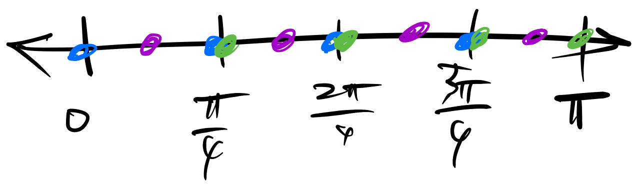
$\int_0^{\pi} \sin\theta \, d\theta$ 4 rectangles



Add up area of rectangles under the curve.

width : $\Delta x = \frac{b-a}{n}$ where b = upper bound a = lower bound n = # of subintervals.

$\Delta x = \frac{\pi - 0}{4} = \frac{\pi}{4}$

if right : $x_0 = \frac{\pi}{4}, \ldots, x_4 = \pi$

height : For left Riemann, start x's from 0

$x_0 = 0,\ x_1 = \frac{\pi}{4},\ x_2 = \frac{2\pi}{4},\ x_4 = \frac{3\pi}{4}$

$\rightarrow h_0 = f(x_0) = \sin(0) = 0$

$h_1 = f(x_1) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$h_2 = f(x_2) = \sin\left(\frac{2\pi}{4}\right) = 1$

$h_3 = f(x_4) = \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

h_i is the height of the i^{th} rectangle.

The integral is approximately the sum of the area of the 4 left rectangles.

$$\int_0^{\pi} \sin(\theta)\, d\theta \approx A_1 + A_2 + A_3 + A_4$$

$$= \Delta x h_0 + \Delta x h_1 + \Delta x h_2 + \Delta x h_3$$

$$= \frac{\pi}{4} \cdot 0 + \frac{\pi}{4} \frac{\sqrt{2}}{2} + \frac{\pi}{4} \cdot 1 + \frac{\pi}{4} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\pi\sqrt{2}}{8} + \frac{2\pi}{8} + \frac{\pi\sqrt{2}}{8}$$

$$= \frac{2\pi + 2\pi\sqrt{2}}{8} = \frac{\pi + \pi\sqrt{2}}{4}$$

$$\frac{d}{dx}\left(\int_2^{x^2-2}\sqrt{3+u}\,du\right)$$

Recall Liebniz

$$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x,t)\,dt\right) = f(x,b(x))\cdot\frac{d}{dx}(b(x)) \quad ①$$

$$\circledast \qquad - f(x,a(x))\cdot\frac{d}{dx}(a(x)) \quad ②$$

$$+ \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)\,dt. \quad ③$$

$b(x) = x^2-2$

$a(x) = 2$

$f(x,u) = \sqrt{3+u}$

$\frac{d}{dx}(b(x)) = 2x$

$\frac{d}{dx}(a(x)) = 0.$

$\frac{\partial}{\partial x} f(x,u) = 0$

$f(x,b(x)) = \sqrt{x^2+1}$

$f(x,a(x)) = \sqrt{5}$

$$\circledast = \sqrt{x^2+1}\cdot(2x) - \cancel{\sqrt{5}(0)}^{\,0} + \cancel{\int_{a(x)}^{b(x)} 0\,dt}^{\,0}$$

$$= \sqrt{x^2+1}\ (2x)$$

$$\int_0^8 f(x)\,dx$$

approximate w/ $n=3$

$$\int_0^8 f(x)\,dx \approx \frac{8-0}{3}\left(f(x_0)+f(x_1)+f(x_2)\right)$$

$n=4$

$$\int_0^8 f(x)\,dx \approx \frac{8-0}{4}\left(f(x_0)+f(x_1)+f(x_2)+f(x_3)\right)$$

n

$$\int_0^8 f(x)\,dx \approx \frac{8-0}{n}\underbrace{\left(f(x_0)+\dots+f(x_{n-1})\right)}_{\text{write in sum notation}}$$

$$= \frac{8}{n}\sum_{i=0}^{n-1} f(x_i)$$

precisely....

$$\int_0^8 f(x)\,dx = \lim_{n\to\infty}\left(\frac{8}{n}\sum_{i=0}^{n-1} f(x_i)\right)$$

$$\sum_{i=0}^{0} f(x_i) = f(x_0)$$

$$\sum_{i=0}^{1} f(x_i) = f(x_0)+f(x_1)$$

$$\sum_{i=0}^{2} f(x_i) = f(x_0)+f(x_1)+f(x_2)$$