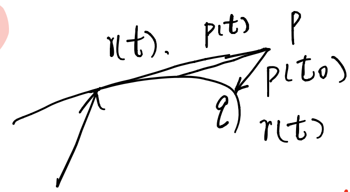
2.1. 2. $r'(t) = (2\cos t,\ 1,\ -\sin t)$. 而 $X(t,u) = r(t) + u r'(t)$

$X(t) = r(t) + u r'(t)$. ← $t = 0, \frac{\pi}{2}$. 是切线面

3.



要用上 $\rho'(t) = 0$ 这一条件

$(\vec{oq} - \vec{op}) \cdot r' = r \cdot r' - \vec{op} \cdot r'$

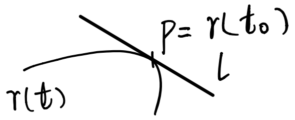
$= -\vec{op} \cdot r' = -\vec{op} \cdot (\underbrace{op + pq})'$

$(0 + 0)$ $= 0$. 证毕.

pq $\rho(t)$是数值, 按说先展成

$|\vec{op}| \cdot |\vec{pq}| \cdot \cos \measuredangle = \sim = 0$.

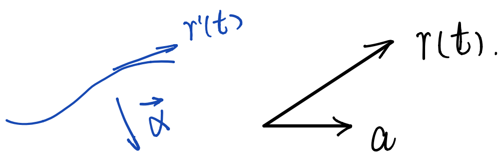
Δ 5.



目标: $L = r(t) + u r'(t)$.

X 不对

6. $\vec{a} \cdot r'(t) \equiv 0$. $\left(a \cdot r(t)\right)' = a r'(t) = 0$.



$\underbrace{a \cdot r(t) = C}$. 就是平面曲线

$\boxed{2.2.}$ 1. (5).

$\int_0^{x_0} \left|\left(x_0, \frac{x_0^2}{2a}, \frac{x_0^3}{6a^2}\right)'\right| dx.$

$= \int_0^{x_0} \left|\left(1, \frac{x}{a}, \frac{x^2}{2a^2}\right)\right| dx$ $\quad 1 + \frac{x^2}{a^2} + \frac{x^4}{4a^4}$

$\frac{1}{2}\ \frac{1}{2a^2}\ \frac{1}{\sqrt{4a^4 + 2a^2x^2 + x^4}}\ (4x^3 + 4a^2 x)\Big|_0^{x_0} \stackrel{x_0}{=} \sqrt{\frac{4a^4 + 2a^2x^2 + x^4}{4a^4}}$

? 2.(3). $r'(t) = (3, 6t, 6t^2)$ $\quad |r'(t)| = \sqrt{9 + 36t^2 + 36t^4}$

注意 ds 与 dt ...

$\frac{dr}{ds} = \frac{r'}{|r'|} = \frac{1}{\sqrt{9 + 36t^2 + 36t^4}}\ (3, 6t, 6t^2)$.

(...就不一样!)

1. 的运动

曲线对弧长求导，得到的一定是单位切向量场。

了解 $r'(t)$ 的意义即可

5. $r(t) = \left(\frac{t^3}{3}, \frac{t^2}{2}, t\right)$. $r'(t) = (t^2, t, 1)$. $\sqrt{9-8}$ $\frac{-3\pm 1}{2}$

$r'(t)\cdot(1,3,2) = 0 \Rightarrow t^2+3t+2=0 \Rightarrow t=-1,-2.$

6. $\frac{dr}{dt} = (t^2, t, e^t)$ $r(t) = \left(\frac{1}{3}t^3+C, \frac{1}{2}t^2+C, e^t+C\right)$

$r(0)=0 \Rightarrow \sim.$

$\boxed{\S 2.3.}$ 1. (2). $r(t) = (3t-t^3, 3t^2, 3t+t^3)$.

$r'(t) = (3-3t^2, 6t, 3+3t^2)$ $\underline{|r'(t)| \neq 1.}$

$$k(t) = \frac{r'(t)\times r''(t)}{|r'(t)|^3}$$

$k = \frac{r''(s)}{|r''(s)|}$ (仅限 s).

$= \frac{r'(t)\times r''(t)}{|r'(t)|^3}$. 记住.

$r''(t) = (-6t, 6, 6t)$

$|r'(t)| = \sqrt{9+9t^4 - \cancel{18t^2} + 36t^2 + 9 + 9t^4 + \cancel{18t^2}}$

$= (18t^4+36t^2+18)^{\frac{1}{2}}$

曲率 k $= 3(2t^4+4t^2+2)^{\frac{1}{2}}$

$27(2t^4+4t^2+2)^{\frac{3}{2}} = |r'(t)|^3$

$r'(t)\times r''(t) = (36t^2-18-18t^2,$ $\cancel{-18t-18t^3-18t+18t^3}$ $-32t$

$\cancel{18-18t^2, 36t^2}$ $18+18t^2$

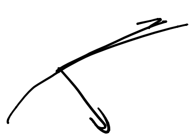
与 $|r'(t)|^3$ 作比

$= (18t^2-18, -32t, 18t^2+18)$

2. (2). $r'(t) = (-a\,\cancel{\cos t}\ \sin t, b\cos t, e^t)\Big|_{t=0}.$

$r'' = (-a\cos t, -b\sin t, e^t)$.

密切平面由 r' r'' 张成.



$X(u,v) = u r'(t) + v(r''(t)) + r(t)$

$= \sim.$

4. 解. ~~$r=(x,\sqrt{12-2x^2},\sqrt{x^2-3})$~~ (参照课本P37).

~~$x^2+y^2+x^2-3=9$~~ ~~$y^2=12-2x^2$~~

因为 x,y,z / x',y',z' / x'',y'',z'' 都在方程中

$\boxed{\text{直接求导解方程}}$ 就能得出

$\begin{cases} r(s) \\ r'(s) \\ r''(s) \end{cases}$

$$\begin{cases} x^2+y^2+z^2-9=0 \\ x^2-z^2-3=0 \end{cases} \to \begin{cases} xx'+yy'+z\cdot z'=0 \\ xx'-zz'=0 \end{cases}$$

$$x'^2+y'^2+z'^2=1$$

$S=0$ 有ですは!!

$(2,2,1)$.

$$\begin{cases} 2x'+2y'+z'=0 \\ 2x'-z'=0 \end{cases} \Rightarrow \begin{matrix} z'=2x' \\ y'=-z' \end{matrix}$$

取 $x'=\frac{1}{3}$, $z'=\frac{2}{3}$, $y'=-\frac{2}{3}$

$$x'^2+4x'^2+4x'^2=1 \quad 9x'^2=1$$

$r'(0)$ $r'=(\frac{1}{3},\frac{2}{3},-\frac{2}{3})$

再导: $\begin{cases} x'x''+y'y''+z'z''=0 \\ x'x''-z'z''=0. \end{cases}$

$2z''-2y''+2z''=0$

$4y''=8z''$

$$\frac{1}{3}x''\cancel{+\frac{2}{3}}-\frac{2}{3}y''+\frac{2}{3}z''=0$$

$$\frac{1}{3}x''-\frac{2}{3}z''=0. \qquad \underline{x''=2z''} \quad \underline{y''=2z''}$$

$$2x+2y+\frac{1}{9}+\frac{4}{9}=\frac{1}{2}x \qquad r'' \to \left(\frac{1}{7},\frac{1}{14},\frac{1}{7}\right)$$

$$\frac{36}{9}x+\frac{5}{9}=\frac{1}{2}x \qquad \frac{35}{9}x+\frac{5}{9}=0$$

$$x=\frac{5}{35}=\frac{1}{7}$$

3.29 (老姐生日)

则 $K(0)=|r''(0)|=$ ~~$\frac{3}{14}$~~

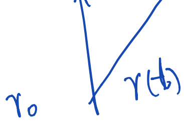
$$\cancel{\frac{1}{49}+\frac{1}{144}}\ \frac{4}{4\times7^2}+\frac{1}{4\times7^2}+\frac{4}{4\times7^2} = \frac{9}{4\times7^2}$$

7. $\alpha(t)$ ~~$X^{(t)}=r(t)+\lambda(t)\,r'(t)=\vec{op}$~~

$$(\vec{op}-r(t))\times r'(t)=\vec{0}$$

$$(r(t)-r_0)\times r'=0$$

$$-r_0\times r'=0$$



$\vec{op}\times r' - r\times r' = 0$ ~~则 $op\times r'$~~ r''

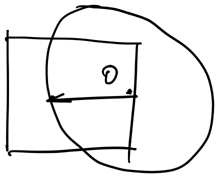
$\vec{op}\times r' = r''$

$\sim = r\times r_0 = C$

平面曲线

8.

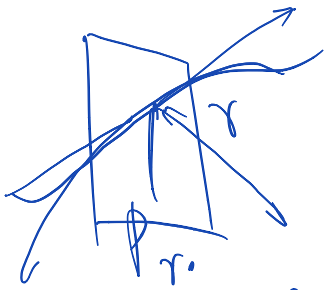
$(r_0 - r)\cdot\alpha = 0$ 目标 $|r_0 - r| = C$.



$-\alpha\cdot\alpha + (r_0 - r)k\beta = 0$ X

~~$r_0 k$~~ $(r_0 - r)k\beta = 1$.

$r_0 - r = 0$ X.



$r - r_0 = \lambda\beta + \mu\gamma$.

$\alpha = \lambda'\beta + \lambda(-k\alpha + \tau\gamma) + \mu'\gamma + \mu\tau\beta$.

$= -\lambda k\alpha + (\lambda' + \mu\tau)\beta + (\lambda\tau + \mu')\gamma$.

$1 = -\lambda k$, $\lambda' + \mu\tau = 0$, $\lambda\tau + \mu' = 0$

$\lambda = -\frac{1}{k}$ $0 + \mu\tau = 0$ $\mu = 0$

$r - r_0 = -\frac{1}{k}\beta$. 则 $|r - r_0| \equiv \frac{1}{k}$ √.

$(r - r_0)\in\beta\times\gamma$

$(r - r_0)\cdot\alpha = 0$

$r\cdot\alpha - r_0\cdot\alpha = 0$

$|r|\cdot|\alpha|\cdot\cancel{\cos\angle\alpha r}$ ✓

$-|r_0||\alpha|\cdot\cos\angle\alpha r_0 = 0$.

是定值

§2.4. 3. $\tilde{r} = \frac{1}{\tau}\beta - \int \gamma\, ds$.

套用Frenet公式即可.

$$\frac{d\tilde{r}}{ds} = \frac{1}{\tau}(-k\alpha + \tau\gamma) - \gamma$$

$$= -\frac{k(s)}{\tau}\alpha \qquad \tilde{\alpha} = \alpha.$$

$$\tilde{r}'' = \left(-\frac{k(s)}{\tau}\alpha\right)' = -\frac{k'}{\tau}\alpha - \frac{k}{\tau}k\beta.$$

$$\tilde{r}' \times r'' = \left(-\frac{k}{\tau}\alpha\right)\times(r'') = \frac{k^3}{\tau^2}\gamma$$

$$\tilde{r}''' = -\frac{1}{\tau}\left(k''\alpha + k^2\beta + 2kk'\beta + k^2(-k\alpha + \tau\gamma)\right)$$

$$= -\frac{1}{\tau}\left((k'' - k^3)\alpha + (k^2 + 2kk')\beta + \tau k^2\gamma\right)$$

$$\tilde{k} = \frac{|r' \times r''|}{|r'|^3} = \frac{k^3}{\tau^2} \Big/ \left(\frac{k}{\tau}\right)^3 = |\tau|$$

$$\tilde{\tau} = \frac{(r'\ r''\ r''')}{|r''|^2} = \frac{k^3}{\tau^2}\gamma \times \cancel{-\frac{1}{\tau}\left(\frac{k^3}{\tau^2}(k''-k^3)\beta + \frac{k^3}{\tau^2}(k^2+2kk')\alpha\right)}$$

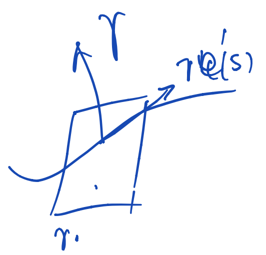
公式

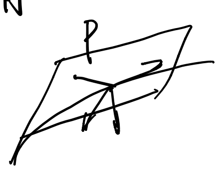
$$-\frac{1}{\tau}\frac{k^3}{\tau^2}\cdot \tau k^2 = -\frac{k^5}{\tau^2} \Big/ \frac{k^6}{\tau^6}$$

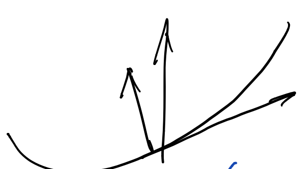
$$= -\frac{k^5}{\tau^2}\frac{\tau^4}{k^6} = \frac{\tau^2}{k}$$

key: $\gamma \to \tau\beta$.

5. 证明:







$(r - r_0)\cdot\gamma = 0.$

$(r - r_0)\boxed{\tau\beta} = 0.$ $\tau = 0$ 或者 $(r - r_0)\perp\beta$.
$\perp\gamma$.

$\tau \equiv 0$ 或者 $(r - r_0)\cdot\beta = 0$

$(r - r_0)\times\alpha = 0.$

曲线恒过一定点

切线 也肯定在一平面

$(r - r_0)\cdot\gamma = 0.$ $|r|\cdot|\gamma|\cos\theta_1$ $-|r_0|\cdot|\gamma|\cos\theta_2 = 0$

$r\cdot\gamma - r_0\cdot\gamma = 0$

$(r - r_0)\times\alpha = 0.$

$(r - r_0)\times k\beta = 0.$ $k\beta = 0$. 则为直线.

6. 解

$$\begin{cases} \alpha'(s) = k\beta = \rho\times\alpha \\ \beta' = -k\alpha+\tau\gamma = \rho\times\beta \\ \gamma' = \tau\beta = \rho\times\gamma \end{cases} \Rightarrow \quad \rho\in\alpha\times\gamma \quad \rho = \lambda\alpha+\mu\gamma$$

要记字公式

$\mu\beta = k\beta$ $\quad \mu = k \quad$ $\lambda\beta = \tau\beta$. 解方程 *1

$\rho = \tau\alpha + k\gamma$. $\quad$ ~~$\tau\gamma + k\alpha = -$~~ ✓

8. 证明. $\quad \alpha' = k\beta \quad \alpha'' = k'\beta + k(-k\alpha+\tau\gamma)$.

~~α'''~~ $\quad = -k^2\alpha + k'\beta + k\tau\gamma$.

$\gamma' = \tau\beta \quad \gamma'' = \tau'\beta + \tau(-k\alpha+\tau\gamma)$

$= -\tau k\alpha + \tau'\beta + \tau^2\gamma$.

$(\alpha\times\alpha' = k\gamma)\cdot\alpha'' = k^2\tau$. $\quad$ 式子左侧 $- \quad -\tau^3k^3$

$(\gamma\times\gamma' = \tau\alpha)\cdot\gamma'' = -\tau^2 k$ $\quad$ 纯代入计算

右侧 $(k\beta)^3 = k^3\beta\cdot\tau^3\beta = k^2\tau^3$ $\quad \varepsilon$ 与 ($\pm$方向有关). ✓

9. 证明,

$\tilde{\gamma}' = \gamma(t)\Big|_{s_0}^{s}$? $\quad$ 记 $\frac{dR(t)}{dt}^{p} = \gamma(t)$ 即可.

$\tilde{\alpha} =$ $\quad$ s_0与s代换关系

$\tilde{\alpha}' = \gamma'(t) = \tau\beta = \tilde{k}\beta$. 则 $\tilde{k} = \tau$ $\quad$ $\boxed{\frac{ds}{d\tilde{s}}}$??

$$\frac{d\tilde{r}(s)}{ds} = \tilde{\alpha}(s) = \frac{d\int_{s_0}^{s}\gamma(t)dt}{ds}$$

$= \gamma(s) - 0$ $\quad$ 全然全知.

$\Rightarrow \tilde{k} = \tau$. ✓

$= $ ~~$k\alpha(s)$~~ $)\tau\beta$

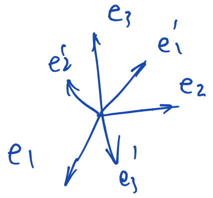
$$\frac{d\tilde{\alpha}(s)}{ds} = \tilde{k}(s)\tilde{\beta}(s) = (k\beta)'$$

$\boxed{\tilde{k}'\tilde{\beta} + \cancel{\tilde{k}}(-\tilde{k}\tilde{\alpha}+\tilde{\tau}\tilde{\gamma})}$ $= \cancel{k\beta}$ $\boxed{\tau'\beta + \tau(-k\alpha+\tau\gamma)}$

$-\tilde{k}^2\tilde{\alpha} + \tilde{k}\tilde{\tau}\tilde{\gamma}$ $\quad -k\tau\alpha + \tau^2\gamma$

$\Rightarrow \tilde{\tau} = k$..

11. 证明,



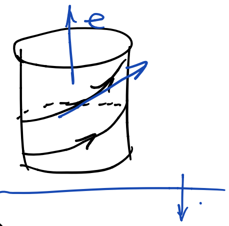
$e_1'\cdot e_2' = \lambda_{11}\cdot\lambda_{21} + \lambda_{12}\cdot\lambda_{22} + \lambda_{13}\cdot\lambda_{23}$

$e_1' = \lambda_{11}e_1 + \lambda_{12}e_2 + \lambda_{13}e_3$

$e_2' = \lambda_{21}e_1 + \lambda_{22}e_2 + \lambda_{23}e_3$

$\lambda_{31}e_1 + \lambda_{32}e_2 + \lambda_{33}e_3$

§2.5. 1. $\alpha(s)\cdot e \equiv C.$



$e\perp\beta.$

$e\cdot k\beta \equiv 0.$ 则 $k=0$? ✗.

求导就是对的

$e\perp\beta.$ $\quad k e\cdot(-k\alpha+\tau\beta\gamma)=0$

$k^2\vec{e}\cdot\vec{\alpha}=k\tau\vec{e}\cdot\vec{\gamma}$

$k\vec{e}\cdot\vec{\alpha}=\tau\vec{e}\cdot\vec{\gamma}$

$kC_1=\tau C_2. \Rightarrow \frac{k}{\tau}=C.$ ✓.

4. 证明: $\gamma'(t)=(1+\sqrt{3}\cos t, -2\sin t, \sqrt{3}-\cos t).$

计算量有点大?

$\gamma_1'(u)=(-\sin\frac{u}{2}, \cancel{\sin}\ \cos\frac{u}{2}, -1).$

$\gamma''(t)=(-\sqrt{3}\sin t, -2\cos t, \sin t)$

$\gamma''(u)=(-\frac{1}{2}\cos\frac{u}{2}, -\frac{1}{2}\sin\frac{u}{2}, 0).$

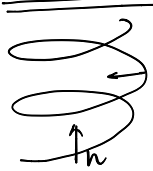
两者曲率与挠率相同,则是同一条曲线

γ''

$k=\frac{\gamma'\times\gamma''}{|\gamma'|^3} \qquad \tau=\frac{(\gamma',\gamma'',\gamma''')}{|\gamma''|^2}.$

分别求出γ_1 γ_2的$k\tau$. 即可.

6. 证明: $\beta\cdot n=0$ $\Rightarrow$. 一般螺线

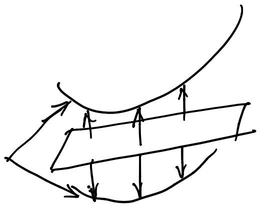


$k(s)\beta(s)\,n=0$

$\alpha'\cdot n=0.$ $\quad \int\alpha'(s)n\,ds=\oint\alpha(s)\cdot n=C.$

即证.

§2.6. 2.



$r(s)$. $r(x(t), y(t), z(t))$.

$\tilde{r}(x(t), y(t), -\tilde{z}(t))$.

$r'(t) = (x', y', z') = \cancel{\alpha}$ $\cancel{z'}$

$r''(t) = (x'', y'', z'') = \cancel{k\beta}$

$r'''(t)$

$\left[k = r' \cdot r'' / |r'|^3\right]$ $\left[\tau = \dfrac{(r'\ r''\ r''')}{|r'|^2}\right]$ ✓

$x' \cdot x'' + y'y'' + z'z''$ (不知有没有简单算法).

3.

$r^{(n+1)}(s) = a_n'(s) \sim \cdots$ 直接导即可

§2.7. 4. $\beta = \tilde{r}$

不太清楚.

$(r-\tilde{r})' = \alpha - \alpha'$

$(r-\tilde{r})'' = k\beta - \tilde{k}\tilde{\beta} = \underline{k\tilde{r} - \tilde{k}\tilde{\beta}}$

$(r-\tilde{r})''' = k'\tilde{r} + k\tilde{\tau}\tilde{\beta} - \tilde{k}'\tilde{\beta} \cancel{- \tilde{k}(-\tilde{k}\tilde{\alpha} + \tilde{\tau}\tilde{r})}$

$= \cancel{k'\tilde{r} + k\tau}$ $+ \tilde{k}^2\tilde{\alpha} - \tilde{k}\tilde{\tau}\tilde{r}$

5. 解：就正常求就好了.

终了~.

$$e_i \cdot e_j = \lambda_{i1} \cdot \lambda_{j1} + \lambda_{i2}\lambda_{j2} + \lambda_{i3}\lambda_{j3}$$

$$\begin{cases} \lambda_{i1} e_1 \quad \lambda_{i2} e_2 \quad \lambda_{i3} e_3 \\ \lambda_{j1} e_1 \quad \lambda_{j2} e_1 \quad \lambda_{j3} e_3 \end{cases}$$