

Part 1: Neuron and Neural Network

Mikhail Romanov, Igor Slinko

Human's Nervous System



Human's Nervous System



Brain
(information processing,
decision making)

Human's Nervous System



Brain
(information processing,
decision making)

↑
Signals from
sensorium

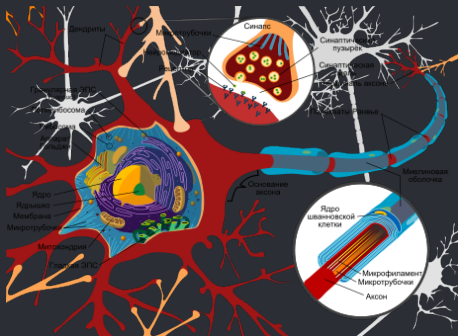
Human's Nervous System



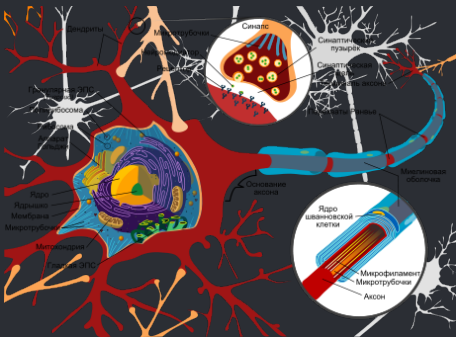
Brain
(information processing,
decision making)

Signals from sensorium Signals to musculoskeletal system

Biological Neuron

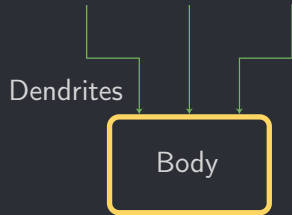
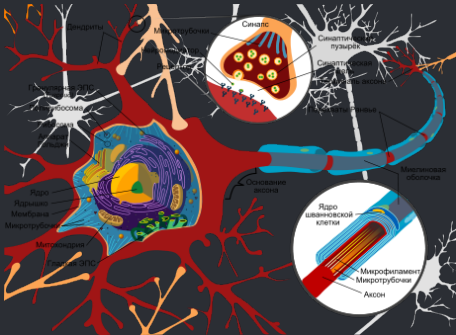


Biological Neuron

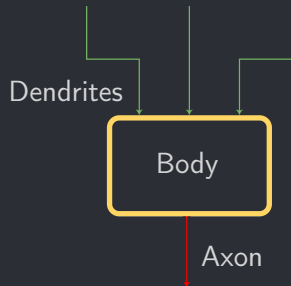
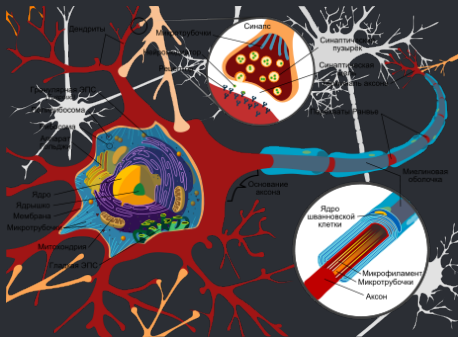


Body

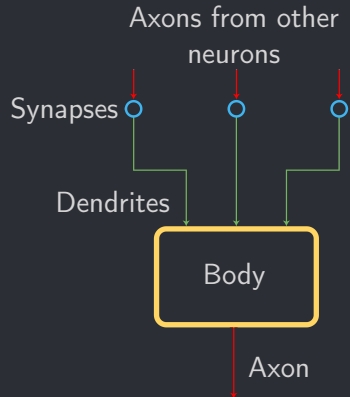
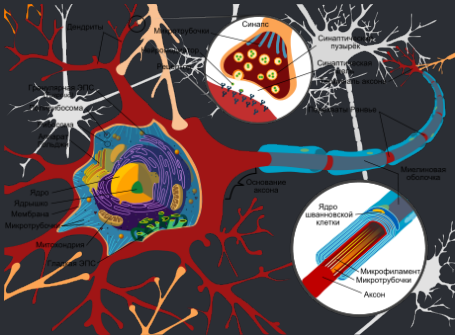
Biological Neuron



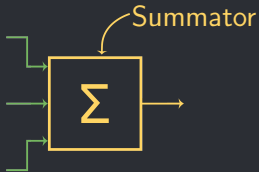
Biological Neuron



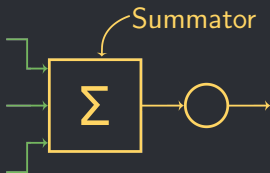
Biological Neuron



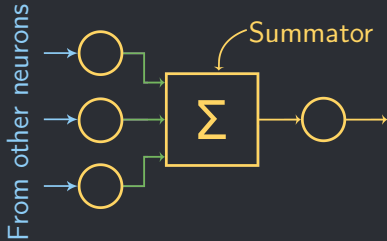
Mathematical Model of the Neuron



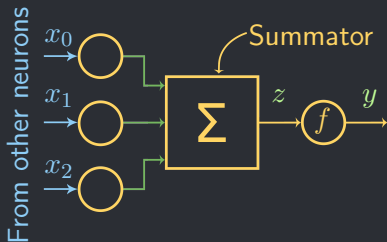
Mathematical Model of the Neuron



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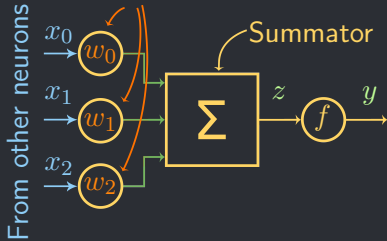
$$y = f(z) = f(x_0 + x_1 + x_2)$$

$f(z)$ – activation function

x_0, x_1, x_2 – inputs

Mathematical Model of the Neuron

Synaptic weights



$$y = f(z) = f(w_0x_0 + w_1x_1 + w_2x_2)$$

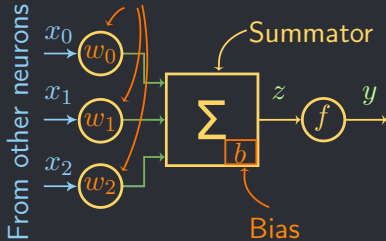
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Mathematical Model of the Neuron

Synaptic weights



$$y = f(z) = f(w_0x_0 + w_1x_1 + w_2x_2 + b)$$

$f(z)$ – activation function

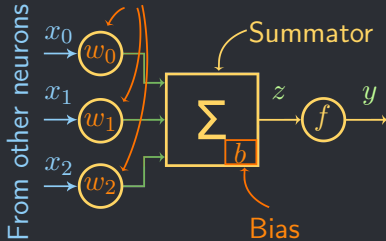
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b – bias

Mathematical Model of the Neuron

Synaptic weights



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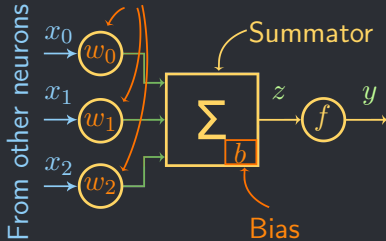
b – bias

$$y = f(z) = f\left(\sum_{i=1}^n w_i x_i + b\right)$$

w_i, b – tunable parameters

Mathematical Model of the Neuron

Synaptic weights



$$y = f(z) = f(w_0x_0 + w_1x_1 + w_2x_2 + b)$$

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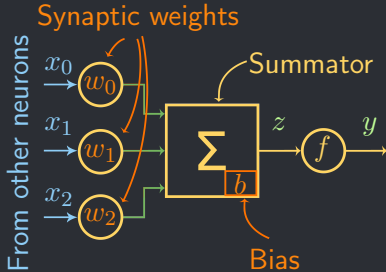
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w_i, b – tunable parameters

$$y = f(z) = f(\langle \mathbf{w}, \mathbf{x} \rangle + b)$$

$\langle \mathbf{w}, \mathbf{x} \rangle$ – scalar product

Mathematical Model of the Neuron



$$y = f(z) = f(w_0x_0 + w_1x_1 + w_2x_2 + b)$$

$f(z)$ – activation function

x_0, x_1, x_2 – inputs

w_0, w_1, w_2 – weights

b – bias

Activation

$$f(z) = \begin{cases} 0, & \text{if } z \leq 0 \\ 1, & \text{if } z > 0 \end{cases}$$

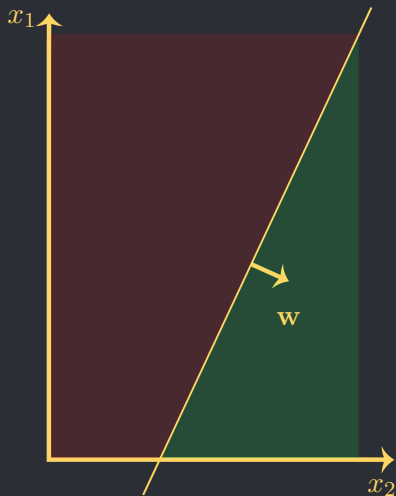
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$$y = f(z) = f(\langle \mathbf{w}, \mathbf{x} \rangle + b)$$

$\langle \mathbf{w}, \mathbf{x} \rangle$ – scalar product

Separator



Separator of a single neuron:
 $x_1 = f(\langle \mathbf{w}, \mathbf{x}_2 \rangle + b)$

Activation Functions

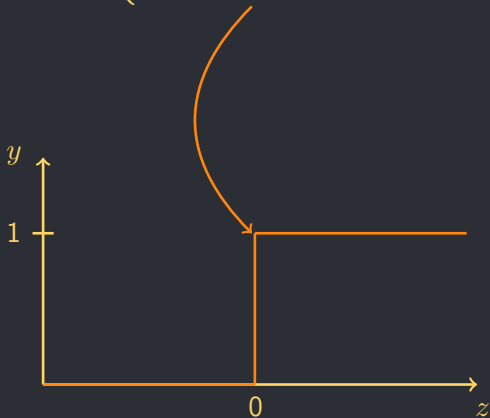
Threshold activation
function:

$$f(z) = \begin{cases} 0, & \text{if } z \leq 0 \\ 1, & \text{if } z > 0 \end{cases}$$

Activation Functions

Threshold activation
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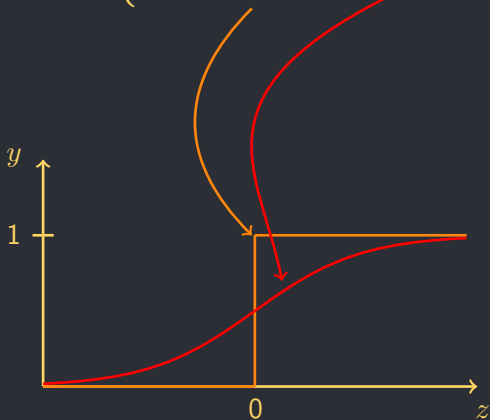
Activation Functions

Threshold activation
function:

$$f(z) = \begin{cases} 0, & \text{if } z \leq 0 \\ 1, & \text{if } z > 0 \end{cases}$$

Sigmoid:

$$\sigma(z) = \frac{1}{1+e^{-z}}$$



Activation Functions

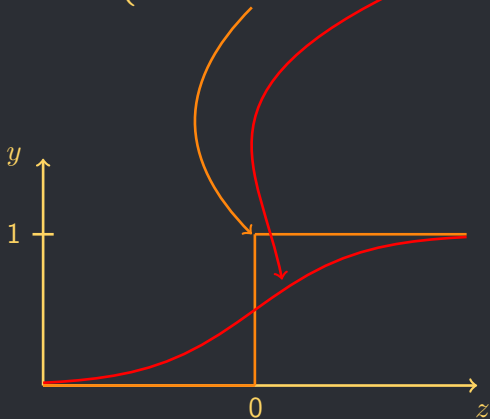
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if $z \rightarrow \infty$ $\sigma(z) \rightarrow$



Activation Functions

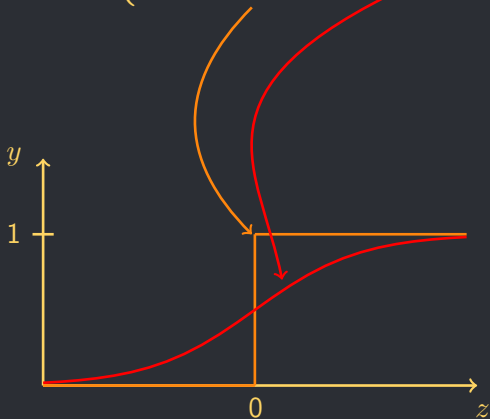
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Activation Functions

Threshold activation
function:

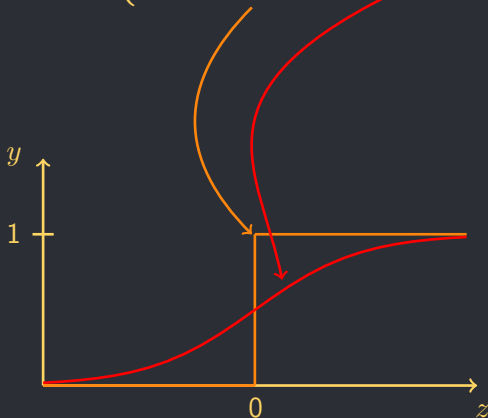
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Sigmoid:

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

if $z \rightarrow \infty$ $\sigma(z) \rightarrow 1$

if $z \rightarrow -\infty$ $\sigma(z) \rightarrow 0$



Activation Functions

Threshold activation
function:

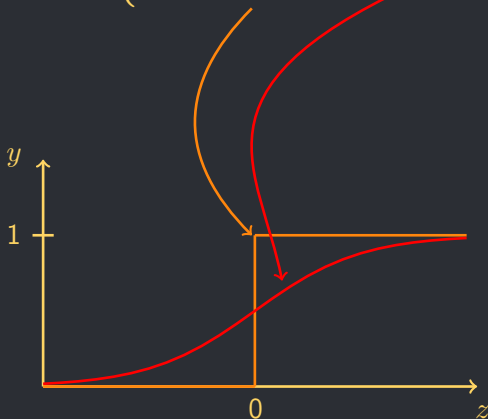
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Activation Functions

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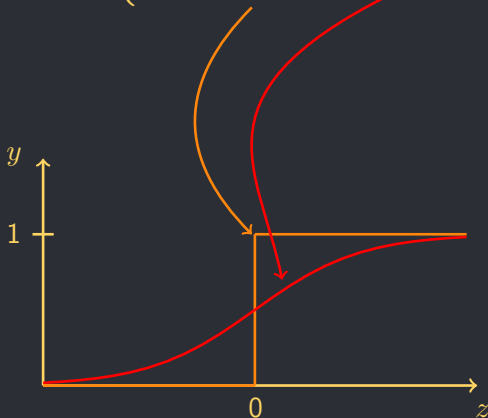
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$$0 < \sigma(z) < 1$$



Activation Functions

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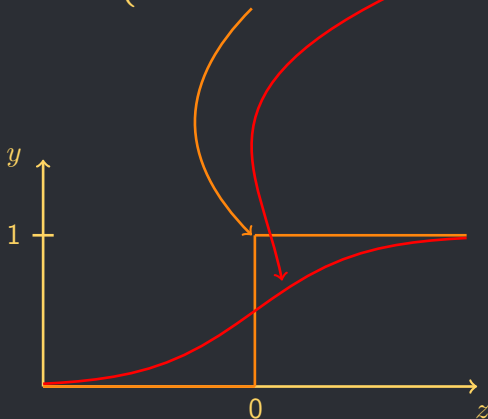
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$$0 < \sigma(z) < 1$$

$$\sigma(0) = 0.5$$



Activation Functions

Threshold activation
function:

$$f(z) = \begin{cases} 0, & \text{if } z \leq 0 \\ 1, & \text{if } z > 0 \end{cases}$$

Sigmoid:

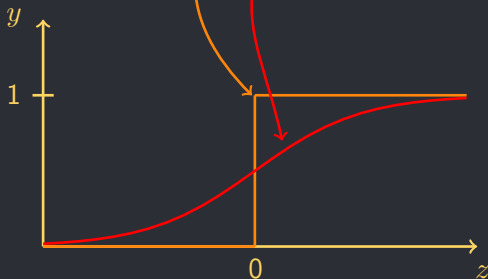
$$\sigma(z) = \frac{1}{1+e^{-z/T}}$$

$$\text{if } z \rightarrow \infty \sigma(z) \rightarrow 1$$

$$\text{if } z \rightarrow -\infty \sigma(z) \rightarrow 0$$

$$0 < \sigma(z) < 1$$

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Activation Functions

Threshold activation
function:

$$f(z) = \begin{cases} 0, & \text{if } z \leq 0 \\ 1, & \text{if } z > 0 \end{cases}$$

Sigmoid:


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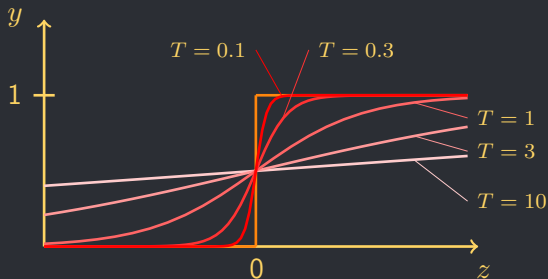
if $z \rightarrow \infty$ $\sigma(z) \rightarrow 1$

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$0 < \sigma(z) < 1$

$\sigma(0) = 0.5$

if $T \rightarrow 0$ $\sigma(z) \rightarrow$ 



Logical Negation

\overline{x}

x	\overline{x}
0	1
1	0

Task: make a neuron
that works as a negation operation

Logical Negation

\bar{x}

x	\bar{x}
0	1
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Task: make a neuron
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Logical Negation

\bar{x}

x	\bar{x}
0	1
1	0

Task: make a neuron that works as a negation operation



Logical "AND"

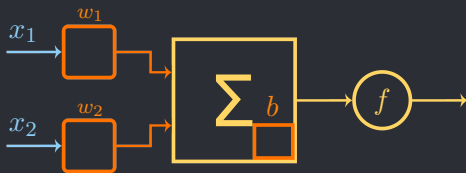
$$x_1 \& x_2$$

x_1	x_2	$x_1 \& x_2$
1	1	1
1	0	0
0	1	0
0	0	0

Logical "AND"

$x_1 \& x_2$

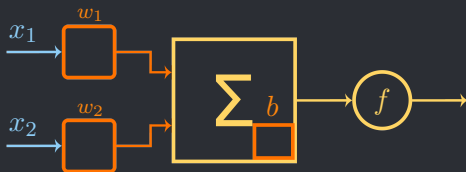
x_1	x_2	$x_1 \& x_2$
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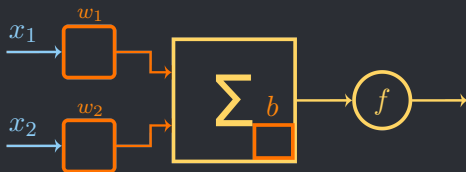


$$\begin{cases} 1w_1 + 1w_2 + b \geq 0 \\ 1w_1 + 0w_2 + b < 0 \\ 0w_1 + 1w_2 + b < 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$

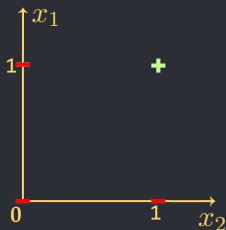
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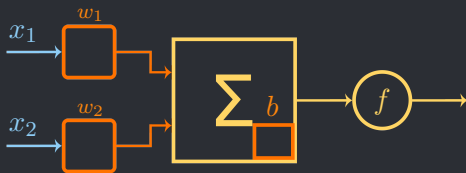
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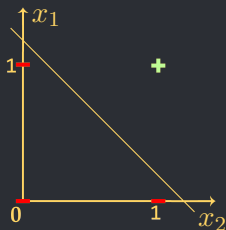
Logical "AND"

x_1 & x_2

x_1	x_2	$x_1 \& x_2$
1	1	1
1	0	0
0	1	0
0	0	0



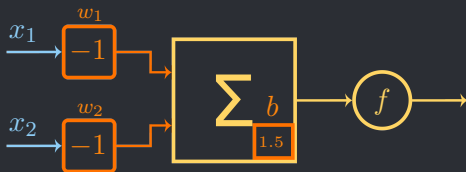
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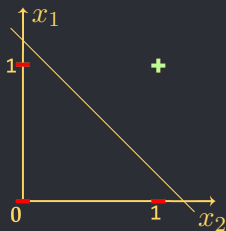
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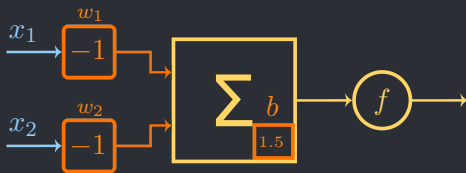
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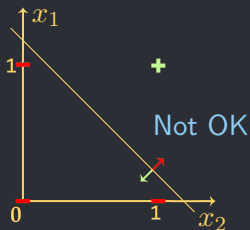
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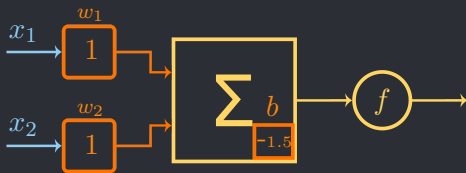
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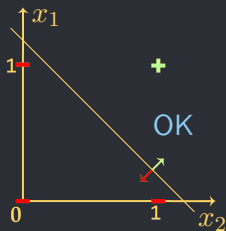
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$$\begin{cases} 1w_1 + 1w_2 + b \geq 0 \\ 1w_1 + 0w_2 + b < 0 \\ 0w_1 + 1w_2 + b < 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$



Logical "OR"

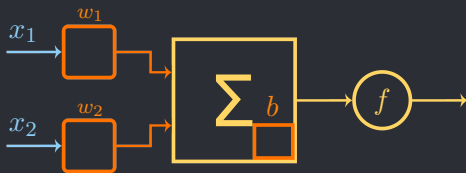
$$x_1 \parallel x_2$$

x_1	x_2	$x_1 \parallel x_2$
1	1	1
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0	1	1
0	0	0

Logical "OR"

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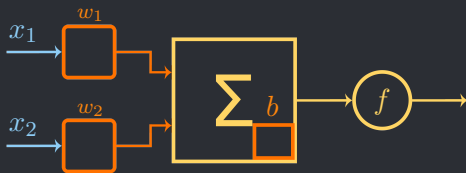
x_1	x_2	$x_1 \parallel x_2$
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Logical "OR"

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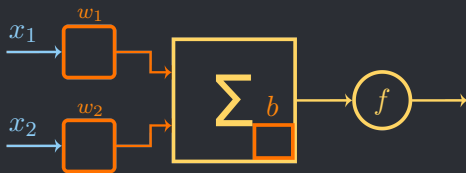


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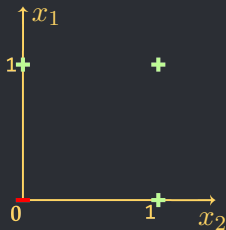
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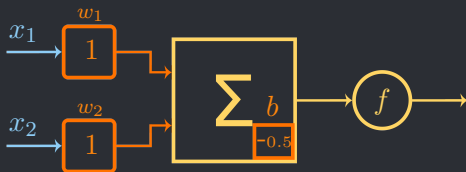
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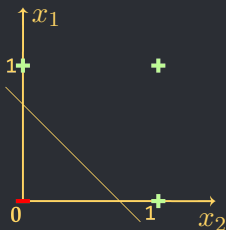
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XOR

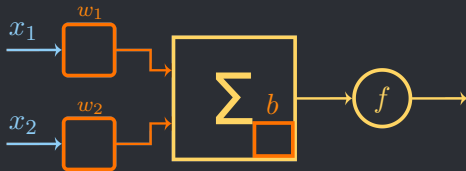
$$x_1 \oplus x_2$$

x_1	x_2	$x_1 \oplus x_2$
1	1	0
1	0	1
0	1	1
0	0	0

XOR

$$x_1 \oplus x_2$$

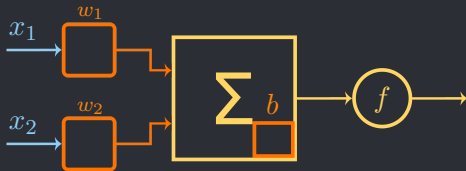
x_1	x_2	$x_1 \oplus x_2$
1	1	0
1	0	1
0	1	1
0	0	0



XOR

$$x_1 \oplus x_2$$

x_1	x_2	$x_1 \oplus x_2$
1	1	0
1	0	1
0	1	1
0	0	0

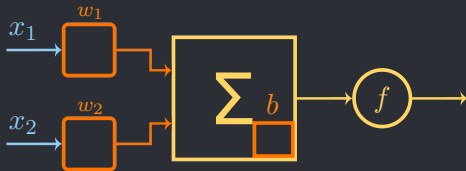


$$\begin{cases} 1w_1 + 1w_2 + b < 0 \\ 1w_1 + 0w_2 + b \geq 0 \\ 0w_1 + 1w_2 + b \geq 0 \\ 0w_1 + 0w_2 + b < 0 \end{cases}$$

XOR

$$x_1 \oplus x_2$$

x_1	x_2	$x_1 \oplus x_2$
1	1	0
1	0	1
0	1	1
0	0	0



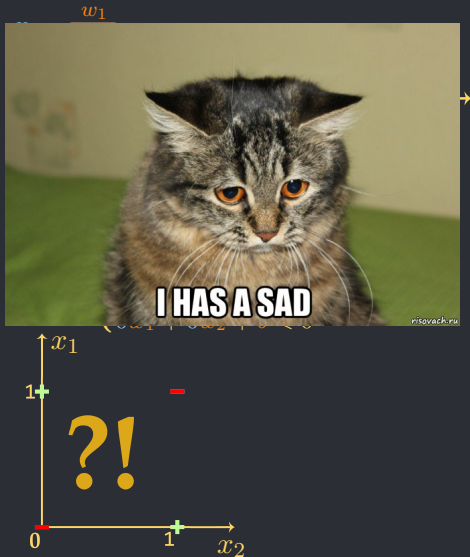
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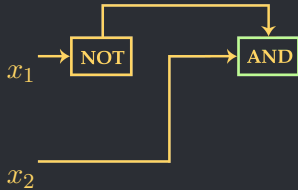


Solution

$$x_1 \oplus x_2 = (\overline{x}_1 \& x_2) \mid (x_1 \& \overline{x}_2)$$

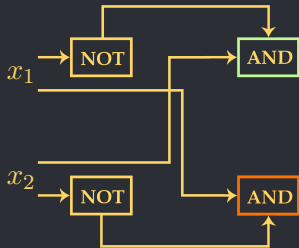
Solution

$$x_1 \oplus x_2 = \underbrace{(\bar{x}_1 \& x_2)}_{\text{AND}} \parallel (x_1 \& \bar{x}_2)$$



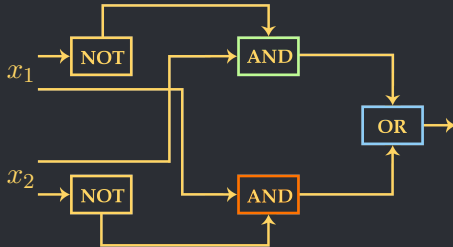
Solution

$$x_1 \oplus x_2 = \underbrace{(\bar{x}_1 \& x_2)} \parallel \underbrace{(x_1 \& \bar{x}_2)}$$



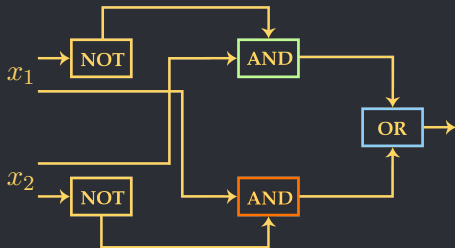
Solution

$$x_1 \oplus x_2 = \underbrace{(\overline{x_1} \& x_2)}_{\text{green}} \parallel \underbrace{(x_1 \& \overline{x_2})}_{\text{orange}}$$



Solution

$$x_1 \oplus x_2 = \underbrace{(\overline{x_1} \& x_2)}_{\text{green}} \parallel \underbrace{(x_1 \& \overline{x_2})}_{\text{orange}}$$



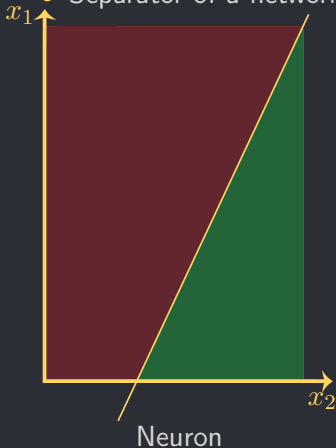
We can make a non-linear operation with a network !

We have used 5 neurons.
But it is possible to use only 3.

Task: do that.

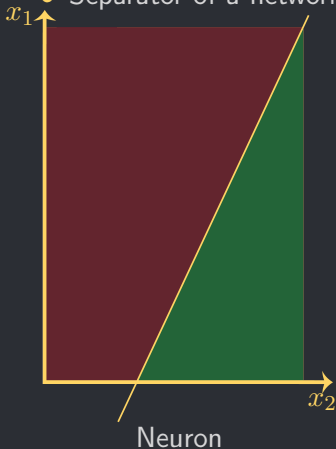
Linearity of a Neuron and Non-Linearity of a Network

- Separator of a single neuron is linear
- Separator of a network may be non-linear



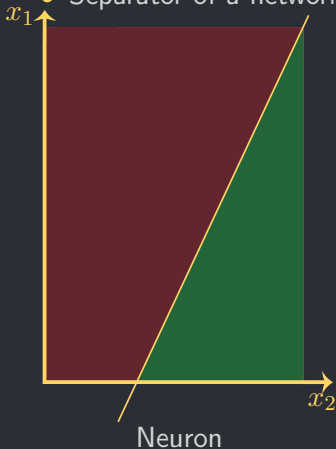
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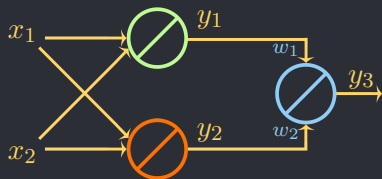


Linearity of a Neuron and Non-Linearity of a Network

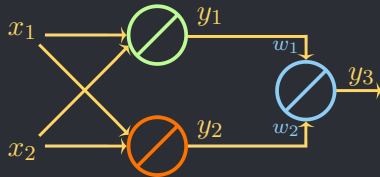
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Network of Neurons with Linear Activations

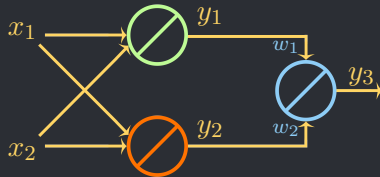


Network of Neurons with Linear Activations



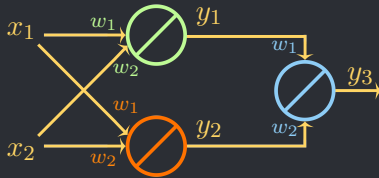
$$y_3 = f(w_1 y_1 + w_2 y_2 + b) =$$

Network of Neurons with Linear Activations



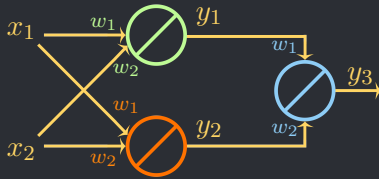
$$\begin{aligned} y_3 &= f(w_1 y_1 + w_2 y_2 + b) = \\ &= w_1 y_1 + w_2 y_2 + b = \end{aligned}$$

Network of Neurons with Linear Activations



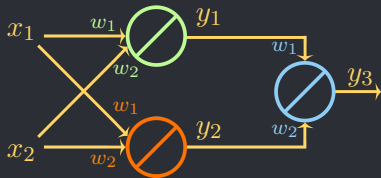
$$\begin{aligned} y_3 &= f(w_1 y_1 + w_2 y_2 + b) = \\ &= w_1 y_1 + w_2 y_2 + b = \\ &= w_1 f(w_1 x_1 + w_2 x_2 + b) + \\ &+ w_2 f(w_1 x_1 + w_2 x_2 + b) + b = \end{aligned}$$

Network of Neurons with Linear Activations



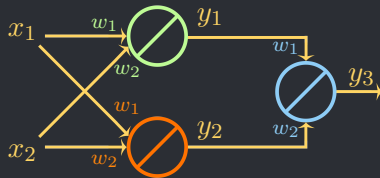
$$\begin{aligned} y_3 &= f(w_1 y_1 + w_2 y_2 + b) = \\ &= w_1 y_1 + w_2 y_2 + b = \\ &= w_1 f(w_1 x_1 + w_2 x_2 + b) + \\ &+ w_2 f(w_1 x_1 + w_2 x_2 + b) + b = \\ &= w_1 w_1 x_1 + w_1 w_2 x_2 + w_1 b + \\ &+ w_2 w_1 x_1 + w_2 w_2 x_2 + w_2 b + b = \end{aligned}$$

Network of Neurons with Linear Activations



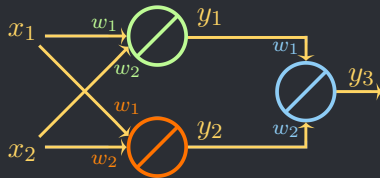
$$\begin{aligned}
 y_3 &= f(w_1 y_1 + w_2 y_2 + b) = \\
 &= w_1 y_1 + w_2 y_2 + b = \\
 &= w_1 f(w_1 x_1 + w_2 x_2 + b) + \\
 &+ w_2 f(w_1 x_1 + w_2 x_2 + b) + b = \\
 &= w_1 w_1 x_1 + w_1 w_2 x_2 + w_1 b + \\
 &+ w_2 w_1 x_1 + w_2 w_2 x_2 + w_2 b + b = \\
 &= x_1 [w_1 w_1 + w_2 w_1] + x_2 [w_1 w_2 + w_2 w_2] + \\
 &+ [w_1 b + w_2 b + b] =
 \end{aligned}$$

Network of Neurons with Linear Activations



$$\begin{aligned}
 y_3 &= f(w_1 y_1 + w_2 y_2 + b) = \\
 &= w_1 y_1 + w_2 y_2 + b = \\
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 &+ w_2 f(w_1 x_1 + w_2 x_2 + b) + b = \\
 &= w_1 w_1 x_1 + w_1 w_2 x_2 + w_1 b + \\
 &+ w_2 w_1 x_1 + w_2 w_2 x_2 + w_2 b + b = \\
 &= x_1 \underbrace{[w_1 w_1 + w_2 w_1]}_{\tilde{w}_1} + x_2 \underbrace{[w_1 w_2 + w_2 w_2]}_{\tilde{w}_2} + \\
 &\quad \underbrace{[w_1 b + w_2 b + b]}_{\tilde{b}} =
 \end{aligned}$$

Network of Neurons with Linear Activations



$$\begin{aligned}
 y_3 &= f(w_1 y_1 + w_2 y_2 + b) = \\
 &= w_1 y_1 + w_2 y_2 + b = \\
 &= w_1 f(w_1 x_1 + w_2 x_2 + b) + \\
 &+ w_2 f(w_1 x_1 + w_2 x_2 + b) + b = \\
 &= w_1 w_1 x_1 + w_1 w_2 x_2 + w_1 b + \\
 &+ w_2 w_1 x_1 + w_2 w_2 x_2 + w_2 b + b = \\
 &= x_1 \underbrace{[w_1 w_1 + w_2 w_1]}_{\tilde{w}_1} + x_2 \underbrace{[w_1 w_2 + w_2 w_2]}_{\tilde{w}_2} + \\
 &\quad \underbrace{[w_1 b + w_2 b + b]}_{\tilde{b}} = \\
 &= x_1 \tilde{w}_1 + x_2 \tilde{w}_2 + \tilde{b}
 \end{aligned}$$

Summary

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- Combining the fully-connected linear neurons is useless.