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The benefit of modeling jumps in realized volatility for risk prediction: Evidence from Chinese mainland stocks

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A B S T R A C T

Recent literature has focused on realized volatility models to predict financial risk. This paper studies the benefit of explicitly modeling jumps in this class of models for value at risk (VaR) prediction. Several popular realized volatility models are compared in terms of their VaR forecasting performances through a Monte Carlo study and an analysis based on empirical data of eight Chinese stocks. The results suggest that careful modeling of jumps in realized volatility models can largely improve VaR prediction, especially for emerging markets where jumps play a stronger role than those in developed markets.

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1. Introduction

The rapid growth of trading activity and financial instability in recent years has led to an expansion in risk management interest and capability. Value-at-Risk (VaR), which refers to an asset or a portfolio's maximum loss over a fixed time horizon with a given probability, has become established as the most commonly used measure of risk exposure. In light of the practical relevance of the VaR concept, the need for reliable VaR estimation and prediction strategies arises.[1]

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[1] Although VaR has been regarded as not being a coherent measure of risk (see Artzner et al., 1997), it has retained its popularity in industry practice because of its simple statistical definition and easy practical calculation.

The methodology on VaR estimation can be loosely divided into non-parametric simulation and parametric model-based calculation. The current study contributes to the parametric group. Rather than using a simulation, we assume that asset returns r_t belong to a location-scale family of probability distributions $r_t = u_t + \sigma_t z_t$, where u_t is the conditional mean, σ_t is the conditional variance and the innovation process z_t has an assumed distribution with zero mean and unit variance. Consequently, VaR with probability α is calculated as the α - quantile of empirical distribution of returns using $VaR_t = u_t + \sigma_t Q_\alpha(z)$. There are three determinant inputs for the VaR calculation. Firstly, the calculation relies on the assumed asset return distribution. Different distributions lead to different quantiles given that other parameters are constant.[2] Additionally, as the location (μ_t) and the scale (σ_t) of return distributions are changing over time, mean and volatility forecasting can significantly influence the accuracy of VaR forecasting. Researchers have uniformly used Autoregressive Moving Average (ARMA) models to capture dynamic persistence of the mean component in the literature. The controversy lies in volatility modeling and forecasting.

The RiskMetrics model is regarded as a benchmark volatility forecasting model in VaR prediction since it is developed in 1994 by J.P. Morgan. However, it is widely documented that the assumptions in this model, such as a normal return distribution and short memory conditional variance, are violated in empirical situations. The empirical return distributions of financial assets have "fat tails" (leptokurtosis) and exhibit volatility clustering, leverage effects or volatility feedback, long range dependence, as well as jumps. Some studies have extended the RiskMetrics model by explicitly accounting for long memory and asymmetry in volatility, and reported improvements in VaR prediction (see So and Yu, 2006; Giot and Laurent, 2004; Härdle and Mungo, 2008). However, these improved models are not yet able to fully describe fat tail behavior observed in actual return series due to the ignorance of jumps.

VaR is essentially a tail point of the asset return distribution, and hence the predictive ability of a VaR model largely depends on how much of the extreme tail behavior of asset returns has been captured by the model. As extreme movements of asset prices, jumps from an important constituent of asset return tail distributions. It is hardly surprising that ignoring jumps will lower the quality of a VaR model and result in underestimating downside risk. Duffie and Pan (2001) incorporate jumps into a stochastic volatility model by employing a multi-factor jump-diffusion process. Maheu and McCurdy (2004) develop a mixed GARCH-Jump model that allows for time-variation and clustering in jump intensity. Chiu et al. (2005) find that these models that allow for jumps are superior to the traditional RiskMetrics model in VaR prediction. With the widespread availability of intraday high-frequency data on financial markets, the notion of realized volatility (RV), formally introduced by Andersen and Bollerslev (1998), along with the bipower variation and jump measures developed by Barndorff-Neilsen and Shephard (2004, 2006), open up a new door to model volatility with jump dynamics. It is hence interesting to investigate whether modeling jumps in realized volatilities can be applied into the risk management in practice.

In general, this paper has two objectives. Firstly, it examines whether a realized volatility model that accounts for jump dynamics as an input towards VaR calculation will improve the accuracy of risk forecasting. Several realized volatility models, including ones which treat realize volatility as a single variable and the others which separately incorporate jumps and continuous sample path into the model, are compared in terms of their VaR forecasting performance by using Monte Carlo simulation. Secondly, this paper studies jump patterns in an emerging market, whereas most of the existing work related to jumps concentrates on developed financial markets. It is well known that stock returns in emerging markets exhibit different characteristics, such as higher volatility, fatter tails, more sudden shifts, and greater predictability. Therefore, jumps in emerging markets are likely to be more frequent, and the ways in which they affect volatility and VaR measures are also likely to differ from developed markets. I use intraday data of eight individual stocks from the Chinese mainland stock market to study jump patterns in this emerging market, and assess how these jumps affect VaR measurement. In addition, a study on a particular market, in this case the Chinese stock market, will be useful for those who wish to invest in this market.

The organization of this article is as follows. Descriptions of four mostly used realized volatility models are provided in Section 2, and VaR forecasting and backtesting procedures are described in Section 3. The set-up and results of the Monte Carlo simulation study are reported in Section 4. The Chinese data description and empirical analysis are presented in Section 5. A conclusion is offered in Section 6.

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[2] A variety of distributions are applied in financial risk management, including the standard normal, Student-t, generalized error distribution (GED), and Skewed Student-t distribution.

2. Realized volatility models

In line with the promotion of the realized volatility concept, a large number of papers discuss the importance of jumps in realized volatility series and develop models to describe it. This section introduces the procedure of extracting jumps from realized volatility, and presents several popular and commonly used realized volatility models with or without jump components.

2.1. Realized volatility decomposition

We assume that the scalar logarithm of the asset price within the active part of the trading day evolves in continuous time as a standard jump-diffusion process

$$dp_s = \mu(s)ds + \sigma(s)dw(s) + \kappa(s)dq(s), \tag{1}$$

where $\mu(s)$ denotes the drift term with a continuous and locally bounded variation, $\sigma(s)$ is a strictly positive spot volatility process, $w(s)$ is a standard Brownian motion and $\kappa(s)dq(s)$ refers to the pure jump part, where $dq(s) = 1$ if there is a jump at time s and 0 otherwise, and $k(s)$ is the size of jump. The corresponding discrete-time within-day geometric return is denoted by

$$r_{t_j} = p_{t-1+j/M} - p_{t-1+(j-1)/M}, \quad j = 1, 2, ...M, \tag{2}$$

where M refers to the number of intraday equally spaced return observations over the trading day t, which depends on the sampling frequency. As such, the daily return of the active part of the trading day has $r_t = \sum_{j=1}^{M} r_{t_j}$. Realized volatility (RV) is defined by the sum of the intraday squared returns as $RV_t \equiv \sum_{j=1}^{M} r_{t_j}^2$. As noted in Andersen and Bollerslev (1998), Comte and Renault (1998), Andersen et al. (2003), and Barndorff-Neilsen and Shephard (2002a, 2002b), RV is a consistent estimator of quadratic variation of a standard jump-diffusion process, which includes both continuous integrated volatility and the sum of squared discontinuous jumps.

In order to disentangle the continuous and jump components, we need a consistent estimator of integrated volatility which is robust even in the presence of jumps. Barndorff-Neilsen and Shephard (2004, 2006) propose realized bipower variation (RBV), defined as the sum of the product of adjacent absolute intraday returns standardized by a constant, to consistently estimate integrated volatility. That is

$$RBV_t \equiv \mu_1^{-2} \left(\frac{M}{M-2} \right) \sum_{j=3}^{M} |r_{t_j}||r_{t_{j-2}}| \rightarrow \int_{t-1}^{t} \sigma^2(s)ds, \tag{3}$$

where $\mu_1 \equiv \sqrt{2/\Pi} \approx 0.79788$ is the expected absolute value of a standard normal random variable. Relative to the original measure considered in Barndorff-Neilsen and Shephard (2004), the bipower variation measure defined above involves an additional stagger value (see Huang and Tauchen, 2005) to correct market microstructure bias. The difference between realized variance and realized bipower variation consistently estimates the part of quadratic variation due to jumps as

$$J_t = RV_t - RBV_t \rightarrow \sum_{t-1<s<t} \kappa^2(s). \tag{4}$$

Here we use a jump detection test to select statistically significant jumps.[3] Under the null hypothesis of no jumps, the test statistic is defined as

$$JS_t = \frac{RV_t - RBV_t}{\sqrt{\left(\mu_1^{-4} + 2\mu_1^{-2} - 5 \right) max\left\{ 1, \frac{TQ_t}{RBV_t} \right\}}} \rightarrow N(0, 1), \tag{5}$$

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[3] See Barndorff-Neilsen and Shephard (2006) and Huang and Tauchen (2005) for more details about this test.

where TQ_t is the realized tripower quarticity which is estimated by

$$TQ_t = \Delta^{-1} \mu_{4/3}^{-3} \sum \left| r_{t_j} \right|^{4/3} \left| r_{t_{j-2}} \right|^{4/3} \left| r_{t_{j-4}} \right|^{4/3} \rightarrow \int_{t-1}^{t} \sigma^4(s) ds, \qquad (6)$$

where $\Delta = 1/M$.

Therefore, non-parametric measures of the jump component and the continuous component of the logarithm price process respectively are

$$C'_t = I(JS_t \leq \Phi_\alpha) \times RV_t + I(JS_t > \Phi_\alpha) \times RBV_t, \qquad (7)$$

and

$$J'_t = I(JS_t > \Phi_\alpha) \times (RV_t - RBV_t), \qquad (8)$$

where $I(.)$ denotes the indicator function and Φ_α refers to the appropriate critical value from the standard normal distribution.

2.2. The Autoregressive Moving Average (ARMA) model

The empirical findings suggest that the logarithm of realized volatility is approximately normal and it appears to be a long-memory process. To deal with this, Andersen et al. (2003) propose the use of an Autoregressive Fractionally Integrated Moving Average Model (ARFIMA) developed initially by Granger and Joyeux (1980), to estimate and forecast realized volatility. However, the estimation of ARFIMA models is non-trivial. As Pong et al. (2004) and Gallant et al. (1999) consider an ARMA(2,1) model as a simple alternative to the long memory ARFIMA model,[4] the first realized volatility model I considered here is an ARMA (2,1) model of the logarithm of realized volatility (RV_t), which is written as

$$\Phi(L)(log(RV_t) - \mu) = \Theta(L)\varepsilon_t, \qquad (9)$$

where $\Phi(L) = 1 - \phi_1 L - \phi_2 L^2$ is the AR polynomial and $\Theta(L) = 1 + \theta_1 L$ is the MA polynomial.

2.3. The Heterogeneous Autoregressive Realized Volatility (HAR) model

Heterogeneous Autoregressive Realized Volatility (HAR) model proposed by Corsi (2009) is an alternative way to describe long memory property of realized volatility. Unlike the ARMA model that has little theoretical motivation, the basic idea behind the HAR model relies on the "Heterogeneous Market Hypothesis".[5] It suggests that the financial market consists of participants with a range of time horizons, which cause different types of volatility components. By assuming that market dynamics are completely determined by the behavior of the participants, current realized volatility and the expectations of volatility over longer horizons can effectively form volatility expectations for the next period. Therefore, the corresponding model for the logarithm of realized volatility is specified as a component model that contains a daily, weekly and monthly logarithm of realized volatility component, and it takes the form as

$$log\left(RV_t^d\right) = \alpha_0 + \alpha_1 log\left(RV_{t-1}^d\right) + \alpha_2 log(RV_{t-1}^w) + \alpha_3 log(RV_{t-1}^m) + \varepsilon_t, \qquad (10)$$

where $log(RV_t^d)$ denotes the logarithm of daily realized volatility, $log(RV_{t-1}^w) = \frac{1}{5} \sum_{i=1}^{5} log(RV_{t-i})$ is the logarithm of weekly realized volatility and $log(RV_{t-1}^m) = \frac{1}{22} \sum_{i=1}^{22} log(RV_{t-i})$ represents the logarithm of monthly realized volatility. Like the ARMA (2,1), the HAR model is parsimonious and easily estimated.

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[4] The special lag structure in the ARMA (2,1) model is related to the fact that the sum of two AR(1) processes gives an ARMA (2,1) process, and an ARMA (2,1) can be interpreted as a component model with a transitory AR(1) component and permanent AR(1) component (see Hamilton, 1994; Gallant et al., 1999).

[5] See Müller et al. (1997) for further details on this hypothesis.

2.4. The HAR-RV-CJ model

In the above two models, realized volatility is treated as a single variable. As many asset prices are best described by a combination of a smooth and very slowly mean-reverting continuous sample path process and a much less persistent jump component, as well as the theory relating to the decomposition of realized volatility into continuous and jump components has been well developed now, Andersen et al. (2007a,b,c) proposed a new HAR-RV-CJ model which explicitly allows for jumps in the realized volatility modeling. In contrast with the above two models, the HAR-RV-CJ model separately incorporates jumps and the continuous sample path of the asset price as two explanatory variables for realized volatility as

$$log(RV)_{t+1} = \beta_0 + \beta_{CD}log(C_t) + \beta_{CW}log\left(C_{t-5,t}\right) + \beta_{CM}log\left(C_{t-22,t}\right)$$

$$+\beta_{JD}log(J_t + 1) + \beta_{JW}log\left(J_{t-5,t} + 1\right) + \beta_{JM}log\left(J_{t-22,t} + 1\right) + \varepsilon_t, \tag{11}$$

where $log\left(C_{t-5,t}\right) = \frac{1}{5}\sum_{i=1}^{5}log(C_{t-i})$ and $log\left(C_{t-22,t}\right) = \frac{1}{22}\sum_{i=1}^{22}log(C_{t-i})$ denote the logarithm of weekly continuous component and the logarithm of monthly continuous component respectively. $log\left(J_{t-5,t} + 1\right) = \frac{1}{5}\sum_{i=1}^{5}log(J_{t-i} + 1)$ and $log\left(J_{t-22,t} + 1\right) = \frac{1}{22}\sum_{i=1}^{22}log(J_{t-i} + 1)$ denote the logarithm of weekly jump component and the logarithm of monthly jump component respectively. As there are zero observations in jumps, and $log(0)$ is undefined, we use $log(J_t + 1)$ to ensure a well-defined measure that equals zero, when jumps are zero. The separation strategy introduced by this model significantly improves the out-of-sample volatility forecast.

2.5. The HAR-CJN model

Following the line of separately considering jumps and continuous sample path in the realized volatility models, Andersen et al. (2011) propose another model, which is referred to as HAR-CJN model, to account for jump dynamics. This model decomposes the total daily return variability into the continuous sample path variance, the variation arising from discontinuous jumps and the overnight return variance, and then model and forecast each component respectively. The HAR-CJN model is actually composed of four separate models, including the HAR-C model for the continuous sample path, the ACH model for the probability of jump occurrence, the HAR-J model for the squared jump size and the GARCH-t model for overnight returns. My current study only focusses on the first three models, and leaves modeling of the overnight returns for further research. Like realized volatility models, the HAR-CJN model is able to effectively incorporate intraday data into a reduced form for daily returns. Meanwhile, the HAR-CJN model can approximate long memory well by simply adding together volatility components for different time horizons. Innovatively, the HAR-CJN model attempts to capture the predictability of jumps and the dynamic persistence of jump intensity and jump size by combining the Autoregressive Conditional Hazard (ACH) model and a log-linear model with GARCH errors.

2.5.1. HAR-C model for continuous sample path variation
Andersen et al. (2011) apply the Heterogeneous Autoregressive Model proposed by Corsi (2009) to the continuous sample path for capturing its long memory dynamic dependencies. The HAR-C model is specified as

$$log\left(C_{t+1}^{'}\right) = \beta_0 + \beta_{CD}log\left(C_t^{'}\right) + \beta_{CW}log\left(C_{t-5,t}^{'}\right) + \beta_{CM}log\left(C_{t-22,t}^{'}\right)$$
$$+\beta_{JD}log\left(J_t^{'} + 1\right) + \beta_{JW}log\left(J_{t-5,t}^{'} + 1\right) + \beta_{JM}log\left(C_{t-22,t}^{'} + 1\right) + \varepsilon_{t+1,C}, \tag{12}$$

where $C_{t-h,t}^{'} \equiv h^{-1}[C_{t-h+1}^{'} + C_{t-h+2}^{'} + ... + C_t^{'}]$ and $J_{t-h,t}^{'} \equiv h^{-1}[J_{t-h+1}^{'} + J_{t-h+2}^{'} + ... + J_t^{'}]$. We work with the logarithm of the continuous sample path and jumps rather than their level, due to the fact that the unconditional distribution of the logarithm of realized volatility is closer to a normal distribution. Furthermore, by using the exponentiation of $log(C_{t+1}^{'})$, we can ensure that the implied continuous variation is always positive. Since $log(0)$ is undefined, we use $log(J_t^{'} + 1)$ to ensure a well-defined measure that equals zero, when jumps are zero. Time-varying volatility-of-volatility has been documented in numerous papers, and hence

there is no reason to neglect it in the context of realized volatility. A conditionally t-distributed GARCH error structure therefore appears in the above model as

$$\varepsilon_{t+1,C} = \sigma_{t+1,C} * \sqrt{\frac{\nu-2}{\nu}} * z_{t+1,C}, \quad z_{t+1,C} \sim t(\nu),$$
$$\sigma_{t+1,C}^2 = \omega_C + \alpha_{1,C}\varepsilon_{t,C}^2 + \beta_{1,C}\sigma_{t,C}^2. \tag{13}$$

2.5.2. ACH model and HAR-J model for jump variation

The models for jump variation are composed of two parts: a model for the probability of jumps coupled with a model for the squared jump sizes. The Autoregressive Conditional Duration (ACD) model proposed by Engle and Russell (1998) is a standard way to model the dynamics of variables that are observed at irregular times. Jumps arrive at irregularly spaced time intervals, and their duration is ideally suited to be modeled by an ACD model. However, the ACD model only updates the conditional expected durations on days when jumps arrive. From a forecasting perspective, it is desirable to continuously incorporate new information as it becomes available. The Autoregressive Conditional Hazard (ACH) model of Hamilton and Jorda (2002) was explicitly designed for this purpose.[6]

Consider a counting process $N(t)$ representing the number of jump days that have occurred up until day t, with the hazard rate of jump occurrence naturally defined as

$$h_t = Pr[N(t) \neq N(t-1)|F_{t-1}], \tag{14}$$

where F_{t-1} is the available information set up to day $t-1$. The simple ACH (1,1) model, which is actually an intensity model, without any information updating between jump days is

$$h_t = \frac{1}{\psi_{N(t)-1}}, \tag{15}$$

with

$$\psi_{N(t)} = \omega + \alpha_1 d_{N(t)-1} + \beta_1 \psi_{N(t)-1},$$

where $d_{N(t)-1}$ is the actual duration, which is the length of time between the $N(t)-2$ and the $N(t)-1$ jump arrival times, that is, $d_{N(t)-1} = t_{N(t)-1} - t_{N(t)-2}$. The expected duration is a weighted average of the last actual duration $d_{N(t)-1}$ and the last expected duration $\psi_{N(t)-1}$.

The augmented ACH (1,1) is designed to update conditional expected durations further by a set of exogenous variables as

$$h_t = \frac{1}{\psi_t},$$
$$\psi_t = \omega + \alpha_1 d_{N(t)-1} + \beta_1 \psi_{t-1} + \delta z_{t-1}, \tag{16}$$

where δz_{t-1} accommodates new information which could significantly influence the hazard rate, such as the latest macroeconomics announcement or company earnings report. The augmented ACH model establishes a natural link between ACD models and intensity models by allowing the ACD model to include time-varying covariates that may change during a duration spell.

In terms of the size of the jump, most continuous-time parametric jump diffusion models assume that they are i.i.d. distributed through time, but the present framework affords us much greater flexibility to model jump sizes. Following the same basic idea underlying the HAR-C model, Andersen et al. (2011) parameterize the conditional jump sizes as a function of the past continuous sample path variations, past jump sizes, as well as raw and expected durations as follows:

$$\log\left(S_{t(i)}\right) = \beta_0 + \beta_{CD}\log\left(C'_{t(i)-1}\right) + \beta_{CW}\log\left(C'_{t(i)-5,t(i)}\right) + \beta_{CM}\log\left(C'_{t(i)-22,t(i)}\right)$$
$$+ \beta_{SD}\log\left(S_{t(i-1)}\right) + \beta_{SW}\log\left(S_{t(i-5),t(i)}\right) + \beta_{SM}\log\left(S_{t(i-22),t(i)}\right) + \lambda\psi_{t(i-1)} + \gamma d_{t(i-1)}, \tag{17}$$

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[6] Details of the ACD and ACH frameworks are explained in Engle and Russell (1998) and Hamilton and Jorda (2002).

where $S_{t(i)}$ is the squared jump size and $t(i)$ records the corresponding trading day t in which the ith jump occurs.

Combining the above three models, the forecast of realized volatility can be expressed as

$$RV_{t|t-1} = Var(r_t|\mathcal{F}_{t-1}) = E(C'_t|\mathcal{F}_{t-1}) + E(J'_t|\mathcal{F}_{t-1}),$$

(18)

where $E\left(C'_t|\mathcal{F}_{t-1}\right)$ represents the conditional mean of the continuous component of volatility calculated from the HAR-C model, and $E\left(J'_t|\mathcal{F}_{t-1}\right)$ represents the conditional mean of the jump component of volatility, which can be calculated by

$$E\left(J'_t|\mathcal{F}_{t-1}\right) = E(S_t|\mathcal{F}_{t-1}, \ I_t = 1) \cdot P(I_t = 1|\mathcal{F}_{t-1}) = E(S_t|\mathcal{F}_{t-1}, \ I_t = 1) \cdot h_t.$$

(19)

$E(S_t|\mathcal{F}_{t-1}, \ I_t = 1)$ is the conditional mean of jump size if there is a jump at time t calculated from the HAR-J model, and h_t is the conditional probability of the occurrence of a jump at time t calculated from the ACH model. I_t is an indicator function which is equal to one if there is jump occurrence at time t, otherwise it is zero. Since the models for C'_t and S_t are formulated in logarithmic terms, the two conditional expectations $E\left(C'_t|\mathcal{F}_{t-1}\right)$ and $E(S_t|\mathcal{F}_{t-1}, \ I_t = 1)$ both involve a Jensen's inequality correction.

3. VaR forecasting and backtesting

This section introduces the procedure of VaR forecasting based on the above realized volatility models and backtesting for evaluating VaR forecasts from these models.

3.1. VaR forecasting

A series of asset returns (r_t) is typically modeled by a location-scale family of probability distributions, as discussed in the introduction. To compute a h-step-ahead VaR with α probability for the daily returns r_t, I use conditional realized volatility obtained from the four different models as an input to estimate a return equation. This is

$$r_t = \mu_t + \sqrt{\sigma^2 RV_{t|t-h}} z_t.$$

(20)

The first parameter needs to be estimated in the above equation as the mean component μ_t, which can be either fitted as an autoregressive process or treated as a constant. σ^2 is an additional parameter to ensure that the rescaled innovation process z_t has a unit variance. According to the distribution assumption, there may be some shape parameters to be estimated. Once the return equation is specified, h-step-ahead VaR can be calculated as

$$VaR_{t|t-h} = \mu_{t|t-h} + \sigma_{t|t-h} Q_\alpha(z),$$

(21)

where $\mu_{t|t-h}$ is the h-step-ahead forecasting of mean of the distribution of r_t, $\sigma_{t|t-h}$ is the h-step-ahead forecasting of standard deviation of the distribution of r_t (i.e., $\sqrt{\sigma^2 RV_{t|t-h}}$), and $Q_\alpha(z)$ is the α quantile of the assumed distribution of z. As the return r_t of my empirical data is not persistent based on their Ljung Box Q-statistics, I set the mean u_t as a constant for simplicity and focus on the forecast of standard deviation σ_t.

3.2. VaR backtesting

The VaR predictive performances of these model candidates can be assessed by a two-step backtesting framework. This framework is designed to evaluate models by not only examining whether the models are able to produce VaR forecasts appropriately, but also by comparing the magnitude of the losses beyond VaR.

In the first stage, I employ the unconditional coverage test (see Kupiec, 1995), independence test (see Christoffersen, 1998) and conditional coverage test to monitor the statistical adequacy of the VaR forecasts from these realized volatility models. These tests examine whether the average number of violations is

statistically equal to the expected coverage rate and whether these violations are independently distributed. In the second stage, the loss function (see Lopez, 1999) based on another risk measure called Expected Shortfall (ES) is utilized to rank each model's predictive ability.

3.2.1. First stage test

Defining a hit sequence as $H_t = I(r_t < -VaR_t^{\alpha})$ and specifying F_{t-1} as the information set up to day $t-1$, it is straightforward to show that an accurate VaR measure at a given level α implies that the hit sequence is identically and independently distributed as a Bernoulli random variable with a hit probability of α. In other words, the VaR violations in each period will occur with the correct conditional and unconditional probability. If the hit sequence generated by a model holds those properties, it should pass the unconditional coverage test, independence test and conditional coverage test.

The unconditional coverage test asks whether the observed number of violations is statistically equal to the desired level by testing

$$
\begin{aligned}
H_0 &: E(H_t) = \alpha \\
&\quad against \\
H_A &: E(H_t) \neq \alpha.
\end{aligned}
$$

The likelihood-ratio test statistic is

$$
LR_{uc} = 2ln\left(\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^N\right) - 2ln\left((1-\alpha)^{T-N}(\alpha)^N\right),
$$

where N is the number of days over a period T in which a violation occurred and α is the desired coverage rate. Under the null hypothesis, $LR_{uc} \sim \chi_1^2$ and the model is rejected if it generates too many or too few deviations from the desired violation rate.

An important shortcoming of the unconditional coverage test is that it will accept a model which generates violation clustering if the overall number of violations is close to the desired coverage level. Violation clustering and associated extreme losses incurred in consecutive periods may cause bankruptcy or at least serious financial problems for institutions or individuals. Therefore, Christoffersen (1998) proposes a relatively strict criterion in order to simultaneously examine the unconditional coverage rate and the independence of the sequence H_t. Under the null hypothesis, the number of violations is not only statistically equal to the desired coverage rate, but also a violation today has no influence on the probability of a violation tomorrow. Namely, the probability of a hit in the next period does not depend on whether a hit occurred in this period. The alternative hypothesis models the "hit" sequence as a binary first-order Markov chain with a transition probability matrix given by

$$
\Pi = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix}, \quad with \ \pi_{ij} = P(H_t = i | H_{t-1} = j).
$$

The appropriate likelihood ratio test statistic is

$$
LR_{in} = 2ln\left((1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}\right) - 2ln\left((1-\pi_1)^{n_{00}+n_{10}}\pi_1^{n_{01}+n_{11}}\right),
$$

where n_{ij} represents the number of transitions from state i to state j, $\pi_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$ denotes the corresponding transition probability from state i to state j, π_1 defines the probability that a hit occurs in the next period whatever state it is in this period, and π_0 defines the probability that no hit occurs in the next period whatever state it is in this period. Under the null hypothesis that the hitting sequence is independent, $LR_{in} \sim \chi_1^2$ and the model is rejected if it generates violation clustering.

The conditional coverage test combines the above two likelihood ratio statistics to test the joint assumption of unconditional coverage and independence of failures. The test statistic is computed as

$$
LR_{cc} = 2ln\left((1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}\right) - 2ln\left((1-\alpha)^{T-N}(\alpha)^N\right).
$$

Under the joint null hypotheses that the observed number of violations is statistically equal to the desired level and the probability of violation in each period is independent, $LR_{in} \sim \chi_2^2$ and the model is rejected if it generates too many or too few deviations from the desired violation rate, or if the violations cluster.

3.2.2. Second stage evaluation

After assessing the statistical adequacy of these model based VaR forecasts, I use Expected Shortfall (ES) based loss function to rank each model's predictive ability. Given that a VaR violation has occurred, Expected Shortfall (ES) is a probability-weighted average of tail loss, which measures the expected value of the loss. The two loss functions employed here are:

$$MSE = \frac{1}{K} \sum_{t=T_0}^{T_0+K} \left(r_{t+h} - E\left(r_{t+h} \middle| r_{t+h} < VaR_{t+h|t^\alpha} \right) \right)^2 \times 1\left[r_{t+h} < VaR_{t+h|t^\alpha} \right]$$

or

$$MAE = \frac{1}{K} \sum_{t=T_0}^{T_0+K} \left| r_{t+h} - E\left(r_{t+h} \middle| r_{t+h} < VaR_{t+h|t^\alpha} \right) \right| \times 1\left[r_{t+h} < VaR_{t+h|t^\alpha} \right]$$

where T_0 is the end of the estimation sample, K is the size of out-of-sample VaR forecasts, and $1[\Lambda]$ is the indicator function of the event Λ.

4. Monte Carlo simulation study

4.1. Data generating process

A small simulation study is conducted to compare the VaR forecasting performance of the above four realized volatility models, given a variety of underlying asset price process specifications. The various price processes and variations of them considered in the simulation are described below.

The initial asset price is set equal to one and the subsequent log prices of asset $Y_t = log(S_t)$ are simulated from a jump-diffusion process as

$$\begin{pmatrix} dY_t \\ dV_t \end{pmatrix} = \begin{pmatrix} \mu \\ \kappa(\theta - V_t) \end{pmatrix} dt + \sqrt{V_t} \begin{pmatrix} 1 & 0 \\ \rho\sigma_v & \sqrt{(1-\rho^2)} \end{pmatrix} \begin{pmatrix} dW_1 \\ dW_2 \end{pmatrix} + \begin{pmatrix} \xi^y dN_t^y \\ \xi^v dN_t^v \end{pmatrix}, \tag{22}$$

where W_1 and W_2 are two independent Brownian motion processes, V_t denotes the volatility of the asset price return at time t and follows a stochastic process, N_t^y and N_t^v are two Poisson processes with constant intensity λ_y and λ_v, and ξ^y and ξ^v are the jump sizes in returns and volatility, respectively.

This specification nests many of the popular asset price models including Eq. (1) When $\lambda_y \neq 0$ but $\lambda_v = 0$, the above equation reduces to Bates' (1996) stochastic volatility model with random jumps occurring in the prices (SVJ). Jump sizes are assumed to be normally distributed as $\xi^y \sim N(\mu_y, \sigma_y^2)$. Essentially, the jump component adds mass to the tail of the return distribution. Increasing σ_y adds tail mass to both tails, while a positive μ_y implies relatively less mass in the left tail and vice versa; Eq. (2). When volatility is also allowed to jump, but jumps to prices and volatility are arrived independently, that is, λ_y and λ_v are two independent Poisson processes, and neither of them are zero, the above equation represents a stochastic volatility independent jumps (SVIJ) model with jump sizes in the price that are normally distributed as $\xi^y \sim N(\mu_y, \sigma_y^2)$ and jump sizes in the volatility are from an exponential distribution $\xi^v \sim exp(\mu_y)$. Eq. (3) When jumps in return and volatility are driven by the same Poisson process, $N_t^y = N_t^v = N_t$, and jump sizes in return and volatility are correlated as $\xi^v \sim exp(\mu_v)$ and $\xi^y | \xi^v \sim N(\mu_y + \rho_j \xi^v, \sigma_y^2)$, the above equation is referred to as stochastic volatility with correlated jump (SVCJ) model. In SVJ model, large price moves stemming from jumps that have no impact on volatility, while the SVCJ specification corrects this shortcoming to allow that price jumps will simultaneously influence both prices and volatility. The added volatility jump component typically adds right skewness in the distribution of volatility, and hence, overall fatten the tails of the return distribution.

Jump intensities are treated as a constant in the above specifications. In the simulation, I also consider a stochastic volatility model with state-dependent or time-varying correlated jumps (SVSCJ) in price and

volatility. This model generalizes the SCVJ model described above to allow the jump intensity to depend on its past values or the current volatility. That is,

$$\lambda_t = \alpha_0 + \alpha_1 \lambda_{t-1} + \alpha_2 V_t, \tag{23}$$

where λ_t is the intensity of a Poisson counting process N_t^y (and N_t^v) at time t, and V_t is the instantaneous volatility at time t. In essence, the SVSCJ model can better describe jump clustering, and allows jumps to be arrived more frequently in high-volatility regimes when α_2 is positive. These generalizations are supported by the empirical facts.

The values of the parameters in my simulation for the above processes are reported in Table 1. For all the data generating processes (DGPs), I simulate the time series of asset prices of length $T = 1500$ days with $K = 14400$ s per day, and the simulation is replicated 1000 times. In each replication, I use five minutes as a sampling frequency to construct the time series of realized volatility, bipower variation and jumps. To assess the VaR forecasting power of all the realized volatility models described in Section 2, I use a rolling window of 500 (or 496) observations to estimate all the models, and obtain out-of-sample one-step-ahead (or five-step-ahead) VaR forecasts with 1% and 5% coverage levels. At the end, I have 1000 VaR forecasts from each model, and the VaR violations can be calculated by comparing the 1000 VaR forecasts with the last 1000 daily return observations.

4.2. Results

The ideal number of 1% (or 5%) VaR violations is 10 or 50 in each replication. I will analyze the distribution of the number of VaR violations from each model to compare their VaR forecasting performance. As there are 1000 replications, such a distribution of the number of VaR violations will have 1000 points.

Panel A and Panel B in Tables 2 and 3 respectively report mean and variance of the number of violations over one-step-ahead and five-step-ahead 1% VaR forecasts from each model. In general, under all the DGPs, the VaR forecasts provided by all the models are positively biased. The ARMA-RV model gives the most biased forecasts with the highest variance, and the HAR-CJN model provides the least biased forecasts with the lowest variance. More importantly, the two realized volatility models (HAR-CJ model and HAR-CJN model) which explicitly model jumps produce less biased and variant VaR forecasts than the other two realized volatility models.

Table 1
Parameter values of data generating processes in Monte Carlo simulation.

Data generating process				
Parameter values	SVJ	SVIJ	SVCJ	SVSCJ
μ	0.050	0.051	0.055	0.055
κ	0.013	0.025	0.026	0.026
θ	0.814	0.559	0.538	0.538
ρ	−0.467	−0.504	−0.484	−0.484
σ_v	0.095	0.090	0.079	0.079
λ^y	0.006	0.005	0.007	0.007
λ^v		0.002		
μ_y	−2.586	−3.085	−1.753	−1.753
σ_y	4.072	2.989	2.886	2.886
ρ_J			−0.601	−0.601
μ_v		1.798	1.483	1.483
α_0				0.002
α_1				0.723
α_2				1.298

Table 2
Descriptive statistics of the number of one-step-ahead VaR (1%) violations for realized volatility models.

	ARMA-RV	HAR-RV	HAR-CJ	HAR-CJN
Mean (bias)				
SVJ	13(3)	12(2)	11(1)	10(0)
SVIJ	15(5)	14(4)	12(2)	11(1)
SVCJ	16(6)	15(5)	13(3)	12(2)
SVSCJ	18(8)	16(6)	13(3)	12(2)
Variance				
SVJ	13.1	12.9	8.7	7.3
SVIJ	15.2	15.1	9.2	9.0
SVCJ	16.7	15.5	10.4	9.2
SVSCJ	19.8	19.3	12.9	9.5
Mean squared error				
SVJ	22.1	16.9	9.7	7.3
SVIJ	40.2	31.1	13.2	10
SVCJ	52.7	40.5	19.4	13.2
SVSCJ	83.8	55.3	21.9	13.5

Note: This table only reports the results associated with 1% VaR violations. The similar tendency is reserved in the results of 5% VaR violations, and they are available upon request.

To assess the trade-off between bias and variance, I also employ the Mean Squared Error (MSE) to compare the VaR forecasting performances of these models. For example, the MSE of 1% VaR is defined as

$$MSE = \frac{1}{1000} \sum_{i=1}^{1000} (X_i - 10)^2,$$

where X_i is the number of violations in the ith replication, and 10 is the ideal number of violations for 1% VaR at each replication. Panel C in Tables 2 and 3 reports MSE of one-step-ahead and five-step-ahead 1% VaR forecasts of all the models. It is clear that under all the DGPs, ARMA model and HAR model perform a bit similarly. However, the HAR-CJ model and HAR-CJN model always have the lower MSEs than the other two models. Moreover, when the jump intensity is time-varying and state-dependent, the HAR-CJN model has lower MSEs than the HAR-CJ model, as the jump intensity dynamics are clearly modeled in the former.

Table 3
Descriptive statistics of the number of five-step-ahead VaR (1%) violations for realized volatility models.

	ARMA-RV	HAR-RV	HAR-CJ	HAR-CJN
Mean (bias)				
SVJ	15(5)	13(3)	12(2)	11(1)
SVIJ	16(6)	15(5)	13(3)	11(1)
SVCJ	16(6)	15(5)	14(4)	12(2)
SVSCJ	18(8)	17(7)	15(5)	12(2)
Variance				
SVJ	13.5	13.5	9.1	7.6
SVIJ	15.6	15.2	9.4	9.1
SVCJ	16.9	15.7	10.5	9.5
SVSCJ	20.1	19.5	13.4	9.9
Mean squared error				
SVJ	38.5	22.5	13.1	8.6
SVIJ	51.6	40.2	18.4	10.1
SVCJ	52.9	40.7	26.5	13.5
SVSCJ	84.1	68.5	38.4	13.9

Note: This table only reports the results associated with 1% VaR violations. The similar tendency is reserved in the results of 5% VaR violations, and they are available upon request.

The similar tendencies are observed at 5% VaR forecasts, and the results associated with this can be provided upon request. These results provide a strong evidence that explicitly modeling jumps in realized volatility model can better describe the tail behavior, and thereby provide better tail point forecast of the asset return distribution.

5. Empirical analysis

5.1. Data description

In this section, I compare VaR forecasts of the four realized volatility models by using empirical data. The analysis is based on intraday data of eight individual stocks on the Chinese Stock Exchange.[7] These stocks are from four different industries, and more details are provided in Table 4. The raw transaction prices were obtained from the China Stock Market & Accounting Research (CSMAR) database provided by the ShenZhen GuoTaiAn Information and Technology Firm (GTA). Trading in the Chinese Stock Exchange is conducted through the electronic consolidated open limit order book (COLOB), and it is carried out in two sessions with a lunch break. The morning session is from 09:30 to 11:30 and the afternoon session is from 13:00 to 15:00. Before the morning session, there is a 10-minute open call auction session from 09:15 to 09:25 to determine the opening price. The afternoon session starts from continuous trading without a call auction. The closing price of the active trading day is generated by taking a weighted average of the trading prices of the final minute. The market is closed on Saturdays and Sundays and other public holidays.

Compared with most Western developed stock markets in the context of institutional setting and trading rules, the Chinese mainland stock market differs in three main ways. First, there is a five-minute break between the periodic auction for the opening price and the normal morning session of continuous trading. In addition, there is a lunch break at midday between the morning and afternoon sessions, as in most other Asian stock markets. Second, the Chinese market is a limit order-driven market using electronic trading without market makers. Floor trading between member brokers and short selling are strictly prohibited. A further difference lies in the relatively immature infrastructure that embodies inadequate disclosure, and the co-existence of an inexperienced regulator with a limited number of informed investors and an enormous number of uninformed investors. Given these differences, data from the Chinese mainland stock exchange is expected to provide a picture of an emerging stock market, which contrasts with western markets in interesting ways.

The actual construction of the series needed for our analysis follows the realized volatility decomposition discussed in Section 2.1. I am only concerned with the active trading period, and overnight information is beyond the scope of this study. Parallelling many previous studies, I use five-minute intervals as the sampling frequency to strike a reasonable balance between accurate measure and microstructure noise.[8] Since there are no transaction records in the first 15-minute interval of many trading days, and also to avoid opening effects, my dataset spans 09:45–11:30 and 13:05–15:00 on each working day (excluding weekends, public holidays and firm-specific trading suspensions) from January 2, 2003 to December 27, 2007. My sample excludes some inactive days that contain only a few transactions to avoid complicating the inference.

Firstly, based on the previous-tick method, I calculate five-minute prices from the tick immediately before the five-minute time stamp throughout each trading day. Secondly, I obtain five-minute intraday returns as the first difference of the logarithmic prices. The open-to-close daily return is naturally defined as the sum of the intraday returns, i.e., $r_t = \sum_{j=1}^{M} r_{t_j} = ln(p_{t_M}) - ln(p_{t_1})$. In addition, realized volatility is constructed as the sum of all squared intraday 5-minute returns $RV_t \equiv \sum_{j=1}^{M} r_{t_j}^2$. The time interval between

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[7] There are two official stock exchanges in the Chinese mainland financial market: the Shanghai Stock Exchange (SSE) and the Shenzhen Stock Exchange (SZSE). Three of the stocks in our analysis (SH600085, SH600351 and SH600050) are from SSE, while five of the stocks (SZ000919, SZ000419, SZ000987, SZ000581 and SZ000088) are from SZSE. All eight stocks are A share stocks.

[8] There is a trade-off between measurement accuracy and microstructure contamination when using high-frequency data to measure financial volatility. For further details concerning the optimal sampling frequency, see Andersen et al. (2007a,b,c).

Table 4
Details of the eight Chinese stocks.

Stock symbol	Company name	Stock exchange	Industry
000919	Jinling Pharmaceutical Co., Ltd	Shenzhen	Pharmacy
600085	Beijing Tongrentang Co., Ltd	Shanghai	Pharmacy
600351	Shanxi Yabao Pharmaceutical Co., Ltd.	Shanghai	Pharmacy
000419	Changsha Tongcheng Holdings Co., Ltd	Shenzhen	Commerce
000987	Guangzhou Friendship Co., Ltd.	Shenzhen	Commerce
000581	Weifu High-Technology Co., Ltd.	Shenzhen	Industrials
000088	Shenzhen Yan Tian Port Holdings Co., Ltd	Shenzhen	Utilities
600050	China United Telecommunications Co., Ltd	Shanghai	Utilities

trades is $\Delta = \frac{1}{M} = 0.023$. Because some days involve fewer than 44 intraday observations, I scale up the variance measure based on the available 5-minute returns. Thirdly, according to jump tests and bipower variation, I decompose the realized volatility into its continuous sample path C'_t, and variation due to jumps J'_t based on a significance level of $\alpha = 0.01$.[9] The duration between two consecutive jumps, $D_{N(t)}$, is calculated as the number of days between the $(N-1)$th and the Nth jump arrival times, that is $D_{N(t)} = t_{N(t)^-} t_{N(t)-1}$, and the jump size $S_{t(N)}$ is the squared jump size within the day when at least one jump occurs. Finally, I adopt an expanding estimation window to specify the models and produce one-step-ahead and five-step-ahead forecasts by starting from the in-sample estimation period covering 694 days from January 2, 2003 to December 30, 2005, and an out-of-sample forecast evaluation period covering 472 days from January 4, 2006 to December 27, 2007.

The descriptive statistics and Ljung-Box Q-statistics for each series are reported in Tables 5 and 6. It is not surprising that realized volatility and the continuous sample path have distinct dynamic dependencies with strong evidence of predictability in all of the eight individual stocks. Compared with developed U.S. stock markets, jumps occur more frequently in this emerging stock market. Approximately 40% of the sample period of each stock (476 out of 1159 days, 469 out of 1166, 460 out of 1162, 471 out of 1164, 450 out of 1164, 336 out of 1150, 329 out of 1137, and 423 out of 1127) contains jumps. Meanwhile, compared with empirical results found by Andersen et al. (2011) for the U.S. stock market, the Ljung-Box Q-statistics reported in Table 6 indicate more significant serial correlation in the jump component, particularly in the jump size. Therefore, jumps play a greater role in this emerging market and exhibit a more predictable pattern than that in the developed US market. The estimated models discussed in the next section provide an attempt to explain the main features of realized volatility and the continuous and jump components of realized volatility in these Chinese stocks.

5.2. Empirical results

5.2.1. Model estimation

Two-step model estimation is carried out in this section. In order to perform the VaR analysis, the four realized volatility models are estimated in the first step to obtain a set of realized volatility forecasts.[10] In the second stage, realized volatility forecasts are used as a proxy for variance in the daily return equation to estimate other parameters needed for VaR calculation. I use the first 694 days of the sample from January 2, 2003 to December 30, 2005 for model development, and then use the last 472 days of the sample from January 4, 2006 to December 27, 2007 for out-of-sample forecast analysis. Once these models have been developed, I use an expanding window to re-estimate their parameters and construct out-of-sample forecasts that incorporate information as it becomes available.

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[9] In practice, the number of detected jumps will depend on the chosen significance level. We follow most of the literature and use a significance level of 0.01.

[10] The ARMA model, HAR-C model, the HAR-J model and HAR-RV model are estimated using Quasi Maximum Likelihood and Eviews 6.0 with a GARCH error structure. The ACH model and the return equation are estimated using maximum likelihood and GAUSS. The standard errors provided for each model are the Newey and West (1987) robust standard errors. All the model specifications use the sample period spanning from January 2, 2003 to December 30, 2005 with a fixed window.

Table 5
Descriptive statistics for eight Chinese stocks.

	r_t	rv_t	$\acute{C}_t$	$\acute{J}_t$	$D_{N(t)}$	$S_{N(t)}$	r_t	rv_t	$\acute{C}_t$	$\acute{J}_t$	$D_{N(t)}$	$S_{N(t)}$
000919							000987					
Mean	−0.0002	0.00073	0.00057	0.00015	2.43	0.00037	0.0023	0.0008	0.0006	0.0001	2.59	0.0003
Std.dev	0.023	0.00076	0.00069	0.0003	2.74	0.0004	0.023	0.0008	0.0007	0.0002	2.75	0.0003
Skewness	−0.57	4.37	5.02	3.89	3.19	2.86	0.23	3.63	4.17	4.05	3.22	2.99
Kurtosis	5.96	38.46	50.2	24.96	15.64	14.89	5.97	23.01	29.70	26.64	16.76	15.59
Obs	1159	1159	1159	1159	476	476	1164	1164	1164	1164	450	450
600085							000581					
Mean	0.0021	0.0005	0.0004	0.00009	2.48	0.0002	0.0012	0.0007	0.0006	0.00008	3.40	0.0003
Std.dev	0.020	0.00065	0.0006	0.0002	3.04	0.00026	0.0238	0.0007	0.0006	0.0002	3.60	0.0003
Skewness	0.14	5.58	6.14	4.94	4.20	3.80	−0.035	3.06	2.72	7.04	2.20	5.08
Kurtosis	7.47	55.88	68.39	41.26	27.98	14.89	5.66	18.30	13.66	89.41	8.11	45.01
Obs	1166	1166	1166	1166	469	469	1150	1150	1150	1150	336	336
600351							000088					
Mean	0.0004	0.0009	0.0008	0.0002	2.52	0.00048	0.0010	0.0006	0.0005	0.00006	3.44	0.0002
Std.dev	0.026	0.0009	0.0008	0.0004	2.88	0.00055	0.022	0.0007	0.0007	0.00016	3.60	0.0002
Skewness	−0.188	3.17	3.11	4.28	3.71	3.08	−0.001	4.35	4.67	4.92	3.67	3.08
Kurtosis	5.71	17.39	17.54	27.89	22.52	15.21	5.25	31.97	36.15	35.46	25.58	14.83
Obs	1162	1162	1162	1162	460	460	1137	1137	1137	1137	329	329
000419							600050					
Mean	0.0017	0.0011	0.0009	0.0002	2.47	0.00050	0.0010	0.0006	0.0005	0.00007	2.67	0.0002
Std.dev	0.026	0.001	0.001	0.0004	3.10	0.00054	0.0189	0.0006	0.0006	0.00018	2.67	0.0002
Skewness	−0.177	4.661	5.493	4.627	5.64	3.68	0.39	4.12	4.11	8.63	3.43	7.22
Kurtosis	5.91	42.04	56.01	35.31	54.05	22.17	7.46	26.51	28.06	116.33	19.63	70.11
Obs	1164	1164	1164	1164	471	471	1127	1127	1127	1127	423	423

Note: r_t denotes the daily return; rv_t denotes the daily realized volatility; $\acute{C}_t$ denotes the continuous sample path variation; $\acute{J}_t$ denotes the jump component; $D_{N(t)}$ denotes the jump duration, and $S_{N(t)}$ denotes the size of jumps. $N(t)$ records the calendar time t when cumulative number of jumps is N.

Table 6
Ljung Box Q-statistics for eight Chinese stocks.

Lags	r_t	rv_t	C'_t	J'_t	$D_{N(t)}$	$S_{N(t)}$	r_t	rv_t	C'_t	J'_t	$D_{N(t)}$	$S_{N(t)}$
000919							000987					
5	3.17(0.670)	499.48(0.000)	596.72(0.000)	280.6(0.000)	117.3(0.000)	108.4(0.000)	5.29(0.381)	1039.2(0.000)	797.79(0.000)	136.35(0.000)	30.158(0.000)	249.10(0.000)
10	12.31(0.265)	669.66(0.000)	764.65(0.000)	531.8(0.000)	259.9(0.000)	192.5(0.000)	14.92(0.135)	1681.7(0.000)	1272.3(0.000)	223.28(0.000)	66.549(0.000)	462.79(0.000)
15	19.95(0.174)	869.58(0.000)	953.67(0.000)	750.5(0.000)	328.8(0.000)	244.2(0.000)	25.27(0.046)	2302.6(0.000)	1729.0(0.000)	315.85(0.000)	72.308(0.000)	610.14(0.000)
20	25.38(0.187)	1045.2(0.000)	1175.0(0.000)	883.1(0.000)	382.4(0.000)	282.9(0.000)	25.76(0.174)	2794.3(0.000)	204.2(0.000)	351.08(0.000)	77.773(0.000)	684.89(0.000)
600085							000581					
5	4.55(0.473)	1072.6(0.000)	1254.7(0.000)	111.8(0.000)	203.8(0.000)	222.9(0.000)	7.36(0.195)	1278.4(0.000)	1379.0(0.000)	42.827(0.000)	76.121(0.000)	55.591(0.000)
10	25.36(0.005)	1463.5(0.000)	1766.1(0.000)	151.1(0.000)	341.2(0.000)	352.9(0.000)	15.29(0.122)	2161.2(0.000)	2434.4(0.000)	59.093(0.000)	113.55(0.000)	80.303(0.000)
15	32.46(0.006)	1883.1(0.000)	2305.1(0.000)	187.3(0.000)	436.1(0.000)	464.4(0.000)	20.14(0.167)	2766.8(0.000)	3137.2(0.000)	75.982(0.000)	146.36(0.000)	99.921(0.000)
20	41.83(0.003)	2360.7(0.000)	2823.6(0.000)	235.8(0.000)	498.2(0.000)	551.7(0.000)	25.27(0.191)	3269.3(0.000)	3772.1(0.000)	79.793(0.000)	161.87(0.000)	111.68(0.000)
600351							000088					
5	2.38(0.795)	605.49(0.000)	705.11(0.000)	246.0(0.000)	90.78(0.000)	154.9(0.000)	3.136(0.679)	1467.6(0.000)	1474.9(0.000)	10.546(0.061)	42.947(0.000)	73.478(0.000)
10	15.13(0.128)	798.45(0.000)	983.63(0.000)	336.9(0.000)	180.01(0.000)	182.8(0.000)	8.998(0.532)	1942.2(0.000)	1941.2(0.000)	16.112(0.096)	70.687(0.000)	113.06(0.000)
15	18.81(0.223)	895.05(0.000)	1178.9(0.000)	372.9(0.000)	233.49(0.000)	195.4(0.000)	17.103(0.313)	2201.0(0.000)	2232.1(0.000)	24.016(0.065)	86.371(0.000)	118.43(0.000)
20	23.96(0.244)	1020.7(0.000)	1417.8(0.000)	398.0(0.000)	283.97(0.000)	247.3(0.000)	20.681(0.416)	2443.6(0.000)	2504.3(0.000)	29.407(0.080)	120.41(0.000)	128.52(0.000)
000419							600050					
5	10.18(0.070)	666.72(0.000)	844.72(0.000)	58.5(0.000)	125.56(0.000)	20.1(0.000)	22.44(0.000)	919.50(0.000)	1052.4(0.000)	11.145(0.049)	135.50(0.000)	44.77(0.000)
10	16.86(0.078)	948.28(0.000)	1216.10(0.000)	102.2(0.000)	217.93(0.000)	47.3(0.000)	27.17(0.002)	1303.7(0.000)	1517.0(0.000)	15.081(0.129)	194.57(0.000)	53.41(0.000)
15	22.55(0.094)	1218.2(0.000)	1633.60(0.000)	138.7(0.000)	287.89(0.000)	53.6(0.000)	48.73(0.000)	1842.2(0.000)	2160.8(0.000)	16.185(0.370)	222.13(0.000)	64.66(0.000)
20	25.08(0.198)	1403.5(0.000)	1936.60(0.000)	156.7(0.000)	335.35(0.000)	80.2(0.000)	52.41(0.000)	2398.8(0.000)	2808.7(0.000)	19.238(0.506)	236.62(0.000)	81.44(0.000)

Note: 1. r_t denotes the daily return; rv_t denotes the daily realized volatility; C'_t denotes the continuous sample path variation; J'_t denotes the jump component; $D_{N(t)}$ denotes the jump duration, and $S_{N(t)}$ denotes the size of jumps. $N(t)$ records the calendar time t when cumulative number of jumps is N. 2. The p-value of Ljung-box Q-statistics is shown in the bracket.

To investigate jump behavior in Chinese market, I report estimation results for the ACH model in Table 7 and estimation results for the HAR-J model in Table 8 for the eight Chinese stocks to analyze their jump intensities and jump sizes. The ACH model presented here only includes the significant explanatory variables and all the insignificant covariates are deleted. I originally experimented with augmenting the ACH (1,1) model by past actual duration and expected duration along with five categories of exogenous variables. The five variables that were added include the logarithm of the number of days until the next news announcements of the Consumer Price Index (CPI) in the model, which turns out to be the most important macroeconomic variable to influence the stock market in this emerging market.[11] CPI announcement dates were collected from the National Bureau of Statistics of China. We also included the logarithm of the number of days until the next announcement of the firm's earnings report, which is released quarterly and collected from the Shanghai Stock Exchange official website. Unscheduled announcements are included in our analysis as well since insiders may know what and when information will be released, and this can have a potentially high effect on stock prices. In order to accommodate day-of-week effects in the jump occurrences, four weekday dummies for Monday, Tuesday, Wednesday, and Thursday were generated and the estimated coefficients in front of them represent the extra probability of jump occurrence relative to Friday. With respect to the specific industry the three individual stocks that belong to, the outbreak of epidemic disease should have an influence on the medicine manufacturing industry. It is well known that Severe Acute Respiratory Syndrome (SARS) affected China at the end of 2002 and lasted until the middle of 2003. Therefore, I generate a dummy variable, which equals 1 from January 2, 2003 to July 31, 2003, to observe its effect in jump occurrence. Finally, Nowak (2008) suggested that the length of time since the latest jump could be an additional information component to predict the chance that the jump will show up in the next period, I incorporate this innovative idea in the ACH model and use a step function to measure the no-jump duration L_t, that is $L_t = 0$ if $J'_t \neq 0$ (a jump occurs), otherwise $L_t = L_{t-1} + 1$ if $J'_t = 0$ (no jump occurs).

The estimated results of the ACH model presented in Table 7 confirm the strong dynamic persistence in jump durations or equivalently the hazard rate of jump occurrence, as the previous literature suggests. The corresponding coefficients of past actual duration and expected duration are both positive and significant. One exciting but not surprising result is that the no-jump duration since the latest jump has a statistically significant positive relationship with the hazard rate of jump occurrence. This suggests that the more no-jump time has passed since the last jump, the more likely that jumps will appear in the next period. However, in contrast to the U.S. fixed-income market in which Andersen et al. (2011) found a statistically significant negative relationship between the hazard rate of jump occurrence and the number of days until the release of macroeconomic announcements, there is no evidence from the estimated results that fundamental information such as released CPI and firm specific news affect the jump hazard rate in these Chinese stocks. Meanwhile, I cannot find a statistically significant day-of-week effect in these three stocks.

Table 8 shows the estimated results for the HAR-J model. These clearly indicate a strong persistence in jump size, which is in contrast to findings in previous literature. Early studies in the stochastic volatility literature assume that jump size is identically and independently distributed with no evidence of predictability, and Andersen et al. (2011) found that jump size is statistically influenced by the past one-month continuous sample path variation, but has no significant own persistence. It is not clear yet what drives this difference in the Chinese stocks.

The estimation results of the return equation with conditional realized volatility from the four models are reported in Table 9. I assume that the return has a Student-t distribution,[12] since this type of distribution can model the fat tail better than the normal distribution by using a shape parameter ν. Smaller values of ν indicate fatter tail behavior. It is clear that the return equation with realized volatility provided by HAR-CJ and HAR-CJN models in all stocks have relatively large values of ν than other models, which is an evidence that modeling of jumps is able to better describe the tail behavior of returns. In terms of the conditional mean in

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[11] Andersen et al (2007) suggest that CPI and employment reports, representing the nominal side and real side of the economy respectively, are the two most important macroeconomic news announcements related to financial markets. However, employment reports in emerging markets are unreliable indicators of the real economy because they do not account for selfemployment and underground jobs.

[12] The skewed Student-t distribution is popular in the recent literature for VaR forecasting due to its ability to capture both kurtosis and skewness, and to distinguish between the long position and short position VaR risk once the distribution is asymmetric (see Giot and Laurent, 2004). However, as we have mentioned in Section 2.4, short selling is not permitted in the Chinese stock market. Therefore, our analysis only needs to focus on long position VaR risk, and the Student-t distribution should be adequate in this case.

Table 7
ACH(1,1) model estimates.

	000919	600085	600351	000419	000987	000581	000088	600050
ω	0.4078(0.1578)	0.1637(0.0552)	0.2811(0.1214)	0.3245(0.1124)	0.2546(0.7865)	0.4124(0.084)	0.2344(0.0567)	0.5321(0.0765)
α_1	0.0976(0.0333)	0.0647(0.0328)	0.0881(0.0172)	0.0547(0.0432)	0.0776(0.0234)	0.0653(0.0124)	0.0564(0.0223)	0.0876(0.0654)
β_1	0.6217(0.1308)	0.8136(0.0459)	0.7659(0.0781)	0.5987(0.0655)	0.6987(0.0321)	0.7655(0.0243)	0.9876(0.0145)	0.8567(0.0543)
δ	0.1290(0.0598)	0.0743(0.0317)	0.1508(0.0698)	0.1193(0.0231)	0.0884(0.0134)	0.1434(0.0504)	0.0645(0.0124)	0.1543(0.0321)
Log likelihood	−0.6450	−0.6430	−0.6449	−0.6396	−0.6567	−0.6390	−0.6535	−0.6321

Note: The ACH(1,1) model is specified as:

$$h_t = \frac{1}{\psi_t},$$
$$\psi_t = \omega + \alpha_1 d_{N(t)-1} + \beta_1 \psi_{t-1} + \delta L_{t-1}.$$

Here we only report the significant coefficients. Newey–West robust standard errors of the corresponding coefficients are reported in brackets.

Table 8
HAR-J model estimates.

	000919	600085	600351	000419	000987	000581	000088	600050
β_0	−1.6713(0.7030)	−3.8053(0.8346)	−1.6589(0.7308)	−2.6461(0.6543)	−1.5436(0.7476)	−4.8761(0.7654)	−3.8765(0.5043)	−2.0842(0.8907)
β_{CD}	0.1051(0.0595)	0.2355(0.0655)	0.1590(0.0768)	0.3215(0.0541)	0.2987(0.0651)	0.1987(0.0432)	0.3576(0.0589)	0.2545(0.0431)
β_{CW}	0.2654(0.1159)	0.0321(0.1293)	0.1735(0.1229)	0.0541(0.1532)	0.1765(0.1452)	0.0786(0.0876)	0.1984(0.1332)	0.2098(0.1567)
β_{CM}	0.4537(0.1462)	0.3137(0.1384)	0.4841(0.1300)	0.5098(0.1432)	0.3876(0.1215)	0.6543(0.1114)	0.2376(0.1265)	0.4178(0.1090)
Log likelihood	−483.7659	−465.6347	−424.8565	−498.6543	−434.7678	−456.9874	−446.8761	−498.766

Note: The HAR-J model is specified as:

$$log\left(S_{t(i)}\right) = \beta_0 + \beta_{CD}log\left(C'_{t(i)-1}\right) + \beta_{CD}log\left(C'_{t(i)-5,t(i)}\right) + \beta_{CD}log\left(C'_{t(i)-22,t(i)}\right)$$
$$+\beta_{CD}log\left(S_{t(i-1)}\right) + \beta_{CD}log\left(S_{t(i-5),t(i)}\right) + \beta_{CD}log\left(S_{t(i-22),t(i)}\right) + \lambda\psi_{t(i-1)} + \gamma d_{t(i-1).}$$

Here we only report the significant coefficients. Newey–West robust standard errors of the corresponding coefficients are reported in brackets.

Table 9
Return equation estimates.

	ARMA-RV	HAR-RV	HAR-CJ	HAR-CJN	ARMA-RV	HAR-RV	HAR-CJ	HAR-CJN
	000919				000987			
μ	−0.0011	−0.0011	−0.0011	−0.0011	0.0009	0.0009	0.0009	0.0009
σ^2	0.7430	0.7327	0.7865	0.8073	0.6687	0.6783	0.6984	0.6991
ν	5.5286	5.3720	5.3987	5.4042	5.4321	5.4467	5.8767	5.8675
	600085				000581			
μ	0.0021	0.0022	0.0021	0.0021	0.0012	0.0012	0.0012	0.0012
σ^2	0.7638	0.7653	0.8076	0.8118	0.6754	0.6899	0.6981	0.6992
ν	6.4290	6.3132	6.3087	6.2545	4.8782	4.8921	4.9876	4.9764
	600351				000088			
μ	−0.0015	−0.0015	−0.0015	−0.0015	0.0007	0.0007	0.0007	0.0007
σ^2	0.8594	0.8315	0.8413	0.8425	0.7251	0.7334	0.7535	0.7549
ν	3.3288	3.3567	3.4321	3.4795	6.8876	6.8991	6.9034	6.9123
	000419				600050			
μ	0.0018	0.0018	0.0018	0.006.18	0.0008	0.0008	0.0008	0.0008
σ^2	0.8976	0.8865	0.9012	0.9005	0.5891	0.5912	0.5945	0.5978
ν	4.9021	4.8766	4.9056	4.9098	5.0123	5.1345	5.2456	5.2571

Note: The return equation with conditional realized volatility is $r_t = \mu_t + \sqrt{\sigma^2 RV_{t|t-1}} z_t$, where $z_t \sim t(0,1,\nu)$. The above table reports the estimation results for the three parameters based on the estimation sample period from January 4, 2003 to December 30, 2005, where μ_t is constant over time.

the return equation, the correlogram of the daily return indicates that there is no evidence of ARMA structure in the return series. I consequently use a constant to measure this component.

5.2.2. Out-of-sample VaR calculation and backtesting

I adopt the VaR calculation methodology introduced in Section 3.1 to compute one-step-ahead and five-step-ahead 1% and 5% VaR forecasts of each stock for all the realized volatility models, and the two-step backtesting procedure presented in Section 3.2 is employed to compare these VaR forecasts.

The statistical adequacy of the VaR forecasts from these realized volatility models is monitored by the unconditional coverage test, independence test and conditional coverage test with a significance level of 5% in the first stage. The percentage of return violations above the VaR forecasts provided by all the models for each stock, and P-values of the three tests on the sequence of violations are presented in Tables 10–13. In terms of the unconditional coverage property, it is clear overall that the HAR-CJ model and the HAR-CJN model are fairly well, while the ARMA model and the HAR model are a bit flawed as the p-values of the unconditional coverage test on the VaR forecasts provided by these two models are below 5% significance level for some stocks. The percentage of VaR violations implies that the ARMA model and the HAR model tend to underestimate the extreme point of the return distribution in these flawed cases. As all the realized volatility models do account for volatility clustering, most of them are able to produce i.i.d VaR violations, causing me to not reject independence of the hitting sequences and the accuracy of the conditional coverage for all the models that survive from the unconditional coverage test.[13]

Based on the loss functions introduced in Section 3.2.2, Table 14 reports the MSE and MAE for all the realized volatility models with respect to one-step-ahead and five-step-ahead VaR predictions at the 1st and 5th percentile. The noticeable feature of these results is that the HAR-CJN model provides VaR forecasts with the lowest MSE and MAE for all the eight stocks. I use Diebold and Mariano (1995) tests (conducted at the 5% level of significance) to examine two specific questions of interest. First, I want to know whether the explicitly modeling jumps in the realized volatility models lead to VaR forecasts that are statistically superior to those obtained from the models which do not separately consider jumps. I

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[13] I do not proceed the independence test and conditional coverage test on the models which do not pass the unconditional test.

Table 10
VaR backtesting results.

| | | One-step-ahead | | | | | | | |
| | | 1% VaR | | | | 5% VaR | | | |
		Percentage of violations	P-value 1	P-value 2	P-value 3	Percentage of violations	P-value 1	P-value 2	P-value 3
000919	ARMA-RV	1.7582%	0.1435	0.1336	0.1063	6.1538%	0.2799	0.2141	0.2392
	HAR-RV	1.7582%	0.1435	0.1444	0.1128	5.9341%	0.3799	0.3153	0.3865
	HAR-CJ	1.5385%	0.2870	0.2733	0.3012	5.4945%	0.6414	0.5778	0.7436
	HAR-CJN	1.0989%	0.8384	0.7950	0.9410	5.4945%	0.6414	0.5778	0.7436
600085	ARMA-RV	1.9027%	0.0793	0.0839	0.0386	6.1311%	0.2749	0.2707	0.2196
	HAR-RV	1.6913%	0.1691	0.1069	0.0893	6.1311%	0.2749	0.2890	0.2296
	HAR-CJ	1.2685%	0.5731	0.4173	0.5726	5.4968%	0.6253	0.3714	0.5102
	HAR-CJN	1.2685%	0.5731	0.4069	0.5641	5.2854%	0.7777	0.5778	0.7436
600351	ARMA-RV	1.7621%	0.1406	0.0963	0.0821	6.1674%	0.2701	0.2174	0.2418
	HAR-RV	1.7621%	0.1406	0.1451	0.1132	5.9471%	0.3680	0.4703	0.4931
	HAR-CJ	1.3216%	0.5116	0.4689	0.6121	5.5066%	0.6258	0.6381	0.7772
	HAR-CJN	1.3216%	0.5116	0.4850	0.6235	5.5066%	0.6258	0.6162	0.7657
000419	ARMA-RV	2.1834%	0.0278			6.3319%	0.2083	0.2633	0.2155
	HAR-RV	2.1834%	0.0278			6.1135%	0.2900	0.2633	0.2768
	HAR-CJ	1.5284%	0.2917	0.5790	0.4706	5.6769%	0.5149	0.3782	0.5160
	HAR-CJN	1.3100%	0.5244	0.5503	0.6657	5.6769%	0.5149	0.3715	0.5103

compare the MSE and MAE from the HAR-CJ model and the HAR-CJN model with those from the ARMA model and the HAR model, and find evidence for this in all the eight stocks studied here. These are indicated by superscript (a) if the HAR-CJ model outperforms the ARMA model and the HAR model, and superscript (b) if the HAR-CJN model outperforms the ARMA model and the HAR model in Table 10. Next, I want to determine whether separate treatment of jump intensities and jump sizes leads to VaR forecasts that are superior to forecasts derived from the model which treats jumps as a single variable. I compare the MSE and MAE from the HAR-CJN model with those that are produced by the HAR-CJ model, and find that the separate treatment of jump intensities and jump size leads to improvement in VaR forecast performance in five out of eight cases. These cases are indicated by superscript (c) in Table 10.

Table 11
VaR backtesting results.

| | | Five-step-ahead | | | | | | | |
| | | 1% VaR | | | | 5% VaR | | | |
		Percentage of violations	P-value 1	P-value 2	P-value 3	Percentage of violations	P-value 1	P-value 2	P-value 3
000919	ARMA-RV	2.1978%	0.0270			6.3736%	0.2003	0.1799	0.1639
	HAR-RV	1.9780%	0.0651	0.0778	0.0363	6.3736%	0.2003	0.2045	0.1801
	HAR-CJ	1.5385%	0.2871	0.2091	0.2494	5.9341%	0.3799	0.4349	0.4717
	HAR-CJN	1.0989%	0.8384	0.8051	0.9441	5.7143%	0.5008	0.3987	0.5328
600085	ARMA-RV	2.3256%	0.0134			6.3425%	0.1976	0.0996	0.0779
	HAR-RV	2.1142%	0.0340			6.1311%	0.2749	0.2498	0.2077
	HAR-CJ	1.4799%	0.3275	0.3203	0.3350	5.7082%	0.4890	0.4689	0.4923
	HAR-CJN	1.2685%	0.5731	0.4752	0.6166	5.7082%	0.4890	0.3922	0.4437
600351	ARMA-RV	1.9824%	0.0636	0.0837	0.0385	6.8282%	0.0896	0.0534	0.0338
	HAR-RV	2.2026%	0.0262			6.6079%	0.1333	0.0984	0.0771
	HAR-CJ	1.7621%	0.1406	0.2074	0.1479	6.1674%	0.2701	0.1881	0.2176
	HAR-CJN	1.5419%	0.2824	0.1575	0.2021	5.9471%	0.3680	0.4512	0.4819
000419	ARMA-RV	2.4017%	0.0106			6.5502%	0.1453	0.1091	0.0837
	HAR-RV	2.4017%	0.0106			6.3319%	0.2083	0.2106	0.1840
	HAR-CJ	0.01746%	0.1463	0.1619	0.1231	5.8952%	0.3921	0.4512	0.4818
	HAR-CJN	1.5284%	0.2917	0.1563	0.2010	5.8952%	0.3921	0.3810	0.4360

Table 12
VaR backtesting results.

| | | One-step-ahead | | | | | | | |
| | | 1% VaR | | | | 5% VaR | | | |
		Percentage of violations	P-value 1	P-value 2	P-value 3	Percentage of violations	P-value 1	P-value 2	P-value 3
000987	ARMA-RV	1.9608%	0.0675	0.1417	0.0583	6.3181%	0.2124	0.2106	0.1840
	HAR-RV	1.9608%	0.0675	0.1381	0.0572	6.3181%	0.2124	0.1708	0.1577
	HAR-CJ	1.5251%	0.2934	0.1371	0.1818	5.4466%	0.6650	0.6124	0.7636
	HAR-CJN	1.3072%	0.5276	0.5690	0.6765	5.4466%	0.6650	0.5797	0.7448
000581	ARMA-RV	2.0225%	0.0569	0.0834	0.0384	6.5169%	0.1598	0.2530	0.2095
	HAR-RV	1.7978%	0.1283	0.1019	0.0859	6.2921%	0.2284	0.2545	0.2703
	HAR-CJ	1.5730%	0.2622	0.1293	0.1737	5.8427%	0.4264	0.4801	0.5928
	HAR-CJN	1.3483%	0.4832	0.4517	0.5994	5.8427%	0.4264	0.4965	0.6037
000088	ARMA-RV	2.2989%	0.0199			6.8966%	0.0853	0.1165	0.0882
	HAR-RV	2.0690%	0.0501	0.1092	0.0476	6.6667%	0.1283	0.2265	0.1939
	HAR-CJ	1.3793%	0.4521	0.4556	0.6023	6.2069%	0.2649	0.4157	0.4594
	HAR-CJN	1.3793%	0.4521	0.4533	0.6006	5.9771%	0.3637	0.4157	0.5462
600050	ARMA-RV	2.0000%	0.0605	0.1094	0.0477	6.4444%	0.1775	0.1973	0.1754
	HAR-RV	2.2222%	0.0248			6.2222%	0.2510	0.2125	0.2379
	HAR-CJ	1.3333%	0.4989	0.4519	0.5996	5.5556%	0.5950	0.1100	0.2421
	HAR-CJN	1.3333%	0.4989	0.4391	0.5898	5.5556%	0.5950	0.0951	0.2155

6. Conclusion

The VaR predictive performance of several recently advanced and some new realized volatility models has been examined in this paper. Some previous studies (Andersen et al., 2007a,b,c, 2011; Liao et al., 2010) reveal that decomposing realized volatility into continuous sample path variation and jumps, and separately modeling each component, outperformed the realized volatility models which treat realized volatility as a single variable with respect to out-of-sample realized volatility forecasting. However, some realized volatility models, particularly the ones which do not account for jumps, may be able to provide good mean point forecast, but fail in tail point forecasting. My analysis here explores the ability of four realized volatility models to predict the tail quantiles of the return distribution. In addition, this paper uses

Table 13
VaR backtesting results.

| | | Five-step-ahead | | | | | | | |
| | | 1% VaR | | | | 5% VaR | | | |
		Percentage of violations	P-value 1	P-value 2	P-value 3	Percentage of violations	P-value 1	P-value 2	P-value 3
000987	ARMA-RV	2.1786%	0.0281			6.5359%	0.1485	0.1133	0.0863
	HAR-RV	2.1786%	0.0281			6.3181%	0.2124	0.2194	0.1895
	HAR-CJ	1.7429%	0.1478	0.1441	0.1126	5.6645%	0.5220	0.4002	0.5340
	HAR-CJN	1.3072%	0.5276	0.4752	0.6166	5.6645%	0.5220	0.3950	0.5298
000581	ARMA-RV	2.2472%	0.0231			6.5169%	0.1598	0.3117	0.2415
	HAR-RV	2.2472%	0.0231			6.5169%	0.1598	0.2461	0.2056
	HAR-CJ	1.5730%	0.2622	0.2667	0.2962	6.0674%	0.3168	0.4511	0.4817
	HAR-CJN	1.3483%	0.4832	0.5708	0.6776	5.8427%	0.4264	0.3809	0.5182
000088	ARMA-RV	2.0690%	0.0501	0.1276	0.0538	7.1264%	0.0551	0.0962	0.0548
	HAR-RV	2.2989%	0.0199			7.1264%	0.0551	0.1132	0.0624
	HAR-CJ	1.6092%	0.2407	0.1565	0.2011	6.4368%	0.1871	0.2193	0.2433
	HAR-CJN	1.6092%	0.2407	0.1451	0.1898	6.4368%	0.1871	0.3091	0.3085
600050	ARMA-RV	2.4444%	0.0093			6.6667%	0.1219	0.1538	0.1093
	HAR-RV	2.2222%	0.0248			6.4444%	0.1775	0.2155	0.1870
	HAR-CJ	1.7778%	0.1351	0.1474	0.1146	6.0000%	0.3447	0.4647	0.4898
	HAR-CJN	1.5556%	0.2734	0.2141	0.2537	5.7778%	0.4596	0.3914	0.5269

Table 14
VaR backtesting results.

		One-step-ahead				Five-step-ahead						One-step-ahead				Five-step-ahead			
		1% VaR		5% VaR		1% VaR		5% VaR				1% VaR		5% VaR		1% VaR		5% VaR	
		MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE			MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
000919	ARMA-RV	2.13	9.01	2.89	10.23	3.21	13.13	3.56	13.58	000987	ARMA-RV	1.79	7.26	2.34	8.95	2.81	10.88	3.09	11.02
	HAR-RV	2.12	9.00	2.90	10.25	3.20	13.14	3.54	13.55		HAR-RV	1.78	7.25	2.31	8.92	2.76	10.87	3.08	10.97
	HAR-CJ	2.01 [a]	8.73 [a]	2.56 [a]	10.05 [a]	3.05 [a]	13.01 [a]	3.21 [a]	13.26 [a]		HAR-CJ	1.68 [a]	7.13 [a]	2.19 [a]	8.68 [a]	2.55 [a]	10.49 [a]	2.93 [a]	10.58 [a]
	HAR-CJN	1.96 [b]	8.67 [b]	2.49 [b]	10.01 [b]	3.02 [b]	12.98 [b]	3.15 [b]	13.03 [b]		HAR-CJN	1.63 [b]	7.03 [b]	2.14 [b]	8.46 [b]	2.49 [b]	10.42 [b]	2.80 [b]	10.53 [b]
600085	ARMA-RV	2.25	9.14	2.94	11.27	3.54	13.69	3.89	13.87	000581	ARMA-RV	2.36	9.58	3.08	11.82	3.71	14.36	4.08	14.54
	HAR-RV	2.23	9.12	2.91	11.23	3.48	13.56	3.83	13.81		HAR-RV	2.33	9.56	3.05	11.78	3.65	14.22	4.02	14.48
	HAR-CJ	2.11 [a]	8.98 [a]	2.76 [a]	10.92 [a]	3.21 [a]	13.20 [a]	3.69 [a]	13.32 [a]		HAR-CJ	2.21 [a]	9.42 [a]	2.89 [a]	11.45 [a]	3.37 [a]	13.84 [a]	3.87 [a]	13.97 [a]
	HAR-CJN	2.05 [b]	8.85 [b]	2.69 [b]	10.65 [b]	3.13 [b]	13.12 [b]	3.53 [b]	13.25 [b]		HAR-CJN	2.15 [b]	9.28 [b]	2.82 [b]	11.17 [b]	3.28 [b]	13.76 [b]	3.70 [b]	13.89 [b]
600351	ARMA-RV	2.18	8.87	2.85	10.93	3.43	13.28	3.77	13.40	000088	ARMA-RV	2.24	9.11	2.93	11.23	3.53	13.64	3.88	13.82
	HAR-RV	2.20	8.89	2.82	10.89	3.38	13.15	3.72	13.40		HAR-RV	2.20	9.09	2.90	11.19	3.47	13.51	3.82	13.76
	HAR-CJ	2.05 [a]	8.71 [a]	2.68 [a]	10.59 [a]	3.11 [a]	12.80 [a]	3.58 [a]	12.92 [a]		HAR-CJ	2.12 [a]	8.95 [a]	2.75 [a]	10.88 [a]	3.20 [a]	13.15 [a]	3.68 [a]	13.27 [a]
	HAR-CJN	2.99 [b]	8.58 [b]	2.61 [b]	10.33 [b]	3.04 [b]	12.73 [b]	3.42 [b]	12.85 [b]		HAR-CJN	2.04 [b]	8.82 [b]	2.68 [b]	10.61 [b]	3.12 [b]	13.07 [b]	3.52 [b]	13.20 [b]
000419	ARMA-RV	2.29	9.31	2.99	11.48	3.61	13.94	3.96	14.13	600050	ARMA-RV	2.15	8.74	2.81	10.78	3.39	13.09	3.72	13.26
	HAR-RV	2.26	9.29	2.96	11.44	3.63	13.81	3.99	14.07		HAR-RV	2.13	8.72	2.78	10.74	3.33	12.97	3.66	13.21
	HAR-CJ	2.15 [a]	9.15 [a]	2.81 [a]	11.12 [a]	3.27 [a]	13.44 [a]	3.76 [a]	13.57 [a]		HAR-CJ	2.02 [a]	8.59 [a]	2.64 [a]	10.44 [a]	3.07 [a]	12.62 [a]	3.53 [a]	12.74 [a]
	HAR-CJN	2.09 [b]	9.01 [b]	2.74 [b]	10.85 [b]	3.19 [b]	13.36 [b]	3.60 [b]	13.50 [b]		HAR-CJN	1.96 [b]	8.46 [b]	2.57 [b]	10.19 [b]	2.99 [b]	12.55 [b]	3.38 [b]	12.67 [b]

Note: The out-of-sample VaR forecast comparison is based on observations from 01/04/2006 to 12/27/2007. All figures in the table have been multiplied by 10^4. The superscript indicates that Diebold and Mariano (1995) tests of the null that VaR forecasts are equal is rejected at the 5% level of significance. The alternatives are: a) HAR-CJ model provides superior forecasts than ARMA model and HAR model; b) HAR-CJN model provides superior forecasts than ARMA model and HAR model; c) HAR-CJN model superior forecasts than HAR-CJ model.

intraday data covering a recent five-year period of eight individual stocks from four different industries in China as an empirical analysis. This provides a good opportunity to examine different jump patterns and how they impact the risk predictive ability of the realized volatility models in an emerging stock market.

From the results of the Monte Carlo simulation and the out-of-sample forecasting results in the real-data studies, I find that in some cases, the ARMA model and the HAR model of realized volatility suffer from excessive VaR violations, implying an underestimation of market risk. Most notably, the other two realized volatility models, HAR-CJ model and HAR-CJN model, in which the jump component is explicitly considered, can produce more accurate VaR forecasts. This finding regarding the importance of the jumps is interesting, because it implies that separately modeling jumps and continuous component in realized volatility do indeed improve the description of the tail behavior in asset return distribution.

Finally, this work finds that jumps in Chinese market are more frequent and account for larger proportion of realized volatility than those of developed markets. Meanwhile, the times between two adjacent significant jumps of these Chinese stocks are highly persistent processes, and a surprising dynamic dependence of jump sizes is also detected in this emerging market. Therefore, modeling jumps in realized volatility plays a more important role in risk prediction of this market. It would be interesting to further study the jump behavior and investigate how jumps affect the risk prediction in other emerging markets.

Acknowledgments

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