椭球：$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

$$f(\varphi,\theta)=\begin{cases} x=a\cos\varphi\cos\theta \\ y=b\cos\varphi\sin\theta \\ z=c\sin\varphi \end{cases}$$

① $\varphi\in(0,2\pi)\quad \theta\in(-\frac{\pi}{2},\frac{\pi}{2})$

② $\varphi\in(-\pi,\pi)\quad \theta\in(-\frac{\pi}{2},\frac{\pi}{2})$.

③ $$f(x_1,y_1)=\begin{cases} x=a\varphi\cos\theta \\ y=b\varphi\sin\theta \\ z=c\sqrt{1-\varphi^2} \end{cases}$$

$\theta\in(0,2\pi)\quad \varphi\in[0,1)$

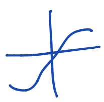
④ $\sim\quad z=-c\sqrt{1-\varphi^2}$

$\theta\in(0,2\pi)\quad \varphi\in[0,1)$

①~②：$\begin{cases} \varphi_2=\varphi_1+\pi \\ \theta_2=\theta_1 \end{cases}$ $(\cancel{\varphi_1\in(-\pi,0)})$. 反了.

$$\frac{\partial(\varphi_2,\theta_2)}{\partial(\varphi_1,\theta_1)}=\begin{vmatrix}1&0\\0&1\end{vmatrix}=1$$

~~①~③：~~ $\cancel{\begin{cases} \varphi_2=\cos\varphi_1 \\ \theta_2=\theta_1 \end{cases}}$



$\begin{pmatrix}-\sin\varphi_1&0\\0&1\end{pmatrix}=\sin\varphi_1\quad \varphi_1\in(\cancel{0,2\pi})$ $-\frac{\pi}{2},\frac{\pi}{2}$.

③ $$f(x_1,y_1)=\begin{cases} x=ax_1 \\ y=by_1 \\ z=c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}} \end{cases}\qquad x_1^2+y_1^2<1$$

①~③ $\begin{cases} x_1=\cos\varphi\cos\theta \\ y_1=\cos\varphi\sin\theta \end{cases}$

$\partial\varphi\sim>0$.

单叶双曲：$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

双叶双曲：$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$.

① $$f(u,v)=\begin{cases} x=a\sqrt{1+v^2}\cos u \\ y=b\sqrt{1+v^2}\sin u \\ z=cv \end{cases}$$

$u\in(0,2\pi)$

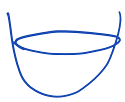
② $u\in(-\pi,\pi)$.

$a\sqrt{1+v^2}$

双叶. $\begin{cases} x=ax_1 \\ y=by_1 \\ z=\sqrt{\frac{x^2}{a^2}+\frac{y^2}{b^2}+1} \end{cases}\qquad x_1^2+y_1^2<1$.

Monge 形式 则是正则的.

椭圆抛物面：$\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$



$\begin{cases} x=ax_1 \\ y=by_1 \\ z=\frac{1}{2}(x_1^2+y_1^2) \end{cases}\qquad x_1,y_1\in R$.

是Monge形式 正则可定向.

双曲抛物面：$\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$

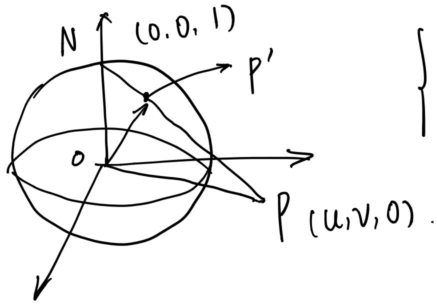
$\begin{cases} x=ax_1 \\ y=by_1 \\ z=\frac{1}{2}(\frac{x^2}{a^2}-\frac{y^2}{b^2}) \end{cases}\qquad x,y\in R$.

高考加油

§3.1. 1. 参考教案

2.

感觉很重要



$$\begin{cases} x^2+y^2+z^2=1 \\ \lambda(x-u,y-v,z)=(-u,-v,1) \end{cases}$$

$$\Rightarrow \begin{cases} \lambda x-\lambda u=-u \\ \lambda y-\lambda v=-v \\ \lambda z=1 \end{cases}$$

用NPP'同一条线来做

$x=\left(\frac{\lambda-1}{\lambda}\right)u=\left(1-\frac{1}{\lambda}\right)u$；$y=\left(1-\frac{1}{\lambda}\right)v$；$z=\frac{1}{\lambda}$

$$\left(1+\frac{1}{\lambda^2}-\frac{2}{\lambda}\right)u^2+\left(1+\frac{1}{\lambda^2}-\frac{2}{\lambda}\right)v^2+\frac{1}{\lambda^2}=1.$$

$$\lambda=\frac{+u^2+v^2+1}{u^2+v^2-1} \Rightarrow P'\left(\frac{2u}{u^2+v^2+1},\frac{2v}{u^2+v^2+1},\frac{u^2+v^2-1}{u^2+v^2+1}\right).$$

$(u\in R, v\in R)$. 球上每一点与N连线肯定过平面且不唯一.

则 p' 坐标可以表示以上.

(2). $S(0,0,-1)$ 与 p 进行同样操作（由对称性）.

$p'=\left(\sim,\sim,-\frac{u^2+v^2+1}{u^2+v^2+1}\right)$. 与 xoy 对称

(3). 公共部分：除去N,S点. 参数变换即 ~~$x=x$ $y=y$ $z=-z$~~.

对 P'_1：$\frac{2u}{u^2+v^2+1}=x$ $\frac{2v}{u^2+v^2+1}=y$

$\Rightarrow$ $xu^2+xv^2+x=2u$

$yu^2+yv^2+y=2v.$

$$\begin{cases} u=\frac{-x}{z-1} \\ v=\frac{-y}{z-1} \end{cases}$$

P'_2：$x=$ — $y=$ —. $z=$ — 代入

有 $u=f(\tilde{u})$ $v=g(\tilde{v})$. 就得到参数变换

不推荐这么求

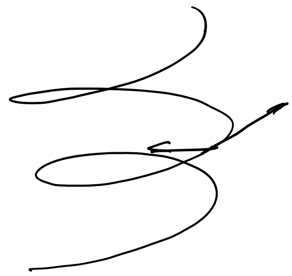
用这种方法：

$$\begin{cases} \frac{2u}{u^2+v^2+1}=\frac{2\tilde{u}}{\tilde{u}^2+\tilde{v}^2+1} \\ \frac{2v}{\sim}=\frac{2\tilde{v}}{\sim} \\ \sim=-\sim \end{cases}$$

$$\Rightarrow \tilde{u}=\frac{u}{u^2+v^2}$$

$\tilde{v} = \frac{v}{u^2+v^2}$ $\qquad \frac{\partial u\, v}{\partial \tilde{u}\, \tilde{v}} > 0$ 成立、切么正则-

4.



圆螺线：$r = (a\cos\theta,\ a\sin\theta,\ b\theta)$.

$r' = (-a\sin\theta,\ a\cos\theta,\ b)$

$r'' = (-a\cos\theta,\ -a\sin\theta,\ 0)$.

~~直线~~ √ $(a\cos\theta - a\lambda\cos\theta\ ,\ a\sin\theta - a\lambda\sin\theta,\ b\theta)$

直纹面.

9. 思考题.

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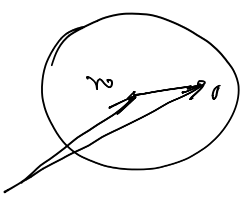
1. √

§3.2. $r - r_0 = \lambda(r_u \times r_v) = \lambda n$.

$\boxed{|r-r_0|^2 = a^2}$ Timechine key 求导



$\Rightarrow 2(r-r_0)\cdot(r_u du + r_v dv) = 0$.

$\Rightarrow r_u du\,(r-r_0) = 0$

$r_v dv\,(r-r_0) = 0$.

$(r-r_0)\times(r_u\times r_v) = 0$.

$r \times r_u \times r_v = r_0 \times r_u \times r_v$

$(r-r_0) r_u = 0$

$r_u \perp r - r_0$
$r_v \perp r - r_0$ $\Big\} \to$ 法线过定点.

$\Leftarrow \quad \lambda^{(u,v)} r_u \times r_v = r - r_0. \qquad (r_u\times r_v)\times(r-r_0) = 0$.

$r - r_0 = \lambda n$

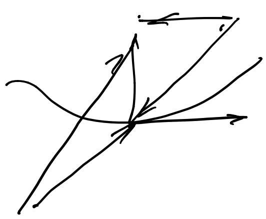
$r_u = \lambda_u n + \lambda n_u$

$r_v = \lambda_v n + \lambda n_v$

$r_u\cdot n = \lambda_u = 0$

$r_v\cdot n = \lambda_v = 0$

则 $\lambda = \lambda_0$ (是常数).



$r_u \times r_v \times r = r_u \times r_v \times r_0$ #

$\boxed{\begin{aligned}(r-r_0)\cdot r_u = 0\\ (r-r_0)\cdot r_v = 0\end{aligned}}$ key.

又 $r_u\ r_v$ 正则~.

$\boxed{r - r_0 = \lambda n}$ 平方即可.

4. ?

5. C: $\cancel{r(t)=(e^t\cos\sqrt{2}t,\ e^t\sin\sqrt{2}t,\ e^t)}$

$\cancel{r'(t)=(e^t\cos\sqrt{2}t-\sqrt{2}e^t\sin\sqrt{2}t.}$

$\cancel{e^t\sin\sqrt{2}t+\sqrt{2}e^t\cos\sqrt{2}t,\ e^t).}$

$\cancel{r_u=(-v\sin u, v\cos v, 0).}$ $\cancel{r'(t)=r_u\frac{du}{dt}+r_v\frac{dv}{dt}.}$

$\cancel{r_v=(\cos u, \sin u, 1)}$

$u=\sqrt{2}t \quad v=e^t$: $\underline{du=\sqrt{2}\,dt}$ $\underline{dv=e^t dt}$

则 $\frac{dr}{dt} = \cancel{dr}\ r_u\frac{du}{dt}+r_v\frac{dv}{dt}=\sqrt{2}r_u+e^t r_v.$ ✓.

~~善~~用全微分公式.

(2) pri: $\dfrac{r\cdot r_u}{|r\cdot r_u|}=\dfrac{r\cdot r_v}{|r\cdot r_v|}$

§3.3. 1.(1) $r=(u\cos v, u\sin v, f(v))$.

$r_u=(\cos v, \sin v, 0)$ $r_v=(-u\sin v, u\cos v, f'(v))$.

$E=r_u\cdot r_u=\cancel{\cos}\ 1.$

$F=r_u\cdot r_v=0$ $\quad I=E\,du^2+2F\,du\,dv+G\,dv^2$

$G=r_v\cdot r_v=f'(v)^2$

$\underline{I=(du)^2+(f'(v))^2(dv)^2.}$ 求就是了.

5. $r(u,v)$, I

$\Rightarrow I=E\,du^2+2F\,du\,dv+G\,dv^2$ $\quad F=0.$ ~~日~~ $P=E\ \ R=G.$

有 $ER-2FQ+GP=EG-GE=0$ ✓

✓

$\Leftarrow$. $ER-2FQ+GP=0$. $\Rightarrow F=0$. ×

$E\,du^2+2F\,du\,dv+R\,dv^2=0$

$P\,du^2+2Q\,du\,dv+R\,dv^2=0 \Rightarrow P\left(\frac{du}{dv}\right)^2+2Q\frac{du}{dv}+R=0$

①根与系数关系，(dv²除过去) $\Rightarrow \frac{du_1}{dv_1}+\frac{du_2}{dv_2}=\frac{Q}{P}$ $\frac{du_1}{dv_1}\cdot\frac{du_2}{dv_2}=-\frac{2Q}{P}$.

则有：

②用正交的原理

第一基本型之积

2个切方向正交：$r_1\ r_2$. 有

$dr_1\cdot dr_2 = (r_u du_1+r_v dv_1)\cdot(r_{u_2}du_2+r_{v_2}dv_2) = E\,du_1\,du_2+F(du_1dv_2+du_2dv_1)+G\,dv_1dv_2$.

$\Leftrightarrow E\left(\frac{du_1}{dv_1}\frac{du_2}{dv_2}\right)+F\left(\frac{du_1}{dv_1}+\frac{du_2}{dv_2}\right)+G$.

$=0$

$\Rightarrow E\frac{Q}{P}+F\left(-\frac{2Q}{P}\right)+G=0$. 证毕.

$r_{u_1}du_1\ r_{u_2}du_2+r_{u_1}du_1\cdot r_{v_2}dv_2+r_{v_1}dv_1\cdot r_{u_2}du_2+r_{v_1}dv_1\ r_{v_2}dv_2$.

~~$r_{u_1}\cdot r_{u_2}$~~ E F F G

§3.4. 1. $r(v,s)=r(s)+v\alpha(s)$.

作参数变换 $r(v,s)=\rho(s)+\tilde{v}\ \alpha$

其中 $\rho(s)=r(s)+l(s)\alpha$

$\tilde{v}=v-l(s)$

有 $r_s=r'(s)\alpha(s)+l(s)\kappa\beta+\tilde{v}\,\kappa\beta(s)$

假装懂了.

2. $r=(u\cos v,\ u\sin v,\ u+v)$.

$r_u=(\cos v,\ \sin v,\ 1)$

$r_v=(-u\sin v,\ u\cos v,\ 1)$.

类型. 改写成正交的参数曲面网

求 I 型. 作正交分解

$E=2\quad F=1\quad G=u+1$.

$I=2du^2+2dudv+(u+1)dv^2$

$=2(du+\frac{1}{2}dv)^2\ \cancel{-\frac{1}{4}dv^2}\ +(u+\frac{3}{4})dv^2$

$\begin{cases}\tilde{u}=u+\frac{1}{2}v.\\ \tilde{v}=v.\end{cases}$

$((u+\frac{1}{2}v)\cos v,\ (u+\frac{1}{2}v)\sin v,\ u+\frac{3}{2}v)$.

§3.5. 1. 证明: $r_1=(a\,\mathrm{ch}\,t\cos\theta,\ a\,\mathrm{ch}\,t\sin\theta,\ at)$

$r_2=(v\cos u,\ v\sin u,\ au)$.

$r_{1t}=(a\,\mathrm{sh}\,t\cos\theta,\ a\,\mathrm{sh}\,t\sin\theta,\ a)$

$\frac{dr_1}{d\theta}=(-a\,\mathrm{ch}t\sin\theta,\ a\,\mathrm{ch}t\cos\theta,\ 0)$.

$E=a^2\mathrm{sh}^2t+a^2\qquad F=0\qquad G=a^2\mathrm{ch}^2t$

对 S_2. $\frac{dr}{du}=(-v\sin u,\ v\cos u,\ a)$

$\frac{dr}{dv}=(\cos u,\ \sin u,\ 0)$.

$E=v^2+a^2\qquad F=0\qquad G=1$

相同的 I 形 ⟺ 保长

$I_1=(a^2\mathrm{sh}^2t+a^2)dt^2+a^2\mathrm{ch}^2t\,d\theta^2$

$I_2 = (v^2 + a^2)\, du^2 + dv^2$

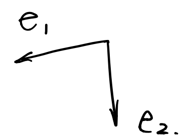
$\underline{u = \tilde{u}}$ $\underline{v = a \sinh t}$ $\Rightarrow I_2 = (a^2 \sinh^2 t + a^2) du^2 \longrightarrow$

△ 这是个正则变换. ✓

3. 证明. $r(u,v) = u e_1 + v e_2$.

$\frac{dr}{du} = e_1 \qquad \frac{dr}{dv} = e_2. \qquad E = 1 \quad F = 0 \quad G = 1.$

$I_1 = du^2 + dv^2$



$I_2 = d\tilde{u}^2 + d\bar{v}^2$

有 I_1 与 I_2 的变换: $\tilde{u} = f(u,v)$ $\tilde{v} = g(u,v)$.

牢记 $I^{*} = I$

$$J = \begin{pmatrix} \frac{\partial f}{\partial u} & \frac{\partial g}{\partial v} \\ \frac{\partial f}{\partial v} & \frac{\partial g}{\partial v} \end{pmatrix}$$

有 $I_2 = (du, dv) J J^{-1} \begin{pmatrix} du \\ dv \end{pmatrix} = I_1$

$\boxed{\text{记住 } J \text{ 的表示形式}}$

$\Rightarrow J \cdot J^{-1} = 1$ 正交的. 则是刚体运动.

$\det J = 1$ 是刚体 $\qquad \det J = -1$ 是关于直线的反射.

4. $r = (f(u) \cos v, f(u) \sin v, g(u))$

$\frac{dr}{du} = (f'(u) \cos v, f'(u) \sin v, g'(u))$

$\frac{dr}{dv} = (-f(u) \sin v, f(u) \cos v, 0)$

$E = f'^2(u) + g'^2(u) \qquad F = 0. \qquad G = f^2(u)$

$$\mathrm{I} = (f'(u)^2 + g'(u)^2)\,du^2 + f(u)^2\,dv^2$$

平面 $r = (\tilde{u}, \tilde{v}, 0)$

$\mathrm{I} = d\tilde{u}^2 + d\tilde{v}^2$

建立对应

$$(du \quad dv)\begin{pmatrix} E & \\ & G \end{pmatrix}\begin{pmatrix} du \\ dv \end{pmatrix} = (d\tilde{u} \quad d\tilde{v})\cancel{\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}}\begin{pmatrix} d\tilde{u} \\ d\tilde{v} \end{pmatrix}$$

$$\begin{cases} d\tilde{u} = \sqrt{f'^2(u) + g'^2(u)}\,du \\ d\tilde{v} = f(u)\,dv \end{cases} \Rightarrow \begin{cases} \tilde{u}_1 = \int \sqrt{f'^2(u) + g'^2(u)}\,du \\ \tilde{v} = \cancel{\int f(u)\,dv} = f(u)\,v. \end{cases}$$

则 $\mathrm{I} = du_1^2 + du_2^2$ ~

直接求即可