Towards an Algebraic Modeling of Emotional States

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***Abstract*—In this study, we present a new vision of modeling emotional states. Indeed, the proposed model is different from traditional approaches like ontological representation. It is based on an algebraic representation of emotions. We represent every emotion as a vector in a space of 8 dimensions where every axis represents a basic emotion. This multidimensional model provides to represent infinity of emotion and provides powerful mathematical tools for the analysis and the processing of these emotions. Therefore, our model permits to model not only the basic emotions (e.g., anger, sadness, fear) but also different types of complex emotions like simulated and masked emotions. Moreover, it permits the exchange of emotional states between heterogeneous applications regardless to the modalities and sensors used in the detection step.**

***Keywords*-modeling emotional states, algebraic representation, multidimensional space, emotional exchanges.**

I. Introduction

Since the late 80s, Emotion-oriented (or ”affective”) computing is becoming a focus in interactive technological systems and more essential for communication, decision-making and behavior. Indeed, affective computing expands human-computer interaction by adapting the experience to the emotional state of the user. There is a rising need for emotional state recognition in several domains, such as health monitoring, video games and human-computer interaction. The lack of a standard in human emotions modeling hinders the sharing of affective information between applications. Current works on modeling and annotation of emotional states (e.g., Emotion Markup Language (EmotionML)[1], Emotion Annotation and Representation Language (EARL)[2], [3]) aim to provide a standard for emotion exchange between applications, but they use natural languages to define emotions. They use words instead of concepts. For example, in EARL, joy would be represented by the following string ”<emotion category=”joy” />”, which is the English word for the concept of joy and not the concept itself, which could be expressed in all languages (e.g., joie, farah, gioia). Our goal is to provide a generic computational model of emotional states that will facilitate inter-systems exchanges and improve the credibility of interactions. Traditional approaches are based on ontologies and natural language, but those are discrete models and their expressivity is limited. For instance, they can express the quality but not the intensity of an emotion, or they cannot model every possible mix of emotions (i.e., complex emotions). In this paper, we present a new computational model of emotional states, based on the Plutchik approach [4], which is set at the conceptual level rather than the language level of emotions. This model is continuous and therefore can model any kind of complex emotion. The remainder of this paper is organized as follows. In Section 2, we give some related psychological and linguistic theories of emotion. In Section 3, we describe our model and we conclude in Section 4.

II. Descriptive Schemes for Emotions

There is no consensus among psychological and linguistic theories on emotions. The word ”emotion” comes from the Latin ”emovere, emotum”, which means movement towards the outside. An emotion is the consequence of a feeling or the grasping of a situation and generates behavioral and physiological changes. Emotion is a complex concept. Darwin [5] said that emotional behavior originally served both as an aid to survival and as a method of communicating intentions. He thought emotions to be innate, universal and communicative qualities. Ekman [6], Izard [7], Plutchik [4], Tomkins [8] and MacLean [9] have developed the theory that there is a small set of basic emotions out of which all others are compounded. The most famous of these basic emotions are the Big Six, used in Paul Ekman's research on multi-cultural recognition of emotional expressions [10]. The Big Six emotions are happiness, sadness, fear, surprise, anger and disgust. There are a theories that describe and explain the nature of emotions. Let us present two of those approaches to emotional synthesis.

A. Dimensional approach

This approach models emotional properties in terms of emotion dimensions. It decomposes emotions over two orthogonal dimensions, namely arousal (from calm to excitement) and valence (from positive to negative) [11]. Complex emotions cannot directly be expressed in this model, which is why we opted for the other one.

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B. Categorical approach

Many theories treat basic emotions as distinct categories (e.g., fear, anger, sadness, happiness) and the number of basic emotions varies from one theory to the next: for instance, there are 6 basic emotions in the Fridja's theory [12], 9 in the Tomkins's theory [8] and 10 in the Izard's theory [7]. In 1962, Plutchik adopted a color metaphor for the combination of basic emotions [4]. We opted for his approach as the basis of our model and will thus describe it in details.

C. Plutchik model

Robert Plutchik proposed a three-dimensional "circumplex model" (Figure 1), which describes the relationships between emotions. He proposed eight primary emotion dimensions arranged as four pairs of opposites: (Joy-Sadness, Fear-Anger, Surprise-Anticipation, Disgust-Trust)[4]. The vertical dimension represents intensity or level of arousal, and the circle represents degrees of similarity among the emotions. He suggested that non-basic emotions are obtained through the addition of basic emotions (color analogy, Plutchik, 1962) [13]. In his model, for instance, Love = Joy + trust and Delight = Surprise + Joy. Plutchik defined rules for building complex emotions out of basic ones. In practice, combination of emotions follows the method "dyads and triads" [14]. He defined the primary dyads emotions as the mixtures of two adjacent basic emotions. Secondary dyad includes emotions that are one step apart on the "emotion wheel", for instance Fear + Sadness = Despair. A tertiary emotion is generated from a mix of emotions that are two steps apart on the wheel (Surprise + Anger = Outrage).

In our work, we chose the Plutchik model because it verifies many important conditions for the elaboration of our model. First, the Plutchik model is based on 8 basic emotions encompassing the common five basic emotions. Then, it takes into account the intensity of emotion i.e., the level of arousal or the feeling degree of each basic emotion for example (terror, fear, apprehension). Finally, the Plutchik model is intuitive, very rich and it is the most complete model in literature because it permits to model complex emotions by using basic ones. Indeed, as we have seen, Plutchik defined the dyads and the triads which are combinations of basic emotions describing complex emotions which are regarded as emotions in usual life.

III. The proposed emotional model

In this work, we present a new approach of modeling emotional states. Indeed, the proposed model is different from traditional approaches like ontological representation. It is based on an algebraic representation using multidimensional vectors. We represent every emotion as a vector in a space of 8 dimensions where every axis represents a basic emotion. This multidimensional model provides the representation of an infinity of emotions and provides also a powerful mathematical tools for the analysis and the processing of these emotions. The proposed model is similar to the RGB colors representation model, which is based on three basic colors (Red, Green, Blue) to build all the others ones. For example, blue and yellow paints mix together to create a green pigment. In order to develop this analogy, it's necessary to define the basic emotions. For this, we will adopt the Plutchik definition of basic emotions, which is a very intuitive and easy model including the idea that complex emotions are obtained by mixing primary ones. This last property is very important on our model because it allows us to define an infinity of combinations using the eight basics emotions defined by Plutchik.

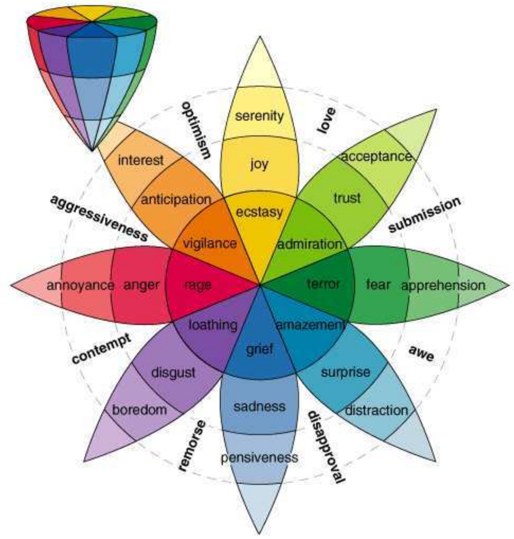

Figure 1. Plutchiks three-dimensional circumplex model

A. Use cases

Certainly, the actual works propose a lot of mechanisms ensuring the capture of the emotional states. But, they present some limitations either on the number of the studied feelings (they treat only the basic ones) or on the capture tools used. The emotional exchange between applications causes many problems due to the application heterogeneity, the complexity of the emotional states, the diversification of the feelings capture tools and the dependence of the treatment to these physical sensors.

Under the research program Emotica [15], between the University of Nice and the Stone trip company, we aim at defining a formal model allowing the communication between various multi-modal applications. In our work, we are interested in the 3D video games especially serious games like flight simulators. In this type of games we are very interested to know at each instant the felt emotion by the player and to adapt in consequence the simulation. We target also to transfer the emotional data between the various players using different platforms and modalities. In the presented work, we capture the physiological features of the player to determine its emotional state and then we transfer these features to his game partner(s) or monitor using the same or other platform(s)/modality(s).

B. Definition

The proposed model consists on the representation of emotions using multidimensional vectors. We represent every emotion as a vector in a space of 8 dimensions where each axis represents a basic emotion. First, we define our Base by (B) = (joy, sadness, trust, disgust, fear, anger, Surprise, anticipation), which are the basic emotions on the Plutchik theory. So every emotion (e) can be expressed as a finite sum (called linear combination) of the basic elements.

$$(e) = \sum_{i=1}^{8} \langle E, u_i \rangle u_i \qquad (1)$$

Thus, $(e) = \alpha_1 Joy + \alpha_2 sadness + \alpha_3 trust + .. + \alpha_7 Surprise + \alpha_8 anticipation$
where α_i are scalars and $u_i (i = 1..8)$ elements of the basis (B). Typically, the coordinates are represented as elements of a column vector E

$$E = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ . \\ . \\ \alpha_8 \end{pmatrix}_B$$

where $\alpha_i \in [0, 1]$ represent the intensity of the respective basic emotion. More the value of α_i get nearer to 1, more the emotion is felt. This value is defined on the emotion recognition step using a Psycho Physiological Emotional Map like the method developed in [16].

In the linear algebra, a basis is a set of vectors that, in a linear combination, can represent every vector in a given vector space or free module, and such that no element of the set can be represented as a linear combination of the others. Thus, we have to prove that (B) = (joy, sadness, trust, disgust, fear, anger, Surprise, anticipation) is a linearly independent spanning set. Then (B) is a basis if it satisfies the following conditions:

- The spanning property: according to Plutchik, there are eight basic physiological states or feeling states, with other states being combinations of these basic emotions. Thus we can obtain all the possible emotion from these eight basic emotions (joy, sadness, trust, disgust, fear, anger, Surprise, anticipation). In our model, each basic emotion is represented by an axis and we can combine many basic emotion by using scalars α_i to get the complex emotions. So, for every emotion (e) it is possible to choose $\alpha_1, \alpha_2, \alpha_3, .., \alpha_8$ such that $(e) = \alpha_1 Joy + \alpha_2 sadness + .. + \alpha_8 anticipation$ can represent a basic or a complex emotion. The scalars α_i are called the coordinates of the vector (e) with are uniquely determined.
- The linear independence property: to prove that they are linearly independent, we suppose that (B) is not a linearly independent, thus, we have two different representations of an emotion in the same base.

$$(e) = \alpha_1 Joy + \alpha_2 sadness + .. + \alpha_8 anticipation \qquad (2)$$

$$(e) = \gamma_1 Joy + \gamma_2 sadness + .. + \gamma_8 anticipation \qquad (3)$$

With

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ . \\ . \\ \alpha_8 \end{pmatrix}_B \neq \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ . \\ . \\ \gamma_8 \end{pmatrix}_B$$

Subtracting the equation(3) from the equation(2), we obtain:
$0 = (\alpha_1 - \gamma_1) Joy + (\alpha_2 - \gamma_2) sadness + .. + (\alpha_8 - \gamma_8) anticipation$
then $\alpha_1 = \gamma_1, \alpha_2 = \gamma_2, .., \alpha_8 = \gamma_8$.
Finally our hypothesis is false, so B is linearly independent.

C. Representation of basic emotions

A vector represents a basic emotion if it verifies the following property:

$$\forall i \in [1..8], \exists \alpha_i \ with \ \frac{\alpha_i}{\sum_{i=1}^{8} \alpha_j} = 1 \qquad (4)$$

A basic emotion is described by a vector, which contains a single non-zero coefficient. The following vectors represent some basic emotions:

$$E_{joy} = \begin{pmatrix} \alpha_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}_B \quad E_{sadness} = \begin{pmatrix} 0 \\ \alpha_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}_B \quad E_{anger} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha_6 \\ 0 \\ 0 \end{pmatrix}_B$$

where $\alpha_1 \neq 0, \alpha_2 \neq 0, and \alpha_6 \neq 0$

The proposed model takes into account the property of the intensity of the emotion. Indeed, each emotion can exist in varying degrees of intensity. The coefficients α_i determine the emotion intensity. According to the value of the coefficients α_i we can make the difference between annoyance, anger and rage or pleasure, joy and ecstasy. So, rage is the basic emotion anger with high intensity. The multidimensional model provides the representation of an infinity of emotions and provides also a powerful mathematical tools for the analysis and the processing of these emotions. The basic algebraic operations, which we are going to detail in our work are the vector addition, the scalar multiplication, the projection and the distance in an Euclidean space.

D. Operations on the emotions

1) Vector addition: We have seen in the previous paragraphs that the mixture of pairs of basic emotions resulted of complex emotion. Joy and trust for example produce the complex emotion "love". "Submission" is a mixture of trust and fear. In this part we define the combination between emotions as the sum of two emotion vectors. This addition is defined as the maximum value of coefficients (term by term). We have chose the maximum instead of the classic sum because the intensity must not exceed 1 and because of the specificity of each modality. For exemple when we have two modalities, each one will give a vector with different coefficients for the same emotion. For the same axis, we keep the highest one because each modality can detect better a specific emotion for exemple with the heart rate modality we can detect the fear component better than the facial expression modality. Let E_{1u} and E_{2u} be two emotional vectors expressed in the basis (B) respectively by $(\lambda_1, \lambda_2, .., \lambda_8)$ and $(\lambda_1', \lambda_2', .., \lambda_8')$. The addition of these two vectors is defined as:

$$E' = E_{1u} \bigoplus E_{2u} = \max(\lambda_i, \lambda_i') for 0 \leq i \leq 8 \qquad (5)$$

In this sense, the vector representing the emotion love, which is mixture of joy and trust, is defined as:

$$E_{love} = E_{Joy} \bigoplus E_{trust}$$

$$E_{love} = \begin{pmatrix} \alpha_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}_B \bigoplus \begin{pmatrix} 0 \\ 0 \\ \alpha_3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}_B = \begin{pmatrix} \alpha_1 \\ 0 \\ \alpha_3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}_B$$

where $\alpha_1 \neq 0$ *et* $\alpha_3 \neq 0$
In the same way we can obtain the "vector form" of the other complex emotions states defined by Plutchik. These emotions combinations are shown on (figure 2).

However, Plutchik's theory suggests that some combinations (opposite emotions), which are: Joy /Sadness, trust/disgust, fear/anger, surprise/anticipation, produce conflict. Nevertheless, we will take it (emotional conflict) into account on our model. Thus, these combinations have an emotional meaning. For example, a person can attempt to regulate the expression of her face to hide the true felt emotion. If we analyse her facial expression we can found joy but he really felt angry.
The figure 3 shows an example of the using of the add operation on application of emotion detection. On this example the detection is done using two modalities. Each modality gives an emotion vector. The vector V_1 is given by the facial modality and the vector V_2 is given by the physiological modality. The final emotion vector V_f is given by the addition of this two vectors using equation 5.

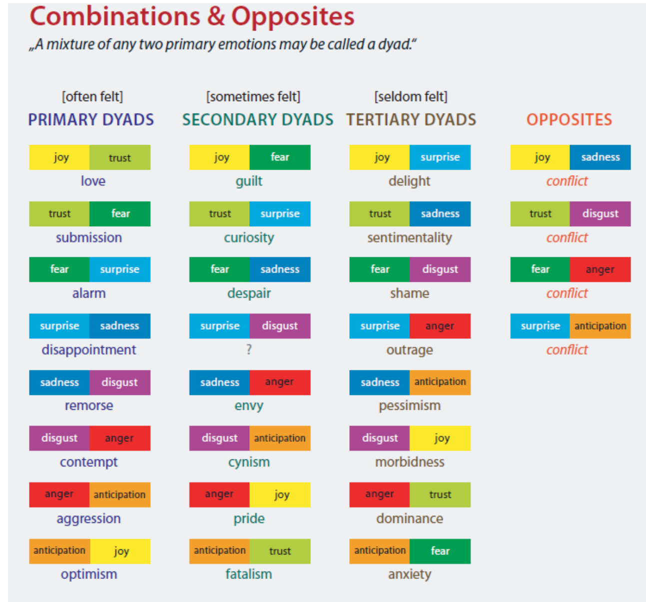

Figure 2. Combination and opposites on the Plutchik's model

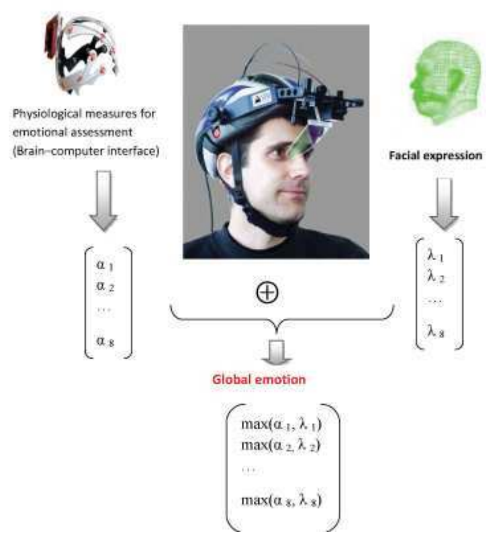

Figure 3. Multi-modality emotion recognition system

2) Scalar multiplication: is one of the basic operations defining a vector space in linear algebra. We can use this operation to the emotion amplification. Indeed, the scalar multiplication provides to change the emotional state by amplifying the intensity, attenuating, stopping an existing emotion. It refers to the multiplication of every entry in the vector by a regular number (called a "scalar") producing a new emotional vector with new valuesα_i. The product of an emotion vector E by a scalar β give a new vector E''.

$$E^{''} = \beta.E = \beta. \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ . \\ . \\ \alpha_8 \end{pmatrix}_B = \begin{pmatrix} \beta.\alpha_1 \\ \beta.\alpha_2 \\ . \\ . \\ \beta.\alpha_8 \end{pmatrix}_B$$

Where the coefficients of the new vector $\beta.E$ are always $\in [0,1]$

3) Decomposition of emotion: The idea is to decompose the complex emotion as a combination of primary or basics emotions to get a true description of the felt emotion. This permits us a pertinent analysis. Therefore we used the projection to decompose the complex emotions.

- Projection

Let E be an emotional vector form $E = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ . \\ . \\ \alpha_8 \end{pmatrix}_B$

The projection of E according to the i component is defined by:

$$\Pi_i(E) = \alpha_i, \quad i \in [1..8] \tag{6}$$

Example:

$$\Pi_4 \begin{pmatrix} 0.2 \\ 0.1 \\ 0 \\ 0.8 \\ 0.3 \\ 0.4 \\ 0 \\ 0.1 \end{pmatrix}_B = 0.8$$

This projection isolates a given component from the vector E to get the intensity according to this axis. This permit to facilitate the interpretation of the emotional vector.

4) Euclidean distance (2-norm distance):

$$d(E,Y) = \sqrt{\sum_{i=1}^{n} (y_i - x_i)^2} \tag{7}$$

With E $\begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix}_B$ and Y $\begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ y_n \end{pmatrix}_B$ are two vectors.

The proposed model is a continuous model providing the representation of infinity of emotions. Thus, to analyse a given vector and determine the nearest emotion from the known ones we need a tool to calculate the similitude from the vector and the known emotions. For this, we propose to use the Euclidean distance (2-norm distance) defined by equation 7. First, we have to generate a data base of emotions composed by the vectors of all emotions proposed by Plutchik given by figure 1 and figure 2. So, our emotion data base is composed by approximately 50 emotions and can be extended by others emotions. Then we have to compute for a given vector V1 the Euclidean distance between it and all the vectors of the data base. Finally we keep the vector of the data base minimizing this distance. This vector represents the nearest emotion of V1 and the computed distance gives an idea of the precision of this interpretation. For example, we can found that the nearest emotion for the vector V1 is "love" with a distance equals to zeros. We can affirm without doubts that V1 represents the emotion "love". More the distance from the nearest vector is important, less the interpretation is accurate. So the proposed method, using the Euclidean distance, permits to analyse automatically a given vector and provides the best interpretation of this vector.

The figure 4 gives an example of an application of emotional exchange . The first user (user1) uses a camera to detect emotions. The emotion detected using the facial expression is represented by the vector E1 and sent to user2. User2 analyse the vector E1 using our algorithm based on the Euclidean distance and find that the felt emotion by user1 is" love" and vice versa.

IV. Conclusion

In this paper, we proposed a new algebraic model, inspired from the Plutchik theory, for the representation and the exchange of emotions. The proposed model is a multidimensional model, where each emotion is represented by a vector in a basis of 8 dimensions. Every axis on this basis represents a basic emotion. Our model provides powerful mathematical tools for the analysis and the processing of emotions. Indeed, using for example the Euclidean distance we can easily interpret an emotional vector and find the correspondent felt emotion. For the future work, we would extend our model to take into account the notion of time by adding a new variable to save the time when the emotion was felt. Using this variable, we can study the evolution of the emotional state of the person on time. Then, we would exploit this information to predict the future emotion state.

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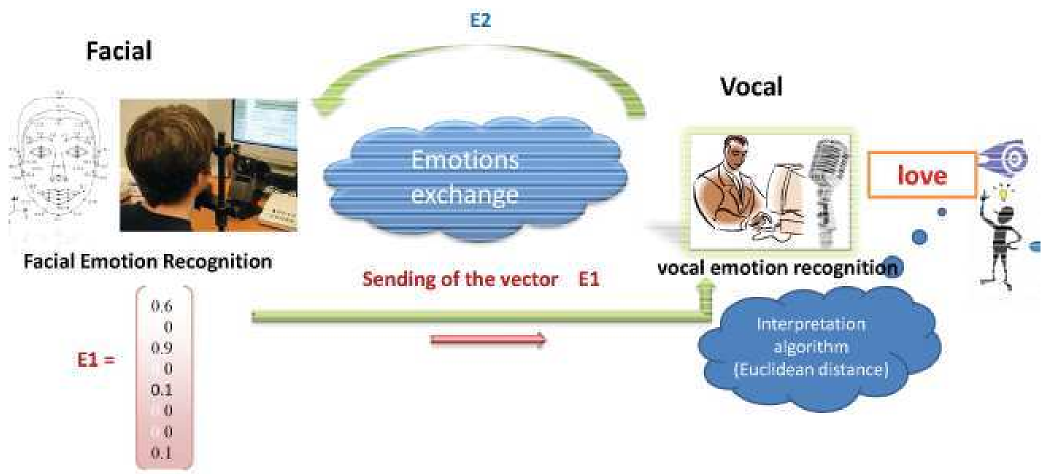

Figure 4. An example of an application of emotional exchange

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