$$\phi + 3g^+$$

Cut-constructible part

We have four different double-cuts which can be divided into two sectors. Computing the double cuts, we obtained the cut-constructible part of the amplitude,

$$A^{2L}_{cc}(\phi;1^+2^+3^+) = A^{1L}(\phi;1^+,2^+,3^+)\sum_{\sigma\in\mathbb{Z}_3} d_{\sigma(i)} I^{1m}_{4,\sigma(1)} + A^{1L}(\phi;1^+,2^+,3^+)\sum_{\sigma\in\mathbb{Z}_3} c_{\sigma(1)} I^{2m}_{3,\sigma(1)}$$
$$- \frac{d_s-2}{6} A^{1L}(\phi;1^+2^+3^+)\, I_2(s_\phi)$$

where

$$d_{\sigma(1)} = -\frac{1}{2} s_{\phi\sigma(1)} s_{\phi\sigma(3)}, \qquad c_{\sigma(1)} = 2\frac{1}{2}(s_{\sigma(1)\phi} - s_\phi).$$

Then the divergent part is

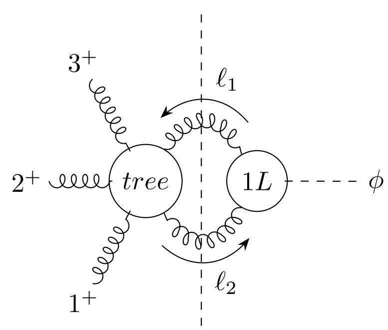
$$\left[A^{2L}_{cc}(\phi;1^+,2^+,3^+)\right]_{IR+UV} = \left[-\frac{1}{\epsilon^2}\sum_{i=1}^{3}(-s_{i,i+1})^{-\epsilon} - \frac{d_s-2}{6\epsilon}\right].$$

The remainder part comes from the finite contribution of one-mass boxes and from the bubble integral. We obtain

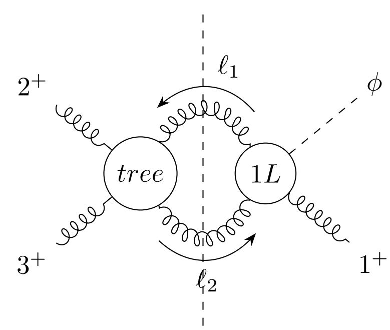
$$\left[A^{2L}_{cc}(\phi;1^+2^+3^+)\right]_{finite} = A^{1L}(\phi;1^+2^+3^+)\left[2\,\mathrm{Li}_2\left(1-\frac{s_\phi}{s_{12}}\right) + 2\,\mathrm{Li}_2\left(1-\frac{s_\phi}{s_{23}}\right) + 2\,\mathrm{Li}_2\left(1-\frac{s_\phi}{s_{31}}\right)\right.$$
$$\left. + \frac{1}{2}\ln^2\frac{s_{12}}{s_{23}} + \frac{1}{2}\ln^2\frac{s_{23}}{s_{31}} + \frac{1}{2}\ln^2\frac{s_{31}}{s_{12}} + \frac{\pi^2}{2} - \frac{d_s-2}{6}\left(2+\ln\left(\frac{\mu_R^2}{-s_\phi}\right)\right)\right].$$

As soon as possible, I will write the cut-constructible pieces in terms of one-loop MI.
In the following sections there are some details about the computation.

1 Cuts with a 1L $\phi + g$ sub-amplitude



(dcut A)



(dcut B)

In s_ϕ-channel, the integrand for the double cut computation is

$$A^{2L}_{int}|_{\text{dcut A}} = -2A^{tree}(\phi^\dagger;\ell_1^+,(-\ell_2)^+)\frac{\langle\ell_1\ell_2\rangle^3}{\langle12\rangle\langle23\rangle\langle3\ell_1\rangle\langle\ell_21\rangle}$$
$$= A^{1L}(\phi;1^+,2^+,3^+)\frac{\mathrm{tr}_-(13\ell_1\ell_2)}{\langle3\ell_13]\langle1\ell_21]}$$

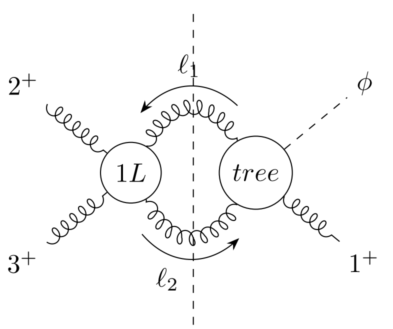
$$\int \mathrm{d}\Phi_2 A^{2L}_{int}|_{\text{dcut A}} = A^{1L}(\phi;1^+,2^+,3^+)\left[\frac{1}{2}(-s_{1\phi}s_{3\phi})I_4^{1m}(s_{1\phi},s_{3\phi};s_\phi)|_{s_\phi\text{-cut}} + \frac{1}{2}(s_{1\phi}-s_\phi)I_3^{2m}(s_{1\phi},s_\phi)|_{s_\phi\text{-cut}} + \frac{1}{2}(s_{3\phi}-s_\phi)I_3^{2m}(s_{3\phi},s_\phi)|_{s_\phi\text{-cut}}\right].$$

In $s_{\phi 1}$-channel the double cut can be easily computed using Schouten identity,

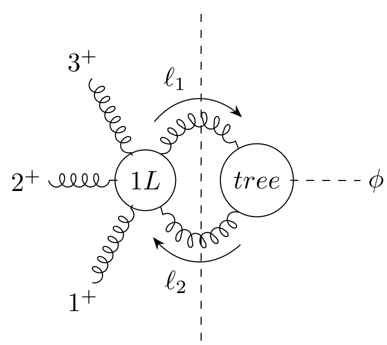
$$\begin{aligned}
A^{2L}_{int}|_{\text{dcut B}} &= -2s_\phi^2 A^{tree}(\phi^\dagger;1^+,\ell_1^+,(-\ell_2)^+)\frac{\langle \ell_1\ell_2\rangle^3}{\langle 23\rangle\langle 3\ell_1\rangle\langle \ell_2 2\rangle}\\
&= A^{1L}(\phi;1^+,2^+,3^+)\left(\frac{\langle 1\ell_2\rangle}{\langle \ell_1 1\rangle}+\frac{\langle \ell_2 3\rangle}{\langle \ell_1 3\rangle}\right)\left(\frac{\langle 1\ell_1\rangle}{\langle \ell_2 1\rangle}+\frac{\langle \ell_1 2\rangle}{\langle \ell_2 2\rangle}\right)\\
&= A^{1L}(\phi;1^+,2^+,3^+)\left[\frac{1}{2}(-s_{1\phi}s_{3\phi})I_4^{1m}(s_{1\phi},s_{3\phi};s_\phi)|_{s_{\phi 1}\text{-cut}}+\right.\\
&\left.\frac{1}{2}(-s_{1\phi}s_{2\phi})I_4^{1m}(s_{1\phi},s_{2\phi};s_\phi)|_{s_{\phi 1}\text{-cut}} + 2\frac{1}{2}(s_{1\phi}-s_\phi)I_3^{2m}(s_{1\phi},s_\phi)|_{s_{\phi 1}\text{-cut}}\right]
\end{aligned}$$

The self-dual Higgs is unordered and this is the motivation of the presence of two boxes with a different configuration of gluons. We do not observe the presence of contributions in this channel that cannot be investigated in the previous one. Hence, we only have one-mass boxes and two-mass triangles with ϕ alone.

2 Cuts with a 1L YM sub-amplitude



(dcut C)



(dcut D)

In $s_{\phi 1}$-channel, the product of sub-amplitude is

$$\begin{aligned}
A^{2L}_{int}|_{\text{dcut C}} &= \left[\frac{\langle \ell_1\ell_2\rangle^3}{\langle 1\ell_1\rangle\langle \ell_2 1\rangle}\right]\left[\frac{d_s-2}{2}\frac{1}{3}\frac{[\ell_1\ell_2][23]}{-\langle \ell_1\ell_2\rangle\langle 23\rangle}\right]\\
&= -\frac{d_s-2}{2}\frac{1}{3}[23]^2\frac{\langle \ell_1\ell_2\rangle}{\langle \ell_1 1\rangle\langle 1\ell_2\rangle} = -\frac{d_s-2}{2}\frac{1}{3}[23]^2\frac{1}{\langle 12\rangle}\left(-\frac{\langle 2\ell_2 1]}{\langle 1\ell_2 1]}+\frac{\langle 2\ell_2 1]}{\langle 1\ell_1 1]}\right)\\
&= -\frac{d_s-2}{2}\frac{1}{3}[23]^2\frac{1}{\langle 12\rangle}\left(-\frac{\langle 2P_{12}1]}{\langle 1P_{12}1]}+\frac{\langle 2P_{12}1]}{\langle 1P_{12}1]}\right) = 0.
\end{aligned}$$

The last passage can be shown using the explicit integrand reduction of 3-pts tensor integrals.

The double cut in s_ϕ-channel was computed in the attached Mathematica notebook. We obtain only a bubble. Because of the colorless of ϕ, we sum over the three possible configurations of gluons and we obtain a coefficient proportional to the one-loop amplitude,

$$-\frac{d_s-2}{6}A^{1L}(\phi;1^+,2^+,3^+)I_2(s_\phi).$$