

# Optimization and Prediction in Natural Gas Networks Using Graph Neural Networks and MPCC-Based Models

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# Energy Context & Relevance of Natural Gas

In Colombia, natural gas is widely used across the residential, commercial, industrial, and thermal power sectors. Its importance becomes evident during dry seasons, when hydroelectric generation is reduced and thermal plants, fueled by natural gas, step in to maintain electricity supply [Promigas, 2021].



# Motivation & Problem Statement

- **Large-scale natural gas networks** require solving complex optimization problems [Sharifi et al., 2022].
- Increasing network size and operational constraints lead to:
  - Long execution times
  - Difficulty in real-time or near real-time decision making
- Classical approaches are accurate but often **computationally expensive**.

*Solving a network with 592 nodes and 460 edges takes 5 min on average, while a larger one with 660 nodes and more than 500 edges requires 30 min using the same MILP-based optimizer [Pfetsch et al., 2014].*

**Main Problem:** There is a need for methods that preserve accuracy while significantly reducing computation time.

# Problem Statement: Specific Challenges

From the general problem of natural gas transportation optimization, three specific challenges can be identified:

- ① **High computational cost:** Solving large-scale optimization problems requires long execution times, limiting the feasibility of real-time or near real-time operation.
- ② **Pipeline modeling complexity:** The Weymouth equation introduces nonlinearities, discontinuities, and nonconvexities that generate approximation errors and numerical instability.
- ③ **Uncertainty in demand:** Variability in hydropower generation and the growth of renewable sources create uncertainty in the demand for natural gas, requiring flexible and stochastic optimization approaches.

**Research Question:** How can an optimization tool be developed that seeks to improve the solution to the problem of natural gas transportation, taking into account computational cost, growing energy demand, and the uncertainty associated with this factor?

# Objectives

## General Objective:

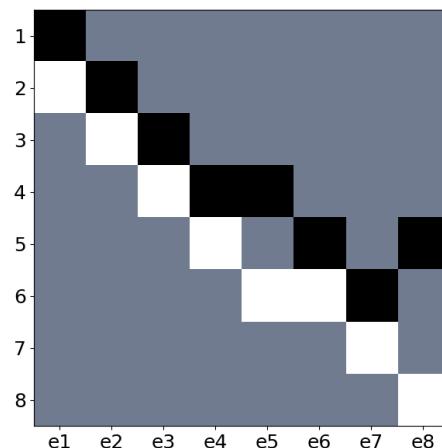
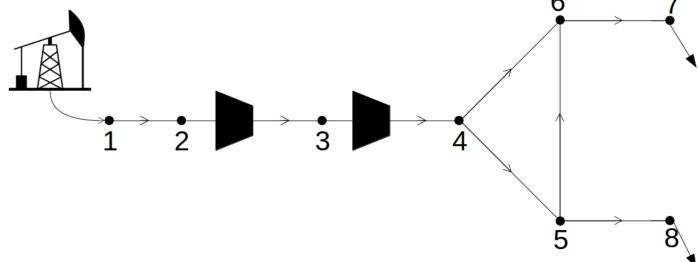
- Develop an optimization tool that integrates knowledge of the gas transportation network topology, a suitable approximation of the Weymouth equation and stochastic optimization techniques to address the gas transportation task taking into account the uncertainties related to hydroelectric generation and the growth of alternative energy sources.

## Specific Objectives:

- ① Design a Graph Neural Networks-based approach of regression that integrates knowledge of natural gas network topology to reduce computational time for operation estimation.
- ② Develop an optimization model for natural gas transportation systems that takes into account the Weymouth equation for reducing that reduces the approximation error in pipeline gas flow calculations.
- ③ Develop a stochastic gas flow dispatch optimization strategy that quantifies the uncertainty in the objective variables and decision variables associated with the operation of the gas system taking into account the constraints of the transportation problem.

# Objective 1 - Natural Gas System as a Graph

A natural gas network can be represented as a directed graph, where nodes correspond to elements such as wells, users, or storage facilities, and edges represent pipelines and compressors.



**Objective:** Achieve near-optimizer accuracy with significant runtime reduction.

# Optimization Problem: Objective and Constraints

The gas network consists of wells  $\mathcal{W}$  (supply nodes), pipelines  $\mathcal{P}$ , compressors  $\mathcal{C}$  and unsupplied demand  $\mathcal{U}$  (demand nodes)

$$\min_{\mathcal{P}, \mathcal{F}} \sum_{t \in \mathcal{T}} \left( \sum_{w \in \mathcal{W}} C_w f_w + \sum_{p \in \mathcal{P}} C_p f_p + \sum_{c \in \mathcal{C}} C_c f_c + \sum_{u \in \mathcal{U}} C_u f_u \right)$$

where  $C_i$  is the cost per unit flow at element  $i$ , and  $f_i$  is the corresponding gas flow.

$$\underline{f}_w \leq f_w \leq \overline{f}_w \quad \forall w \in \mathcal{W} \quad \text{Well capacity} \tag{1}$$

$$-\overline{f}_p \leq f_p \leq \overline{f}_p \quad \forall p \in \mathcal{P} \quad \text{Pipeline limits} \tag{2}$$

$$0 \leq f_u \leq \overline{f}_u \quad \forall u \in \mathcal{U} \quad \text{Demand satisfaction} \tag{3}$$

$$\sum_{m: (m, n) \in \mathcal{A}} f_m = \sum_{m': (n, m') \in \mathcal{A}} f_{m'} \quad \text{Nodal gas balance} \tag{4}$$

# CensNet Layers: Integrating Node & Edge Features

**Motivation:** Traditional GCNs focus mainly on *node features* and ignore *edge features*, missing part of the graph's information.

## CensNet innovation:

- Alternates between **node layers** and **edge layers**.
- Each layer type updates its own features while using information from the other:
  - **Node layer:** Node embeddings are updated using both node adjacency and transformed edge features.
  - **Edge layer:** Edge embeddings are updated using edge adjacency and transformed node features.
- Uses an *incidence matrix*  $\mathbf{T}$  to switch between node and edge domains.

## Benefits:

- Captures both *structural* (adjacency) and *relational* (edge) information.
- Enhances long-range dependencies through alternating propagation.

# Node layer propagation

$$\mathbf{H}_v^{(l+1)} = \sigma \left( (\mathbf{T} \Phi(\mathbf{H}_e^{(l)} \mathbf{p}_e) \mathbf{T}^\top \odot \tilde{\mathbf{A}}_v) (\mathbf{H}_v^{(l)} \mathbf{W}_v) \right)$$

## Definitions:

- $N_v$ : number of nodes.
- $D_v, D'_v$ : input and output node feature dimensions.
- $\mathbf{H}_v^{(l)} \in \mathbb{R}^{N_v \times D_v}$ : node features at layer  $l$ .
- $\mathbf{H}_e^{(l)} \in \mathbb{R}^{N_e \times D_e}$ : edge features at layer  $l$ .
- $\mathbf{p}_e \in \mathbb{R}^{D_e}$ : learnable projection vector for edges.
- $\mathbf{T} \in \{0, 1\}^{N_v \times N_e}$ : incidence matrix.
- $\tilde{\mathbf{A}}_v \in \mathbb{R}^{N_v \times N_v}$ : normalized node adjacency.
- $\mathbf{W}_v \in \mathbb{R}^{D_v \times D'_v}$ : learnable weight matrix.
- $\sigma(\cdot)$ : non-linear activation, applied element-wise.
- $\Phi(\cdot)$ : diagonalization operator.

# Edge layer propagation

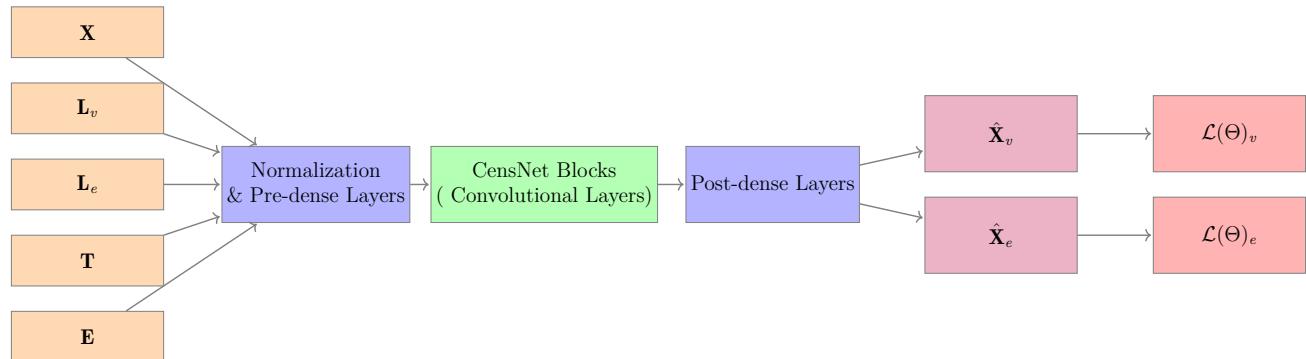
$$\mathbf{H}_e^{(l+1)} = \sigma \left( (\mathbf{T}^\top \Phi(\mathbf{H}_v^{(l)} \mathbf{p}_v) \mathbf{T} \odot \tilde{\mathbf{A}}_e) (\mathbf{H}_e^{(l)} \mathbf{W}_e) \right)$$

## Definitions:

- $N_e$ : number of edges.
- $D_e, D'_e$ : input and output edge feature dimensions.
- $\mathbf{H}_e^{(l)} \in \mathbb{R}^{N_e \times D_e}$ : edge features at layer  $l$ .
- $\mathbf{H}_v^{(l)} \in \mathbb{R}^{N_v \times D_v}$ : node features at layer  $l$ .
- $\mathbf{p}_v \in \mathbb{R}^{D_v}$ : learnable projection vector for nodes.
- $\tilde{\mathbf{A}}_e \in \mathbb{R}^{N_e \times N_e}$ : normalized edge adjacency (line graph).
- $\mathbf{W}_e \in \mathbb{R}^{D_e \times D'_e}$ : learnable weight matrix.
- $\sigma(\cdot)$ : non-linear activation, applied element-wise.

**Core idea:** Information “switches” between node and edge spaces at each step, enriching representations.

# CensNet-based Model Architecture



**Inputs:** Encapsulate both physical attributes and topological information of the network:

- Node features:  $\mathbf{X} \in \mathbb{R}^{N_v \times 3}$ , injection limits and demand.
- Node Laplacian:  $\mathbf{L}_v \in \mathbb{R}^{N_v \times N_v}$ , node connectivity.
- Edge Laplacian:  $\mathbf{L}_e \in \mathbb{R}^{N_e \times N_e}$ , edge connectivity.
- Edge features:  $\mathbf{E} \in \mathbb{R}^{N_e \times 5}$ ,  $K$ ,  $\beta$ , flow limits.

$$\mathcal{L}(\Theta) = \sum_{r=1}^R \| Y_r - \hat{Y}_r \|_2^2$$

# Dataset and Experimental Setup

Two different gas network configurations were used to evaluate the proposed CensNet-based model.

Case	Nodes	Pipes	Comps.	Users
8-node	8	6	2	2
Colombia	63	48	14	27

Different scenarios were generated by adding 5%–25% noise to user demands and solving with APOPT. These outputs were used as training targets for the CensNet model.

## Experimental Setup

- **Samples:** 2000 (8-node), 2400 (63-node)
- **Data split:** 60% train, 20% val, 20% test

- **Training:** 1500 epochs, Adam optimizer, Leaky ReLU ( $\alpha = 0.2$ )
- **Learning rate:**  $1 \times 10^{-2}$ , exponential decay (0.9 every 1000 steps)
- **Hyperparameters:**
  - $N_{\text{channels}}$ : 16–64
  - $N_{\text{layers}}$ : 1–5 (conv)
  - $N_{\text{dense}}$ : 2–32 (post-dense)
- **Losses tested:**
  - Nodal loss only
  - Nodal + edge loss

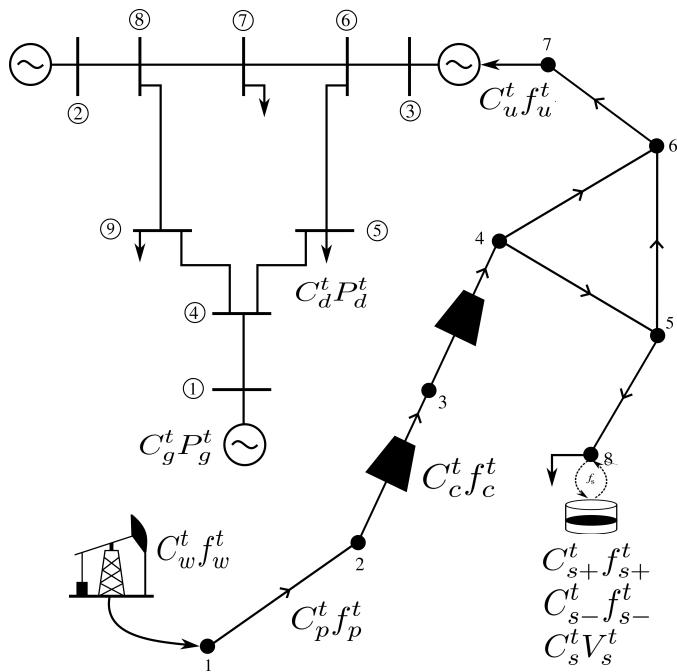
# Performance Comparison: 8-Node and 63-Node Networks

Model	Nodal $R^2$	Edge $R^2$	Balance Error	Time (s)	T-test
<b>8-node Network</b>					
CensNet (N)	0.99	-2.82	$5.7 \pm 16.86$	$0.65 \pm 0.4$	$T = 15$
CensNet (N+E)	0.99	0.99	$0.08 \pm 1.17$	$0.64 \pm 0.4$	$p = 1.3 \times 10^{-34}$
<b>63-node Network</b>					
CensNet (N)	0.99	-5.88	$0.04 \pm 60.83$	$4.9 \pm 5.6$	$T = 47$
CensNet (N+E)	0.99	0.99	$0.01 \pm 23.4$	$4.9 \pm 5.6$	$p = 4.9 \times 10^{-110}$

The CensNet-based model captures the network's topological information, enabling it to satisfy the gas balance constraint naturally to some extent. Time tests, performed on a single case, confirm that CensNet is significantly faster than APOPT; moreover, being a network-based model, it can efficiently propagate entire batches of scenarios forward.

# Interconnected system - Definition

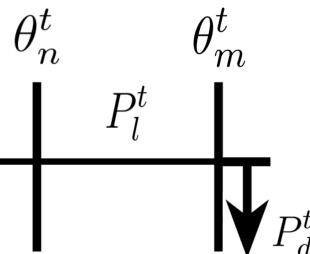
A power-gas interconnected system is a hybrid infrastructure that integrates natural gas and power networks, enhancing overall system efficiency [Duan et al., 2022].



$$\begin{aligned} \min_{\mathcal{P}, \mathcal{F}} \quad & \sum_{g \in \mathcal{G}} C_g^t P_g^t + \sum_{d \in \mathcal{D}} C_d^t P_d^t + \\ & \sum_{w \in \mathcal{W}} C_w^t f_w^t + \sum_{p \in \mathcal{P}} C_p^t f_p^t + \\ & \sum_{c \in \mathcal{C}} C_c^t f_c^t + \sum_{u \in \mathcal{U}} C_u^t f_u^t + \quad (5) \\ & \sum_{s \in \mathcal{S}} C_{s+}^t f_{s+}^t + \sum_{s \in \mathcal{S}} C_{s-}^t f_{s-}^t + \\ & \sum_{s \in \mathcal{S}} C_s^t V_s^t \end{aligned}$$

# Power system constraints

The constraints of the electrical system represent the technical limits of its components, as well as the physical laws that govern them. In this work, the DC model is adopted, which has been shown to be sufficient for similar applications [Yang et al., 2019].



$$\underline{P}_g^t \leq P_g^t \leq \overline{P}_g^t \quad \forall g \in \mathcal{G}, \quad (6a)$$

$$-\overline{P}_l^t \leq P_l^t \leq \overline{P}_l^t \quad \forall l \in \mathcal{L}, \quad (6b)$$

$$P_l^t = B_{nm}(\theta_n - \theta_m) \quad \forall l = (n, m) \in \mathcal{L}, \quad (6c)$$

$$0 \leq P_d^t \leq \overline{P}_d^t \quad \forall d \in \mathcal{D}, \quad (6d)$$

$$-\overline{\theta}_n^t \leq \theta_m^t \leq \overline{\theta}_n^t \quad \forall n \in \mathcal{N}_P, \quad (6e)$$

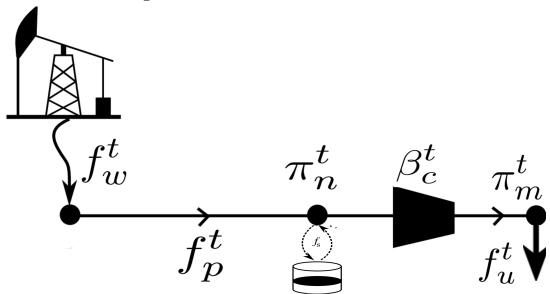
$$\sum_{\substack{l \in \mathcal{L}_{n+} \\ g=n}} P_l^t + P_g^t = \sum_{\substack{l \in \mathcal{L}_{n-} \\ d=n}} P_l^t + P_d^t \quad \forall n \in \mathcal{N}_P \quad (6f)$$

The second set of restrictions interconnects the natural gas and electric power systems through gas-fired power plants that generate electricity.

$$f_n^t = P_n^t \cdot \text{HR}_n, \quad \forall n \in \mathcal{N}_I, \quad (7)$$

# Gas system constraints

This set of constraints ensures that wells, pipelines, nodal pressures, compressors, unsupplied demand and storage facilities operate within proper operating limits. [García-Marín et al., 2022].



$$\pi_m^t \leq \beta_c^t \pi_n^t \quad \forall c = (n, m) \in \mathcal{C} \quad (8a)$$

$$0 \leq f_{s+}^t \leq V_{0s} - \underline{V}_s \quad \forall s \in \mathcal{S} \quad (8b)$$

$$0 \leq f_{s-}^t \leq \overline{V}_s - V_{0s} \quad \forall s \in \mathcal{S} \quad (8c)$$

$$V_s^t = V_s^{t-1} + f_{s-}^{t-1} - f_{s+}^{t-1} \quad \forall s \in \mathcal{S} \quad (8d)$$

$$\text{sgn}(f_p^t)(f_p^t)^2 = K_{nm}((\pi_n^t)^2 - (\pi_m^t)^2) \quad \forall p = (n, m) \in \mathcal{P} \quad (9)$$

Modeling the Weymouth equation as a nonconvex and discontinuous equality constraint presents significant challenges for optimization algorithms, leading to multiple local optima [Jiang et al., 2021].

# Proposed complementarity-based approach

Complementarity constraints can be used to represent non-uniform or discontinuous operators, such as absolute value, sign, and min/max [Baumrucker et al., 2008]. The sign equation can be represented by an optimization problem whose constraints avoid the use of binary variables.

$$\mathcal{O}_\epsilon : \min_{y_p^t} -y_p^t f_p^t \quad (10a)$$

$$\text{s.t. } y_p^t (f_p^t)^2 = K_{nm} \left( (\pi_n^t)^2 - (\pi_m^t)^2 \right) \quad (10b)$$

$$f_p^t = f_{p+}^t - f_{p-}^t \quad (10c)$$

$$-1 \leq y_p^t \leq 1 \quad (10d)$$

$$f_{p+}^t (y_p^t + 1) \leq \epsilon \quad (10e)$$

$$f_{p-}^t (1 - y_p^t) \leq \epsilon \quad (10f)$$

The regularized optimization model  $\mathcal{O}_\epsilon$  offers several key properties that justify its use in tackling challenging MPCC problems [Ralph and Wright, 2004].

# Databases

Each of the mentioned databases was selected based on its unique characteristics and the significant contributions it could make to the study.

	<b>Topology</b>	<b>Connection points</b>	<b>Closed loops</b>	<b>Problem</b>
<b>Case 1</b>	9-bus 8-system	1	1	Small system with one loop
<b>Case 2</b>	118-bus 48-system	9	7	Contains several interconnected loops
<b>Case 3</b>	96-bus 63-system	10	0	Fully radial but considers bidirectional flows.

Table 1: Databases used in the study

# Experimental setup

In order to establish a baseline for comparison, two alternative methodologies were employed:

- Taylor series approximation [Fodstad et al., 2015].
- Second Order Cone Programming (SOC) [Schwele et al., 2019].

The objective function value for each of the mentioned databases was computed by solving the optimization problem using:

- The IPOPT [Wächter and Biegler, 2005] solver.
- The GEKKO [Beal et al., 2018] package.

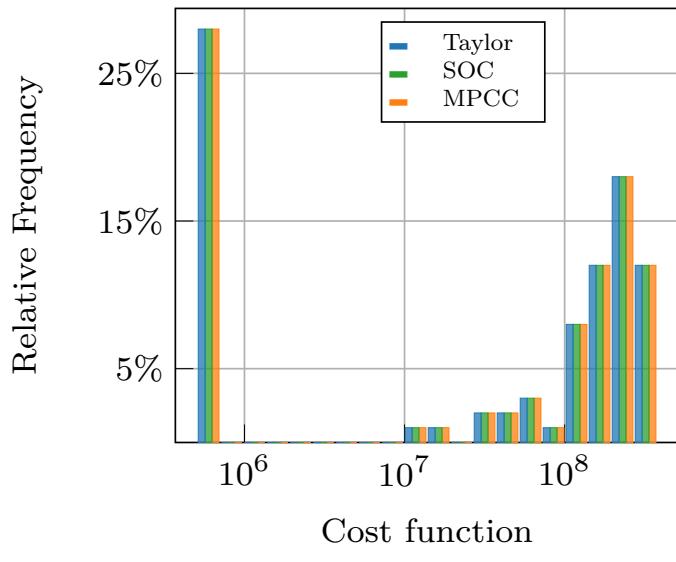
Since understanding and quantifying the inherent errors introduced by any constraint approximation approach supports its real-world pertinence, the considered validation trades off the reached cost function and constraint error values.

$$WE_p^t = |f_p^t - \left( K_{nm} |(\pi_n^t)^2 - (\pi_m^t)^2| \right)^{1/2}|, \quad \forall p = (n, m) \in \mathcal{P} \quad (11)$$

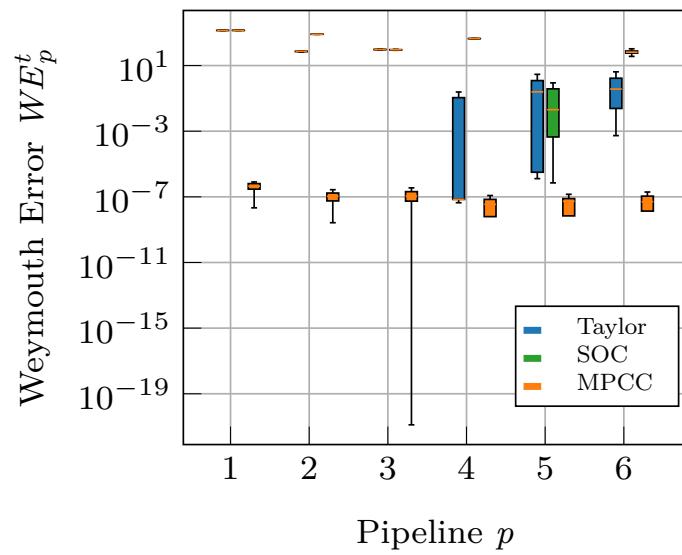
Hence,  $WE_p^t$  metric explains the inherent sensitivity of tested approaches and validates the significance of their differences.

# Results - Case 1

To evaluate the accuracy of different Weymouth equation approximations in the 8/9 system, we conducted a Monte Carlo experiment. The optimization problem was solved one hundred times for a single day, with natural gas demands sampled uniformly.

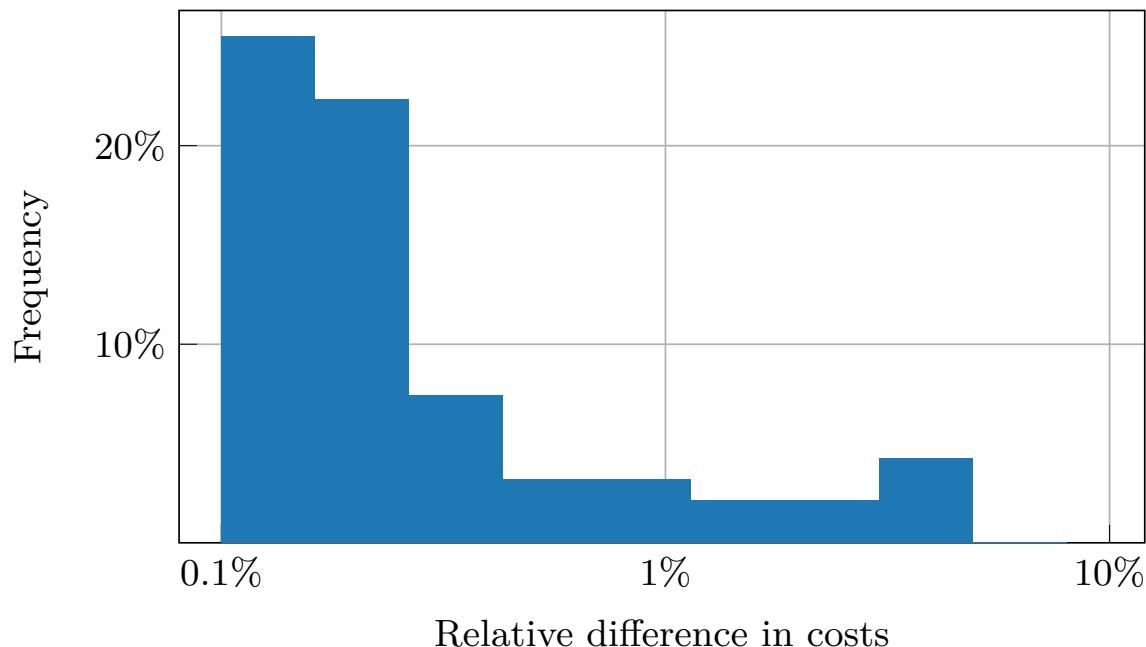


The analysis of the error distributions shows that the complementarity-based formulation (MPCC) produces considerably lower deviations in key arcs compared to the Taylor and SOC approaches.



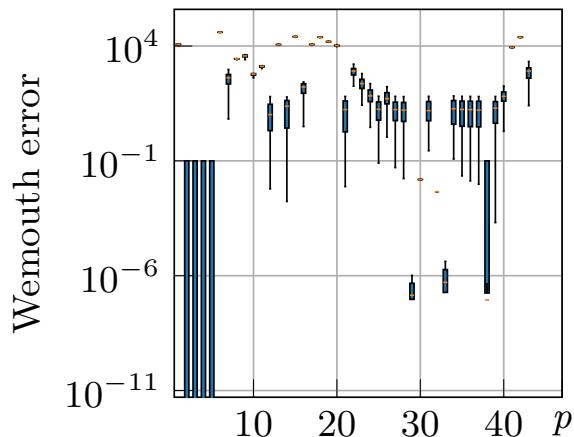
## Results – Case 2: Cost Comparison

For the 118/48 system, both baseline approaches produced identical objective function values. The relative cost difference, however, was always positive, indicating that the MPCC formulation systematically leads to higher cost outcomes compared with the Taylor and SOC approximations.

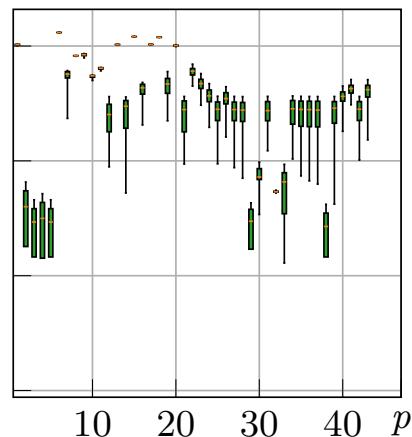


# Results – Case 2: Weymouth Error Analysis

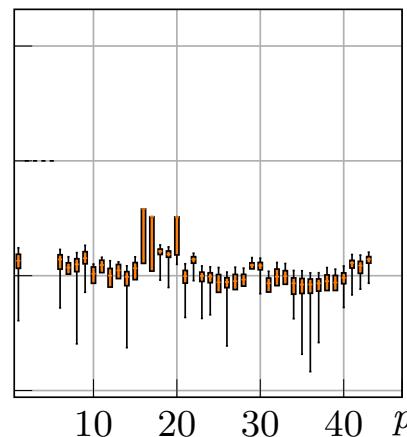
In contrast, the analysis of Weymouth approximation errors highlights the advantages of MPCC: the proposed formulation reduces deviations by nearly seven orders of magnitude, from around  $10^1$  to  $10^{-6}$ , compared with the Taylor and SOC approaches.



(a) Taylor



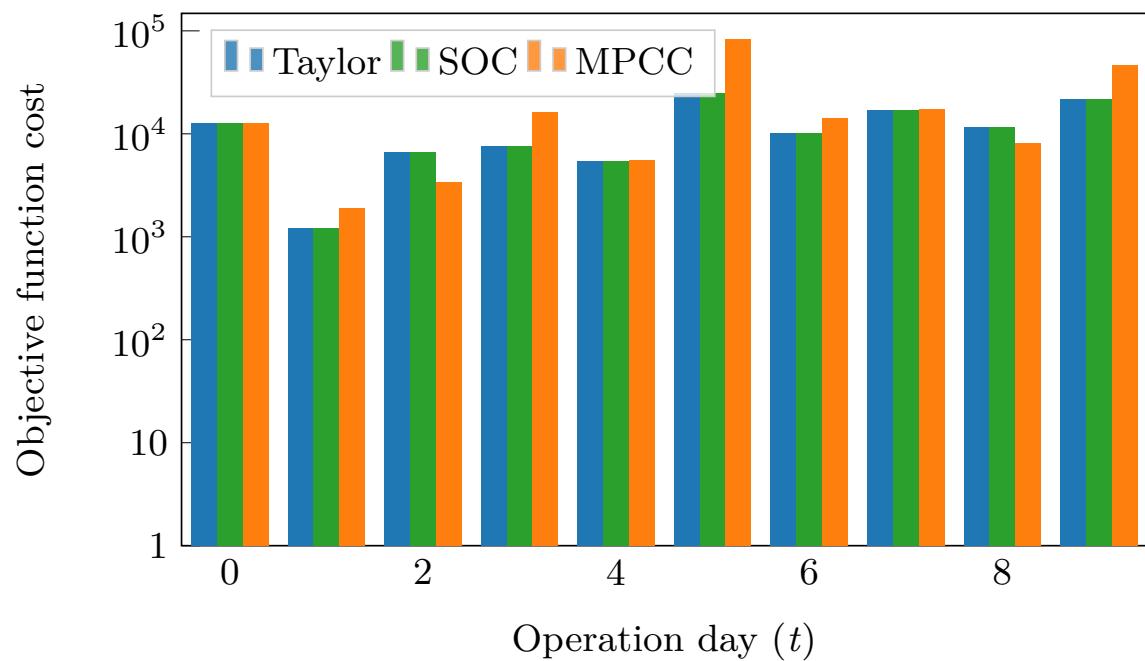
(b) SOC



(c) MPCC

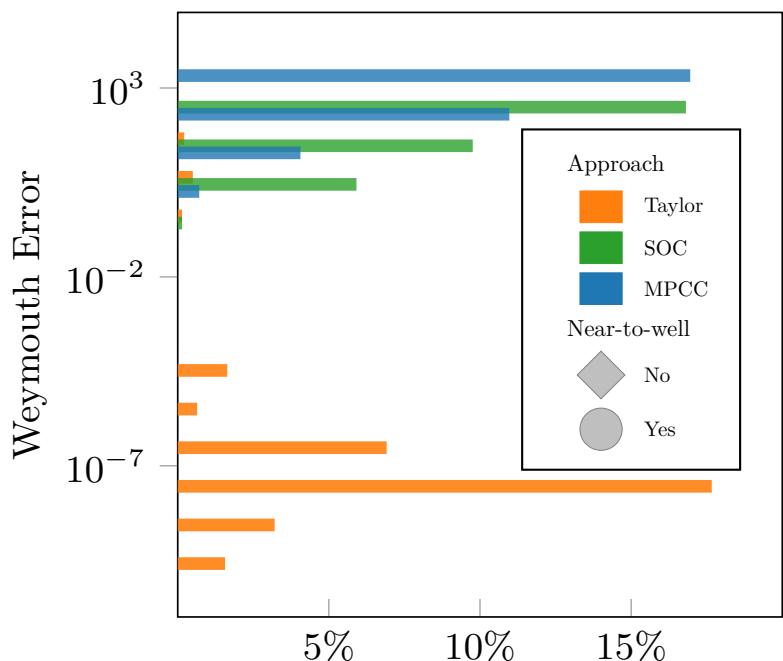
## Results - Case 3 (Part 1)

For the 96/63 system, the evaluation differs from the previous cases. The Weymouth approximations were tested in an operational setting covering ten consecutive days ( $|\mathcal{T}| = 10$ ), with gas extraction costs varying randomly each day.

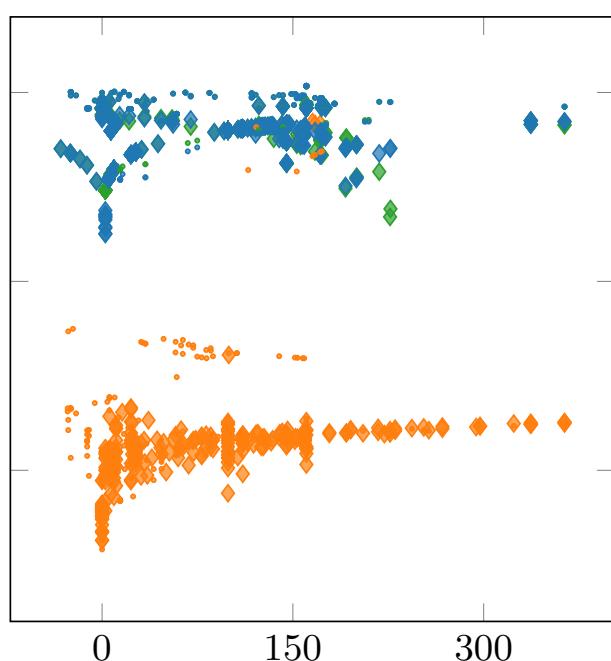


# Results - Case 3 (Part 2)

The analysis of Weymouth errors reveals their distribution across pipelines, as well as their relationship with gas flow magnitudes and the operation day. Results are compared across the Taylor, SOC, and MPCC formulations.



(a) Relative frequency



(b) Pipeline flow ( $f_m^t$ )

# Objective 3 - Uncertainty in Gas Network Operation

Gas transportation systems are exposed to multiple sources of uncertainty:

- **Demand variability:** Thermoelectric generation depends on hydropower availability, which fluctuates with rainfall and droughts.
- **Supply fluctuations:** Variations in extraction fields and import conditions impact gas availability.
- **Operational constraints:** Pressure limits, compressor capacities, and pipeline flow limits must always be satisfied.

**Challenge:** Stochastic conditions require repeated re-optimizations. Purely data-driven models may fail to capture the physics of the system, leading to unreliable predictions under uncertainty.

# Physics-Guided Neural Networks

Traditional neural networks fit patterns in data but ignore system physics, which increases the risk of producing unrealistic or infeasible predictions when faced with uncertain scenarios.

## Physics-Informed Neural Networks (PINNs):

- Integrate *physical laws* directly into the training process.
- Loss function penalizes violations of:
  - **Gas balance constraint** – nodal mass conservation.
  - **Weymouth equation** – pressure–flow relation in pipelines.
- Act as a *regularizer*, enhancing generalization under stochastic conditions.

# Methodology: Physics-Guided CensNet

**Overall loss function:**

$$\mathcal{J}(\Theta) = \mathcal{J}_{\text{data}} + \mathcal{J}_{\text{balance}} + \mathcal{J}_{\text{weymouth}}$$

**Gas balance loss:**

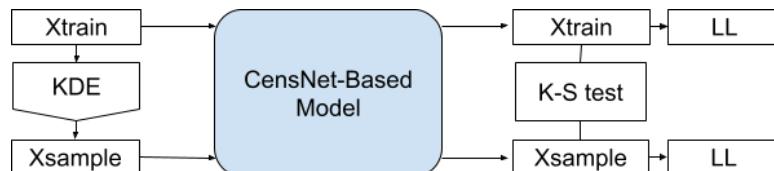
$$\mathcal{J}_{\text{balance}} = \mathbf{T} \cdot \hat{\mathbf{f}}_e - \mathbf{d} + \hat{\mathbf{f}}_n$$

**Weymouth loss:**

$$\mathcal{J}_{\text{weymouth}} = \mathbf{M}_{\mathcal{P}} \left( \hat{\mathbf{f}}_e^{\circ 2} - \mathbf{K} \circ (\mathbf{T} \cdot \hat{\boldsymbol{\pi}}^{\circ 2}) \right)$$

**Stochastic analysis:**

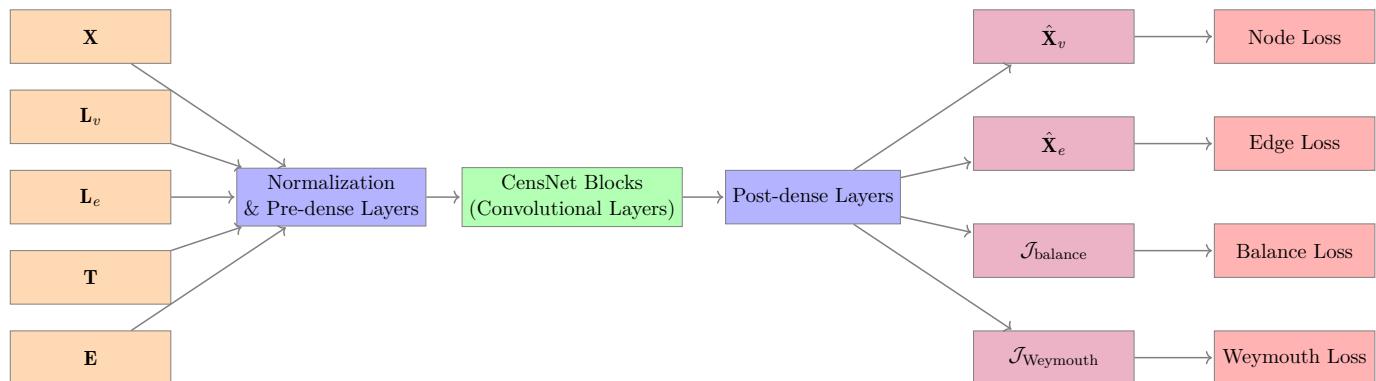
- KDE fitted to training inputs to generate synthetic samples.
- Samples propagated through CensNet to obtain outputs.
- Log-likelihoods and K-S tests used to assess distributional similarity.



# Methodology: Physics-Guided CensNet

## Implementation:

- Built on CensNet architecture from deterministic setup.
- Training data from nonlinear gas network optimization (MPCC).
- Noise (5%–25%) injected to emulate operating uncertainty.



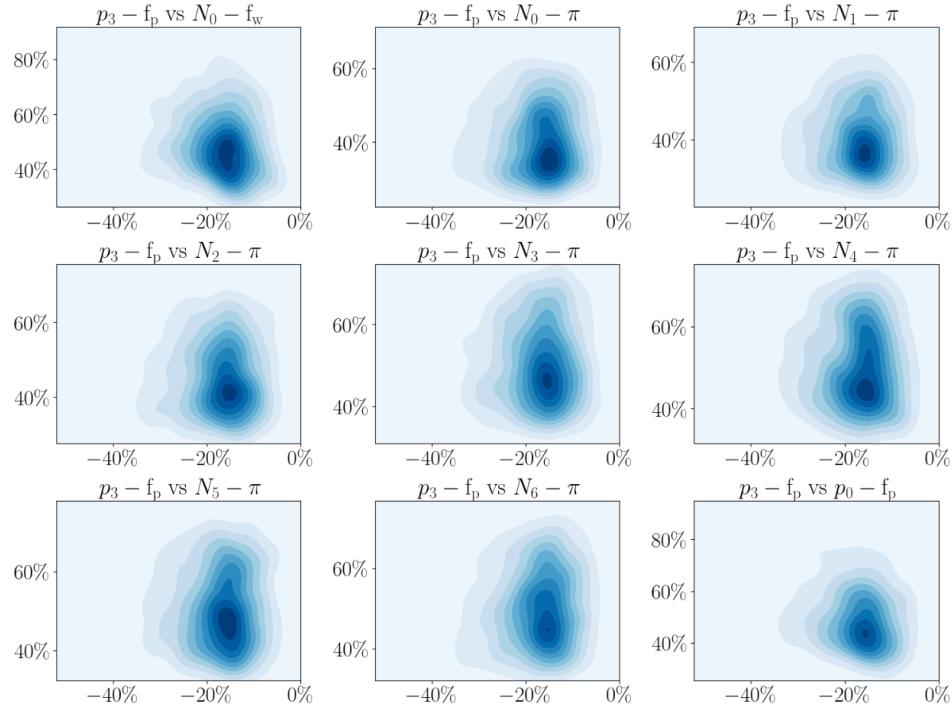
# Case Study I: 8-Node Network – Performance Comparison

Method	Node Error	Edge Error	Balance Error	Time (s)
CensNet (N)	$0.00 \pm 17.99$	$22.76 \pm 15.43$	$-0.01 \pm 17.45$	$0.86 \pm 0.50$
CensNet (N+E)	$-0.11 \pm 18.27$	$0.22 \pm 21.65$	$-0.12 \pm 1.70$	$0.85 \pm 0.50$
CensNet (N+E+B)	$-0.02 \pm 18.00$	$-0.02 \pm 21.30$	$-0.03 \pm 0.90$	$0.85 \pm 0.50$
CensNet (N+E+W)	$-0.07 \pm 17.56$	$2.60 \pm 20.03$	$-0.07 \pm 2.21$	$0.86 \pm 0.50$
CensNet (N+E+B+W)	$0.05 \pm 17.91$	$0.25 \pm 21.14$	$0.05 \pm 1.69$	$0.85 \pm 0.50$

- Using only node loss leads to accurate nodal flows but large errors in edge predictions.
- Adding edge loss (**N+E**) reduces edge error from 22.76 to 0.22 and improves global balance.
- Including balance loss (**N+E+B**) further minimizes balance error, achieving best overall consistency.
- Weymouth loss (**N+E+W**) introduces slight trade-offs, slightly increasing edge/balance errors but retaining nodal accuracy.
- All CensNet variants are over an order of magnitude faster than IPOPT, with **N+E+B** offering the best accuracy-speed compromise.

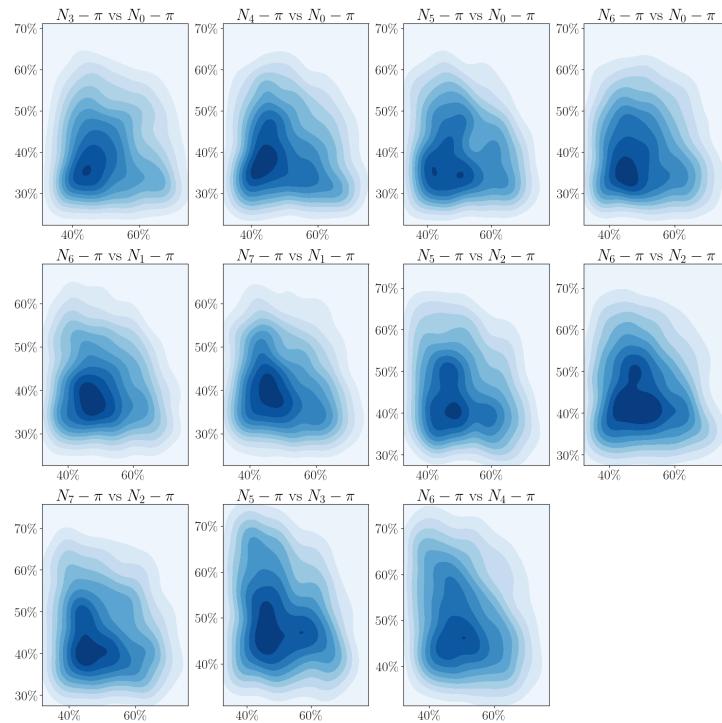
# Negative Flows in Pipeline $p_3$

- Flow in  $p_3$  consistently negative across scenarios and KDE samples.
- Caused by orientation assumption → optimizer adjusts direction.
- Operability unaffected, but  $p_3$  supports cost minimization.



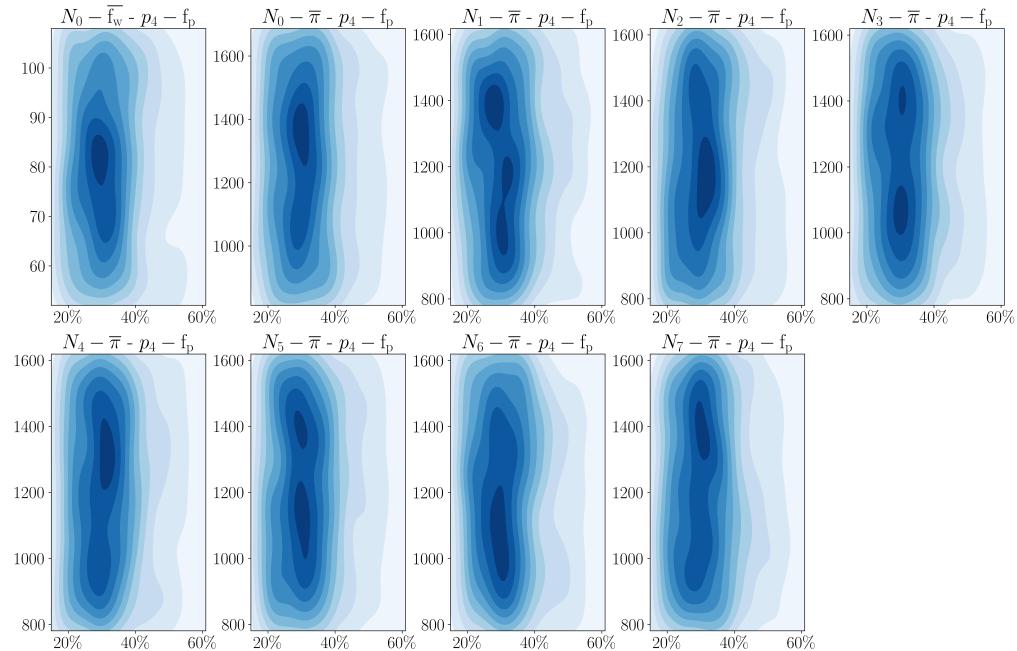
# Pressure Concentration in Mid-Range

- Nodal pressures cluster at 30–40% of normalized limits.
- Avoids extremes, ensuring reliable operation.
- Implies optimizer favors efficiency + stability.



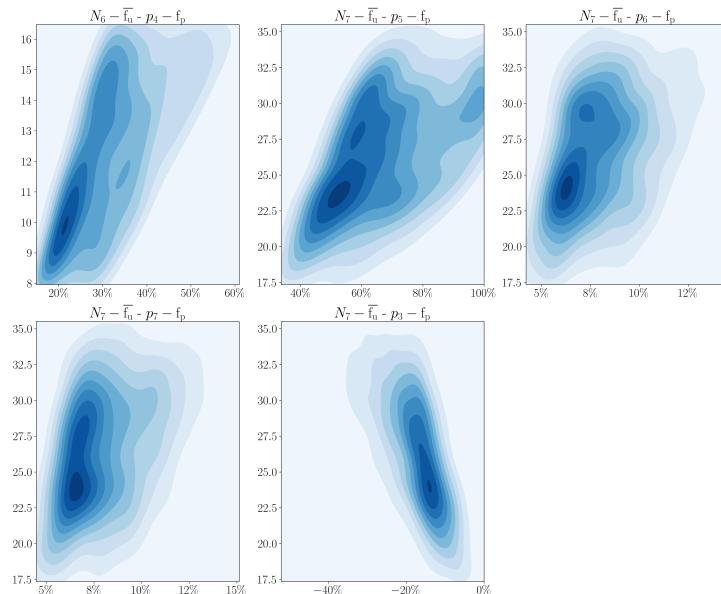
# Stable Utilization of Pipeline $p_4$

- Despite wide dispersion in inputs,  $p_4$  flow remains 20–40%.
- Reflects robustness: optimizer stabilizes this delivery path.
- Behavior consistently learned and reproduced in KDE samples.



# Linear Input–Output Relations

- Positive correlation: demand bounds ( $N_6, N_7$ ) vs.  $p_4-p_7$  flows.
- Negative correlation:  $N_7$  demand bound vs.  $p_3$  flow.
- Shows network internalized optimizer's allocation strategies.



## Results:

- Log-likelihoods:  $y_{\text{sample}} = -6.696 \times 10^6$ ,  $\bar{y}_{\text{train}} = -6.657 \times 10^6$ .
- K–S test:  $p > 0.44$  across all alternatives.
- ⇒ Synthetic outputs statistically consistent with training outputs.

# Conclusions

- **GNN-based prediction:** CensNet achieved accurate and scalable predictions for natural gas networks, with significant reductions in computation time compared to traditional optimization.
- **Optimization with MPCC:** Proposes a formulation to approximate Weymouth's restrictions, improving accuracy of pressure–flow representation and showing robustness in real-world scheduling tasks.
- **Stochastic modeling:** Physics-guided neural networks preserved structural dependencies and generalized reliably under uncertainty, validated through KDE sampling and K–S testing.
- **Overall:** Integrating physical constraints with GNNs yields efficient, accurate, and robust surrogates for optimization in natural gas systems.

# Future Work

- Extend GNN-based models to capture **transient dynamics** and **operational uncertainty**.
- Develop **stochastic MPCC formulations** to handle variability from renewables and uncertain demand.
- Integrate with **hydrological series prediction tools** to anticipate gas consumption linked to hydro–thermal coordination.

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