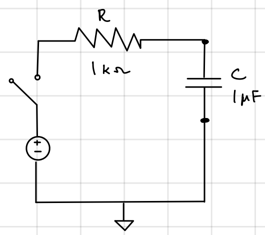
Simple RC Circuit   but opening & closing switch every 1.02 ms :)



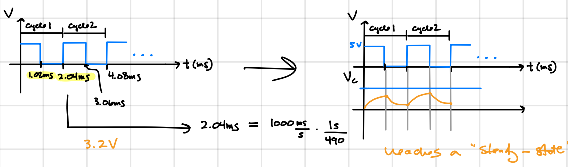
$\tau = RC = 1000\,\Omega \times \frac{1}{1{,}000{,}000\ F}$

$\tau = \frac{1}{1000\,s} = 1\ ms$

We use voltage across the capacitor at any time t, for derivation see next page

$V_c(t) = \underbrace{V_{source}}_{0V \to 5V} + (\underbrace{V_{initial}}_{changes} - V_{source})e^{-t/RC}$

$f = 490\,Hz \rightarrow t = \frac{s}{490} = 2.04\ ms$



reaches a "steady-state"

set of values of 3.7 V max → 1.3 V min

Now, with PWM pin at 50% duty cycle and 490 Hz.

cycle 1

Charging   0 s → 1.02 ms | $V(1.02ms) = 5V + (0V - 5V)\underbrace{e^{-1.02}}_{0.36059} = \underline{3.19703\,V}$

discharging   1.02 ms → 2.04 ms | $V(2.04ms) = 0V + (3.19703\,V - 0V) \times 0.36059 = \underline{1.15282\,V}$ 1.2V

cycle 2

Charging   2.04 ms → 3.06 ms | $V(3.06ms) = 5V + (1.15282V - 5V) \times 0.36059 = \underline{3.61274V}$ 3.6V

discharging   3.06 ms → 4.08 ms | $V(4.08ms) = 0V + (3.61274V - 0V) \times 0.36059 = \underline{1.30272V}$ 1.3V

cycle 3

Charging   4.08 ms → 5.10 ms | $V(5.10\,ms) = 5V + (1.30272V - 5V) \times 0.36059 = \underline{3.66680V}$ 3.7V

discharging   6.12 ms → 7.14 ms | $V(6.12ms) = 0V + (3.66680V - 0V) \times 0.36059 = \underline{1.32221\,V}$ 1.3V

cycle 4

Charging   7.14 ms → 8.16 ms | $V(8.16\,ms) = 5V + (1.32221\,V - 5V) \times 0.36059 = \underline{3.67383V}$ 3.7V

discharging   8.16 ms → 9.18 ms | $V(9.18\,ms) = 0V + (3.67383V - 0V) \times 0.36059 = \underline{1.32475V}$ 1.3V

cycle 5

Charging   9.18ms → 10.20 ms | $V(10.20\,ms) = 5V + (1.32475\,V - 5V) \times 0.36059 = \underline{3.67474V}$ 3.7V

discharging   10.20 ms → 11.22 ms | $V(11.22ms) = 0V + (3.67474V - 0V) \times 0.36059 = \underline{1.32507V}$ 1.3V

cycle 6

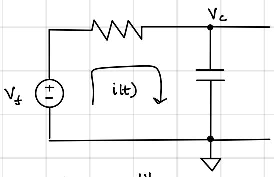
Charging   11.22ms → 12.24 ms | $V(12.24\,ms) = 5V + (1.32507\,V - 5V) \times 0.36059 = \underline{3.67486V}$ 3.7

discharging   12.24 ms → 13.26 ms | $V(13.26\,ms) = 0V + (3.67486V - 0V) \times 0.36059 = \underline{1.32512\,V}$ 1.3V

cycle 7

Charging   13.26 ms → 14.28 ms | $V(14.28\,ms) = 5V + (1.32512\,V - 5V) \times 0.36059 = \underline{3.67488V}$ 3.7V

discharging   14.28 ms → 15.30 ms | $V(15.30\,ms) = 0V + (3.67488V - 0V) \times 0.36059 = \underline{1.32512\,V}$ 1.3V



$i(t) = C\frac{dV_c}{dt}$

$\boxed{V_c(t) = V_{source} + (V_{initial} - V_{source})e^{-t/RC}}$

Derivation of Voltage equations

initial conditions: V_i = initial voltage across capacitor

Start: Kirchoff's voltage eqn

$V_f = V_R(t) + V_c(t)$

$V_f = i(t)R + V_c(t)$ — where current through C is $i(t) = \frac{C\,dV_c(t)}{dt}$

$V_f = \frac{C\,dV_c(t)}{dt}R + V_c(t)$ — rearrange

$\frac{dV_c(t)}{dt} = \frac{V_f - V_c(t)}{RC}$ — Substitute $u(t) = V_f - V_c(t)$, $-\frac{du(t)}{dt} = \frac{dV_c(t)}{dt}$

$-\frac{du(t)}{dt} = \frac{u(t)}{RC}$

$\int \frac{du(t)}{u(t)} = -\int \frac{dt}{RC}$

$\ln|u(t)| = -\frac{t}{RC} + m_1$

$u(t) = e^{-t/RC + m}$

$u(t) = me^{-t/RC}$

$V_f - V_c(t) = me^{-t/RC}$

$V_c(t) = V_f - me^{-t/RC}$

$V_c(0) = V_i = V_f - m$ — where V_i = initial voltage over capacitor

$m = V_f - V_i$

$V_c(t) = V_f - (V_f - V_i)e^{-t/RC}$

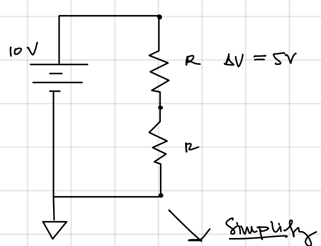
$\boxed{V_c(t) = V_f + (V_i - V_f)e^{-t/RC}}$

Voltage Divider

used to lower source voltage

Seen everywhere, are useful circuit snippets, need to be familiar with

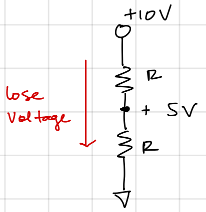
Consider series circuit w/ two identical resistors. Ohms law & Kirchoff's voltage law tell us:



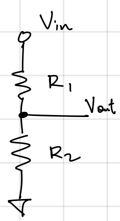
① Voltage drops over resistor as $V = IR$

② Sum of voltage drops over each R is 10V.

Simplify

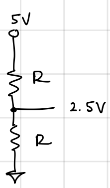


more general



So,

1:1



Derivation of Voltage divider eqn.

① total current through circuit

$V_{in} = I(R_1 + R_2)$

$I = \frac{V_{in}}{(R_1 + R_2)}$

② Voltage across R_2

$V_{out} = IR_2$

Subst. in I

$V_{out} = V_{in}\frac{R_2}{R_1 + R_2}$