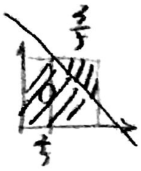
§1

§1.2 11 $p=\frac{\binom{6}{1}\binom{5}{1}}{\binom{10}{6}}=\frac{30}{\frac{10!}{4!6!}}=\frac{1}{7}$ $p=\frac{\binom{5}{1}\binom{4}{1}}{\binom{10}{4}}=\frac{20}{3\times7\times10}=\frac{2}{21}$

$\cancel{\frac{C_6^4(5\times4\times3)}{210}}$ $\frac{5!\cdot 4\cdot 5}{3!\,21}$ $\frac{\ }{10}$

12 $p=\frac{5\times5\times5}{6^3}$; $p=\frac{\binom{3}{1}5\times5}{6^3}=\frac{3\times5\times5}{6^3}$

23. $\begin{cases}x,y\in(0,1)\\ x+y<7/5\end{cases}$  $p=\frac{S_1}{S}=\frac{1}{2}\times(\frac{3}{5})^2$ 对了.

24.  $\begin{cases}X-Y\le 2\\ Y-X\le 1\end{cases}$ $p=\frac{S_1}{S}$

§1.3 1. $P(A\cup B)=\cancel{P(A)+P(B)-P(AB)}=0.8$ $\cancel{P(A\bar{B})}$ $P(A-B)=P(A)-P(AB)=0.3$

$P(AB)=0$

9 $\left(\frac{C_{n-1}^1}{C_n^1}\right)^{k-1}\cdot\frac{1}{C_n^1}$ 10. $P(AB)$, $P(A\cup B)=1=\overset{1}{P(A)}+P(B)-\overset{0}{P(AB)}$

16. $P(AB)=P(A)+P(B)-P(A\cup B)=P(\bar{A}\cap\bar{B})=1-P(A\cup B)$

17. $\cancel{P(\bar{A}\bar{B})=P(\bar{A})+P(\bar{B})-P(\bar{A}\cup\bar{B})}$ $\cancel{P(\overline{A\cap B})=1-P(AB)}$

$P(A-B)=P(A)-P(AB)=0.4$ $P(\overline{AB})$

§1.4 1. (1) $P(B|A)=\frac{P(AB)}{P(A)}=\frac{3\%}{15\%}=\frac{1}{5}$ (2) $P(A|B)=\frac{3\%}{5\%}=\frac{3}{5}$

2. $P(A|\bar{C})=\frac{P(A\bar{C})}{P(\bar{C})}=\frac{0.6}{0.95}=\frac{12}{19}$

12. $p=\frac{\binom{8}{1}}{\binom{10}{1}}\cdot\frac{\binom{7}{1}}{\binom{9}{1}}+\frac{\binom{2}{1}}{\binom{10}{1}}\cdot\frac{\binom{8}{1}}{\binom{9}{1}}$

15. $P(E|A)+P(F|B)+P(G|C)=\cdots$

16. (1) $2n$, n $p=\frac{0.97(2n)+0.94n}{3n}$

0.03 0.06

(2).

17. 全概率公式 $P(B)=P(A_1)\cdot P(B|A_1)+P(A_2)\cdot P(B|A_2)$.

§1.5 1. $P(A\cup B\cup C)=\underset{\frac{1}{5}}{P(A)}+\underset{\frac{1}{2}}{P(B\cup C)}-\underset{\frac{1}{5}}{P(A)}\cdot\underset{\frac{1}{2}}{P(B\cup C)}$ $\Rightarrow\frac{3}{5}$.

$P(B\cup C)=P(B)+P(C)-P(B)\cdot P(C)$.

$\cancel{\frac{1}{3}+\frac{1}{4}-\frac{1}{12}}$ $\frac{1}{2}$ $\frac{7}{10}-\frac{1}{10}$

2. (1) $P(AB)=P(A)\cdot P(B)$ (2) $P(A\cup B)=P(A)+P(B)-P(A)\cdot P(B)$

(3) $P(A\cup B)-P(AB)$

▲ 9. (1) $P(AB)=x^2$ $ABC\neq\emptyset$

$P(AB\cup AC)=P(AB)+P(AC)-P(ABC)=2x^2$

$P(AB\cup AC)\leq P(A)=x \Rightarrow \underline{x\leq 0.5}$ 0.5可取到

(2) ...

§2.

§2.1 2. $1\sim6$ $P=\frac{56}{36}$, $\cancel{0\ 1\ 2\ 3\ 4\ 5}$ $\frac{6}{36}$ $\frac{5}{36}$ $(4\ \ 3\ \ 2\ 1)$ $\times 2$

3. $X=n$ $P(X=n)=\frac{3}{10}\times\frac{2}{9}\times\frac{1}{8}$ ——

$1\ 2\ 3\ 4$

(2) $P(X=n)=$ ✓.

8. $\cancel{P(X<3)=\frac{1}{3}+\frac{1}{4}=P(X\le 3)}$.

9 $\cancel{\int_{-\infty}^{\ln x} \ln t\, dt}$ $\Rightarrow \ln x$ $\int_2^{2.5} p(x)\,dx$.

12. $x\in(-1,0)$ $F=\int_{-1}^{x}(1+t)dt=t+\frac{1}{2}t^2\Big|_{-1}^{x}$

$F=\int_0^x 1-t\,dt=t+\frac{1}{2}t^2\Big|_0^x$

15. $\lim_{x\to1} Ax^2=1$. $A=1$. $P(0.3\le X\le 0.7)=\int_{0.3}^{0.7}p(x)\,dx = F(0.7)-F(0.3)$.

$\cancel{P(x_1}$

§2.2. 1. $E(X)=-2\times0.4+0\times0.3+2\times0.2$.

$E(3X+5)=3E(X)+5$.

13. $E(X)=\int_0^\infty x\cdot e^{-x}dx=\int_0^\infty -x\,de^{-x}=-xe^{-x}\Big|_0^\infty+\int_0^\infty +e^{-x}dx$

$=-e^{x}\Big|_0^\infty=1$

15. $E(X)=\int_0^1(a+bx^2)x\,dx=\frac{1}{2}ax^2+\frac{1}{4}bx^4\Big|_0^1=\frac{1}{2}a+\frac{1}{4}b=\frac{2}{3}$ $\begin{cases}2a+b=\frac{8}{3}\\3a+b=3\end{cases}$ $\begin{cases}a=\frac{1}{3}\\b=2\end{cases}$

$\int_0^1 a+bx^2\,dx=1$. $\longrightarrow ax+\frac{1}{3}bx^3\Big|_0^1=a+\frac{b}{3}=1$.

§2.3. 1 $\left.\begin{array}{l}E(X^2-3X+2)=1\\E(X^2)-3E(X)=-1\\\lambda+\lambda^2-3\lambda=-1\end{array}\right|\Rightarrow\lambda$

$E(X^2)-\lambda^2=\lambda$.

3. $Var(1-3X)=9Var(X)=9\lambda$

5 $E(X)=$~ $E(X^2)=\int(x^2p(x))dx$.

$$p(x)=\begin{cases}\frac{e^x}{2}, & x<0\\0, & x\in[0,1)\\\frac{1}{4}e^{-\frac{1}{2}(x-1)}, & x\ge1.\end{cases}$$

§2.4. 1. $X \sim b(n,p)$ $P(X=0)=\binom{n}{0}(0.1)^0(0.9)^3$
$P(X=1)=\binom{n}{1}0.1\,(0.9)^2$ $\Big\} = 0.9^3+0.3\,0.9^2$

△ ~~10~~. $X\sim P(\lambda)$. $P(A|\text{有效}) = \frac{P(AB)}{P(B)} = \frac{P(3)\text{下}X=2}{0.75} = \frac{\cancel{9}}{\cancel{2}}e^{-3}\cdot\frac{4}{\cancel{3}}^{2} = 6e^{-3}$

A=感冒≠2

$P(X=2)=\frac{\cancel{3^2}\,9}{2!}\cdot e^{-3}$

12. $P(X=k)=(1-p)^{k-1}p=\left(\frac{n}{m+n}\right)^{k-1}\cdot\frac{m}{m+n}$ $\frac{m+n}{m}$次 (几何分布) $\to$ ~~why -1~~ 黑球数要减 $\boxed{\text{读题}}$

15. $P(X=0)=0.9^{10}$. $P(X=1)=\binom{10}{1}0.9^9\cdot 0.1=0.9^9$

$1-0.9^{10}-0.9^9=P$检检. $4\times P\cdots$

(10). $P(A|X=2)=\frac{P(A)P(X=2|A)}{P(X=2)}$ ← 生过下的2次 → 分为接触与未的

§2.5. 3 $K\sim U(1,6)$ $K^2-4\geq 0$. $K\geq 2$. $=\frac{4}{5}$

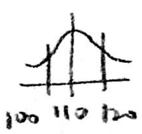
(6) $R\sim U(a,b)$ $\bar R=\frac{a+b}{2}$, $S=\pi\bar R^2=\pi\frac{(a+b)^2}{4}$ ✗ 构造面积的函数

11. $X\sim Exp(\lambda)=\lambda\cdot e^{-\lambda x}$ $(x>0)$

$P(X\leq 10)=\int_0^{10}p(x)dx=-e^{-\frac{x}{5}}\Big|_0^{10}=-e^{-2}+1=1-e^{-2}$

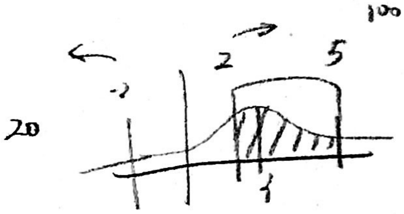
$P(Y\geq 1)=1-P(Y=0)=1-(e^{-2})^5$

16. $X\sim N(110,12^2)$



查表求 $\Phi\left(\frac{120-110}{12}\right)-\Phi\left(\frac{100-110}{12}\right)=$

$2\Phi(0.83333)-1$



$\Phi\left(\frac{15-3}{\sigma}\right)\cdot-\Phi\left(\frac{12-3}{2}\right)$ 方差

30 计算题

§2.6 10. (1) $X\sim U(0,1)$

$F_Y(y)=F(-2\ln x<y)$
$=F(x>\cancel{\ }e^{-2y})$
$=1-e^{-2y}$, $y\in(0,\infty)$

(2) $F_Y(y)=F(3X+1<y)$
$=F(x<\frac{y-1}{3})$
$=\frac{y-1}{3}$, $y\in(1,4)$

(3). $F(e^x<y)=F(x<\ln y)$
$=\ln y$ $(y\in(1,\infty))$

(4). $|\ln x|<y$
$e^{-y}<x<e^{y}$
$=e^{y}$

§3.

§3.1 2. $P(X=Y)=P(X=Y=1)+P(X=Y=2)=\frac{\binom{3}{1}\binom{2}{1}}{\binom{7}{4}}+\frac{\binom{3}{2}}{\binom{7}{4}}$

5. $\int_0^2\int_2^4 k(6-x-y)\,dy\,dx=1 \Rightarrow k=1/8.$

$\int_0^1 dx\int_2^3 \sim dy \to$ dx积过

(2). $P(X<1,Y<3)=\int_0^1\int_2^3 \frac{1}{8}(6-x-y)\,dy\,dx=\sim$

(3). $P(X<1.5)=\int_0^{1.5}\int_2^4 \frac{1}{8}(6-x-y)\,dy\,dx$ $\quad P_X(x)=\int_2^4 \sim dy.$

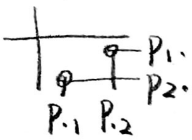
(4) $P(X+Y\leq 4)=\int_0^2\int_2^{4-x} \sim dy\,dx \longrightarrow$

6. (1) $\int_0^\infty\int_0^\infty ke^{-(3x+4y)}\,dx\,dy=1\Rightarrow k=12$

(2). $F(x,y)=\int_0^x\int_0^y ke^{-(3x+4y)}\,dy\,dx$

(3) $P(0<X\leq 1,0<Y\leq 2)=\int_0^1\int_0^2 p(x)\,dy\,dx = F(1,2)-F(0,0)$

§3.2 (10) $P(x_1,y_2)=\frac{1}{9}=\cancel{P_X(x_1)\cdot P_Y(y_2)=}$



$P_{1\cdot}=a+\frac{1}{9}+c$ $\quad P_{\cdot1}=a+\frac{1}{9}$ $\quad$: $P_{22}=P_{2\cdot}\cdot P_{\cdot2}$

$P_{2\cdot}=\frac{1}{9}+b+\frac{1}{3}$ $\quad P_{\cdot2}=\frac{1}{9}+b$

$P_{\cdot3}=\frac{1}{3}+c$

11. $Exp(1)=e^{-x}$ $\quad Exp(\lambda)=\lambda e^{-\lambda x}.$

$p(x,y)=P_X(x)\cdot P_Y(y)=e^{-y}$ $\quad x\in(0,1).\ y>0$

(2) $\int_0^1 dx\int_0^x e^{-y}\,dy=\int_0^1 -e^{-y}\big|_0^x\,dx\ \cdots$

(3). $P(X+Y\leq 1)=\int_0^1 dx\int_0^{1-x} e^{-y}\,dy$

14. (1) $P_X(x)=\int_0^\infty xe^{-(x+y)}\,dy$; if $P(x,y)=P_X(x)\cdot P_Y(y)$. 则独立.

$P_Y(x)=\int_0^\infty x\cdot e^{-(x+y)}\,dx$

§3.3. (3)、从X,Y分布列构造 $X\backslash Y$ (联合分布列).

X \ Y	0	1	
-1	1/4	0	1/4
0	0	1/2	1/2
1	1/4	0	1/4
	1/2	1/2	

$P(XY\neq 0)\Rightarrow P(XY\neq 0)=0.$

$\max\{X,Y\}=0,1$

★6. $(X+Y)/2=Z$

$z\le 0$ 时 $F_Z(z)=0$

$z>0$ 时 $F_Z(z)=\int_0^{2z}dx\int_0^{2z-x}e^{-(x+y)}dy=\sim=1-(2z+1)e^{-2z}$ 求导即可

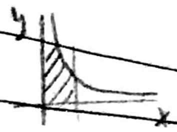
(2). $Z=Y-X$ $Y=X+Z$ $\boxed{\text{分段点 } 0}$

~~$F(z)$~~ $z\le 0$ 时 $F_Z(z)=\int_{-z}^{\infty}dx\int_0^{x+z}\sim dy.$

step1 $Y=f(x,z)$，由 x,y Range 找分段点

step2 画图，在分段中确定 $F_Z(z)$

step3 求导 $F_Z(z)$.



9. $X=\begin{cases}1\\0\end{cases}$ $Y=e^{-y}\ (y>0)$ $y=\frac{x}{z}$. $z<0$ 时 $F_Z(z)=0$ 不是仅 $\iint$...

~~$z>0$ 时 $F_Z(z)=\int_0^1 dx\int_0^{\frac{x}{z}}\sim dy$~~

√ 12. $Z=|X_1-X_2|$ 求 $p(x,y)$ 同 6

16. $p(x,y)=\begin{cases}e^{-(x+y)}\\0\end{cases}$ $p\ \boxed{\text{Jacob}}=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}=-u.$

$p(u,v)=p(uv,u\cdots)\cdot|u|\ \longrightarrow$

§3.4 2 ~~$E_n=nE\sim=3n$~~ ~~$n^2\ E_1=\sim$~~ $E(X_i)=\frac{7}{2}$ $Y\ Var(X)=\sum Var(X_i)$

△ 4. $p(x,y)=\begin{cases}1, & x\in(0,1),\ y\in(0,1)\\0, & \text{other.}\end{cases}$

7.

$F(x,y)=\int_{1-x}^{1}\int_x^1 2\,dy\,dx=$ 正规计算即可

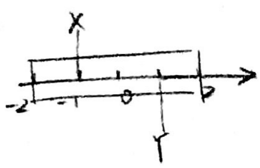
$E(X+Y)=\int_0^1 dx\int_{1-x}^{1}(x+y)\cdot 2dy=\frac{4}{3}$

$E\ \vee\ E((X+Y)^2)$ ———— $=\sim$ $Var\cdots$

12. $F_Y(y)=P(Y=\max\{X_1,X_2\cdots X_5\})=P\{X_1\le y\}\cdot P\{X_2\le y\}\cdots P\{X_5\le y\}=[F(y)]^5$

$=y^{10},\ 0\le y<1$

16.



$X+Y=$	-2	0	2
$p=$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

§4

§4.3 大数定律 7 $E(X_n)=\sum_{k=1}^{\infty}\frac{1}{\cancel{2^k}}\cdot\frac{\cancel{2^k}}{k^2}$ 有界即存在 由辛钦 … 满足~

9. ~~$X\sim P(\sqrt{n})=\frac{(\sqrt{n})^k}{k!}e^{-\sqrt{n}}$ $E=\frac{1}{\sqrt{n}}$. $E(X_n)$ 存在.~~

$\frac{1}{n^2}\mathrm{var}(\sum_{i=1}^{n}X_i)=\frac{1}{n^2}\sum\sqrt{k}\leq\frac{1}{\sqrt{n}}\to 0$

$\sqrt{npq}$

§4.4 2. $X_n\sim(100,0.8)$. $E(X_n)=80$ $Var(X_n)=16$. 100·0.8·0.2

$P(X_n\leq 85)\approx\phi\left(\frac{85+0.5-80}{4}\right)=\sim$

12. $E=0.5$. $Var=0.01$.

$\phi\left(\frac{2510-0.5-2500}{0.1}\right)\sim$.

20. △ ~~$Cov(X,Y) = E(XY) - E(X)E(Y)$~~ §3

~~$E(X) = \frac{n}{6} = E(Y)$~~

24. $Cov(U,V) = Cov(2X+Y, 2X-Y) = \underline{4Cov(X,X) - Cov(Y,Y)}$

§3.5

2. $X \sim Ga(p)$ $P_X(x) = (1-p)^{x-1}p$ 联合分布 $\boxed{P_{ij} = P\{X=i, Y=j\} = p^2(1-p)^{j-2}}$

$Z = Ga(p)$ $P_Z(Z) = (1-p)^{Z-1}p$

$Y = X+Z$.

$F(X,Y) =$

$P(X=i \mid Y=j) = \frac{P_{ij}}{P_{\cdot j}}$

$P(Y=j \mid X=i) = \frac{P_{ij}}{P_{i\cdot}}$

3.

X \ Y	1	2
1	1/8	1/4
2	1/8	1/2

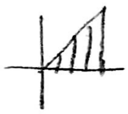
$Y=1$:

X	1	2
	1/2	1/2

$Y=2$: $\frac{1}{3}$ $\frac{2}{3}$

不独立

5. $p(x,y)$



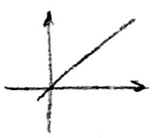
$p(y|x) = \frac{p(x,y)}{p(x)} = \frac{3x}{3x^2}$.

$P_X(x) = p(x,\infty) = \boxed{\int_{-\infty}^{\infty} p(x,y)dy} = 3x^2$

$p(x,y)$

$(3x+1)\lambda^2 e^{-\lambda(x+y)}$

16. $X \sim \underline{\lambda e^{-\lambda x}}$ $E(Z) = \int_0^{\infty} dx \int_x^{\infty} \quad dy$ ✓



§5.

§5.3

1. $\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i=\frac{30}{10}=3$, $S_n^2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2=\cdots$, S_n 标准.

5. $\bar{x}=\frac{1}{m+n}\cdot\left(\sum_{i=1}^{n}x_{1i}+\sum_{i=1}^{m}x_{2i}\right)=\frac{n\bar{x}_1+m\bar{x}_2}{m+n}$

$$s^2=\frac{1}{m+n}\left(\sum_{i=1}^{n}(x_{1i}-\bar{x})^2+\sum_{i=1}^{m}(x_{2i}-\bar{x})^2\right)=\cancel{\frac{1}{m+n}\left(nS_1^2+mS_2^2\right)}$$

8. $U(-1,1)$, $E(\bar{x})=\nu E(U)=0$, $Var(\bar{x})=\frac{1}{n^2}Var(U)=\frac{1}{3n}$

$\cancel{\int_{-1}^{1}x^2dx=\frac{1}{3}x^3\Big|_{-1}^{1}}$ $\frac{(b-a)^2}{12}=\frac{1}{3}$

$Var(X)=E(X^2)-(E(X))^2$

§5.4☆

7. $X\sim F(n,n)$, $P(X<1)=0.5$

$X\sim F(n,n)\Rightarrow\frac{1}{X}\sim F(n,n)$, $X>0$.

$P(X>1)=P(\frac{1}{X}>1)=P(X<1)=\frac{1}{2}$

△9. $X_1+X_2\sim N(0,2\sigma^2)$
$X_1-X_2\sim N(0,2\sigma^2)$
$\Rightarrow$ $\frac{X_1+X_2}{\sqrt{2}\sigma}$, $\frac{X_1-X_2}{\sqrt{2}\sigma}\sim N(0,1)$

$\chi_1^2=\frac{(X_1+X_2)^2}{2\sigma^2}\sim\chi^2(1)$

$\chi_1^2=\frac{(X_1-X_2)^2}{2\sigma^2}\sim\chi^2(1)$

$\Rightarrow Cov(X_1+X_2,X_1-X_2)=Cov(X_1,X_1)-Cov(X_2,X_2)=Var(X_1)-Var(X_2)=0$

独立 则

$Y=\frac{(X_1+X_2)^2}{(X_1-X_2)^2}\sim\boxed{F(1,1)}$ ☆此处指的(X_1+X_2)与(X_1-X_2)是χ^2分布的

确定法

$Y=\frac{X/n}{Y/m}\sim F(n,m)$, $X\sim\chi^2(n)$, $Y\sim\chi^2(m)$

step 1. 求 χ^2分布，求n,m
step 2. 判定2个χ^2分布独立
step 3. 写出$F(n,m)$

△12. $\bar{X}_n\sim N(\mu,\frac{\sigma^2}{n})$, $X_{n+1}\sim N(\mu,\sigma^2)$.

$x_{n+1}-\bar{x}_n\sim N(0,\sigma^2+\frac{\sigma^2}{n})$

$$t\sim\frac{\frac{x_{n+1}-\bar{x}_n}{\sqrt{\sigma^2+\frac{\sigma^2}{n}}}}{\sqrt{\frac{(n-1)S_n^2}{\sigma^2}/(n-1)}}=\cdots\sim t(n-1)$$

$\frac{x_{n+1}-\bar{x}_n}{\sqrt{\sigma^2+\frac{\sigma^2}{n}}}\sim N(0,1)$. $\boxed{\frac{(n-1)S_n^2}{\sigma^2}\sim\chi^2(n-1)}$

14. $\frac{X_1^2+\cdots+X_{10}^2}{\sigma^2} \sim \chi^2(10)$

$\frac{X_{11}^2+\sim+X_{15}^2}{\sigma^2} = \chi^2(5)$

$\frac{\chi^2(10)/10}{\chi^2(5)/5} = F(10,5)$ ✓

15. $\boxed{\frac{\bar{X}-\mu}{S/\sqrt{17}} \sim t(16)}$

$P(\bar{X} > \mu + kS) = P\left\{\underbrace{\frac{\bar{X}-\mu}{S/\sqrt{17}}}_{t(16)} > \sqrt{17}k\right\} = 0.95$

~~$t_{0.05}(16) = \sqrt{17}k$~~ ✓ 0.95 $\boxed{1.745}$

$(= t_{0.95}(16))$

16. 常规加减 $\mu . \sigma^2$

§5.5 ~（不考）

§6.

§6.1. 3. $E_\theta(\hat\theta)=\theta$ $E(\hat\theta^2)$ 有 $Var(\hat\theta)=E(\hat\theta^2)-E(\hat\theta)^2=E(\hat\theta^2)-\theta^2>0$

即 $E(\hat\theta^2)>\theta^2$ 不是无偏

7. $E(a\bar x_1+b\bar x_2)=(a+b)\mu=\mu$. 即为$\underline{\mu}$的无偏估计.

$E(\bar x_1)=\mu=E(\bar x_2)$ $\quad Var(Y)=a^2\sigma^2+b^2\sigma^2=(a^2+b^2)\sigma^2$, s.t. $\underline{a^2+b^2 最小}$

8. $Corr(\bar x,T)=\dfrac{Cov(\bar x,T)}{\sqrt{Var\,\bar x}\sqrt{Var\,T}}=\dfrac{\cancel{E(\bar xT)+E(\bar x)\cdot E(T)}\,\mu^2}{\sim}$ $\quad \underline{Coxxx}$

~~$E(T)=\mu=E(\bar x)$~~ ✓ $\quad$ Q $\boxed{设\ T(X_1\cdots X_n)=\sum_{i=1}^n a_iX_i}$ $\quad ,\sum_{i=1}^n a_i=1.$

$Cov(\frac{1}{n}\sum X_i,\sum a_iX_i)$ $\quad$ * $\underline{Cov(aX,bY)=ab\,Cov(X,Y)}$ $\quad Var(\bar X)=\frac{1}{n}Var(X)$

§6.2. 2. ~~$EX=\frac{\theta}{2}$ 则 $\frac{\hat\theta}{2}=E(X)=\frac{1}{10}\sum_{i=1}^{10}x_i$~~ ✓ $\quad$ ✓ 标准写法↓

$EX=\frac{\theta}{2}$ $\quad \theta=2EX\Rightarrow \underline{\hat\theta=2\bar x}\ \cdots$

3. $EX=\sum_{i=0}^{N-1}i\cdot\frac{1}{N}=\frac{\cancel{N}}{\cancel{N}}\cdot\frac{N-1}{2}\Rightarrow N=2EX+1$

$\hat N=2\bar x+1=\cdots$

(2) $EX=\sum_{k=2}^{\infty}(k-1)\theta^2(1-\theta)^{k-2}\cdot k.=\frac{2}{\theta}$ (求和

$\sum_{k=2}^{\infty}k(k-1)\theta^2(1-\theta)^{k-2}=\theta^2\sum_{k=2}^{\infty}\frac{d^2}{d\theta^2}(1-\theta)^k=\boxed{\theta^2\frac{d^2}{d\theta^2}}\left(\sum_{k=2}^{\infty}(1-\theta)^k\right)$ ✓

§6.3. 1. 最大似然估计. $L(\theta)=\prod_{i=1}^n P(x_i;\theta)$ $\quad I_{0<x_i<1}\longrightarrow\begin{cases}x\in(0,1)=1\\ x\bar\in(0,1)=0.\end{cases}$

$=\prod_{i=1}^n\sqrt\theta\,x_i^{\sqrt\theta-1}$ $\quad = \theta^{\frac{n}{2}}\cdot(x_1\cdots x_n)^{\sqrt\theta-1}$

$\ln L(\theta)=\frac{n}{2}\ln\theta+(\sqrt\theta-1)\ln(x_1\cdots x_n)$

$\frac{d\ln L(\theta)}{d\theta}=0\Rightarrow\theta=\sim$ 即为最大似然.

§6.6. 2. $\alpha = 0.05$.

σ^2已知，求μ. $\dfrac{u_{1-\alpha/2}\,\sigma}{\sqrt{n}} \leq \dfrac{k}{2} \Rightarrow n \geq \left(\dfrac{2u_{1-\alpha/2}\,\sigma}{k}\right)^2$

3. (1). $Y \sim N(\mu, 1)$. $\bar{Y} \pm \dfrac{u_{0.975}}{\sqrt{4}\,2} = \sim$

$\bar{Y} = 0$

(2) $x = e^Y$?

4. $\bar{X} \sim N(\cancel{56.32}\ \mu, \dfrac{\sigma^2}{n})$ $\sigma \sim \left(\dfrac{8s^2}{\chi^2_{0.975}(8)}, \dfrac{8s^2}{\chi^2_{0.025}(8)}\right)$

$\dfrac{8 \cdot s^2}{\sigma^2} \sim \chi^2(8)$

(2) μ : $\bar{x} \pm t_{0.995} \times 8s/\sqrt{9}$

9. (1). $\bar{x} - \bar{y} \pm u_{0.975}\sqrt{\dfrac{\sigma_1^2}{m} + \dfrac{\sigma_2^2}{n}}$

$\sqrt{\dfrac{(m-1)S_x^2 + (n-1)S_y^2}{m+n-2}}$

(2). $\bar{x} - \bar{y} \pm \sqrt{\dfrac{m+n}{mn}}\, S_w t_{0.975}(m+n-2)$

(4) $\left(\dfrac{S_1^2}{S_2^2}\dfrac{1}{F_{0.975}(10,15)}, \dfrac{S_1^2}{S_2^2}\dfrac{1}{F_{0.025}(10,15)}\right)$.

§7

§7.1. ① (1). ~~$\alpha = P_0\{X \in W\}$~~ $\alpha = P\{\bar{X} \in W | H_0\} = P(\bar{X} \geq 2.6 | \mu = 2) \Rightarrow P\left\{\frac{\bar{x}-\mu}{6/\sqrt{n}} \geq 2.\cancel{6} \middle| \mu = 2\right\}$ $\quad \frac{2.6-\mu}{6/\sqrt{n}} = 2.68$

$\mu = \frac{1}{n}\sum x_i = \bar{x}$ ~~$= 1 - \Phi(\sim) = 0.0037$~~

化成标准正态分布

$\Rightarrow P\{ \ 1 - \Phi(2.68)$

(2). $\beta = P\{X \in \bar{W} | H_1\} = P\{\bar{x} < 2.6 | \mu = 3\} = \boxed{P\left\{\frac{\bar{x}-\mu}{6/\sqrt{n}} < -1.79\right\}}$

$= \Phi(-1.79) = \sim$

(3). $\alpha = P\{\bar{X} \geq 2.6, \mu = 2) = — 1 - \Phi(0.6\sqrt{n}) \to 0.$

$\beta = \Phi(-0.4\sqrt{n}) \to 0.$

④ $\alpha = P\{x_{(n)} \in W | H_0\} = P\{x_{(n)} \leq 2.5 | \theta \geq 3.\}$

$X_n: \ P_n(x) = \frac{n x^{n-1}}{\theta^n} \quad = \int_0^{2.5} \frac{n \cdot x^{n-1}}{3^n} dx = \left(\frac{5}{6}\right)^n.$

$\alpha \leq 0.05 \Rightarrow \left(\frac{5}{6}\right)^n \leq 0.05$ ——

§7.2. 2. $\bar{x} = 4.484. \ n = 9 \quad u = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{4.484 - 4.55}{0.108/3}$

$H_0: \mu = \mu_0 = 4.55$ vs $H_1 \ \mu \neq \mu_0$

$W = \{|u| \geq u_{0.975}\} = \sim$

设服在 $X \sim N(\mu, 0.108^2)$

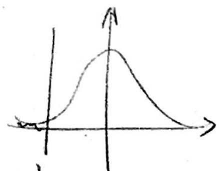
$H_0: \mu = 4.55$ vs $H_1 \ \mu \neq 4.55$

选定统计量 $U = \frac{\bar{x} - \mu}{6/\sqrt{n}} \sim N(0,1)$

$P = 2(1 - \Phi(\ ))$

⑤ ~~$u = \frac{\bar{x}13 - \mu_0}{}$~~ ~~$P\{\mu \leq 13 | \mu = 15\}$~~

$X \sim N(\mu, 2.5). \quad H_0: \mu =$ ——



$U = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1). \quad \alpha = 0.05 \Rightarrow W: \{u \leq -1.645\}$

左侧拒绝

$= \frac{\bar{x} - 15}{\sqrt{2.5}/\sqrt{n}}.$

$\beta = P\left\{\frac{\bar{X} - 15}{\sqrt{2.5}/\sqrt{n}} > -1.65 \middle| \mu \leq 13\right\} = P\left\{\frac{\bar{x} - \mu}{\sqrt{2.5}/\sqrt{n}} > -1.65 + \frac{15-\mu}{\sqrt{2.5}/\sqrt{n}}\right\}$

因为是二类错误

$= 1 - \Phi(-1.65 + \sim\sqrt{n}) \leq 0.05$

记这个

错判对

$\beta = P\{X \in \bar{W}\}$

13. $X_1 \sim N(\mu_1, \sigma^2)$

$X_2 \sim N(\mu_2, \sigma^2)$

$H_0: \mu_1 = \mu_2$ vs

$H_1: \mu_1 \neq \mu_2$.

$T = \dfrac{\bar{X}_1 - \bar{X}_2}{S_w \sqrt{\frac{1}{9} + \frac{1}{8}}}$ 统计量 $S_w = \sqrt{\sim} = \ldots$

左侧拒绝 $W = \{ |t| \geq t_{1-0.025}(15) \}$

15. $\bar{X} \sim N(\mu_1, \dfrac{\sigma_1^2}{n})$

$\bar{Y} \sim N(\mu_2, \dfrac{\sigma_2^2}{m})$

$H_0: \mu_1 = 2\mu_2$

$H_1: \mu_1 \neq 2\mu_2$.

I类

$\underline{\bar{X} - 2\bar{Y}} \sim N(\mu_1 - 2\mu_2, \dfrac{\sigma_1^2}{n} + \dfrac{4\sigma_2^2}{m})$

化成一个量

$U = \dfrac{(\bar{X} - 2\bar{Y}) - (\mu_1 - 2\mu_2)}{\sqrt{\frac{\sigma_1^2}{n} + \frac{4\sigma_2^2}{m}}} \sim N(0,1)$.

$X \sim N(\mu, \sigma^2)$

$\bar{X} \sim N(\mu, \dfrac{\sigma^2}{n})$

什么时候加？

22. $X \sim N(\mu, \sigma^2)$

$H_0: \sigma^2 = 400$ vs $H_1: \sigma^2 \neq 400$

$\chi^2 = \dfrac{(n-1)S^2}{\sigma_0^2} \sim \chi^2(n-1)$

$W \{ \chi^2 \leq \chi^2_{0.025}(n-1)$ or $\chi^2 \geq \chi^2_{0.975}(n-1) \}$.

25. $X \sim N(\mu, \sigma_1^2)$

$Y \sim N(\mu, \sigma_2^2)$

H_0 —— H ——

$F = \dfrac{S_x^2}{S_y^2} \sim F(m-1, n-1)$

$W =$ ——　代入.