AT_4_23_22

> Hi, I am trying out Inkdrop to document a tutoring session with AT

Sigma Notation

First, we need to understand the notation

$$\sum_{i=0}^{n} f(i)$$

This means to sum up all the function values evaluated at i from i=0,...,n

Function Notation

Arithmetic Sequences

> $$t(n) = d(n-1) + t_1$$

t(n) gives us the nth value of the arithmetic sequence.

Arithmetic Series

> $$S(n) = \frac{n}{2}(t_1 + t_n)$$

S(n) gives us the sum of the first n values in the arithmetic sequence.

Example 1

Consider the following *arithmetic* sequence

$$\{2, 4, 6, 8, \ldots\}$$

A series of this sequence would be the sum of the first n values. For example,

$$S(3) = 2 + 4 + 6$$

Instead of using the notation in example 1, I could ~~right~~ write it in function notation.

$$t(n) = 2(n-1) + 2$$

Notice that took sequence notation and changed it into function notation.

Just like how a sequence can be represented in different ways, the sum can also. These are difference representations.

$$\begin{aligned} S(3) &= 2+4+6 \\ &= \frac{3}{2}(2+6) \\ &= \sum_{n=1}^{3} 2(n-1)+2 \end{aligned}$$

Geometric Sequences

> $$t(n) = t_1 r^{n-1}$$

t(n) gives us the nth value of the *geometric* sequence.

Geometric Series

> $$S(n) = \frac{rt_n - t_1}{r-1}$$

S(n) gives us the sum of the first n values in the *geometric* sequence.

Example 2a

Consider the following geometric sequence

$$\{5, 10, 20, \ldots\}$$

Ofcourse, I can write this sequence in function notation.

$$t(n) = 5(2)^{n-1}$$

Now, a geometric *series* of the first 16 values of the above sequence can be expressed as

$$\begin{aligned} S(16) &= 5 + 10 + \cdots + t_{16} \\ &= \frac{2(5 * 2^{15}) - 5}{2 - 1} \\ &= \sum_{i=1}^{16} 5(2)^{i-1} \end{aligned}$$

Example 2b

Let us practice reverse engineering. Evaluate the following sum.

$$\sum_{t=0}^{15} 5(2)^t$$

❗ Crucial Step:

We can manipulate the above sum to make it look like part 2a.

$$\sum_{t=0}^{15} 5(2)^t = \sum_{i=1}^{16} 5(2)^{i-1}$$

Can you see why?

Notice that we shifted the index up by one, but *also* we subtracted one from the index in the argument.

So now we realize that the sum given at the beginning of 2b is really the sum of the first 16 values of the geometric sequence from 2a.

Example 3

Evaluate the following sum

$$\sum_{p=1}^{179}[16-4(p-1)]$$

What type of sequence do you feel? Arithmetic.

Let us switch around some numbers

$$\sum_{p=1}^{179}[-4(p-1)+16]$$

Do you remember what the sum notation looked like for arithmetic sequences?

> $$t(n) = d(n-1) + t_1$$
> $$S(n) = \frac{n}{2}(t_1 + t_n)$$

Good luc.