25 + 75（期末）

1.4. 条件概率

Def. 1.4.1, $A, B \in \Omega$, $P(B)>0$: $P(A|B) = \dfrac{P(AB)}{P(B)}$

↳ $P(AB) = P(B) \cdot P(A|B)$

$P(A_1, A_2 \cdots A_n) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1A_2) \cdots P(A_n|A_1 \cdots A_{n-1})$

ex. $A_1 \cdots A_n$ 互不相容: $P(\bigcup_{n=1}^{\infty} A_n | B) = \sum_{n=1}^{\infty} P(A_n|B)$

（全概率公式）$B_1, B_2 \cdots B_n$ 分割 Ω. 有: $P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A|B_i)$.

↳ $P(A) = P(B)P(A|B) + P(\bar{B})P(A|\bar{B})$

（叶贝斯公式）$P(B_i|A) = P(B_i)P(A|B_i) / \sum_{j=1}^{n} P(B_j)P(A|B_j)$ $\quad i=1,2,\cdots n.$

1.5 独立性.

$P(AB) = P(A) \cdot P(B)$: A、B 相互独立. $\quad$ * 独立 ≠ 互不相容.

$P(ABC) = P(A) \cdot P(B) \cdot P(C)$: ABC 相互独立

---

① 大数定律.

- Chebychev. $Var(X_i) \leq c$.
- 马尔可夫: $\frac{1}{n^2} Var(\sum_{i=1}^{n} X_i) \longrightarrow 0 \ (n \to \infty)$
- 辛钦. $\{X_n\}$ 独立同. X_i 期望存在.

② 中心极限定理.

③ $Cov(X, Y) = E(XY) - E(X) \cdot E(Y)$.

$Corr(X, Y) = Cov(X, Y) / \sqrt{VarX}\sqrt{VarY}$.

④ 分清样本均值与总体 X, $\bar{x}$, S, σ

$$\boxed{P(B_i|A) = \frac{P(B_i)P(A|B_i)}{\sum_{j=1}^{n} P(B_j)P(A|B_j)}}$$

⑤ 次序统计量. $p_k(x) = \cancel{\binom{n}{k-1}} \cdot (F(x))^{k-1} \cdot (1-F(x))^{n-k} \cdot p(x)$.

No.k 的量 $x_{(k)} = x$.

⑥ 三大抽样分布.

	E	Var.
$\chi^2 = X_1^2 + \cdots + X_n^2 \sim \chi^2(n)$. $\boxed{X_i \sim N(0,1)}$	n	$2n$.

$$F(m,n) = \frac{\chi^2(m)/m}{\chi^2(n)/n}.$$

$X \sim F(n,n)$, $\frac{1}{X} \sim F(n,n)$.

$$t = \frac{N(0,1)}{\sqrt{\chi^2(n)/n}} \sim t(n).$$

$$\frac{N(0,1)}{\sqrt{\chi^2(n)/n}} \sim t(n)$$

$X \sim N(\mu, \sigma^2)$, $\frac{X-\mu}{\sigma} \sim N(0,1)$

$\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$, $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$.

$\{x_n\} \sim N(\mu, \sigma^2)$, $\boxed{\frac{\bar{x}-\mu}{s/\sqrt{n}} \sim t(n-1)}$

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1)$$

假设检验.

对 μ. 正态. (单个).

σ^2 已知. $U = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$

$\{u \geq u_{1-\alpha}\}$

$\{u \leq u_\alpha\}$

$\{|u| \geq u_{1-\alpha/2}\}$.

σ^2 未知. $T = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} \sim t(n-1)$

$\{t \geq t_{1-\alpha}(n-1)\}$

$\{t \leq t_\alpha(n-1)\}$

$\{|t| \geq t_{1-\alpha/2}(n-1)\}$.

(两个)

σ_1, σ_2 已知. $U = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}}$

$\{u \geq u_{1-\alpha}\}$

$\{u \leq u_\alpha\}$

$\{|u| \geq u_{1-\alpha/2}\}$.

$\sigma_1 = \sigma_2 = \sigma$, 未知. $T = \frac{\bar{x}-\bar{y}}{S_w\sqrt{\frac{1}{m}+\frac{1}{n}}} \sim t(m+n-2)$

$\{t \geq t_{1-\alpha}(m+n-2)\}$

$\{t \leq t_\alpha(m+n-2)\}$

$\{|t| \geq t_{1-\alpha/2}(m+n-2)\}$.

$$S_w^2 = \frac{(m-1)S_x^2 + (n-1)S_y^2}{(m+n-2)}$$

	H_0	H_1
I	$\mu_1 \leq \mu_2$	$\mu_1 > \mu_2$
II	$\mu_1 \geq \mu_2$	$\mu_1 < \mu_2$
III	$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$

t 样本. $u = \dfrac{\bar{X} - \bar{Y}}{\sqrt{\dfrac{S_X^2}{m} + \dfrac{S_Y^2}{n}}}$

(即用 $S_X^2 \Leftrightarrow \sigma_X^2$

$S_Y^2 \Leftrightarrow \sigma_Y^2$).

对 σ.

- 单个 χ^2 检验. $\chi^2 = \dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$
  - $\{\chi^2 \geq \chi^2_{1-\alpha}(n-1)\}$
  - $\{\chi^2 \leq \chi^2_{\alpha}(n-1)\}$
  - $\{\chi^2 \leq \chi^2_{\alpha/2}(n-1)$ or $\chi^2 \geq \chi^2_{1-\alpha/2}(n-1)\}$.
- 两个 F 检验 $F = \dfrac{S_X^2}{S_Y^2} \sim F(m-1, n-1)$
  - $\{F \geq F_{1-\alpha}(m-1, n-1)\}$
  - $\{F \leq F_{\alpha}\}$
  - $\{F \leq F_{\alpha/2}$ or $F \geq F_{1-\alpha/2}\}$.

正态总体检验.

- 对 μ
  - 单个
    - σ 知 $\rightarrow u$
    - σ 不知 $\rightarrow t$
  - 2个
    - σ_1, σ_2 知 $\rightarrow U$
    - $\sigma_1 = \sigma_2 = \sigma$ 不知 $\rightarrow T$
- 对 σ
  - 单个 $\rightarrow \chi^2$
  - 2个 $\rightarrow F$

区间估计

- 单个 (正态)
  - μ
    - σ 知 $\rightarrow G = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$ $\quad \bar{X} \pm u_{1-\alpha/2} \boxed{\sigma/\sqrt{n}}$ 分母
    - σ 不知. $\rightarrow t = \dfrac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1)$ $\quad \bar{X} \pm t_{1-\alpha/2} S/\sqrt{n}$
  - $\sigma \rightarrow \dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$ $\quad \left[\dfrac{(n-1)S^2}{\chi^2_{\alpha/2}}, \dfrac{(n-1)S^2}{\chi^2_{1-\alpha/2}}\right]$.
- 两个
  - $\mu_1 - \mu_2$
    - σ_1^2, σ_2^2 知 $\rightarrow \dfrac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{m} + \dfrac{\sigma_2^2}{n}}}$
    - $\sigma_1^2 = \sigma_2^2 = \sigma^2$ 不知 $\rightarrow t = \dfrac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{S_w\sqrt{\dfrac{1}{m} + \dfrac{1}{n}}}$
    - $\sigma_1^2/\sigma_2^2 = c$ 知 $\rightarrow t = \dfrac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{S_w\sqrt{\dfrac{1}{m} + \dfrac{1}{n}}}$ $\quad$ 但 S_w^2 中 $\dfrac{(m-1)S_X^2 + (n-1)S_Y^2/c}{(m+n-2)}$
  - $\sigma_1^2/\sigma_2^2 \rightarrow F = \dfrac{S_X^2/\sigma_1^2}{S_Y^2/\sigma_2^2} \sim F(m-1, n-1)$

ex. t 样本

$$\left(\frac{S_x^2}{S_y^2}\cdot\frac{1}{F_{1-\alpha/2}},\ \frac{S_x^2}{S_y^2}\cdot\frac{1}{F_{\alpha/2}}\right).$$

$$\frac{\bar{x}-\mu}{S/\sqrt{n}} \sim t(n-1)$$

$$X\sim N(\mu,1)\quad \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

§7.3. other 分布

指数 $Exp(\frac{1}{\theta})$. $\chi^2 = \frac{2n\bar{x}}{\theta_0} \sim \chi^2(2n)$.

$\{\chi^2 \geq \chi^2_{1-\alpha}(2n)\}$

$\{\chi^2 \leq \chi^2_{\alpha}(2n)\}$

$\{\chi^2 \leq \chi^2_{\alpha/2}$ or $\chi^2 \geq \chi^2_{1-\alpha/2}\}$.

大样本: $u = \frac{\bar{x}-\theta}{\sqrt{\sigma^2(\hat{\theta})}/\sqrt{n}} \sim N(0,1)$

eg. $B(n,p)$下. $\hat{p} \sim p(1-p)$

$\{u \geq u_{1-\alpha}\}$

§7.4. χ^2拟合.

Ⅱ类 $\chi^2 = \sum_{i=1}^{r} \frac{(n_i - np_i)^2}{np_i} \sim \chi^2(r-1)$ $\chi^2 \geq \chi^2_{1-\alpha}(r-1)$

列联表. $\chi^2 = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{(n_{ij} - np_{ij})^2}{np_{ij}} \sim \chi^2(m-1)(n-1)$ $\chi^2 \geq \chi^2_{1-\alpha}(m-1)(n-1)$

7. H_0: $p_i = \frac{\lambda^i}{i!}e^{-\lambda}$, $i=1\sim5$. $\sum_{i=6}^{\infty}\frac{\lambda^i}{i!}e^{-\lambda}$

选取 $\chi^2 = \sum_{i=1}^{r-1}\frac{(n_i - np_i)^2}{np_i} \sim \chi^2(r-k-1)$

(有一个未知数)

拒绝域 $W. \geq \chi^2_{1-\alpha}(r-k-1)$

计算. (其中 $\lambda = \bar{x} = 1$.)

§8.

来源	平方和	自由度	均方	F比
因子	S_A	$f_A = r-1$	$MS_A = S_A / f_A$	$F = MS_A / MS_e$
误差	S_e	$f_e = n-r$	$MS_e = S_e / f_e$	
Sum	S_T	$f_T = n-1$		

$S_A = m \sum_{i=1}^{r} (\bar{y}_{i\cdot} - \bar{y})^2$ 行 行均值减总均值 y是 r行 m列的数据

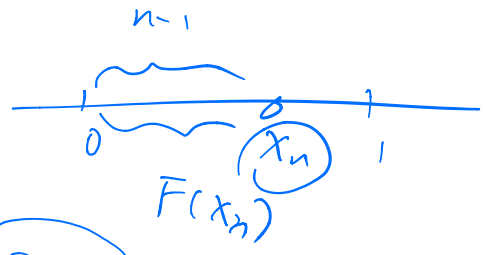
$S_e = \sum_{i=1}^{r} \sum_{j=1}^{m} (y_{ij} - \bar{y}_{i\cdot})^2$ 元素减行均值. $n = r \cdot m$

$S_T = S_A + S_e$.

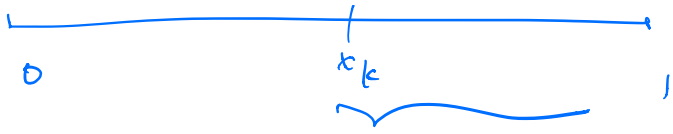
☆ if. $F \geq F_{1-\alpha}(f_A, f_e)$. 则认为因子A显著

$X \sim U(0,1) \leftarrow X_1 \text{—} X_n$

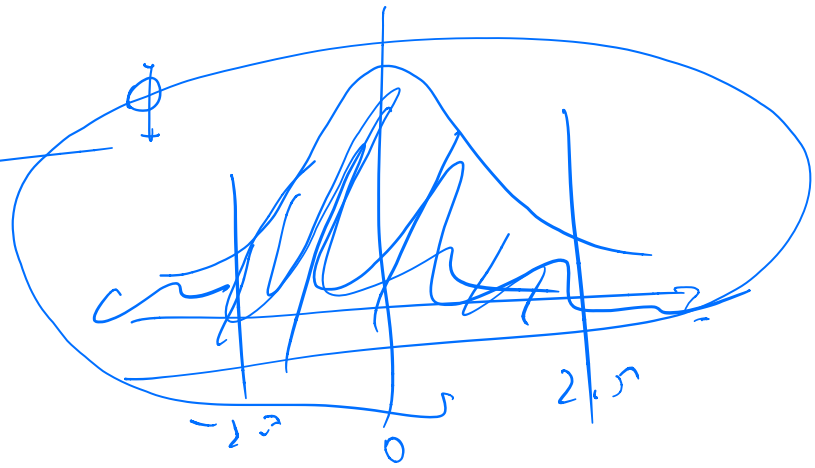
$p(x) = 1$ $(X_{(n)}$



$$P_n(x) = \underline{\binom{n}{1} (F(x))^{n-1}} \cdot p(x).$$



$$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



§2.4. 4)

§2.6. (2)

§3.2 边缘分布 独立

§3.3. $Z=g(X,Y)$ 求密度函数

§3.4. 4 20 ☆

§3.5

$U \sim N(0,1)$

F

$-2 \le U \le 2$

$\Phi(2)-\Phi(-2)$

$$P_Z(u,v) = P_X(x(u,v), y(u,v)) \cdot |J_{(u,v)}|$$

$X_{(1)}$ $X_{(2)}$

$X_i \begin{cases} 1 \\ 0 \end{cases}$

X Y	0	1	
0	1/6	1/6	$\frac{1}{6}$
1	1/6	0	$\frac{1}{6}$

$Y \quad 0$

$\partial x \frac{\partial}{\partial} + 1 \times S \quad \frac{\partial}{\partial}$

$u - \mathrm{cov}(X_i, Y_i)$

$\cancel{\sum X_i Y_i - \mathrm{Cov}}$

$\mathrm{Cov}(X, Y)$

$\left(\sum X_i, \sum Y_j\right)$

$(i \neq j)$

$\sum_{i=1}^{n} \mathrm{Cov}(X_i, Y_i)$

$X \sim N(\mu, \sigma^2)$

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

$$\int u' v \, dx = uv - \int u v' \, dx$$

$\sim N(0, n)$

χ^2. $\quad \chi^2 = X_1^2 + \cdots + X_n^2 \quad \chi^2(n)$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

$$F \quad \frac{\chi^2(m)/m}{\chi^2(n)/n} = \frac{X_1/m}{X_2/n} \sim F(m,n)$$

$$t \quad \frac{X_1}{\sqrt{\chi^2/n}} \sim t(n)$$

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$$

$\chi^2_{1-\alpha}$ $\bar{F}_\alpha$

$\bar{X} \sim N(\mu, \frac{\sigma^2}{\sigma n})$ χ^2_α t_α u

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

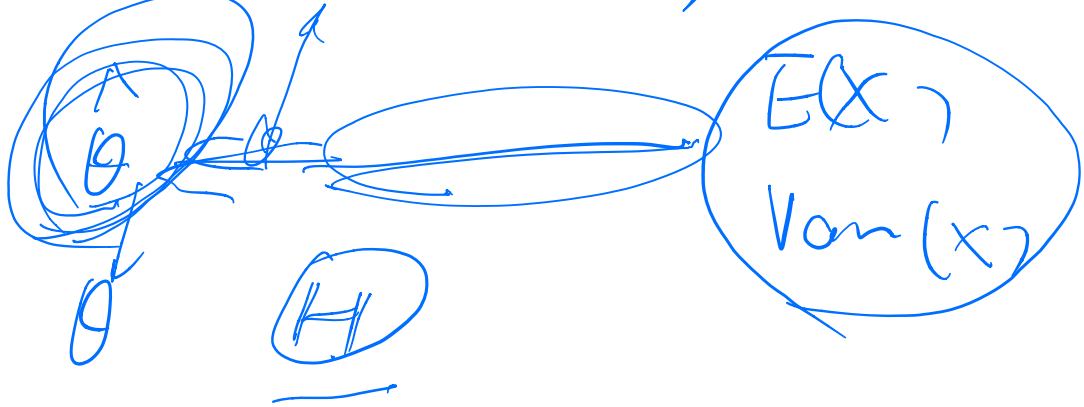
$$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$$

$t > \sim(\sqrt{n})$

σ^2 t

σ^2

$P(t(n) > \sim) \geq 0.95$



$$\boxed{E(\hat{\theta}) = \theta}$$

$$\sum a_i X_i$$

$$\sum_{k=1}^{N-1} k \cdot \frac{1}{N} = E(X)$$

$$= \frac{1}{N} \cdot \frac{N(N-1)}{2} = E(X)$$

$$\hat{N} = \frac{2\overline{X}}{\theta} + 1$$

$$f(\theta) = \cancel{E(X)}$$

$$\begin{cases} \bar{x} = \frac{\sum x_i}{n} = f(\mu, \theta) \\ s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = f(\mu, \theta) \end{cases}$$

[illegible] k

[illegible] 2 —

极大 & 估计

step1 $$L(\theta) = \prod_{i=1}^{n} f(x_i; \theta)$$

$$= (\sqrt{\theta})^n (x_1 \cdots x_n)^{\sqrt{\theta}-1}$$

step 2 $$\ln L(\theta) = \frac{n}{2} \ln \theta + (\sqrt{\theta}-1) \ln (x_1 \cdots x_n)$$

step 3 $$\frac{d \ln L(\theta)}{d\theta} = \frac{n}{2\theta} + \frac{1}{2\sqrt{\theta}} \ln \underline{\quad}$$

step $$= 0 \Rightarrow \underline{\theta} =$$

μ σ ✓ $\quad G = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$

μ σ ✗ $\quad \dfrac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

μ σ ✓ $\quad G = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$

μ_1, μ_2

$\bar{X} \quad \bar{Y}$

$G = \dfrac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{}$

$\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$

$\bar{X} - \bar{Y} \sim N\left(\mu_1 - \mu_2, \dfrac{\sigma_1^2}{m} + \dfrac{\sigma_2^2}{n}\right)$

$$\alpha = P\{X \in W \mid H_0\}$$

$$= P\{\bar{X} \geq 2.6 \mid \mu = 2\}$$

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \geq \cancel{\frac{2\sigma-\mu}{\sigma/\sqrt{n}}}$$

$N(0,1)$

$\cancel{f(x)}$ n $\sqrt{(n)}$

$$U \sim N(0,1)$$

$$\cancel{\mu} \geq u_{1-\alpha}$$

$$\cancel{u \leq u_{\alpha}}$$

$$\begin{cases} u \leq u_{\alpha/2} \\ u \geq u_{1-\alpha/2} \end{cases}$$

$$U = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$$

$$u \geq u_{1-\alpha}$$

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \geq u_{1-\alpha}$$

$\sigma/\sqrt{n}$

$$\mu \leq \boxed{\bar{x} - u_{1-\alpha}\frac{\sigma}{\sqrt{n}}}$$