Computational Metaphysics: The Virtues of Formal Computer Proofs Beyond Maths

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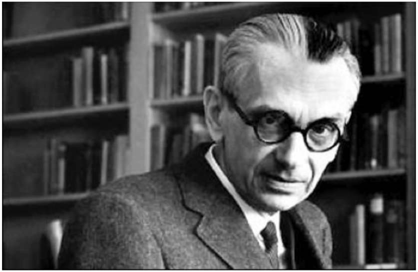

"There is a scientific (exact) philosophy and theology, which deals with concepts of the highest abstractness; and this is also most highly fruitful for science."

- Kurt Gödel (Wang, 1996)[p. 316]

2nd CLE4Science, UNICAMP, Brasil, 2017

Presentation Structure

A Logic Zoo

B **Universal Logic Reasoning in Classical HO Logic**

C **Computational Metaphysics**

D Conclusion

Part A
Logic Zoo

"If we had it [a *characteristica universalis*], we should be able to reason in metaphysics and morals in much the same way as in geometry and analysis."

(Leibniz, 1677)

(A) Logic Zoo

Classical Logic, of order

0. Propositional Logic
1. First-order Logic
2. Second-order Logic

...

n. Higher-order Logic

Non-Classical Logics

- Intuitionistic/Constructive Logics (incl. Univalent Foundations)
- Modal Logics, Conditional Logics, Temporal Logis, Spatial Logics
- Many-valued Logics
- Paraconsistent Logics
- Free Logics, Inclusive Logics
- Logics for special applications: Ethics, Social Choice, Legal Reasoning, ...
- ...

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Can the set $N = \{1, 2, \ldots, n\}$ be divided into two parts such that no part contains a Pythagorean triple $(\mathbf{a}, \mathbf{b}, \mathbf{c})$ with $\mathbf{a}^2 + \mathbf{b}^2 = \mathbf{c}^2$?

$n = 40$

$$3^2 + 4^2 = 5^2$$
$$5^2 + 12^2 = 13^2$$
$$8^2 + 15^2 = 17^2$$
$$7^2 + 24^2 = 25^2$$
$$20^2 + 21^2 = 29^2$$
$$12^2 + 35^2 = 37^2$$

(choose color of the other numbers arbitrarily)

Example Application in Maths:

- Pythagorean Triples Problem solved by SAT-Solving in 2016 (Heule, Kullmann, Marek)

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Example Applications in Maths:

- First-order Set Theories: ZF, ZFC, ...

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Example Applications in Maths:

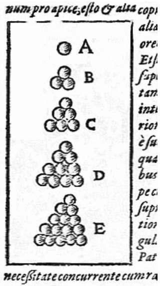
- Four-Colour Theorem: Formal verification by Gonthier in 2005 (Coq)
- Kepler's Conjecture: Formal verification by Hales in 2014 (HOL-light)

(A) Logic Zoo: Keplers Conjecture — Higher-order Logic —

Kepler (1571-1630)

The most compact way of stacking balls of the same size in space is a pyramid.

$$V\_ = \frac{\pi}{\sqrt{18}} \approx 74\%$$



- Proved in 1998 by Hales: 300 page proof, with code and data
- Submitted to the Annals of Mathematics: referees gave up to verify it all
- **Flyspeck project (completed in 2014):**
  - A formal verification of the proof in HOL Light
  - 27,223 proved theorems, 228 definitions, 30 person-years
- **Recent work of Cezary Kalyszik & Josef Urban:**
  Combination of **Machine Learning & Automated Theorem Proving**
  Result: more than 50% of the proofs can already be fully automated

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Example Application in Metaphysics/Philosophy: Rest of this talk!

Necessarily, God exists: $\Box\exists x.Gx$

Kurt Gödel's definition of God: $Gx \; := \; \forall\Phi.Positive\ \Phi \rightarrow \Phi x$

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Own Research: Utilise Higher-order Logic (HOL) as Meta-Logic

This turns

- HOL into a (quite) universal logic
- HOL provers into (quite) universal reasoners

Applications in Maths, CS, AI, Philosophy, Computational Linguistics, ...

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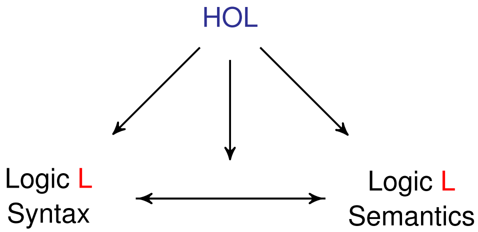
Higher-order Logic: Don't be afraid of it!

Paradoxes (e.g. Russel's Paradox) eliminated by **Types**
Incompleteness avoided by **Henkin Semantics**

Part B
Universal Logic Reasoning in HOL via Shallow Semantical Embeddings

jww:
Larry Paulson (Cambridge, UK), Bruno Woltzenlogel-Paleo (ANU, Australia),
Alex Steen and Max Wisniewski (both FU Berlin)

(B) Universal Logic Reasoning in HOL



Examples for L we have already studied:
Modal Logics, Description Logics, Conditional Logics, Intuitionistic Logics, Access Control Logics, Hybrid Logics, Multivalued Logics, Logics based on Neighborhood Semantics, (Mathematical) Fuzzy Logics, Paraconsistent Logics, Free Logic, Dyadic Logic, ...

Embedding works also for quantifiers (first-order & higher-order)

HOL provers become universal logic reasoning engines!

interactive:	Isabelle/HOL, PVS, HOL4, Hol Light, Coq/HOL, ...
automated:	TPS, LEO-II, Satallax, Nitpick, Isabelle/HOL, ...

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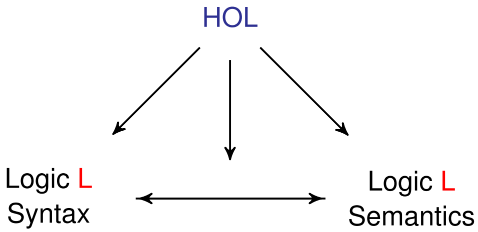

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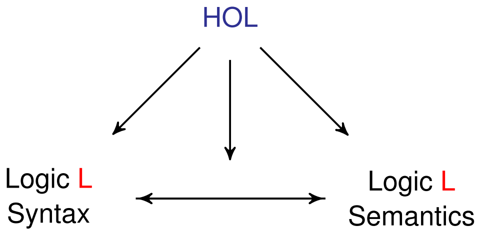

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(B) Universal Logic Reasoning in HOL

HOL (meta-logic) $\varphi ::=$

L (object-logic) $\psi ::=$

Embedding of in

$=$

$=$

$=$

$=$

$=$

Pass this set of equations to a HOL theorem prover

(B) Universal Logic Reasoning in HOL

HOL $\quad s,t \quad ::= \quad c_\alpha \mid x_\alpha \mid (\lambda x_\alpha s_\beta)_{\alpha\to\beta} \mid (s_{\alpha\to\beta}\, t_\alpha)_\beta \mid \neg s_o \mid s_o \vee t_o \mid \forall x_\alpha\, t_o$

HOML $\quad \varphi,\psi \quad ::= \quad \ldots \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \to \psi \mid \Box\varphi \mid \Diamond\varphi \mid \forall x_\gamma\, \varphi \mid \exists x_\gamma\, \varphi$

HOML in HOL: HOML formulas φ are mapped to HOL predicates $\varphi_{\mu\to o}$ (explicit representation of labelled formulas)

$$
\begin{array}{lcl}
\neg & = & \lambda\varphi_{\mu\to o}\lambda w_\mu \neg\varphi w \\
\wedge & = & \lambda\varphi_{\mu\to o}\lambda\psi_{\mu\to o}\lambda w_\mu(\varphi w \wedge \psi w) \\
\to & = & \lambda\varphi_{\mu\to o}\lambda\psi_{\mu\to o}\lambda w_\mu(\neg\varphi w \vee \psi w) \\
\Box & = & \lambda\varphi_{\mu\to o}\lambda w_\mu \forall u_\mu\, (\neg \boldsymbol{r} wu \vee \varphi u) \\
\Diamond & = & \lambda\varphi_{\mu\to o}\lambda w_\mu \exists u_\mu\, (\boldsymbol{r} wu \wedge \varphi u) \\
\forall & = & \lambda h_{\gamma\to(\mu\to o)}\lambda w_\mu \forall d_\gamma\, hdw \\
\exists & = & \lambda h_{\gamma\to(\mu\to o)}\lambda w_\mu \exists d_\gamma\, hdw \\
\text{valid} & = & \lambda\varphi_{\mu\to o}\forall w_\mu \varphi w
\end{array}
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Ax (polymorphic over γ)

The equations in Ax are given as axioms to the HOL provers!

(B) Universal Logic Reasoning in HOL

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(B) Universal Logic Reasoning in HOL

Standard translation is defined as follows:

- $ST_x(p) \equiv P(x)$, where p is an atomic formula; P(x) is true when p holds in world x.
- $ST_x(\top) \equiv \top$
- $ST_x(\bot) \equiv \bot$
- $ST_x(\neg\varphi) \equiv \neg ST_x(\varphi)$
- $ST_x(\varphi \wedge \psi) \equiv ST_x(\varphi) \wedge ST_x(\psi)$
- $ST_x(\varphi \vee \psi) \equiv ST_x(\varphi) \vee ST_x(\psi)$
- $ST_x(\varphi \to \psi) \equiv ST_x(\varphi) \to ST_x(\psi)$
- $ST_x(\Diamond_m \varphi) \equiv \exists y(R_m(x,y) \wedge ST_y(\varphi))$
- $ST_x(\Box_m \varphi) \equiv \forall y(R_m(x,y) \to ST_y(\varphi))$

$| \; \forall x_\alpha \, t_o$

$| \; \exists x_\gamma \, \varphi$

$\varphi_{\mu\to o}$

$$\Box \;=\; \lambda\varphi_{\mu\to o}\lambda w_\mu \forall u_\mu\,(\neg \mathbf{r} wu \vee \varphi u)$$
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(B) Universal Logic Reasoning in HOL

Example

HOML formula	$\Diamond\exists x G x$
HOML formula embedded in HOL	valid $(\Diamond\exists x G x)$
expansion	$(\lambda\varphi\forall w_\mu \varphi\, w)(\Diamond\exists x G x)$
$\beta\eta$-normalisation	$\forall w_\mu((\Diamond\exists x G x)\, w)$
expansion	$\forall w_\mu(((\lambda\varphi\lambda w_\mu \exists u_\mu\,(rwu \wedge \varphi u))\exists x G x)\, w)$
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What are we doing?

In order to prove that φ is valid in HOML,
–> we instead prove that valid φ can be derived from Ax in HOL.

This can be done with interactive or automated HOL theorem provers.

(B) Universal Logic Reasoning in HOL

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$\beta\eta$-normalisation	$\forall w_\mu((\Diamond \exists x G x)\, w)$
expansion	$\forall w_\mu(((\lambda \varphi \lambda w_\mu \exists u_\mu\, (\boldsymbol{r} w u \wedge \varphi u)) \exists x G x)\, w)$
$\beta\eta$-normalisation	$\forall w_\mu \exists u_\mu(\boldsymbol{r} w u \wedge (\exists x G x) u)$
syntactic sugar	$\forall w_\mu \exists u_\mu(\boldsymbol{r} w u \wedge (\exists(\lambda x G x)) u)$
expansion	$\forall w_\mu \exists u_\mu(\boldsymbol{r} w u \wedge ((\lambda h_{\gamma\to(\mu\to o)} \lambda w_\mu \exists d_\gamma\, h d w)(\lambda x G x)) u)$
$\beta\eta$-normalisation	$\forall w_\mu \exists u_\mu(\boldsymbol{r} w u \wedge \exists x G x u)$

What are we doing?

In order to prove that φ is valid in HOML,
–> we instead prove that valid φ can be derived from Ax in HOL.

This can be done with interactive or automated HOL theorem provers.

(B) Universal Logic Reasoning in Isabelle/HOL

GodProof.thy

GodProof.thy (~/chris/trunk/tex/talks/2016-BMS/)

```
theory GodProof imports Main
begin
 typedecl i -- "type for possible worlds"
 typedecl μ -- "type for individuals"
 type_synonym σ = "(i⇒bool)"

(* Shallow embedding modal logic connectives in HOL *)
 abbreviation mneg ("¬_"[52]53)     where "¬φ    ≡ λw. ¬φ(w)"
 abbreviation mand (infixr"∧"51)    where "φ∧ψ   ≡ λw. φ(w)∧ψ(w)"
 abbreviation mor (infixr"∨"50)     where "φ∨ψ   ≡ λw. φ(w)∨ψ(w)"
 abbreviation mimp (infixr"→"49)    where "φ→ψ  ≡ λw. φ(w)⟶ψ(w)"
 abbreviation mequ (infixr"↔"48)    where "φ↔ψ  ≡ λw. φ(w)⟷ψ(w)"
 abbreviation mnegpred ("¬_"[52]53) where "¬Φ    ≡ λx.λw. ¬Φ(x)(w)"

(* Shallow embedding of generic box and diamnond operators *)
 abbreviation mboxgen ("□") where "□r φ ≡ λw. ∀v.  r w v ⟶ φ(v)"
 abbreviation mdiagen ("◇") where "◇r φ ≡ λw. ∃v. r w v ∧ φ(v)"

(* Shallow embedding of constant domain quantifiers in HOL *)
 abbreviation mall_const ("∀c") where "∀c Φ ≡ λw.∀x. Φ(x)(w)"
 abbreviation mallB_const (binder"∀c"[8]9) where "∀c x. φ(x) ≡ ∀c φ"
 abbreviation mexi_const ("∃c") where "∃c Φ ≡ λw.∃x. Φ(x)(w)"
 abbreviation mexiB_const (binder"∃c"[8]9) where "∃c x. φ(x) ≡ ∃c φ"
```

Documentation | Sidekick | State | Theories

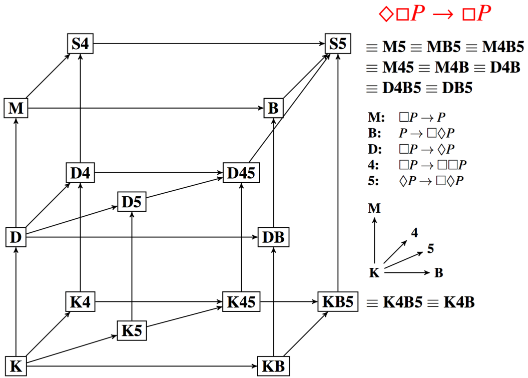
Proof state | Auto update | Update | Search: | 100%

Output | Query | Sledgehammer | Symbols

7,33 (185/4922) (isabelle,isabelle,UTF-8-Isabelle) N m r o UG 427/708MB 8:28 AM

(B) Universal Logic Reasoning in HOL

Which modal logic?



Which notion of quantification?

- possibilist quantifiers — constant domain semantics
- actualist quantifiers — varying domain semantics

"If we had it [a *characteristica universalis*], we should be able to reason in metaphysics and morals in much the same way as in geometry and analysis."

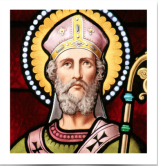
(Leibniz, 1677)

Part C
Computational Metaphysics

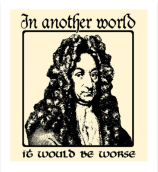
jww: Bruno Woltzenlogel-Paleo (ANU, Australia), and others

Ontological Proofs of God's Existence

A Long and Continuing Tradition in Philosophy

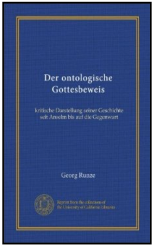

St. Anselm

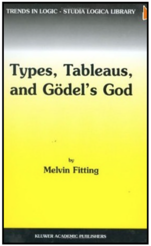

Descartes

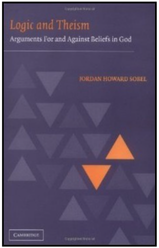


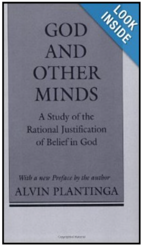
Leibniz

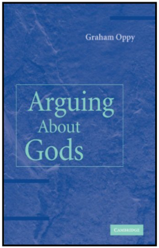
Gödel



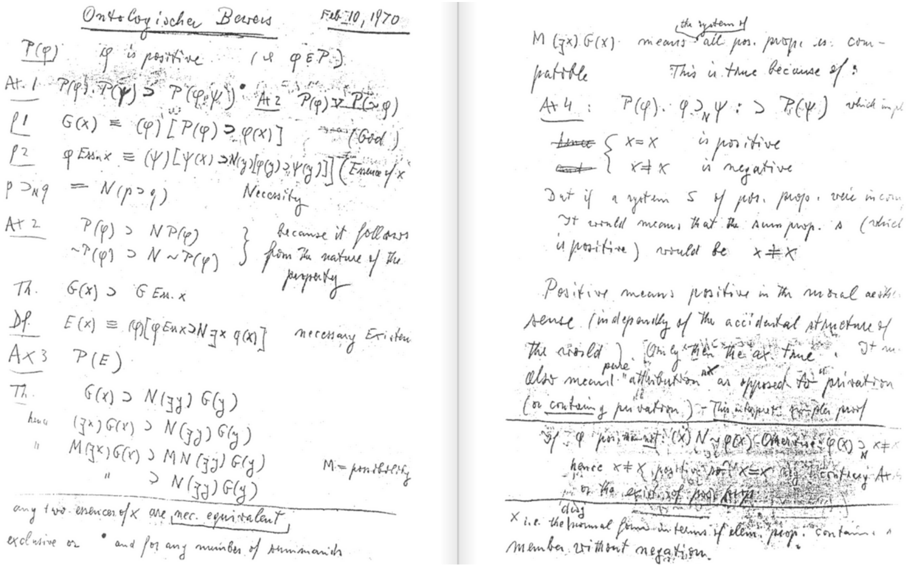








(C) Computational Metaphysics: Kurt Gödel's Ontological Argument



Ontologischer Beweis Feb 10, 1970

$P(\varphi)$ φ is positive ($\equiv \varphi \in P$)

At. 1 $P(\varphi).P(\psi) \supset P(\varphi.\psi)$* At 2 $P(\varphi) \vee P(\sim\varphi)$

P1 $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ (God)

P2 $\varphi \text{ Ess. } x \equiv (\psi)[\psi(x) \supset N(y)[\varphi(y) \supset \psi(y)]]$ (Essence of x)

$p \supset_N q = N(p \supset q)$ Necessity

At 2 $P(\varphi) \supset NP(\varphi)$, $\sim P(\varphi) \supset N\sim P(\varphi)$ } because it follows from the nature of the property

Th. $G(x) \supset G \text{ Ess. } x$

Df. $E(x) \equiv (\varphi)[\varphi \text{ Ess } x \supset N\,\exists x\, \varphi(x)]$ necessary Existence

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" $\supset N(\exists y)G(y)$ M = possibility

any two essences of x are nec. equivalent

exclusive or * and for any number of summands

$M(\exists x)G(x)$ means the system of all pos. prop. is compatible. This is true because of:

At 4: $P(\varphi).\varphi \supset_N \psi :\supset P(\psi)$ which impl

$x=x$ is positive; $x \neq x$ is negative

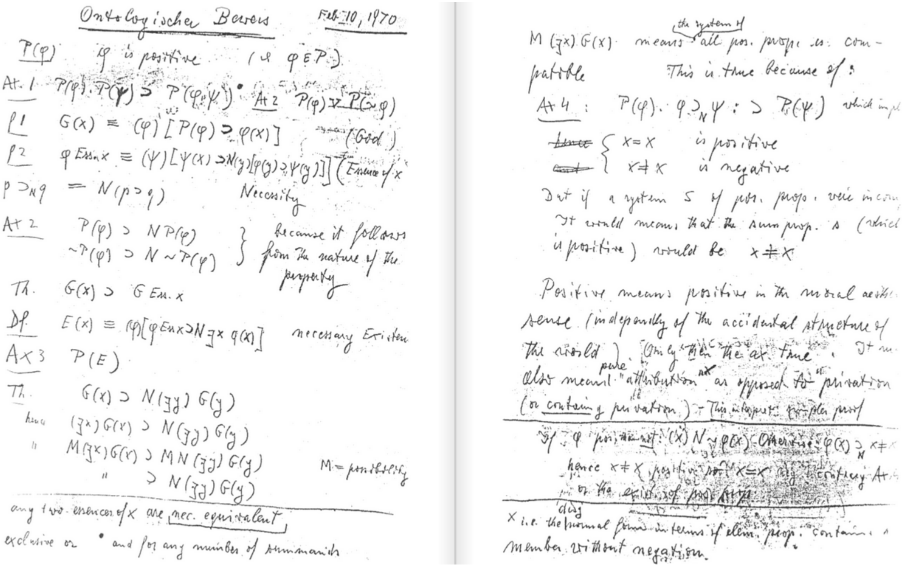
But if a system S of pos. prop. were incon. It would mean that the sum prop. s (which is positive) would be $x \neq x$

Positive means positive in the moral aesth. sense (independently of the accidental structure of the world). Only then the at. true. It may also mean pure "attribution"[x] as opposed to "privation" (or containing privation). This interpret. simpler proof

If φ pos. not: $(x) N \sim\varphi(x)$. Otherwise: $\varphi(x) \supset_N x \neq x$ hence $x \neq x$ positive not $x=x$ only contrary At. or the exist. of pos. prop.

x i.e. the disj. normal form in terms of elem. prop. contains a member without negation.

(C) Computational Metaphysics: Kurt Gödel's Ontological Argument


Ontologischer Beweis Feb 10, 1970

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Nachrichten > Wissenschaft > Mensch > Mathematik > Formel von Kurt Gödel: Mathematiker bestätigen Gottesbeweis

Formel von Kurt Gödel: Mathematiker bestätigen Gottesbeweis

Von Tobias Hürter

picture-alliance/ Imagno/ Wiener Stadt- und Landesbibliothek

Kurt Gödel (um das Jahr 1935): Der Mathematiker hielt seinen Gottesbeweis jahrzehntelang geheim

Ein Wesen existiert, das alle positiven Eigenschaften in sich vereint. Das bewies der legendäre Mathematiker Kurt Gödel mit einem komplizierten Formelgebilde. Zwei Wissenschaftler haben diesen Gottesbeweis nun überprüft - und für gültig befunden.

Montag, 09.09.2013 – 12:03 Uhr

Drucken | Versenden | Merken

Jetzt sind die letzten Zweifel ausgeräumt: Gott existiert tatsächlich. Ein Computer hat es mit kalter Logik bewiesen - das MacBook des Computerwissenschaftlers Christoph Benzmüller von der Freien Universität Berlin.

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Front Page World Europe Germany Business Zeitgeist Newsletter

English Site > Germany > Science > Scientists Use Computer to Mathematically Prove Gödel God Theorem

Holy Logic: Computer Scientists 'Prove' God Exists

By David Knight

picture-alliance/ Imagno/ Wiener Stadt- und Landesbibliothek

Austrian mathematician Kurt Gödel kept his proof of God's existence a secret for decades. Now two scientists say they have proven it mathematically using a computer.

Two scientists have formalized a theorem regarding the existence of God penned by mathematician Kurt Gödel. But the God angle is somewhat of a red herring -- the real step forward is the example it sets of how computers can make scientific progress simpler.

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DIE WELT

FREITAG, 18. OKTOBER 2013

Zippert zappt

Teuer erkaufte Harmonie

Mindestlohn lockt SPD in dritte Ehe mit der Union

Mittwoch soll über eine große Koalition verhandelt werden. Votum des SPD-Konvents steht aus

THEMEN

Sport
Bundestrainer Löw unterschreibt beim DFB bis 2016

Wirtschaft
Der Jumbojet wird bei Boeing zum Ladenhüter

Karriere
Stress im Job
So legen Sie sich einen dicken Panzer zu

Aus aller Welt
Mietprozess in Paris: Dürfen 15 Quadratmeter 330 Euro kosten?

Dax
Im Minus

Göttliche Mathematik

Haben Forscher mit einem Computerprogramm die Existenz Gottes bewiesen?

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Is God Real? Scientists 'Prove' His Existence With Godel's Theory And MacBooks

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Researchers say they used MacBook to prove Goedel's God theorem

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HOME / SCIENCE NEWS

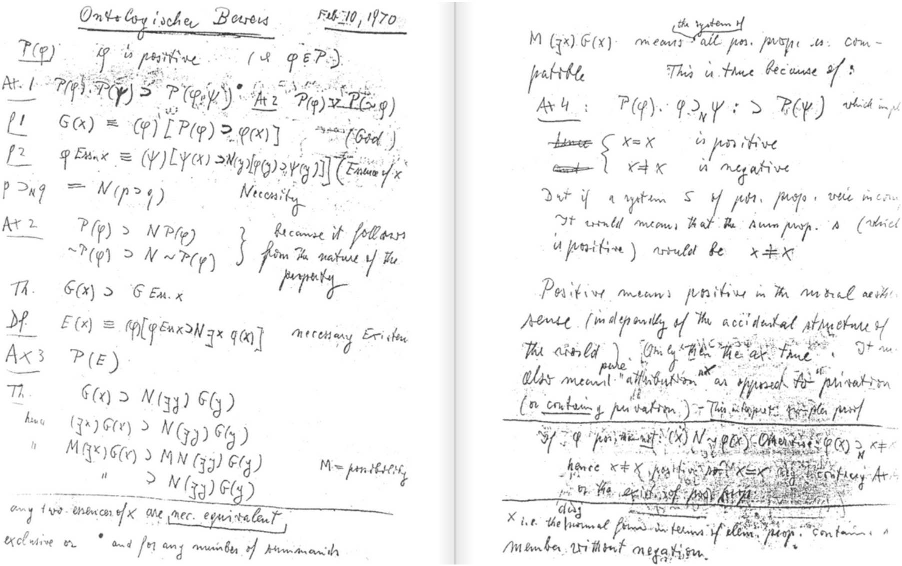
Researchers say they used MacBook to prove Goedel's God theorem

God exists, say Apple fanboy scientists

With the help of just one MacBook, two Germans formalize a theorem that confirms the existence of God.

See more serious and funny news links at
`https://github.com/FormalTheology/GoedelGod/blob/master/Press/LinksToNews.md`

(C) Computational Metaphysics: Kurt Gödel's Ontological Argument



Ontologischer Beweis   Feb 10, 1970

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$\left\{ \begin{array}{ll} x = x & \text{is positive} \\ x \neq x & \text{is negative} \end{array} \right.$

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* i.e. the disj. normal form in terms of elem. prop. contains a member without negation.

(C) Computational Metaphysics: Scott's Variant of Gödel's Ontological Argument

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2 A property necessarily implied by a positive property is positive:
$\forall\phi\forall\psi[(P(\phi) \land \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$

Thm. T1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

Def. D1 A *God-like* being possesses all positive properties: $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

Axiom A3 The property of being God-like is positive: $P(G)$

Cor. C Possibly, God exists: $\Diamond\exists x G(x)$

Axiom A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$

Def. D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \leftrightarrow \phi(x) \land \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Thm. T2 Being God-like is an essence of any God-like being: $\forall x[G(x) \rightarrow G\ ess.\ x]$

Def. D3 *Necessary existence* of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$

Axiom A5 Necessary existence is a positive property: $P(NE)$

Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

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Difference to Gödel (who omits this conjunct)

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Modal operators are used

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Thm. T3 Necessarily, God exists: $\Box\exists xG(x)$

second-order quantifiers

(C) Computational Metaphysics: Vision of Leibniz (1646–1716) — *Calculemus!*

Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo ... dicere: calculemus. (Leibniz, 1684)

If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other ...: Let us calculate.

(Translation by Russell)

Required:
characteristica universalis and **calculus ratiocinator**

(C) Computational Metaphysics: Scott's and Gödel's Variants — Demo

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2 A property necessarily implied by a positive property is positive:
$$\forall\phi\forall\psi[(P(\phi) \land \Box\forall x[\phi(x) \to \psi(x)]) \to P(\psi)]$$

Thm. T1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \to \Diamond\exists x\phi(x)]$

Def. D1 A *God-like* being possesses all positive properties: $G(x) \leftrightarrow \forall\phi[P(\phi) \to \phi(x)]$

Axiom A3 The property of being God-like is positive: $P(G)$

Cor. C Possibly, God exists: $\Diamond\exists x G(x)$

Axiom A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \to \Box P(\phi)]$

Def. D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \leftrightarrow \phi(x) \land \forall\psi(\psi(x) \to \Box\forall y(\phi(y) \to \psi(y)))$

Thm. T2 Being God-like is an essence of any God-like being: $\forall x[G(x) \to G\ ess.\ x]$

Def. D3 *Necessary existence* of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \to \Box\exists y\phi(y)]$

Axiom A5 Necessary existence is a positive property: $P(NE)$

Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

(C) Computational Metaphysics: Scott's and Gödel's Variants — Demo

Axiom A1 $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2

$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Thm. T1 $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

Def. D1 $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

Axiom A3 $P(G)$

Cor. C $\Diamond\exists x G(x)$

Axiom A4 $\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$

Def. D2

$$\phi\ ess.\ x \leftrightarrow \phi(x)\ \wedge\ \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$$

Thm. T2 $\forall x[G(x) \rightarrow G\ ess.\ x]$

Def. D3

$$NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$$

Axiom A5 $P(NE)$

Thm. T3 $\Box\exists x G(x)$

(C) Computational Metaphysics: Scott's and Gödel's Variants — Demo

Axiom A1 $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2

$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Def. D1 $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

Axiom A3 $P(G)$

Axiom A4 $\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$

Def. D2

$$\phi\ ess.\ x \leftrightarrow \phi(x)\ \wedge\ \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$$

Def. D3

$$NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$$

Axiom A5 $P(NE)$

Thm. T3 $\Box\exists xG(x)$

(C) Computational Metaphysics: Scott's and Gödel's Variants — Demo

GodProof.thy (~/chris/trunk/tex/talks/2016-BMS/)

```
 axiomatization where
  A1a: "⌊∀ᵖ Φ. P(¬Φ) ⟶ ¬P(Φ)⌋" and A1b: "⌊∀ᵖ Φ. ¬P(Φ) ⟶ P(¬Φ)⌋" and
  A2:  "⌊∀ᵖ Φ Ψ. P(Φ) ∧ □(∀x. Φ(x) ⟶ Ψ(x)) ⟶ P(Ψ)⌋"
theorem T1: "⌊∀ᵖΦ. P(Φ) ⟶ ◇(∃x. Φ(x))⌋" using A1a A2 by blast
 definition G where "G(x) = (∀ᵖ Φ. P(Φ) ⟶ Φ(x))"
 axiomatization where A3: "⌊P(G)⌋"
corollary C: "⌊◇(∃x. G(x))⌋" by (metis A3 T1)
 axiomatization where A4: "⌊∀ᵖ Φ. P(Φ) ⟶ □(P(Φ))⌋"
 definition ess (infixr "ess" 85) where "Φ ess x = Φ(x) ∧ (∀ᵖ Ψ. Ψ(x) ⟶ □(∀y. Φ(y) ⟶ Ψ(y)))"
theorem T2: "⌊∀x. G(x) ⟶ G ess x⌋" by (smt A1b A4 G_def ess_def)
 definition NE where "NE(x) = (∀ᵖ Φ. Φ ess x ⟶ □(∃x. Φ(x)))"
 axiomatization where A5:  "⌊P(NE)⌋"

(* T3: Necessarily, God exists *)
 theorem T3: "⌊□(∃x. G(x))⌋"

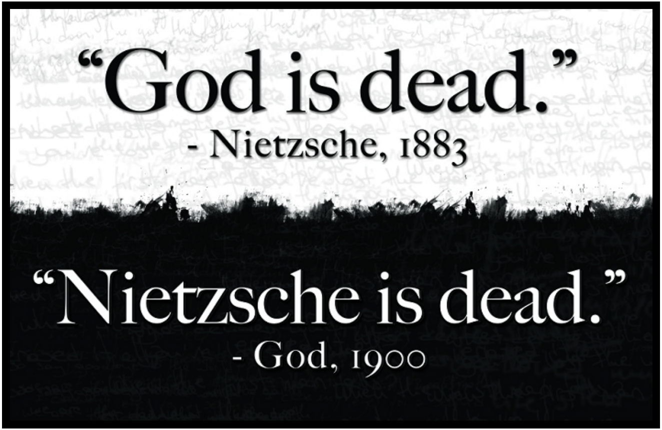

(* Check for Consistency *)
 lemma True nitpick [satisfy, user_axioms] oops
(* Check for Inconsistency *)
lemma False sledgehammer [remote_leo2,verbose]
```

Proof state | Auto update | Update | Search: | 100%

```
proof (prove)
goal (1 subgoal):
 1. ⌊(λw. ⌊S5 w ⟶ mexi_prop G⌋)⌋
```

Output | Query | Sledgehammer | Symbols

Documentation | Sidekick | State | Theories



Results of Experiments

see e.g. [ECAI-2014, IJCAI-2016]

(C) Computational Metaphysics: Results of Experiments

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma}\text{.}\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu\text{.}\, \dot{\neg}(\phi X)) \dot{\equiv} \dot{\neg}(p\phi)]$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma}\text{.}\, \dot{\forall}\psi_{\mu\to\sigma}\text{.}\, (p_{(\mu\to\sigma)\to\sigma}\phi \,\dot{\wedge}\, \dot{\Box}\dot{\forall}X_\mu\text{.}\, (\phi X \dot{\supset} \psi X)) \dot{\supset} p\psi]$						
T1	$[\dot{\forall}\phi_{\mu\to\sigma}\text{.}\, p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Diamond}\dot{\exists}X_\mu\text{.}\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu\text{.}\, \dot{\forall}\phi_{\mu\to\sigma}\text{.}\, p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot{\Diamond}\dot{\exists}X_\mu\text{.}\, g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot{\forall}\phi_{\mu\to\sigma}\text{.}\, p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Box}p\phi]$						
D2	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}\text{.}\, \lambda X_\mu\text{.}\, \phi X \,\dot{\wedge}\, \dot{\forall}\psi_{\mu\to\sigma}\text{.}\, (\psi X \dot{\supset} \dot{\Box}\dot{\forall}Y_\mu\text{.}\, (\phi Y \dot{\supset} \psi Y))$						
T2	$[\dot{\forall}X_\mu\text{.}\, g_{\mu\to\sigma}X \dot{\supset} (\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, g X)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathbf{NE}_{\mu\to\sigma} = \lambda X_\mu\text{.}\, \dot{\forall}\phi_{\mu\to\sigma}\text{.}\, (\mathbf{ess}\,\phi X \dot{\supset} \dot{\Box}\dot{\exists}Y_\mu\text{.}\, \phi Y)$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathbf{NE}_{\mu\to\sigma}]$						
T3	$[\dot{\Box}\dot{\exists}X_\mu\text{.}\, g_{\mu\to\sigma}X]$	D1, C, T2, D3, A5	K	CSA	—/—	—/—	3.8/6.2
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \dot{\supset} \dot{\Box}s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot{\forall}\phi_{\mu\to\sigma}\text{.}\, \dot{\forall}X_\mu\text{.}\, (g_{\mu\to\sigma}X \dot{\supset} (\dot{\neg}(p_{(\mu\to\sigma)\to\sigma}\phi) \dot{\supset} \dot{\neg}(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot{\forall}X_\mu\text{.}\, \dot{\forall}Y_\mu\text{.}\, (g_{\mu\to\sigma}X \dot{\supset} (g_{\mu\to\sigma}Y \dot{\supset} X \doteq Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2’	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}\text{.}\, \lambda X_\mu\text{.}\, \dot{\forall}\psi_{\mu\to\sigma}\text{.}\, (\psi X \dot{\supset} \dot{\Box}\dot{\forall}Y_\mu\text{.}\, (\phi Y \dot{\supset} \psi Y))$						
CO’	$\emptyset$ (no goal, check for consistency)	A1(⊃), A2, D2’, D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2’, D3, A5	KB	UNS	—/—	—/—	—/—

(C) Computational Metaphysics: Results of Experiments

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg(\phi X)) \dot\equiv \dot\neg(p\phi)]$						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \dot\wedge \dot\Box\dot\forall X_\mu.\, (\phi X \dot\supset \psi X)) \dot\supset p\psi]$						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Diamond\dot\exists X_\mu.\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
		A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \phi X$						
A3	$[p_{(\mu\to\sigma)\to\sigma} g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Box p\phi]$						
D2	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \phi X \dot\wedge \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \dot\supset \dot\Box\dot\forall Y_\mu.\, (\phi Y \dot\supset \psi Y))$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma}X \dot\supset (\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma}\, gX)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathrm{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, ($e						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathrm{NE}_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \dot\supset \dot\Box s_\sigma]$						
FG	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall X_\mu.\, (g\ \ X \dot\supset ($						
MT	$[\dot\forall X_\mu.\, \dot\forall Y_\mu.\, (g_{\mu\to\sigma}X \dot\supset (g_\mu$						
CO	∅ (no goal, check for cons						
D2'	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda$						
CO'	∅ (no goal, check for cons						

Automation of Scott's Variant

Summary

- Proof verified and automated
- Logic KB is sufficient (the often critised S5 not needed!)
- Possibilist & actualist quantification (for ind.)
- Exact dependencies determined
- Theorem provers found alternative Proofs
  e.g. self-identity $\lambda x(x = x)$ not needed

(C) Computational Metaphysics: Results of Experiments

Consistency: Gödel vs. Scott

- **Scott's assumptions are consistent; shown by Nitpick**
- **Gödel's assumptions are inconsistent; shown by LEO-II (new philosophical result)**

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot\forall \phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg(\phi X)) \dot\equiv \dot\neg(p\phi)]$						
A2	$[\dot\forall \phi_{\mu\to\sigma}.\, \dot\forall \psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\wedge\, \dot\Box\dot\forall X_\mu.\, (\phi X \dot\supset \psi X)) \dot\supset p\psi]$						
T1	$[\dot\forall \phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Diamond\dot\exists X_\mu.\, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall \phi_{\mu\to\sigma}.\, p_{(\mu-}$						
A3	$[p_{(\mu\to\sigma)\to\sigma} g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma} X]$						
A4	$[\dot\forall \phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \dot\supset \dot\Box p$						
D2	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda$						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma} X \dot\supset (\mathbf{ess}_{(\mu\to\sigma)}$						
D3	$\mathbf{NE}_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall \phi_{\mu\to\sigma}.\, (\mathrm{e}$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathbf{NE}_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma} X]$						
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \dot\supset \dot\Box s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot\forall \phi_{\mu\to\sigma}.\, \dot\forall X_\mu.\, (g_{\mu\to\sigma} X \dot\supset (\dot\neg(p_{(\mu\to\sigma)\to\sigma}\phi) \dot\supset \dot\neg(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot\forall X_\mu.\, \dot\forall Y_\mu.\, (g_{\mu\to\sigma} X \dot\supset (g_{\mu\to\sigma} Y \dot\supset X \dot= Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \dot\supset \dot\Box\dot\forall Y_\mu.\, (\phi Y \dot\supset \psi Y))$						
CO'	$\emptyset$ (no goal, check for consistency)	A1(⊃), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

(C) Computational Metaphysics: Results of Experiments

	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu. \dot{\neg}(\phi X)) \dot{\equiv} \dot{\neg}(p\phi)]$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma}. \dot{\forall}\psi_{\mu\to\sigma}. (p_{(\mu\to\sigma)\to\sigma}\phi \dot{\wedge} \dot{\Box}\dot{\forall}X_\mu. (\phi X \dot{\supset} \psi X)) \dot{\supset} p\psi]$						
T1	$[\dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Diamond}\dot{\exists}X_\mu. \phi X]$	A1($\supset$), A2	K	THM	0.1/0.1	0.0/0.0	—/—
D1	$g_{\mu\to\sigma} = \lambda X_\mu. \dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu-}$						
A3	$[p_{(\mu\to\sigma)\to\sigma} g_{\mu\to\sigma}]$						
C	$[\dot{\Diamond}\dot{\exists}X_\mu. g_{\mu\to\sigma} X]$						
A4	$[\dot{\forall}\phi_{\mu\to\sigma}. p_{(\mu\to\sigma)\to\sigma}\phi \dot{\supset} \dot{\Box}p$						
D2	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to}$ … $\lambda\phi_{\mu\to\sigma}. \lambda$						
T2	$[\dot{\forall}X_\mu. g_{\mu\to\sigma}X \dot{\supset} (\mathbf{ess}_{(\mu\to\sigma)}$						
D3	$\mathbf{NE}_{\mu\to\sigma} = \lambda X_\mu. \dot{\forall}\phi_{\mu\to\sigma}. (e$						
A5	$[p_{(\mu\to\sigma)\to\sigma}\mathbf{NE}_{\mu\to\sigma}]$						
T3	$[\dot{\Box}\dot{\exists}X_\mu. g_{\mu\to\sigma} X]$						
		A1, A2, D1, A3, A4, D2, D3, A5	K	CSA	—/—	—/—	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
MC	$[s_\sigma \dot{\supset} \dot{\Box}s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
FG	$[\dot{\forall}\phi_{\mu\to\sigma}. \dot{\forall}X_\mu. (g_{\mu\to\sigma}X \dot{\supset} (\dot{\neg}(p_{(\mu\to\sigma)\to\sigma}\phi) \dot{\supset} \dot{\neg}(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot{\forall}X_\mu. \dot{\forall}Y_\mu. (g_{\mu\to\sigma}X \dot{\supset} (g_{\mu\to\sigma}Y \dot{\supset} X \dot{=} Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2'	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}. \lambda X_\mu. \dot{\forall}\psi_{\mu\to\sigma}. (\psi X \dot{\supset} \dot{\Box}\dot{\forall}Y_\mu. (\phi Y \dot{\supset} \psi Y))$						
CO'	$\emptyset$ (no goal, check for consistency)	A1($\supset$), A2, D2', D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	—/—	—/—	—/—

Further Results

- Monotheism holds
- God is flawless
- ...

(C) Computational Metaphysics: Results of Experiments

Modal Collapse (detected first by Sobel)

$$\forall\varphi(\varphi \rightarrow \Box\varphi)$$

- proved by LEO-II and Satallax
- already in logic KB
- for possibilist and actualist quantification (ind.)

Modal Collapse can be read as:

- There are no contingent truths
- Everything is determined / there is no free will

	HOL encoding						
A1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}(\lambda X_\mu.\, \dot\neg$ …						
A2	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall\psi_{\mu\to\sigma}.\, (p_{(\mu\to\sigma)\to}$ …						
T1	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\supset\, \dot\Diamond\dot\exists$ …						
D1	$g_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu-}$ …						
A3	$[p_{(\mu\to\sigma)\to\sigma}\, g_{\mu\to\sigma}]$						
C	$[\dot\Diamond\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
A4	$[\dot\forall\phi_{\mu\to\sigma}.\, p_{(\mu\to\sigma)\to\sigma}\phi \,\dot\supset\, \dot\Box p$ …						
D2	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda$ …						
T2	$[\dot\forall X_\mu.\, g_{\mu\to\sigma}X \,\dot\supset\, (ess_{(\mu\to\sigma)-}$ …						
D3	$NE_{\mu\to\sigma} = \lambda X_\mu.\, \dot\forall\phi_{\mu\to\sigma}.\, (e$ …						
A5	$[p_{(\mu\to\sigma)\to\sigma}\, NE_{\mu\to\sigma}]$						
T3	$[\dot\Box\dot\exists X_\mu.\, g_{\mu\to\sigma}X]$						
MC	$[s_\sigma \,\dot\supset\, \dot\Box s_\sigma]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
FG	$[\dot\forall\phi_{\mu\to\sigma}.\, \dot\forall X_\mu.\, (g_{\mu\to\sigma}X \,\dot\supset\, (\dot\neg(p_{(\mu\to\sigma)\to\sigma}\phi) \,\dot\supset\, \dot\neg(\phi X)))]$	A1, D1	KB	THM	16.5/—	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	—/—
MT	$[\dot\forall X_\mu.\, \dot\forall Y_\mu.\, (g_{\mu\to\sigma}X \,\dot\supset\, (g_{\mu\to\sigma}Y \,\dot\supset\, X \doteq Y))]$	D1, FG	KB	THM	—/—	0.0/3.3	—/—
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	—/—	—/—	—/—
CO	$\emptyset$ (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	—/—	—/—	7.3/7.4
D2’	$ess_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda\phi_{\mu\to\sigma}.\, \lambda X_\mu.\, \dot\forall\psi_{\mu\to\sigma}.\, (\psi X \,\dot\supset\, \dot\Box\dot\forall Y_\mu.\, (\phi Y \,\dot\supset\, \psi Y))$						
CO’	$\emptyset$ (no goal, check for consistency)	A1($\supset$), A2, D2’, D3, A5	KB	UNS	7.5/7.8	—/—	—/—
		A1, A2, D1, A3, A4, D2’, D3, A5	KB	UNS	—/—	—/—	—/—

(C) Computational Metaphysics: Avoiding the Modal Collapse

SOME EMENDATIONS OF GÖDEL'S ONTOLOGICAL PROOF

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Kurt Gödel's version of the ontological argument was shown by J. Howard Sobel to be defective, but some plausible modifications in the argument result in a version which is immune to Sobel's objection. A definition is suggested which permits the proof of some of Gödel's axioms.

Gödel's Ontological Proof Revisited *

C. Anthony Anderson and Michael Gettings

University of California, Santa Barbara
Department of Philosophy

Gödel's version of the modal ontological argument for the existence of God has been criticized by J. Howard Sobel [5] and modified by C. Anthony Anderson [1]. In the present paper we consider the extent to which Anderson's emendation is defeated by the type of objection first offered by the Monk Gaunilo to St. Anselm's original Ontological Argument. And we try to push the analysis of this Gödelian argument a bit further to bring it into closer agreement with the details of Gödel's own formulation. Finally, we indicate what seems to be the main weakness of this emendation of Gödel's attempted proof.

Magari and others on Gödel's ontological proof

Petr Hájek
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182 07 Prague, Czech Republic
e-mail: hajek@uivt.cas.cz

1 Introduction

This paper is a continuation of my paper [H] and concentrates almost exclusively to mathematical properties of logical systems underlying Gödel's ontological proof [G] and its variant by Anderson [A], with special care paid to Magari's criticism [M]. Since [H] is written in German, we shall try to summarize its content in such a way that knowledge of [H] will be not obligatory for reading the present paper (even it remains advantageous). Here we describe

Der Mathematiker und die Frage der Existenz Gottes

(betreffend Gödels ontologischen Beweis)

Es ist gut, daß wir nicht wissen,
sondern glauben, daß ein Gott sei.
(Kant, Nachlaß)

1. Einführung

Gödels zu Lebzeiten unveröffentlichter Beweis für die notwendige Existenz eines Gott-ähnlichen Wesens hat sowohl philosophisches als auch mathematisches Interesse geweckt. Zweck der vorliegenden Arbeit ist es, zu einer Deutung des Gödelschen Textes beizutragen, 1. durch Kommentierung der einschlägigen Literatur und 2. durch Bereitstellung von etwas Modelltheorie. Die Arbeit enthält keinen philosophischen Beitrag. Während der letzten Jahre habe ich etliche Male über Gödels Gottesbeweis vorgetragen, insbesondere auf dem Symposium zur Feier von Professor Gert Müller (Heidelberg, Jänner 1991), doch habe ich niemals beabsichtigt, eine Veröffentlichung über das Thema zu machen. Da ich wiederholt um eine schriftliche Version gebeten wurde, entschloß ich mich, schnell eine „erweiterte Kurzfassung"[1] zu schreiben, ohne aus ihr einen

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A New Small Emendation of Gödel's Ontological Proof

Keywords: Ontological proof, Gödel, modal logic, comprehension, positive properties.

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Gödel's ontological proof of necessary existence of a godlike being was finally published in the third volume of Gödel's collected works [7]; but it became known in 1970 when Gödel showed the proof to Dana Scott and Scott presented it (in fact a variant of it) at a seminar at Princeton. Detailed history is found in Adams' introductory remarks to the ontological proof in [7]. The proof uses modal logic and its analysis is an exciting exercise in systems of formal modal logic. Needles to say, formal modal logic has found several

Understanding Gödel's Ontological Argument

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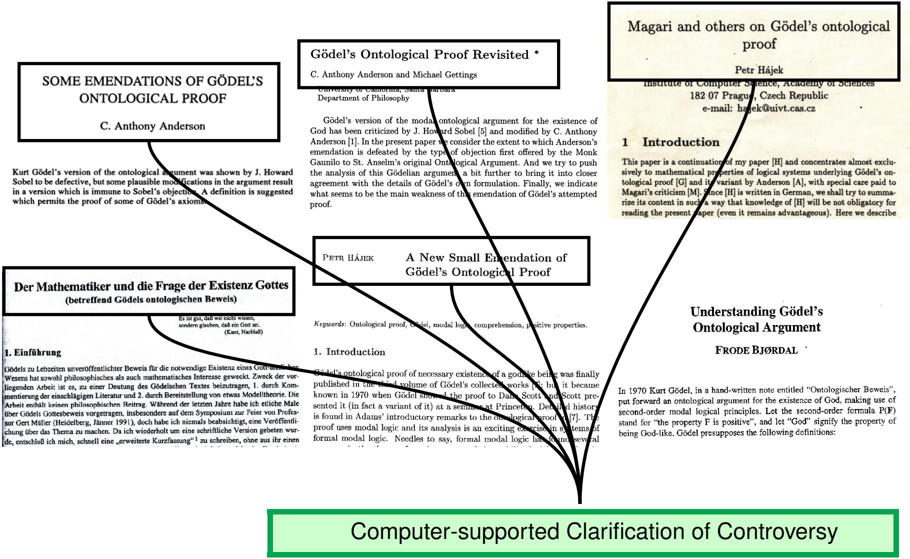
In 1970 Kurt Gödel, in a hand-written note entitled "Ontologischer Beweis", put forward an ontological argument for the existence of God, making use of second-order modal logical principles. Let the second-order formula P(F) stand for "the property F is positive", and let "God" signify the property of being God-like. Gödel presupposes the following definitions:

(C) Computational Metaphysics: Avoiding the Modal Collapse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(C) Computational Metaphysics: Avoiding the Modal Collapse

A controversy between Magari, Hájek and Anderson regarding the redundancy of some axioms

Proof	D1'	A:A1'	A2'	A3'	A4	A4'	H:A4	A5	A5'	H:A5	T3	T3'	MC
Scott (const)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (K4B)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (K4B)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

S/I = superfl. & indep.; R = superfl. & redund.; S/U = superfl. & unknown whether redund. or indep.; N/I = non-superfl. & indep.; P = provable; CS = counter-satisfiable

(C) Computational Metaphysics: Avoiding the Modal Collapse

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Scott (var)	-	-	-	-	N/I	-	-	N/I	-	-	P	-	P
Anderson (const)	-	-	-	-	R (K4B)	-	-	-	R	-	-	P	CS
Anderson (var)	-	-	-	-	R (K4B)	-	-	-	R	-	-	P	CS
Anderson (mix)	-	-	-	-	R (K4B)	-	-	-	I	-	-	CS	CS
Hájek AOE' (var)	-	-	CS		S/I	-	-	-	S/I	-	-	P (KB)	CS
Hájek AOE'_0 (var)	-	-	-	CS	R	-	-	-	S/U	-	-	P (KB)	CS
Hájek AOE'' (var)	-	-			-	-	S/I	-	-	S/I	-	P (KB)	CS
Anderson (simp) (var)	-	R	R			R (K4B)	-	-		-	-		
Bjørdal (const)	R (K4)	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS
Bjørdal (var)	CS	-	R	R	-	R (KT)	-	-	N/I	-	-	P (KB)	CS

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Leibniz (1646–1716)

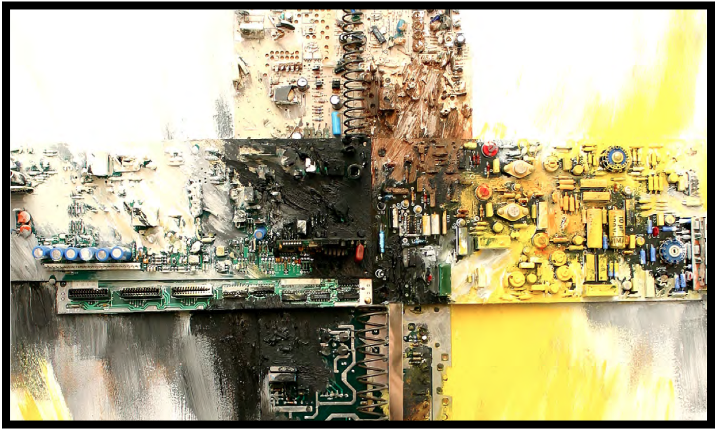
characteristica universalis *and* ***calculus ratiocinator***

If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other . . . : Let us calculate.

But: **Intuitive proofs/models are needed to convince philosophers**

(C) Computational Metaphysics: See our Recent Publications

- **Automating Emendations of the Ontological Argument in Intensional Higher-Order Modal Logic** KI 2017, Springer, LNCS, 2017.
- **Computer-Assisted Analysis of the Anderson-Hájek Controversy**, In Logica Universalis, 2017.
- **Analysis of an Ontological Proof Proposed by Leibniz**, Chapter in Death and Anti-Death, Volume 14, Ria University Press, 2017.
- **An Object-Logic Explanation for the Inconsistency in Gödel's Ontological Theory**, KI 2016, Springer, LNCS, 2016.
- **The Inconsistency in Gödel's Ontological Argument: A Success Story for AI in Metaphysics**, IJCAI 2016, AAAI Press, 2016.
- **The Modal Collapse as a Collapse of the Modal Square of Opposition**, Chapter in The Square of Opposition: A Cornerstone of Thought (Collection of papers related to the World Congress on the Square of Opposition IV, Vatican, 2014), Springer, Studies in Universal Logic, 2016.
- **On Logic Embeddings and Gödel's God**, WADT 2014, Springer, LNCS, 2015.
- **Experiments in Computational Metaphysics: Gödel's Proof of God's Existence**, In Science & Spiritual Quest, 9th All India Students' Conference, Bhaktivedanta Institute, Kolkata, 2015.
- **Invited Talk: On a (Quite) Universal Theorem Proving Approach and Its Application in Metaphysics**, TABLEAUX 2015, Springer, LNAI, 2015.
- **Automating Gödel's Ontological Proof of God's Existence with Higher-order Automated Theorem Provers**, ECAI 2014, IOS Press, Frontiers in Artificial Intelligence and Applications, 2014.
- ...



Rational Reconstruction of the Inconsistency of Gödel's Axioms

[Benzmüller&WoltzenlogelPaleo, IJCAI-2016]

(C) Computational Metaphysics: Rational Reconstruction of Inconsistency

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2 A property necessarily implied by a positive property is positive:
$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Thm. T1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

Def. D1 A *God-like* being possesses all positive properties: $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

Axiom A3 The property of being God-like is positive: $P(G)$

Cor. C Possibly, God exists: $\Diamond\exists x G(x)$

Axiom A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$

Def. D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \leftrightarrow \boxed{\cancel{\phi(x) \wedge}}\ \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Thm. T2 Being God-like is an essence of any God-like being: $\forall x[G(x) \rightarrow G\ ess.\ x]$

Def. D3 *Necessary existence* of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$

Axiom A5 Necessary existence is a positive property: $P(NE)$

Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

Difference to Gödel (who omits this conjunct)

(C) Computational Metaphysics: Rational Reconstruction of Inconsistency

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 thf(109,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false) | (((p@SV9)@SV4)=$false))),inference(sim,[status(thm)],[10
5])).
 thf(110,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@SV9)@SV4)=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true))),inference(sim,[status(thm)],[101]
)).
 thf(111,plain,(![SV3:$i,SV8:(mu>($i>$o))]: ((((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: $true))@SV3)=$true))),inference(sim,[status(thm)],[76])).
 thf(112,plain,(![SV11:(mu>($i>$o)),SV3:$i]: ((((p@(^[SX0:mu,SX1:$i]: $false))@SV3)=$false) | (((p@SV11)@SV3)=$true))),inference(sim,[status(thm)],[80])).
 thf(113,plain,((($false)=$true)),inference(fo_atp_e,[status(thm)],[25,112,111,110,109,108,107,84,83,82,75,74,73,72,71,70,69,68,67,66,65,62,57,56,51,42,29])).
 thf(114,plain,($false),inference(solved_all_splits,[solved_all_splits(join,[])],[113])).
% SZS output end CNFRefutation

%***** End of derivation protocol ****
%***** no. of clauses in derivation: 97 ****
%***** clause counter: 113 ****

% SZS status Unsatisfiable for ConsistencyWithoutFirstConjunctinD2.p : (rf:0,axioms:6,ps:3,u:6,ude:false,rLeibEQ:true,rAndEQ:true,use_choice:true,use_extuni:true,use_
extcnf_combined:true,expand_extuni:false,foatp:e,atp_timeout:25,atp_calls_frequency:10,ordering:none,proof_output:1,clause_count:113,loop_count:0,foatp_calls:2,transl
ation:fof_full)
ontoleo:DemoMaterial cbenzmueller$
```

(C) Computational Metaphysics: Rational Reconstruction of Inconsistency

[BenzmüllerWoltzenlogelPaleo, IJCAI, 2016]

Def. D2* $\phi\ ess.\ x \leftrightarrow \cancel{\phi(x) \wedge}\ \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

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Theorem T1 Positive Properties are possibly exemplified. $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

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- by T1, A5: $\Diamond\exists x[NE(x)]$

Def. D3 $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$

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Inconsistency $\bot$

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Axiom A5 Necessary existence is a positive property.

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Def. D3 Necessary existence of an entity is the necessary exemplification of all its essences.

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Def. D2* $\phi\ ess.\ x \leftrightarrow \cancel{\phi(x) \wedge}\ \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Lemma L1 $\boxed{\text{Selfdifference (empty property) is an essence of every entity.}}$

Theorem T1 Positive properties are possibly exemplified.

Axiom A5 Necessary existence is a positive property.

- by T1, A5: Necessary existence is possibly exemplified.

Def. D3 Necessary existence of an entity is the necessary exemplification of all its essences.

- by D3: Possibly there exists an entity whose essences are necessarily exemplified.
- by L1, simpl.: Possibly there exists an entity, such that necessarily there is an selfdifferent entity.
- selfdiff.: Possibly there exists an entity, such that necessarily Falsum.
- simplification Possibly, necessarily Falsum.

Inconsistency Falsum.

(C) Computational Metaphysics: Intuitive Proofs are needed!

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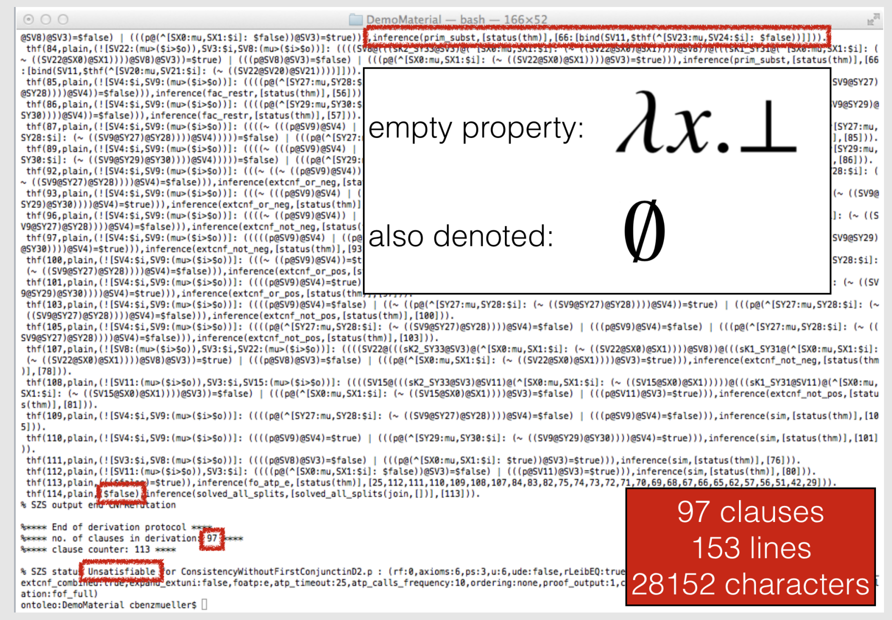
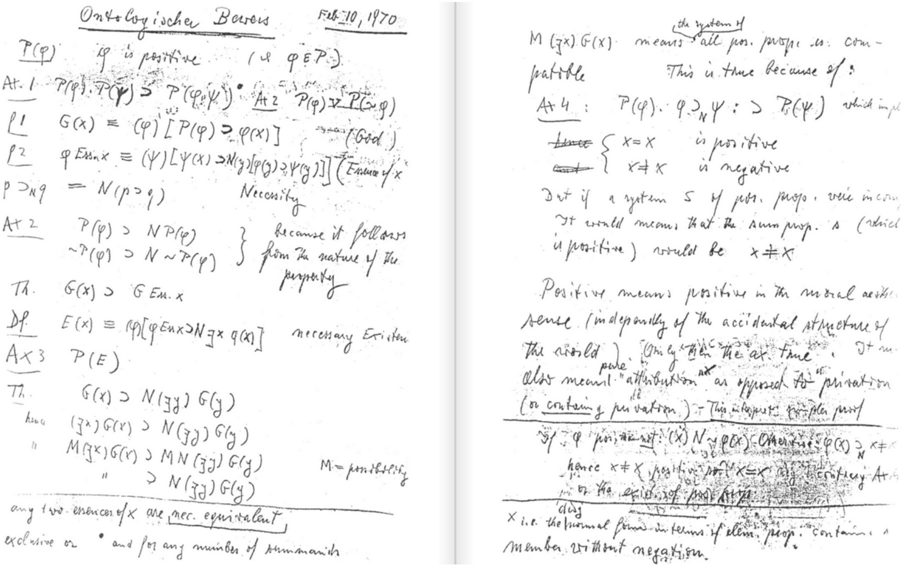
Inconsistency Falsum.

(C) Computational Metaphysics: Rational Reconstruction of Inconsistency



(C) Computational Metaphysics: Gödel's Manuscript — Inconsistent Axioms



Ontologischer Beweis — Feb 10, 1970

$P(\varphi)$: φ is positive (or $\varphi \in P$)

At. 1 $P(\varphi).P(\psi) \supset P(\varphi.\psi)$ * — At 2 $P(\varphi) \vee P(\sim\varphi)$

P1 $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ (God)

P2 $\varphi \, Ess. \, x \equiv (\psi)[\psi(x) \supset N(y)[\varphi(y) \supset \psi(y)]]$ (Essence of x)

$p \supset_N q = N(p \supset q)$ Necessity

At 2 $P(\varphi) \supset N P(\varphi)$, $\sim P(\varphi) \supset N \sim P(\varphi)$ — because it follows from the nature of the property

Th. $G(x) \supset G \, Ess. \, x$

Df. $E(x) \equiv (\varphi)[\varphi \, Ess \, x \supset N \exists x \, \varphi(x)]$ — necessary Existence

Ax 3 $P(E)$

Th. $G(x) \supset N(\exists y) G(y)$

hence $(\exists x) G(x) \supset N(\exists y) G(y)$

" $M(\exists x) G(x) \supset MN(\exists y) G(y)$

" $\supset N(\exists y) G(y)$ — M = possibility

any two essences of x are nec. equivalent

exclusive or * and for any number of summands

$M(\exists x) G(x)$ means the system of all pos. props. is compatible. This is true because of:

At 4 : $P(\varphi). \varphi \supset_N \psi : \supset P(\psi)$ which impl

$x = x$ is positive

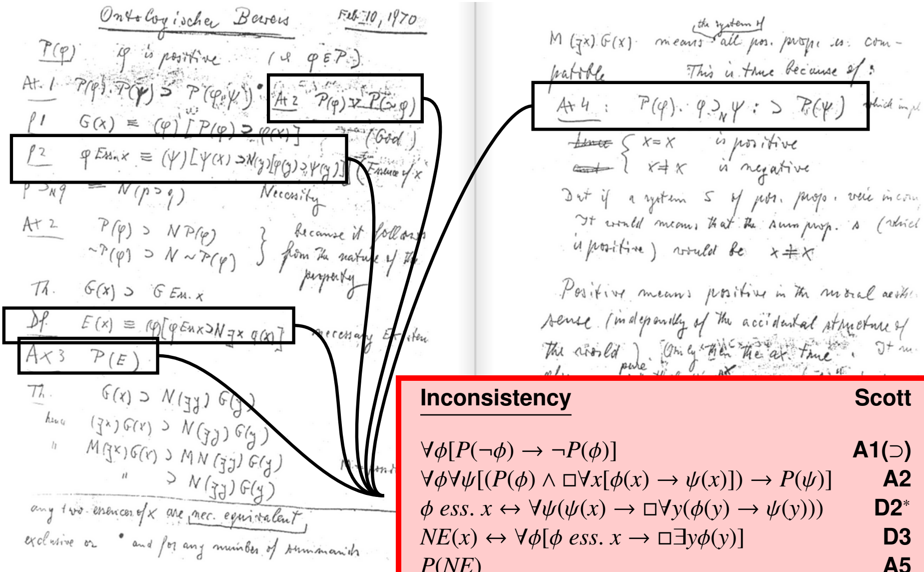
$x \neq x$ is negative

But if a system S of pos. props. were incon[sistent] it would mean that the sum prop. s (which is positive) would be $x \neq x$

Positive means positive in the moral aesth[etic] sense (independently of the accidental structure of the world). Only then the ax. true. It may also mean pure "attribution" as opposed to "privation" (or containing privation). This interpret. simpler proof

If φ pos. not: $(x) N \sim \varphi(x)$. Otherwise: $\varphi(x) \supset_N x \neq x$ hence $x \neq x$ positive not $x = x$ nor the contrary At. or the existence of pos. prop.

X i.e. the disj. normal form in terms of elem. prop. contains a member without negation.



Inconsistency **Scott**

$\forall\phi[P(\neg\phi) \rightarrow \neg P(\phi)]$	**A1(⊃)**
$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$	**A2**
$\phi\ ess.\ x \leftrightarrow \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$	**D2***
$NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$	**D3**
$P(NE)$	**A5**

Computational Metaphysics
— Recent and Ongoing Work —

jww: Ed Zalta (Stanford, US), Dana Scott (Berkeley, US), Bruno Woltzenlogel Paleo (Canberra, Australia), Alex Steen and Max Wisniewski (Berlin), many others

(C) Computational Metaphysics: Lecture Course on Computational Metaphysics

Metaphysics: *Foundational Branch in Philosophy*, that ...

... studies the fundamental nature of **being** and **the world** that encompasses it

... looks **beyond experience** in the real world

Flammarions Holzstich – in L'atmosphère, Paris 1888

Adresses **ultimate questions**, such as:

- What is there?
- What is it like?
- **Is there a God?**
- What can I know?

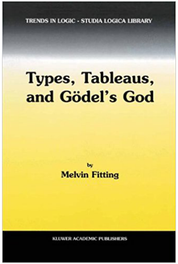
Method: **Rational Argumentation**

Lecture Course won the 2015 Central Teaching Award of FU Berlin

(jww: Alex Steen, Max Wisniewski, and others)

- MSc students in Maths, CS, Philosophy and Physics from FU, TU and HU
- Invited Lectures by Philosophers (Zalta & Lenzen) and Computer Scientists
- First course of this kind worldwide!

Further Experiments



Melvin Fitting (New York)

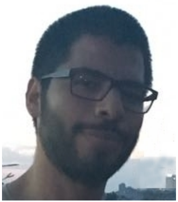
Philosophy/Metaphysics

Ontological Argument
(avoids modal collapse)

Intensional higher-order modal logic

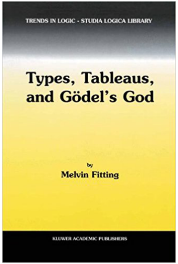
Verified (main chapters)

`https://www.isa-afp.org/entries/Types_Tableaus_and_Goedels_God.shtml`



David Fuenmayor
(Philosophy, FU Berlin)

Further Experiments



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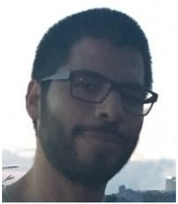


David Fuenmayor
(Philosophy, FU Berlin)

Ed Zalta (Stanford)

Philosophy/Metaphysics

Principia Logico-Metaphysica
(Foundations for Metaphysics & Mathematics)

Hyperintensional higher-order modal logic

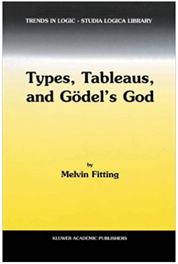
Inconsistency/Paradox revealed (comparable to Russel's Paradox for Frege's Begriffsschrift)

Emendations proposed

`https://github.com/ekpyron/TAO`

Daniel Kirchner
(Mathematics, FU Berlin)

Further Experiments



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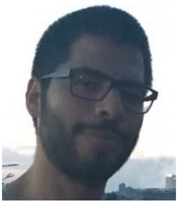

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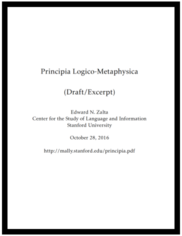

Principia Logico-Metaphysica

(Draft/Excerpt)

Edward N. Zalta
Center for the Study of Language and Information
Stanford University

October 28, 2016

http://mally.stanford.edu/principia.pdf

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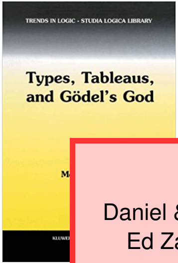


Melvin Fit

Philosophy/Metaphysics

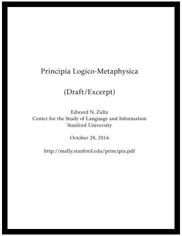
Ontological Argument
(avoids modal collapse)

vid Fuenmayor
sophy, FU Berlin)

Kirchner Paradox

Daniel & Isabelle/HOL have become close advisors of Ed Zalta in the search for a repair of the paradox.

This is *Computational Metaphysics* par excellence!!!



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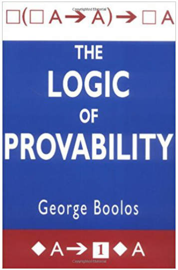
Weitere Experimente



Philosophy & Mathematics

Textbook on Provability Logic

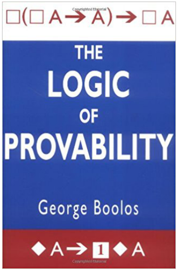
Provability Logic

Verified (most parts)

`Url:soon`

David Streit
(Mathematics, FU Berlin)

Weitere Experimente



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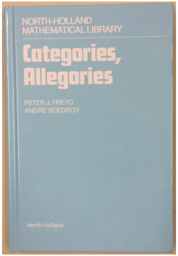


Mathematics

Textbook on Category Theory

Free first-order logic

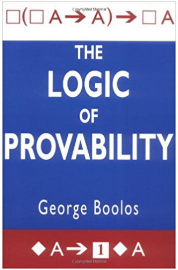
(Constricted) Inconsistency detected

Emendation proposed

`http://arxiv.org/abs/1609.01493`

D. Scott
(UC Berkeley)

Weitere Experimente



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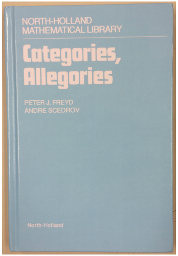

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`http://arxiv.org/abs/1609.01493`

Irina Makarenko
(Computer Science, FU Berlin)

Other Completed Studies

Variants of the Ontological Argument	jww
E. J. Lowe	David Fuenmayor Alexander Steen Max Wisniewski
G.W. Leibniz	Matthias Bentert David Streit Bruno Woltzenlogel-Paleo
G. Eder und E. Ramharter	Lukas Grätz Fabian Schütz
G. Oppy	Eberhard Höpfner Magdalena Haselsteiner Jose Antonio Akieme Rodriguez Pablo Agustin Martin Torres
C.A. Anderson, P. Hájek, F. Bjordal	Leon Weber Bruno Woltzenlogel-Paleo

Conclusion

- **Universal Logic Reasoning in HOL via Shallow Semantic Embeddings** (exploits/reuses significant recent improvements in ITP and ATP in HOL)
- **Deep Analysis of Rational Arguments on the Computer**
- **Computational Metaphysics**

• even **novel results** contributed **by theorem provers**
• **works well in practice**: matches granularity of human arguments
• related work (mainly on Anselm's original argument):
  – with first-order ATP PROVER9 [OppenheimerZalta, 2011]
  – with interactive proof assistant PVS [Rushby, 2013, 2017]

Outlook: Many relevant other applications (but I need resources!)

- Maths, Computer Science, Artificial Intelligence
- Philosophy, Natural Language Processing
- Legal-, Ethical- and Moral-Reasoning in Intelligent Machines
- Rational Argumentation in general

Research

Two decades of related own research

- Classical Higher-Order Logic (HOL) (since 1995)
- Intelligent Proof Assistant Systems & User Interfaces (since 1996)
- Automation of HOL: LEO Theorem Provers (since 1997)
- Agent-Based Architectures in Theorem Proving (since 1998)
- Integration of Interactive and Automated Theorem Proving (since 1999)
- Natural Language Dialog about Proofs (since 2001)
- International TPTP Infrastructure for HOL (since 2006)
- Expressive Ontologies in HOL (since 2008)
- **HOL as a Universal Logic via Semantical Embeddings** (since 2012)
- **Applications: Metaphysics, Maths, AI, Computer Science** (since 2012)

Research Profile

HOL — AI — ATP/ITP — Universal Logic — Interdisciplinary Applications

Research Methods

Theory — Modeling/Design — Implementation — Empirical Research

Two decades of related own research

- Classical Higher-Order Logic (HOL) (since 1995)

Latest Interests

- Rational Argumentation (between Humans and Intelligent Machines)
- Machine Ethics
- Expressive Upper Level Ontologies
- Industry 4.0
- ATP and Machine Learning
- ...

- **HOL as a Universal Logic via Semantical Embeddings** (since 2012)
- **Applications: Metaphysics, Maths, AI, Computer Science** (since 2012)

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What about the devil?

Roman Kuznets

May 25 at 5:05pm ·

Question after the talk on Gödel's ontological proof of God: "Can such a reasoning be used to prove the non-existent of Satan?" Speaker: "I believe that is still an open problem." Pure gold.

Bruno, are you up for the challenge?

Common Beliefs regarding the Question

Hans de Nivelle I always assumed that the Anselm ontological argument applies to Satan without change. Satanity is the worst possible property. From that follows that the notion must be realized in the world, because not-realized it would be better.

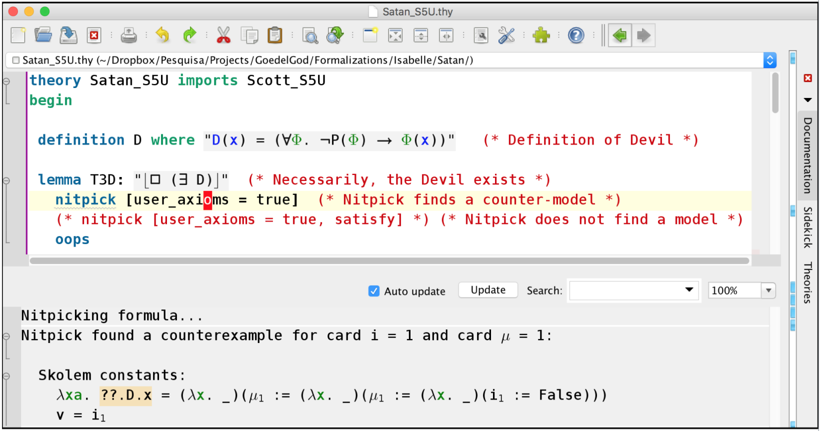
Like · Reply · 2 · May 25 at 5:16pm · Edited

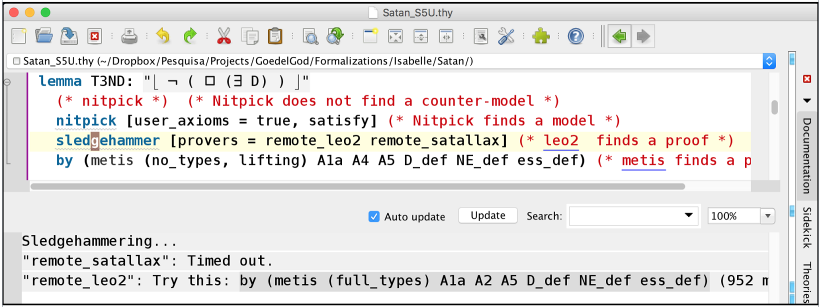
Roman Kuznets Roy Dyckhoff, would this be a satisfactory answer?

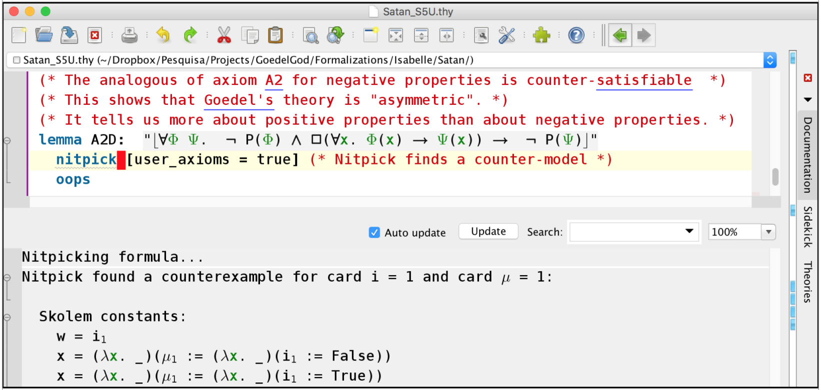
Like · Reply · 1 · May 25 at 6:38pm · Edited

> **Roy Dyckhoff** Worst possible for us, perhaps, but not for Satan himself...
>
> Like · Reply · 1 · May 25 at 8:04pm
>
> **Roy Dyckhoff** Non-existence is a thoroughly negative property; so, defining S as the maximally negative being, one might argue that S has the property of non-existence... (necessarily).
>
> Like · Reply · May 25 at 9:57pm

Bruno Woltzenlogel Paleo I have just spent a few moments playing with this in Isabelle. Here is what I can tell you:


Satan_S5U.thy
Satan_S5U.thy (~/Dropbox/Pesquisa/Projects/GoedelGod/Formalizations/Isabelle/Satan/)
theory Satan_S5U imports Scott_S5U
begin
definition D where "D(x) = (∀Φ. ¬P(Φ) ⟶ Φ(x))" (* Definition of Devil *)
lemma T3D: "⌊□ (∃ D)⌋" (* Necessarily, the Devil exists *)
nitpick [user_axioms = true] (* Nitpick finds a counter-model *)
(* nitpick [user_axioms = true, satisfy] *) (* Nitpick does not find a model *)
oops
Auto update
Update
Search:
100%
Nitpicking formula...
Nitpick found a counterexample for card i = 1 and card μ = 1:
Skolem constants:
λxa. ??.D.x = (λx. _)(μ1 := (λx. _)(μ1 := (λx. _)(i1 := False)))
v = i1
Documentation
Sidekick
Theories


Satan_S5U.thy
Satan_S5U.thy (~/Dropbox/Pesquisa/Projects/GoedelGod/Formalizations/Isabelle/Satan/)
lemma T3ND: "⌊ ¬ (□ (∃ D)) ⌋"
(* nitpick *) (* Nitpick does not find a counter-model *)
nitpick [user_axioms = true, satisfy] (* Nitpick finds a model *)
sledgehammer [provers = remote_leo2 remote_satallax] (* leo2 finds a proof *)
by (metis (no_types, lifting) A1a A4 A5 D_def NE_def ess_def) (* metis finds a p
Auto update
Update
Search:
100%
Sledgehammering...
"remote_satallax": Timed out.
"remote_leo2": Try this: by (metis (full_types) A1a A2 A5 D_def NE_def ess_def) (952 m
Documentation
Sidekick
Theories


Satan_S5U.thy
Satan_S5U.thy (~/Dropbox/Pesquisa/Projects/GoedelGod/Formalizations/Isabelle/Satan/)
(* The analogous of axiom A2 for negative properties is counter-satisfiable *)
(* This shows that Goedel's theory is "asymmetric". *)
(* It tells us more about positive properties than about negative properties. *)
lemma A2D: "⌊∀Φ Ψ. ¬ P(Φ) ∧ □(∀x. Φ(x) → Ψ(x)) → ¬ P(Ψ)⌋"
nitpick [user_axioms = true] (* Nitpick finds a counter-model *)
oops
Auto update
Update
Search:
100%
Nitpicking formula...
Nitpick found a counterexample for card i = 1 and card μ = 1:
Skolem constants:
w = i1
x = (λx. _)(μ1 := (λx. _)(i1 := False))
x = (λx. _)(μ1 := (λx. _)(i1 := True))
Documentation
Sidekick
Theories

Reading “P” as “Negative” and “G" as “Devil-like”

```
Scott_S5U.thy
theory Scott_S5U imports QML_S5U
begin
 consts P :: "(μ⇒σ)⇒σ"
 axiomatization where
  A1a: "⌊∀Φ. P(¬Φ) → ¬P(Φ)⌋" and
  A1b: "⌊∀Φ. ¬P(Φ) → P(¬Φ)⌋" and
  A2:  "⌊∀Φ Ψ. P(Φ) ∧ □(∀x. Φ(x) → Ψ(x)) → P(Ψ)⌋"
 definition G where
  "G(x) = (∀Φ. P(Φ) → Φ(x))"
 axiomatization where
  A3: "⌊P(G)⌋" and
  A4: "⌊∀Φ. P(Φ) → □(P(Φ))⌋"
 definition ess (infixr "ess" 85) where
  "Φ ess x = Φ(x) ∧ (∀Ψ. Ψ(x) → □(∀y. Φ(y) → Ψ(y)))"
 definition NE where
  "NE(x) = (∀Φ. Φ ess x → □(∃ Φ))"
 axiomatization where
  A5:  "⌊P(NE)⌋"

 theorem T3: "⌊□ (∃ G)⌋" -- {* LEO-II proves T3 in 2,5sec *}
  sledgehammer [provers = remote_leo2]
  by (metis (lifting, full_types)
     A1a A1b A2 A3 A4 A5 G_def NE_def ess_def)

 lemma True nitpick [satisfy,user_axioms,expect=genuine] oops
 -- {* Consistency is confirmed by Nitpick *}

 theorem T2: "⌊∀x. G(x) → G ess x⌋"
  sledgehammer [provers = remote_leo2]
  by (metis A1b A4 G_def ess_def)

 lemma MC: "⌊∀Φ. Φ → (□ Φ)⌋"  -- {* Modal Collapse *}
  sledgehammer [provers = remote_satallax, timeout=600]
  by (meson T2 T3 ess_def)
end
```

Do the 5 axioms still make sense?

Do they capture our intuition of negativity?

Are we willing to accept “P(NE)” ?

Are we willing to accept “P(λx.x=x)” ?

Are we willing to accept “P(λx.T)” ?

Proof in S5

Scott's version

$$\textbf{D2: } \varphi\ ess\ x \equiv \cancel{\varphi(x)} \land \forall\psi.(\psi(x) \to \Box\forall x.(\varphi(x) \to \psi(x)))$$

Gödel's version did not have this conjunct !

$$\textbf{D3: } E(x) \equiv \forall\varphi.[\varphi\ ess\ x \to \Box\exists y.\varphi(y)]$$

$$\dfrac{\dfrac{\textbf{A3}}{P(G)} \qquad \dfrac{\dfrac{\textbf{A2}}{\forall\varphi.\forall\psi.[(P(\varphi) \land \Box\forall x.[\varphi(x) \to \psi(x)]) \to P(\psi)]} \qquad \dfrac{\textbf{A1a}}{\forall\varphi.[P(\neg\varphi) \to \neg P(\varphi)]}}{\textbf{T1: } \forall\varphi.[P(\varphi) \to \Diamond\exists x.\varphi(x)]}}{\textbf{C1: } \Diamond\exists z.G(z)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\textbf{A1b}}{\forall\varphi.[\neg P(\varphi) \to P(\neg\varphi)]} \qquad \dfrac{\textbf{A4}}{\forall\varphi.[P(\varphi) \to \Box P(\varphi)]}}{\textbf{T2: } \forall y.[G(y) \to G\ ess\ y]} \qquad \dfrac{\textbf{A5}}{P(E)}}{\textbf{L1: } \exists z.G(z) \to \Box\exists x.G(x)}}{\Diamond\exists z.G(z) \to \Diamond\Box\exists x.G(x)} \qquad \dfrac{\textbf{S5}}{\forall\xi.[\Diamond\Box\xi \to \Box\xi]}}{\textbf{L2: } \Diamond\exists z.G(z) \to \Box\exists x.G(x)}$$

$$\dfrac{\textbf{C1: } \Diamond\exists z.G(z) \qquad \textbf{L2: } \Diamond\exists z.G(z) \to \Box\exists x.G(x)}{\textbf{T3: } \Box\exists x.G(x)}$$

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26 May 2016

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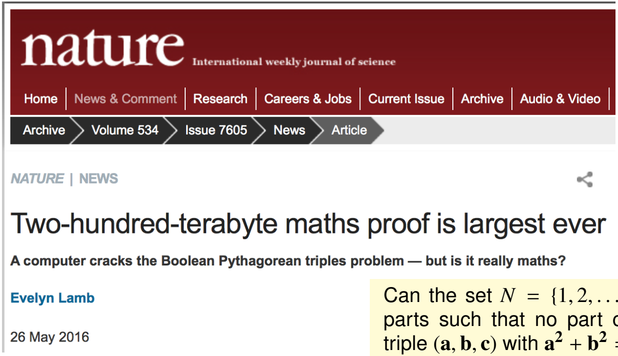

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$n = 10$

$$3^2 + 4^2 = 5^2$$

(choose color of the other numbers arbitrarily)

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Unicoloring forbidden!

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$n = 20$

$$3^2 + 4^2 = 5^2$$
$$5^2 + 12^2 = 13^2$$
$$8^2 + 15^2 = 17^2$$

(choose color of the other numbers arbitrarily)

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$n = 30$

$$3^2 + 4^2 = 5^2$$
$$5^2 + 12^2 = 13^2$$
$$8^2 + 15^2 = 17^2$$
$$7^2 + 24^2 = 25^2$$

(choose color of the other numbers arbitrarily)

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$n = 40$

$$3^2 + 4^2 = 5^2$$

$$5^2 + \boxed{12}^2 = 13^2$$

$$8^2 + 15^2 = 17^2$$

$$7^2 + 24^2 = 25^2$$

$$20^2 + 21^2 = 29^2$$

$$\boxed{12}^2 + 35^2 = 37^2$$

(A) Logic Zoo: Pythagorean Triples Problem — Propositional Logic —

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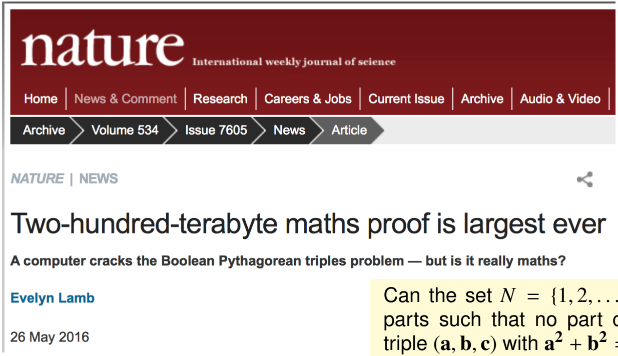

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Shown by SAT-Solver:

For $n \geq 7825$ consistent bi-coloring becomes impossible.

$\Diamond\Box\bot$ Implies Inconsistency: Syntactical Proof in Modal Logic K

- Syntactical proof in proof assistent Coq
- Assume: $\Diamond\Box\bot$ (holds globally)
- Show: there exist a formula φ (in current world) s.t. φ and $\neg\varphi$
- Choose $\varphi := \Diamond\Diamond\bot$
- $\Diamond\Diamond\bot$ follows from $\Diamond\Box\bot$
- $\neg\Diamond\Diamond\bot$ holds: $\neg\Diamond\Diamond\bot \equiv \Box\Box\top$, which easily follows by necessitation
- Hence, $\Diamond\Diamond\bot$ and $\neg\Diamond\Diamond\bot$
- Hence, $\bot$

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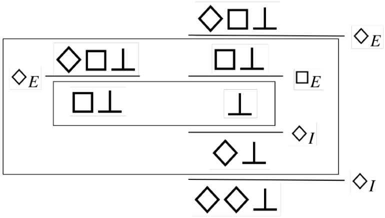

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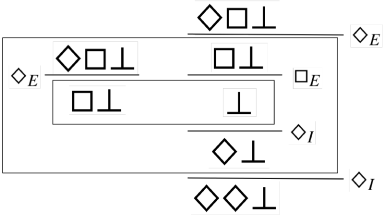

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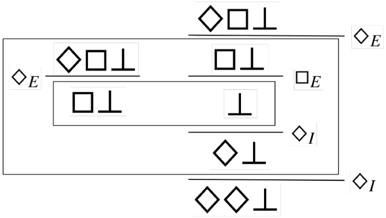

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