§4.1.

1. (1). 解. $r=(a\cos\varphi\cos\theta,\ a\cos\varphi\sin\theta,\ b\sin\theta)$.

$r_\varphi=(-a\sin\varphi\cos\theta,\ -a\sin\varphi\sin\theta,\ 0)$

$r_\theta=(-a\cos\varphi\sin\theta,\ a\cos\varphi\cos\theta,\ b\cos\theta)$.

$n=r_\varphi\times r_\theta=\big(-ab\sin\varphi\sin\theta\cos\theta,\ ab\sin\varphi\cos^2\theta,$

$-a^2\sin\varphi\cos\varphi\cos^2\theta-a^2\sin\varphi\cos\varphi\sin^2\theta\big)$.

$n_\varphi=(-ab\cos\varphi\sin\theta\cos\theta,\ ab\cos\varphi\cos^2\theta,$

$\boxed{\begin{matrix}-a^2\cos^2\varphi\cos^2\theta+a^2\sin^2\varphi\cos^2\theta\\ -a\cos^2\varphi\sin^2\theta+a^2\sin^2\varphi\sin^2\theta\end{matrix}})$

$-a\cos^2\varphi+a^2\sin^2\varphi$

$n_\theta=(-ab\sin\varphi\cos^2\theta+ab\sin\varphi\sin^2\theta.\ ,$

$-2ab\sin\varphi\cos\theta\sin\theta,\quad 0)$

$\cancel{+2a^2\sin\varphi\cos\varphi\cos\theta\sin\theta-a^2\sin\varphi\cos\varphi\,2\sin\theta\cos\theta}$

$\cancel{L=n_\varphi\cdot r_\varphi=\sim}$

牢记I、II型的计算方法.

$$\begin{cases} L=r_{\varphi\varphi}\cdot n=-r_\varphi n_\varphi=\sim \\ M=r_{\varphi\theta}\cdot n=-r_\varphi n_\theta=-r_\theta n_\varphi=\sim \\ N=r_{\theta\theta}\cdot n=-r_\theta n_\theta=\sim \end{cases}$$

(3). $r=(a(u+v),\ a(u-v),\ 2uv)$. 双曲.

$r_u=(a,\ a,\ 2v)$ $\quad r_{uu}=(0,0,0)$ $\quad r_{uv}=(0,0,2)$

$r_v=(a,\ -a,\ 2u)$ $\quad r_{vv}=(0,0,0)$

$n=r_u\times r_v=\big(2au+2a\overset{v}{\cancel{u}},\ 2av-2au,\ \overset{-2a^2}{\cancel{-a^2+a^2}}\ 0\big)$. $\cdot\ \dfrac{1}{|n|}$ (单位).

$\cancel{L=r_{uu}\cdot n=0\quad M=r_{uv}\cdot n=0\quad N=0\ \text{what?}}$

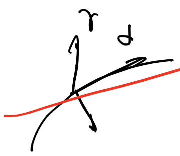
$(u,v,\ \ldots =0)\quad \dfrac{1}{\ldots}$

$$(u+v,\ v-u,\ a)\ \frac{\quad}{\sqrt{2u^2+2v^2+a^2}}$$

$$M=\frac{-2a}{\sqrt{2u^2+2v^2+a^2}} \qquad L=N=0. \qquad ▨$$

3. 高概率.

$r(s)$. 切线面. $\alpha=r'(s)$. $\beta=\dfrac{r''(s)}{|r''(s)|}$ $\gamma=\dfrac{r'\times r''}{|r''|}$.



$X=r+\lambda\alpha+\mu\gamma$

$$r(s,v)=r(s)+v\,r'(s)$$

$$r_s=\alpha(s)+vk\beta \qquad r_v=r'(s)=\alpha$$

$$n=r_s\times r_v=\cancel{\alpha\times\alpha}+vk\beta\times\alpha=vk\gamma \quad \frac{1}{vk}=\gamma$$

$$\begin{cases} r_{ss}=k\beta+vk'\beta+vk\cancel{\beta}'(-k\alpha+\tau\gamma) \\ r_{sv}=k\beta \\ r_{vv}=0 \end{cases}$$

$$L=r_{ss}\cdot n=vk\tau \qquad M=r_{sv}\cdot n=0 \qquad N=0.$$

$$\mathrm{II}=\boxed{vk\tau\,ds^2}$$

$(x(u_0,v_0),\ y(u_0,v_0),\ z(u_0,v_0))$

5. 证明: $r(u,v)$ $r_2=r(u,v)\cdot\sigma+\underbrace{(u_0,v_0)}$ key

たすげで!

其中 σ 是正交矩阵 $\sigma\cdot\sigma^{-1}=1$ (回忆刚体变换).

① $\dfrac{\partial r_2}{\partial u}=r_u\sigma$; $\dfrac{\partial r_2}{\partial v}=r_v\sigma$ $\Rightarrow E_2F_2G_2=EFG$.

② $\dfrac{\partial^2 r_2}{\partial u^2}=r_{uu}\sigma$; $\dfrac{\partial^2 r_2}{\partial v^2}=r_{vv}\sigma$ $\Rightarrow L_2M_2N_2=LMN$

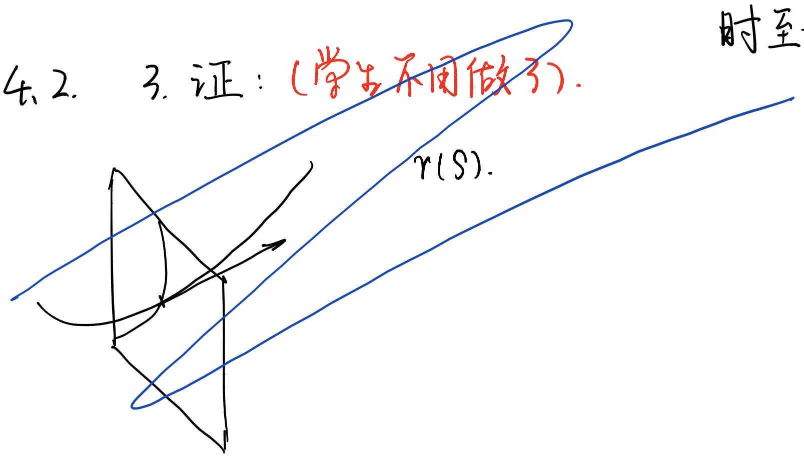
$$\frac{\partial r_2}{\partial u}\times\frac{\partial r_2}{\partial v}=n_2=n\sigma$$

key: σ 分解成正交系 $\{\sigma_1,\sigma_2,\sigma_3\}$.

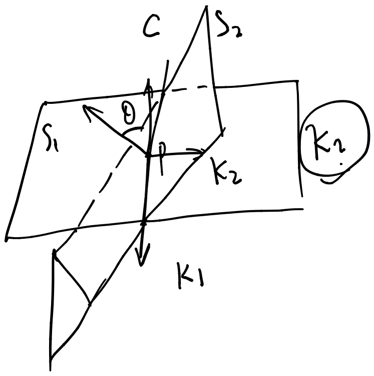
$$\begin{vmatrix} i & j & k \\ r_u\sigma_1 & r_u\sigma_2 & r_u\sigma_3 \\ r_v\sigma_1 & r_v\sigma_2 & r_v\sigma_3 \end{vmatrix}$$

时至今日，你依然是我的光.

§4.2. 3. 证：(学生不用做了).



4.



k_1 k_2.

$$\begin{cases} \boxed{k\cos\angle(\beta, n_1) = k_1} \\ k\cos\angle(\beta, n_2) = k_2. \end{cases}$$

β与n_1，n_2必在一个平面上

$$\underbrace{\angle\beta, n_1}_{\theta_1} + \underbrace{\angle\beta, n_2}_{\theta_2} = \underbrace{\angle n_1 n_2}_{\theta_3}.$$

$$k\cos\angle(\beta, n_2) = k\cos(\angle n_1 n_2 - \angle\beta n_1)$$

$$= k\cos\theta_3\cos\theta_1 - k\sin\theta_3\sin\theta_1$$

$$= k_1\cos\theta_3 - k\sin\theta_1\sin\theta_3.$$

$$\Rightarrow k\sin\theta_1\sin\theta_3 = k_1\cos\theta_3 - \cancel{k\cos\theta_2}\, k_2$$

$$k^2(\sin^2\theta_1\sin^2\theta_3) = (k_1\cos\theta_3 - k_2)^2$$

$$(k^2 - k_1^2)\sin^2\theta_3 = \sim$$

$$\Rightarrow k.$$

? 千古难题

5. (1). 求法曲率: $r=(a\cos u\cos v,\ a\cos u\sin v,\ a\sin u)$.

$$r_u=(-a\sin u\cos v,\ -a\sin u\sin v,\ a\cos u)$$

$$r_v=(-a\cos u\sin v,\ a\cos u\cos v,\ 0)$$

$$r_{uu}=(-a\cos u\cos v,\ -a\cos u\cos v,\ -a\sin u)$$

$$r_{uv}=(a\sin u\sin v,\ -a\sin u\cos v,\ 0)$$

$$r_{vv}=(-a\cos u\cos v,\ -a\cos u\sin v,\ 0)$$

$$n=\frac{r_u\times r_v}{|r_u\times r_v|}=\Big(+\cancel{a^2}\cos^2 u\cos v,\ +\cancel{a^2}\cos^2 u\sin v,\ \cancel{-a^2\sin u\cos u\cos^2 v-a^2\sin u\cos u\sin^2 v}+\cancel{a^2}\sin u\cos u\Big)\frac{1}{|\cos^3 u|\,a^2}\cos^2 u$$

$E=r_u\cdot r_u=a \qquad F=r_u\cdot r_v=0 \qquad G=\cancel{a^2c^2u\,s^2v+a^2\ldots c^2v}$

$\cancel{\cos^4 u\cos^2 v+\cos^4 u\sin^2 v}$

$\cancel{+\sin^2 u\cos^2 u+\cos^4 u}$

$\cancel{(1-\cos^2 u)\cos^2 u}$

$\cos^2 u\ \cancel{=\cos}$

$$\mathrm{I}=a\,du^2+(a^2\cos^2 u)\,dv^2$$

$$\mathrm{II}=a\,du^2+a\cos^2 u\,dv^2$$

$$\Rightarrow k_n=\frac{\mathrm{II}}{\mathrm{I}}=\frac{1}{a}\ ✓$$

但是 $u+v=\frac{\pi}{2}$ 什么图？

6. (1). $r=(u\cos v,\ u\sin v,\ bv)$. 渐近曲线的求法。

$$r_u=(\cos v,\ \sin v,\ 0)$$

$$r_v=(-u\sin v,\ u\cos v,\ b)$$

$$r_{uu}=(0,0,0) \qquad r_{uv}=(-\sin v,\ \cos v,\ 0)$$

$$r_{vv}=(-u\cos v,\ -u\sin v,\ 0).$$

总结常见面的I、II型.

$n=r_u\times r_v=(b\sin v,\ -b\cos v,\ u\ \cancel{\cos^2 v+u\sin^2 v})$ ...

$n = r_u \times r_v = (b\sin v, -b\cos v, \cancel{u\cos v + u\sin v}) \cdot \frac{1}{\sqrt{b^2+u^2}}$

$L = r_{uu} \times n = 0 \quad M = \cancel{-b\sin^2 v - b\cos^2 v} \frac{-b}{\sqrt{b^2+u^2}} \quad N = 0.$

$\mathrm{II} = -\frac{2b}{\sqrt{b^2+u^2}} \boxed{du\,dv} = 0 \Rightarrow u = u_0, \ v = v_0$ 均是 ✓ $\quad 2M\,du\,dv. = 0.$

渐近曲面

??. 看答案.

7. 证明: $S: \mathrm{II} = L\,du^2 + 2M\,du\,dv + N\,dv^2 = 0$

$= L(u'(s))^2 + 2M\,u'(s) \cdot v'(s) + N(v'(s))^2 = 0.$

$\begin{cases} r(u,v) \Rightarrow. \\ r(s) \end{cases}$

$\mathrm{I} = E\,du^2 + 2F\,du\,dv + G\,dv^2 \cancel{= 0.}$ dei!

①

C: C是渐近 则 $\gamma \perp n(s).$ 定理2.4.

γ与n共线 $\gamma = n$

$\tau = -\gamma' \cdot \beta = -n\text{的}.(\alpha \times \gamma)$

§4.3. 1. $z = \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2}$ (0,0). 法曲率与主曲率.

高概率

$s: r(x, y, \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2})$

$r_x = (1, 0, \frac{2}{\alpha^2}x) \quad r_{xx} = (0, 0, \frac{2}{\alpha^2}) \quad r_{xy} = (0,0,0).$

$r_y = (0, 1, \frac{2}{\beta^2}y) \quad r_{yy} = (0, 0, \frac{2}{\beta^2})$

II: $n=\left(-\frac{2}{\alpha^2}x,\ -\frac{2}{\beta^2}y,\ 1\right)\cdot\frac{1}{\sim}$　　$(0,0,1)$

$\mathrm{I}|_0 = \cancel{1+\alpha^2}\ dx^2+dy^2$　　$k_n|_0=\dfrac{\frac{2}{\alpha^2}dx^2+\frac{2}{\beta^2}dy^2}{dx^2+dy^2}$

$\mathrm{II}|_0=\frac{2}{\alpha^2}dx^2+\frac{2}{\beta^2}dy^2$

(主曲率)、 $n(x,y)=\left(-\frac{2}{\alpha^2}x,\ -\frac{2}{\beta^2}y,\ 1\right)\cdot\frac{1}{\sim}$

$n_x=\left(-\frac{2}{\alpha^2},0,0\right)$　　$n_y=\left(0,-\frac{2}{\beta^2},0\right)$.

主曲率的求法. $\cancel{W|_{x=0}}$ $W(n_x|_0)=-\frac{2}{\alpha^2}$

$W(n_y|_0)=-\frac{2}{\beta^2}$　　这是啥　　$\Sigma: x^2+y^2+z^2=1$

$\begin{vmatrix} L-\lambda E, & M-\lambda F \\ N-\lambda F, & N-\lambda G \end{vmatrix}$ 关系. 如何?

映射到球面上的投影值

$dn=W(dr)$

Eulor 公式简单的应用.

2. 证:  选曲率: 用Eular公式:

两个正交基向量: $e_1 e_2$. 主曲率为 $k_1\ k_2$

$\forall$方向, $\theta_1\theta_2$ 有 $k_n(\theta_1)=k_1\cos^2\theta_1+k_2\sin^2\theta_1$
$k_n(\theta_2)=k_1\cos^2\theta_2+k_2\sin^2\theta_2$. $\Rightarrow \theta_1=\theta_2+\frac{\pi}{2}$.

$\Rightarrow k_n(\theta_1)+k_n(\theta_2)=k_1+k_2$ ✓

5. 证明、

$k_n(\theta)=k_1\cos^2\theta+k_2\sin^2\theta$.

$\frac{1}{2\pi}\int_0^{2\pi}k_n(\theta)\,d\theta=\frac{k_1+k_2}{2}$ ✓

§4.4 2. 解. $r=(g(s),\theta,f(s))$. 旋转曲面. (把曲面写出来)

$$K=\frac{LN-M^2}{EG-F^2}=\sim.$$

$$r=(g\cos\theta,\ \cancel{f}\ g\sin\theta,\ f(s))$$

$r\sim$

$$\boxed{\begin{aligned}k_1+k_2&=2H=\frac{LG-2MF+NE}{EG-F^2}\\ k_1\cdot k_2&=K=\frac{LN-M^2}{EG-F^2}.\end{aligned}}$$

よい:

4. 解. 主曲率: n_x n_y 的 W 映射

9.(2). $\begin{pmatrix}L&M\\M&N\end{pmatrix}\cdot\begin{pmatrix}E&F\\F&G\end{pmatrix}^{-1}\cdot\begin{pmatrix}L&M\\M&N\end{pmatrix}=\begin{pmatrix}e&f\\f&g\end{pmatrix}$.

记 $-(n_u,n_v)=A(r_u,r_v)\cancel{A}$.

则有 $(n_u,n_v)\begin{pmatrix}n_u\\n_v\end{pmatrix}=A(r_u,r_v)\begin{pmatrix}r_u\\r_v\end{pmatrix}A^T$

$$=A\begin{pmatrix}E&F\\F&G\end{pmatrix}A^T$$

又 $\underset{\sim\sim\sim}{\begin{pmatrix}L&M\\M&N\end{pmatrix}=\overset{r_ur_u}{\begin{pmatrix}E&F\\F&G\end{pmatrix}}A}$. key

就有 $A=\begin{pmatrix}E&F\\F&G\end{pmatrix}^{-1}\begin{pmatrix}L&M\\M&N\end{pmatrix}$

则 $\begin{pmatrix}e&f\\f&g\end{pmatrix}=\left(\begin{pmatrix}E&F\\F&G\end{pmatrix}^{-1}\begin{pmatrix}L&N\\M&N\end{pmatrix}\right)^{-1}\begin{pmatrix}E&F\\F&G\end{pmatrix}\overset{A}{(\sim)}$

$$= \sim = \begin{pmatrix} L & M \\ M & N \end{pmatrix} \begin{pmatrix} E & F \\ F & G \end{pmatrix} \begin{pmatrix} h & w \end{pmatrix}$$