
- **novel results** contributed by **HOL-ATPs**
- infrastructure can be adapted for **other logics and logic combinations**
- **basic technology works well**; however, improvements still needed

Relevance (wrt foundations and applications)

- Philosophy, AI, Computer Science, Computational Linguistics, Maths

Little related work: only for Anselm's simpler argument

- first-order ATP PROVER9 [OppenheimerZalta, 2011]
- interactive proof assistant PVS [Rushby, 2013]

Ongoing/Future work

- Landscape of verified/falsified ontological arguments
- You may consider to contribute:
  `https://github.com/FormalTheology/GoedelGod.git`

(Interim) Culmination of two decades of related own research

- Theory of classical higher-order logic (HOL) (since 1995)
- Automation of HOL / own LEO provers (since 1998)
- Integration of interactive and automated theorem proving (since 1999)
- International TPTP infrastructure for HOL (since 2006)
- HOL as a universal logic via semantic embeddings (since 2008)
- jww Bruno Woltzenlogel-Paleo:
  Application in Metaphysics: Ontological Argument (since 2013)

. . . success story (despite strong criticism/opposition on the way!) . . .
. . . huge media attention . . .

(Interim) Own standpoint

- I am not fully convinced (yet) by the ontological argument.
- However, it seems to me that **the belief in a (God-like) supreme being is at least not necessarily irrational/inconsistent**.