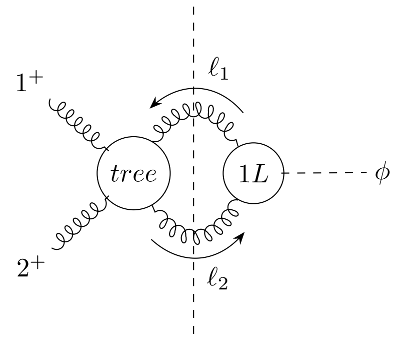
$$\phi + 2g^+$$

Cut-constructible part

Since $s_\phi = s_{12}$ is the only invariant in this 3pt amplitude, we have to compute only two different double cuts which present a different loop level of the sub-amplitudes.

1 Cut with a 1L $\phi + g$ sub-amplitude



(dcut A)

The integrand for the double cut computation is

$$\begin{aligned}
A^{2L}_{int}|_{\text{dcut A}} &= A^{1L}(\phi; \ell_1^+, (-\ell_2)^+) A^{tree}(1^+, 2^+, \ell_2^-, (-\ell_1)^-) \\
&= -2A^{tree}(\phi^\dagger; \ell_1^+, (-\ell_2)^+) \frac{\langle \ell_1 \ell_2 \rangle^3}{\langle 12 \rangle \langle 2\ell_1 \rangle \langle \ell_2 1 \rangle} \\
&= \frac{2s_\phi^2}{\langle 12 \rangle} \frac{\langle \ell_1 \ell_2 \rangle}{\langle 1\ell_2 \rangle \langle \ell_1 2 \rangle} = \frac{-2s_\phi^2}{\langle 12 \rangle \langle 21 \rangle} \frac{\langle 12\ell_1\ell_2 1]}{\langle 1\ell_2 1] \langle 2\ell_1 2]} \\
&= A^{1L}(\phi; 1^+, 2^+) \frac{\mathrm{tr}_-(12\ell_1\ell_2)}{\langle 1\ell_2 1] \langle 2\ell_1 2]} \\
&= A^{1L}(\phi; 1^+, 2^+) \frac{\frac{1}{2}\,\mathrm{tr}(12\ell_1\ell_2)}{\langle 1\ell_2 1] \langle 2\ell_1 2]} + \text{spurious terms}
\end{aligned}$$

If we expand the trace, we obtain

$$\tfrac{1}{2}\,\mathrm{tr}(12\ell_1\ell_2) = 2(p_1 \cdot p_\phi)(p_2 \cdot \ell_1) - 2(p_1 \cdot \ell_2)(p_2 \cdot p_\phi).$$

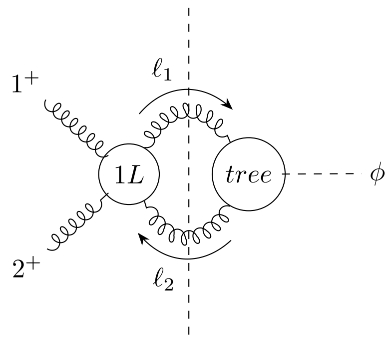
We only have contributions proportional to the propagators $(p_2 \cdot \ell_1)$ and $(p_1 \cdot \ell_2)$. For this reason we don't have boxes, but only triangles.

$$\begin{aligned}
A^{2L}_{int}|_{\text{dcut A}} &= A^{1L}(\phi; 1^+, 2^+) \left[-\frac{p_1 \cdot p_2}{2(1 \cdot \ell_2)} + \frac{p_2 \cdot p_1}{2(p_2 \cdot \ell_1)} \right] \\
\int \mathrm{d}\Phi_2 A^{2L}_{int}|_{\text{dcut A}} &= A^{1L}(\phi; 1^+, 2^+) \left[-(p_1 \cdot p_2) I_3^{1m}(s_{12}) - (p_1 \cdot p_2) I_3^{1m}(s_{12}) \right] \\
&= -s_{12} A^{1L}(\phi; 1^+, 2^+) I_3^{1m}(s_{12}).
\end{aligned}$$

The self-dual Higgs is unordered, then we have two equivalent cuts connected by a permutation $\mathbb{Z}_2$ of the two gluons.
From this sector, we obtain the expected IR structure

$$A^{2L}_{cc(I)} = -2s_{12} A^{1L}(\phi; 1^+, 2^+) I_3^{1m}(s_{12}) = A^{1L}(\phi; 1^+, 2^+) \left[-\frac{1}{\epsilon^2} \sum_{i=1}^{2} (-s_{i,i+1})^{-\epsilon} \right]$$

2 Cuts with a 1L YM sub-amplitude



(dcut B)

The product of sub-amplitudes is

$$\begin{aligned} A_{int}^{2L}|_{\text{dcut B}} &= A^{tree}(\phi; (-\ell_1)^-, \ell_2^-) A^{1L}(1^+, 2^+, (-\ell_2)^+, \ell_1^+) \\ &= -\langle \ell_1 \ell_2 \rangle \frac{1}{3} \frac{[\ell_1(-\ell_2)][12]}{\langle \ell_1(-\ell_2)\rangle \langle 12 \rangle} = \frac{1}{3} \frac{s_\phi^2}{\langle 12 \rangle \langle 21 \rangle} = -\frac{1}{6} A^{1L}(\phi; 1^+, 2^+) \end{aligned}$$

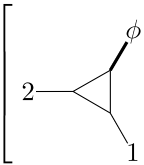
Using the fact that ϕ is unordered, we have two bubble contributions,

$$\int \mathrm{d}\Phi_2 A_{int}^{2L}|_{\text{dcut B}} + (1 \leftrightarrow 2) = -\frac{1}{6} \sum_{\sigma} A^{1L}(\phi; 1^+, 2^+) I_2(s_{12}) = -\frac{1}{3} A^{1L}(\phi; 1^+, 2^+) I_2(s_{12}).$$

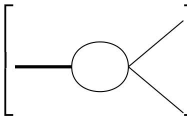
3 Summary

Computing the double cuts, we obtained the cut-constructible part of the amplitude,

$$A_{cc}^{2L}(\phi; 1^+, 2^+) = -\, 2 s_{12} A^{1L}(\phi; 1^+, 2^+) \Bigg[$$



$$\Bigg] - \frac{1}{3} A^{1L}(\phi; 1^+, 2^+) \Bigg[$$



$$\Bigg]$$

Then the divergent part is

$$\left[A_{cc}^{2L}(\phi; 1^+, 2^+, 3^+)\right]_{IR+UV} = \left[-\frac{1}{\epsilon^2} \sum_{i=1}^{2} (-s_{i,i+1})^{-\epsilon} - \frac{1}{3\epsilon} \right].$$

The remainder part comes from the bubble integral. We obtain

$$\left[A_{cc}^{2L}(\phi; 1^+ 2^+ 3^+)\right]_{finite} = -\frac{1}{3} A^{1L}(\phi; 1^+ 2^+ 3^+) \left(2 + \ln \left(\frac{\mu_R^2}{-s_\phi} \right) \right).$$

In order to match the normalisation from AMflow, we can use

```
A2ccFR=A0*9/4*(2+wgt*Log[-mu^2/(s[4]+I*delta)])+A0*2*Pi^2/12*wgt^2
```

where the last contribution comes from the difference between the normalisations at $O(\epsilon^2)$ multiplied by the IR structure of the poles.