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Abstract

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Chapter 1

Introduction

1.1 Problem statement

Natural gas transportation is an integral part of the natural gas industry, relying on a pipeline network to transfer natural gas from various sources to consumers, fulfilling their demand. This network is divided into two main types: transmission and distribution. Transmission networks transport large volumes of gas at high pressure over long distances from gas sources to distribution centers. On the other hand, distribution networks deliver gas to individual consumers [2]. In general, natural gas transmission systems are composed of four fundamental elements: injection fields, responsible for injecting the hydrocarbon from extraction fields or regasification plants into the system; pipelines, which transport the gas from a sending node to a receiving node; compressors, which are responsible for raising the pressure at the outlet node relative to the inlet node; and end user [3]. Ensuring gas flow to meet end-user demand, minimizing network operating costs, and maintaining system elements within appropriate technical operating limits are critical factors in natural gas transportation. Coordinating these factors requires formulating optimization problems, which must be solved efficiently, taking into account the numerous variables and the nature of these variables [4].

The optimization problem is determining the best operational configurations to meet consumer demand while ensuring the technical and physical constraints of the natural gas transportation system. It must also be considered that these transportation systems are usually interconnected with the electricity systems since the latter usually require natural gas as fuel for the thermal power plants. These power plants are significant natural gas consumers, relying on a steady supply to generate electricity [5].

The Colombian case is no exception; although the Colombian energy matrix comprises 70% hydroelectric plants, the remainder consists primarily of thermoelectric plants [6]. These thermoelectric plants are crucial for complementing the hydro plants to meet energy demand, especially during periods of drought. They become essential during events like the El Niño phenomenon when reduced water availability limits hydroelectric generation [7]. As other studies have shown, variations in rainfall, droughts, or floods in countries with high hydroelectric power plants can significantly affect water availability for hydropower production [8].

The above situation necessitates solving the optimization problem multiple times to ensure the system's correct operation across various scenarios. Consequently, this process takes considerable time and is both resource-intensive and time-consuming. Despite the high computational cost of each model execution, the resulting solutions are not utilized in subsequent optimization processes, even in similar operational scenarios. Therefore, there is a pressing need to develop a machine learning strategy that leverages historical solutions to provide faster responses to different operational scenarios by learning from past optimization outcomes.

Although production fields, compressors, and end users of natural gas are well-represented, modeling transmission pipelines remains complex due to the nonlinear relationship between flow and pressures at its ending nodes. This complexity arises from the Weymouth equation, which includes a nonconvex and discontinuous sign function that determines flow direction based on differential pressure. These nonconvexities introduce discontinuities lead to numerical issues and optimization instability [9, 10]. Various authors have approached the challenge posed by the Weymouth equation differently. One of the most widely accepted methods involves approximating this equation due to its inherent complexity and nonconvex nature. However, since it is an approximation, this solution introduces errors that impact the accuracy of optimization outcomes. Mitigating these errors remains critical for further research and development in natural gas transportation systems [3].

1.2 Justification

Natural gas is an energy source that has gained great relevance worldwide, and this can be attributed to two fundamental causes. Firstly, it has been observed that a country's economic growth is closely related to its energy consumption [11]. Therefore, as nations develop and grow economically, it is expected that they will seek energy security to meet their own demand and continue their progress without interruptions. The second major motivation for the use of natural gas is its lower greenhouse gas emissions compared to other fuels, making it a favorable option, especially in a context where there is a growing interest in environmental care. Natural gas emits fewer greenhouse gases compared to other fossil fuels, making it a favorable option for climate change mitigation [12]. In this context, the natural gas system plays a crucial role in providing clean and versatile energy [13]. It is an efficient and less polluting energy source compared to conventional fossil fuels, such as coal and oil [14]. Its use is essential for electricity generation, residential and industrial heating, and also for supplying energy-intensive industrial sectors.

According to figures from the U.S. Energy Information Administration (EIA), global natural gas consumption in 2015 reached 124.24 trillion cubic feet, with a projected increase of 43% by 2040, where 75% is associated with the industrial sector and electricity generation based on thermal power plants. This pronounced increase in consumption contrasts with the total volume of proven reserves worldwide, which reached 154.53 trillion cubic feet in 2015 and projects a growth of about 52% by 2014 [15]. Colombia does not have interests different from those mentioned above, especially when considering the potential environmental impacts [16]. Therefore, it is necessary for the country not only to have a national gas transportation system but also to ensure that it is operated in the best possible way, so that natural resources are maximized. In the Colombian context, natural gas is a very important energy source as it is used in various sectors such as residential, commercial, industrial, and thermal [17]. It is especially in the latter sector that this fuel becomes more relevant during dry seasons, as it is when reservoir levels drop and thus hydroelectric power generation decreases. This problem is exacerbated in years when the El Niño phenomenon occurs [18], making it of great interest to have tools that allow for the optimal injection and transportation of natural gas to fully meet demand.

Although most of the country's electricity demand is commonly met by hydroelectric plants [19], this type of generation presents a significant source of uncertainty in the energy system since its effectiveness and generation capacity are directly linked to the country's climatic and meteorological conditions, especially in extreme cases such as the El Niño phenomenon [20]. Variations in precipitation, droughts, or floods can have a significant impact on the availability of water for hydroelectric power production, affecting the balance between supply and demand in the electrical system [21]. Additionally, the increase in energy demand and the transition to renewable energy sources pose significant challenges in the efficient and reliable transportation of gas. Optimizing the natural gas transportation system, considering the uncertainty associated with renewable energy generation and demand variability, is essential to ensure a reliable, sustainable, and environmentally friendly energy supply [22].

Chapter 2

Power System - Censnet

2.1 Formulation of Power System

Chapter 3

Optimization Using Mathematical Programs with Complementarity Constraints

3.1 Formulation of Interconnected Power and Gas Systems

An interconnected system can be effectively represented by a directed graph denoted as $\{\mathcal{N}, \mathcal{E}\}$, where the sets of units $\mathcal{N}$ and edges $\mathcal{E}$ consider all power and gas components along with their interconnections. On the electrical power side, the system holds power units $\mathcal{N}_P \subset \mathcal{N}$, termed buses, and power edges $\mathcal{B} \subset \mathcal{E}$ or branches. The power buses comprise generators $\mathcal{G} \subset \mathcal{N}_P$ injecting power and users $\mathcal{D} \subset \mathcal{N}_P$ demanding power [23]. The branches $\mathcal{B} = \{b = (n, m) \mid n, m \in \mathcal{N}_P\}$ connect the buses to make the electrical power flow from the generators to the users. Although the physical power flow is alternating current, the system is accurately modeled using a linear direct current (DC) approximation. The DC model ignores reactive power flows and voltage magnitude fluctuations and approximates active power flows using linear transfer distribution factors [24]. Further, the linear characteristics allow stating linear programming problems. Thus, the DC model serves as an appropriate approximation for many power system operations and planning studies, providing a balance of accuracy and computational tractability [25]. On the natural gas side, the system denotes the units as gas nodes $\mathcal{N}_f \in \mathcal{N}$, including gas supply nodes or wells $\mathcal{W} \subset \mathcal{N}_f$, gas demand nodes or users $\mathcal{U} \subset \mathcal{N}_f$, and gas storage facilities $\mathcal{S} \subset \mathcal{N}_f$. Similarly, the set of directed gas adjacency edges $\mathcal{A} = \{(n,m) \mid n,m \in \mathcal{N}_f\} \subset \mathcal{E}$ delineates the network structure through two kinds of transmission elements: transport pipelines $\mathcal{P} = \{p = (n,m) \mid n,m \in \mathcal{N}_f\}$ and compressing stations $\mathcal{C} = \{c = (n,m) \mid n,m \in \mathcal{N}_f\}$, so that $\mathcal{P} \cup \mathcal{C} = \mathcal{A}$ and $\mathcal{P} \cap \mathcal{C} = \emptyset$.

Then, the optimization problem of the interconnected system seeks to minimize the operation costs for satisfying the demands of the interconnected system while encompassing the power and gas constraints. Specifically, the following cost function linearly combines the flows of power and gas through the operation costs of the interconnected system elements:

$$\begin{aligned} \min_{\mathcal{P},\mathcal{F}} \quad & \sum_{g\in\mathcal{G}} C_g^t P_g^t + \sum_{d\in\mathcal{D}} C_d^t P_d^t + \sum_{w\in\mathcal{W}} C_w^t f_w^t + \\ & \sum_{p\in\mathcal{P}} C_p^t f_p^t + \sum_{c\in\mathcal{C}} C_c^t f_c^t + \sum_{u\in\mathcal{U}} C_u^t f_u^t + \\ & \sum_{s\in\mathcal{S}} C_{s+}^t f_{s+}^t + \sum_{s\in\mathcal{S}} C_{s-}^t f_{s-}^t + \sum_{s\in\mathcal{S}} C_s^t V_s^t \end{aligned} \tag{3.1}$$

where C_g^t denotes the generation cost by the g-th bus and C_d^t the unsupplied power demand for the d-th user. For the natural gas system, Equation (3.1) integrates the costs for injecting gas into the system using the well w, for transporting gas through the pipeline p, and for the pressure boosting of compressor c, at the time instant t, namely, C_w^t, C_p^t, and C_c^t, respectively. C_u^t denotes the penalty cost for not supplying the demanded gas to the user u. Lastly, C_{s+}^t, C_{s-}^t, and C_s^t represent the costs of injecting, extracting, and storing gas at the s-th storage station. Therefore, the decision variables for the optimization problem are P_g^t for the generated power, P_d^t for the unsupplied power, f_w^t for the inject gas flow, f_p^t and f_c^t for the transported gas through pipeline p and compressor c, f_u^t for the unsupplied gas demand, f_{s+}^t, f_{s-}^t, and f_s^t for injecting, extracting, and storing gas. Traditionally, a transported gas with a positive value of $f_p^t > 0$ moves in the predefined direction, while a negative value flows in the opposite one, with no impact on the optimization process. On the other hand, compressor stations solely allow unidirectional gas flow, expressed as $f_c^t \geq 0$. By optimizing this integrated cost function while adhering to the system's operational constraints, the proposed methodology effectively balances the demands of both energy systems, leading to a comprehensive solution that minimizes costs while ensuring reliable and efficient operation.

Optimization of the integrated cost function in Equation (3.1) while adhering to the system's operational constraints must lead to a comprehensive solution balancing the demands of both energy systems while ensuring reliable and efficient operation. Three sets of operational constraints describe the within and between power and gas interplay.

The first constraint set guarantees a stable power system operation: Equation (3.2) ensures that the generated power P_g^t lies between the technical minimum $\underline{P_g^t}$ and maximum $\overline{P_g^t}$. Equation (3.3) bounds the power flow through the transmission line P_l^t, preventing damages, such as overheating. Equation (3.4) models the power flow over the electrical network through the reactance-based relationship of the power flow P_l^t, the line susceptance B_{nm}, and the voltage angles θ_n, θ_m at buses n, m. Equation (3.5) limits the unsupplied power P_d^t to the user demand $\overline{P_d^t}$. Equation (3.6) ensures stable operating conditions within the interconnected power grid by restricting the bus voltage angles. Equation (3.7) defines the power balance at each bus, i.e., the total input and generated power must equal the total output and unsupplied power, being $\mathcal{L}_{n+} = \{(m, n') \in \mathcal{L} : n' = n\}$ and $\mathcal{L}_{n-} = \{(n', m) \in \mathcal{L} : n' = n\}$ the set of inflow and outflow transmission lines at the n-th bus, respectively.

$$
\begin{aligned}
\underline{P_g^t} \leq P_g^t \leq \overline{P_g^t} \quad & \forall\, g \in \mathcal{G}, && (3.2)\\
-\overline{P_l^t} \leq P_l^t \leq \overline{P_l^t} \quad & \forall\, l \in \mathcal{L}, && (3.3)\\
P_l^t = B_{nm}(\theta_n - \theta_m) \quad & \forall\, l = (n, m) \in \mathcal{L}, && (3.4)\\
0 \leq P_d^t \leq \overline{P_d^t} \quad & \forall\, d \in \mathcal{D}, && (3.5)\\
-\overline{\theta_n^t} \leq \theta_n^t \leq \overline{\theta_n^t} \quad & \forall\, n \in \mathcal{N}_P, && (3.6)\\
\sum_{\substack{l \in \mathcal{L}_{n+} \\ g = n}} P_l^t + P_g^t = \sum_{\substack{l \in \mathcal{L}_{n-} \\ d = n}} P_l^t + P_d^t \quad & \forall\, n \in \mathcal{N}_P && (3.7)
\end{aligned}
$$

The second constraint set interconnects natural gas and electrical power systems through gas-fired power plants generating electricity, as expressed by Equation (3.8) , where f_n^t stands for the natural gas fuel consumption to generate a power P_n^t at generator bus $n \in \mathcal{N}_I$, the heat-rate HR_n defines the generator efficiency, and the set $\mathcal{N}_I = \mathcal{G} \cap \mathcal{U}$ holds all the units in the interconnected system belonging to both the power generator and gas demand sets.

$$
f_n^t = P_n^t \cdot \mathrm{HR}_n, \quad \forall\, n \in \mathcal{N}_I, \tag{3.8}
$$

The third constraint set models the gas transportation system: Equation (3.9) forces each production well to inject the flow f_w^t over the technical minimum $\underline{f_w^t}$ and under the maximum capacity $\overline{f_w^t}$. Equation (3.10) upper-bounds the gas flow through pipelines f_p^t to the structural capacity $\overline{f_p^t}$. Equation (3.11) fixes safe operating limits for the pressure on the n-th node π_n^t as $[\underline{\pi_n^t}, \overline{\pi_n^t}]$. The constraint in Equation (3.12) asserts that the compression ratio π_m^t/π_n^t cannot physically exceed the compressor's design limitation $\beta_c \geq 1 \; \forall c = (n,m) \in \mathcal{C}$, enabling the representation of different compressors by adjusting the values of $\beta_c \geq 1$. Equation (3.13) ensures that the unsupplied demand f_u^t is lower than the corresponding user demand $\overline{f_u^t}$. The nodal gas balance in Equation (3.14) guarantees that the gas entering the node n equals the gas leaving it. Equations (3.15) and (3.16) limit the gas injection f_{s+} and extraction f_{s-} rates at storage facilities according to the feasible operating range determined by the currently stored volume V_s^t, respectively. In turn, Equation (3.17) balances the gas storage unit such that gas volume at operation period t V_s^t equals the volume from period V_s^{t-1} plus the difference between injected f_{s+}^{t-1} and extracted f_{s+}^{t-1} gas flow, a fundamental constraint for modeling the dynamics of gas storage over time. Lastly, Equation (3.18), known as the Weymouth equation, summarizes the physical behavior of gas flow through pipelines by relating the gas flow through the pipeline f_p^t to the pressures at the ends of the pipeline $\pi_n^t, \pi_m^t \; \forall \; p = (n,m) \in \mathcal{P}$. The Weymouth equation defines a nonlinear, nonconvex, disjunctive flow-pressure relationship that hampers the optimization of the gas transport system.

$$\underline{f_w^t} \leq f_w^t \leq \overline{f_w^t} \quad \forall \; w \in \mathcal{W} \tag{3.9}$$

$$-\overline{f_p^t} \leq f_p^t \leq \overline{f_p^t} \quad \forall \; p \in \mathcal{P} \tag{3.10}$$

$$\underline{\pi_n^t} \leq \pi_n^t \leq \overline{\pi_n^t} \quad \forall \; n \in \mathcal{N}_f \tag{3.11}$$

$$\pi_m^t \leq \beta_c^t \pi_n^t \quad \forall c = (n,m) \in \mathcal{C} \tag{3.12}$$

$$0 \leq f_u^t \leq \overline{f_u^t} \quad \forall \; u \in \mathcal{U} \tag{3.13}$$

$$\sum_{m:(m,n)\in\mathcal{A}} f_m^t = \sum_{m':(n,m')\in\mathcal{A}} f_{m'}^t \quad \forall \; n \in \mathcal{N}_f \tag{3.14}$$

$$0 \leq f_{s+}^t \leq V_{0s} - \underline{V_s} \quad \forall \; s \in \mathcal{S} \tag{3.15}$$

$$0 \leq f_{s-}^t \leq \overline{V_s} - V_{0s} \quad \forall \; s \in \mathcal{S} \tag{3.16}$$

$$V_s^t = V_s^{t-1} + f_{s-}^{t-1} - f_{s+}^{t-1} \quad \forall \; s \in \mathcal{S} \tag{3.17}$$

$$sgn(f_p^t)(f_p^t)^2 = K_{nm}((\pi_n^t)^2 - (\pi_m^t)^2) \quad \forall \; p = (n,m) \in \mathcal{P} \tag{3.18}$$

3.2 Mathematical Programming with Complementarity Constraints for Weymouth Approximation

The Weymouth equation is the fundamental model for gas flow through pipelines. However, it presents a challenge for optimal interconnected operation due to its nonlinearity, which arises from the signum function determining the gas flow direction. This nonlinearity results from the complex physics of gas flow, making it challenging to find optimal solutions for gas transportation systems [26]. Traditional optimization approaches struggle to handle the non-convex terms within the Weymouth equation. However, recent advances in optimization techniques, particularly mathematical programs with complementary constraints (MPCC), offer a promising solution to address this issue. MPCC specializes in handling complementarity constraints and non-convexities, making it well-suited to tackle the intricacies of the Weymouth equation [27]. This type of formulation involves optimization problems of the general form:

$$\mathcal{O}: \min \; f(x,y) \tag{3.19a}$$
$$\text{s.t. } h_i(x,y) = 0 \tag{3.19b}$$
$$g_j(x,y) \geq 0 \tag{3.19c}$$
$$0 \leq G_k(x) \perp H_k(y) \geq 0 \tag{3.19d}$$

where $f(x,y)$ is the cost function, $h(x,y)$ and $g(x,y)$ capture equality and inequality constraints in the optimization problem $\mathcal{O}$. Equation (3.19d) represents the complementarity conditions, with the operator $\perp$ indicating that at a solution, either x or y must be zero while the other must remain non-negative. These conditions turn MPCC into a modeling tool for scenarios with variables exhibiting complementarity relationships, such as economic equilibrium [28], variational inequalities [29], and the intricate dynamics of natural gas transportation systems [30]. To deal with the non-convexity, this work rewrites the Weymouth equation as the following mathematical program with two complementarity constraints:

$$
\begin{aligned}
\mathcal{O}_W : \ & \min_{y_p^t} -y_p^t f_p^t \\
& \text{s.t. } y_p^t (f_p^t)^2 = K_{nm}((\pi_n^t)^2 - (\pi_m^t)^2) \\
& -1 \leq y_p^t \leq 1 \\
& f_p^t = f_{p+}^t - f_{p-}^t \\
& 0 \leq f_{p+}^t \perp (y_p^t + 1) \geq 0 \\
& 0 \leq f_{p-}^t \perp (1 - y_p^t) \geq 0
\end{aligned}
\tag{3.20}
$$

where $f_{p+}^t \geq 0$ and $f_{p-}^t \geq 0$ hold the positive and negative components of the gas flow in the p-th pipeline at operation period t, for assessing directional flow.

Solving MPCC presents a unique set of challenges distinguishing it from traditional optimization problems. One notable challenge is the need for regularity properties, making MPCC more complex [31]. Compared to smooth optimization problems, where gradients and Hessians provide valuable information for optimization algorithms, MPCC often lacks these properties, leading to difficulties in devising efficient numerical methods.

3.2.1 Linear Independence Constraint Qualification (LICQ)

The Linear Independence Constraint Qualification (LICQ) is a critical condition in optimization, particularly in nonlinear programming problems (NLPs). LICQ ensures the existence and uniqueness of Lagrange multipliers, simplifying their interpretation and enhancing the clarity of their role in constrained optimization. Additionally, LICQ provides a robust framework for local analysis, guaranteeing that the KKT conditions are sufficient for optimality when satisfied at a specific point. [32].

LICQ is a constraint qualification used in optimization problems to ensure that the gradients of the active inequality constraints and the gradients of the equality constraints are linearly independent at the minimizing point x^* of the original constrained optimization problem $\mathcal{P}$, understanding the set of active constraints as,

$$
I(x^*) := \{1 \leq l \leq p \,|\, g_l(x^*) = 0\}, \tag{3.21}
$$

i.e., the inequality constraints at the point x^* that lie on its boundary. The above indicates that this constraint qualification is fulfilled when the elements of the set $\mathcal{F}$ are linearly independent at the point x^*.

$$\mathcal{F} = \{(\nabla h_1(x^*)), \ldots, (\nabla h_m(x^*)), (\nabla g_n(x^*), \forall n \in I(x^*))\} \tag{3.22}$$

3.2.2 Mangasarian-Fromovitz Constraint Qualification (MFCQ)

When an optimization problem does not meet the LICQrequirements, it is possible to resort to a second,less stringent criterion to check whether the KKT conditions are satisfied. This second criterion is known as Mangasarian-Fromovitz Constraint Qualification (MFCQ). The LICQ focuses on ensuring linear independence of the gradients of the active inequality and equality constraints [33]. On the other hand, the main objective of MFCQ is to guarantee that the gradients of the equality constraints are linearly independent at the optimal point $\mathbf{x}^*$, and furthermore that there exists a vector $\mathbf{d} \in \mathbb{R}^n$ such that

$$\nabla h_i(\mathbf{x}^*)^\top \mathbf{d} < 0 \tag{3.23}$$

for all equality constraints.

$$\nabla g_j(\mathbf{x}^*)^\top \mathbf{d} < 0 \tag{3.24}$$

and for all active inequality constraints.

It is widely recognized that conventional constraint qualifications in nonlinear programming, such as LICQ and MFCQ, are typically not satisfied in the case of MPCC. As a result, KKT conditions commonly associated with MPCC may not be applicable or valid at a local minimization point [34]. Therefore, posing relaxed nonlinear programs (RNLP) deals with the numerical resolution of MPCC by introducing a positive regularization parameter $\epsilon \in \mathbb{R}^+$ that simplifies the solution and properly handles the inequalities [35]. These programs typically satisfy constraint qualifications, making them more amenable to efficient optimization techniques. Relaxing MPCC ensures that inequalities are appropriately treated as inactive, particularly when $G_k(x)H_k(y) \leq \epsilon$, enhancing their structural integrity. Besides, relaxed programs reliably approximate the original problem as $\epsilon \to 0$ [36]. Hence, instead of working with the original problem

$\mathcal{O}_W$, the relaxed problem $\mathcal{O}_\epsilon$ is considered:

$$\mathcal{O}_\epsilon : \min_{y_p^t} -y_p^t f_p^t \tag{3.25a}$$

$$\text{s.t. } y_p^t (f_p^t)^2 = K_{nm}((\pi_n^t)^2 - (\pi_m^t)^2) \tag{3.25b}$$

$$f_p^t = f_{p+}^t - f_{p-}^t \tag{3.25c}$$

$$-1 \leq y_p^t \leq 1 \tag{3.25d}$$

$$f_{p+}^t (y_p^t + 1) \leq \epsilon \tag{3.25e}$$

$$f_{p-}^t (1 - y_p^t) \leq \epsilon \tag{3.25f}$$

Theoretically, the relaxed problem offers fundamental properties that tackle challenging MPCC problems [37]. Firstly, the relaxed approach guarantees the convergence to the true MPCC solution as $\epsilon \to 0$. Additionally, the boundedness of Lagrange multipliers ensures numerical stability and avoids issues with infinitely large values during optimization. Lastly, the local uniqueness of the $\mathcal{O}_\epsilon$ solution under specific conditions guarantees a single and well-defined solution. Therefore, the proposed relaxed optimization problem deals with the non-convexity in the Weymouth equation while guaranteeing the KKT conditions around ϵ, posing a standard optimization problem, and avoiding ambiguity in interpreting results.

3.3 Case studies

The current section validates the proposed MPCC approach by comparing its performance against two well-established methods for approximating the Weymouth equation: i) The Taylor series approach that piecewise approximates Weymouth with line segments [38] and ii) The SOC programming that introduces a two-stage optimization, namely, flow direction estimation and cost minimization [39]. The validation contrasts Taylor, SOC, and MPCC approaches in three case studies of interconnected systems with different complexities.

The considered validation aims to quantify the inherent errors and the cost–error trade-off of the contrasted approaches to support its real-world pertinence. Therefore, this work reports two performance metrics: the cost function in Equation (3.1) that assesses the capacity for optimally operating an integrated system and the Weymouth error metric ($WE_p^t \in \mathbb{R}^+$) for quantifying the required flow to guarantee equality for pipeline p at time instant t in Equation (3.18), as follows:

$$WE_p^t = \left| f_p^t - \left(K_{nm}|(\pi_n^t)^2 - (\pi_m^t)^2|\right)^{1/2} \right|, \quad \forall\, p = (n,m) \in \mathcal{P}. \tag{3.26}$$

Hence, the WE_p^t metric, measured in million standard cubic feet per day (MMSCFD), explains the approximations' inherent sensitivity and validates the significance of their differences.

3.3.1 Case Study I: 9/8 System

The network depicted in Figure 3.1 interconnects a nine-bus power system and an eight-node natural gas network. The small size of case 9/8 enables fast execution, efficient analysis, and rigorous validation of the contrasted approaches. The 9/8 network also features a closed trajectory and bidirectional pipelines, allowing looped infrastructure with potential flow reversals.

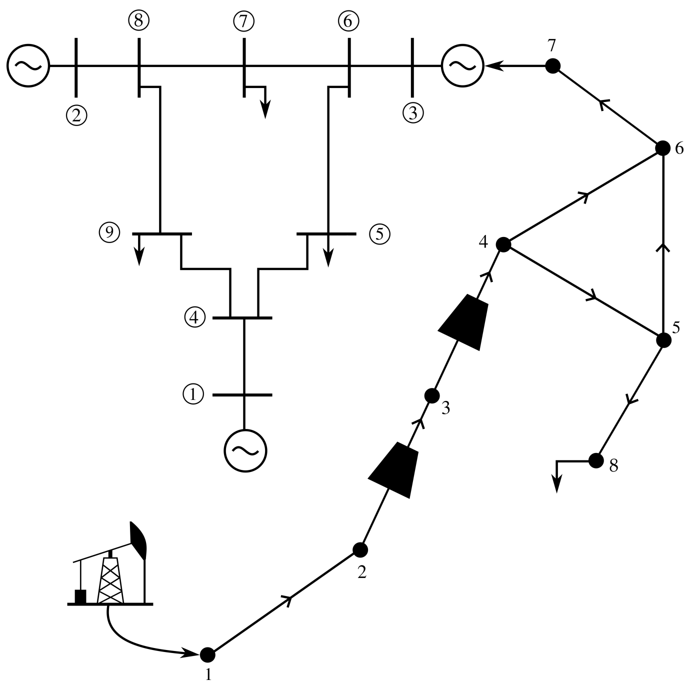

Figure 3.1: Integrated system 9/8 used in Case Study I, modified from the MPNG software. [1]

To assess the performance of Weymouth approximation approaches on the 9/8 system, a Monte Carlo experiment estimates the cost function and Weymouth error distributions by solving the optimization problem for one day ($\mathcal{T} = \{1\}$) one hundred times with uniformly sampled natural gas demands. Further network parameter details can be found in the publicly available repository OptiGasFlow (`https://github.com/cblancom/optigasflow`, accessed on 05 April 2024). **??** depicts the cost function histogram for Taylor, SOC, and MPCC approaches. Remarkably, the three histograms evidence identical distribution patterns, leading to regular solutions across approaches.

The boxplots in **??** show the Weymouth approximation error distribution for each pipeline using three approaches. The error distributions, including median and interquartile range, indicate that MPCC consistently maintains accuracy throughout the network. In contrast, the widely varying errors of the Taylor and SOC approaches suggest a lack of consistency in the achieved solution. Therefore, in a small network, the proposed MPCC approach converges to identical operational costs as Taylor and SOC, even in rationing, while meeting all linear constraints and improving the Weymouth approximation.

Chapter 4

Gas System - Censnet

4.1 Formulation of Gas System

Chapter 5

Conclusions

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