6. (a) Show that the Lagrange's bracket is invariant under canonical transformation.

(b) Show that the transformation defined by $q = \sqrt{2P}\sin Q,\ p = \sqrt{2P}\cos Q$ is canonical by using Poisson bracket.

7. (a) A body of unit mass is constained to move on the path $y = \cosh x$ under a potential $V = \frac{1}{2}x^2$. Set up the Hamilton-Jacobi equation and finally solve it.

(b) State and prove Whittaker's equations.

8. (a) A self attracting sphere of uniform density ρ and radius 'a' changes to one of uniform density and radius 'b'. Show that the work done by its mutual attractive forces is given by $\frac{3}{5}M^2\left(\frac{1}{b}-\frac{1}{a}\right)$ where M is the mass of the sphere.

(b) Discuss the potential of a uniform solid sphere.

S.C.No.—M/22/21703103

M. Sc. EXAMINATION, 2022

(First Semester)

(Batch 2021)

MATHEMATICS

21MTH-103

Mechanics

Time : 3 *Hours* *Maximum Marks :* 80

Note : Attempt *Five* questions in all. All questions carry equal marks.

1. (a) Define Moment of Inertia for a system consisting of n particles.

(b) What do you mean by coplanar distribution ?

(c) State Donkin's theorem.

(d) What do you mean by Hamilton canonical variables ?

(e) Define Poisson's Identity and principle of least square.

(f) State Jacobi's equations and Jacobi theorem.

(g) What do you mean by surface and solid harmonics ?

(h) Define cyclic coordinates.

2. (a) Discuss and state the theorems of parallel and perpendicular axes.

(b) A square of side 'a' has particles of masses m, $2m$, $3m$, $4m$ at its vertices. Show that the principal moment of inertia at the center of square are $2ma^2$, $3ma^2$, $5ma^2$, find the directions of the principal axes.

3. (a) A uniform solid of rectangular block is of mass M and dimensions $2a \times 2b \times 2c$. Find the equation of the momental ellipsoid for a corner O of the block,

referred to the edges through O as coordinate axes and hence determine the moment of inertia about OO', where O' is the point diagonally opposite to O.

(b) Find the kinetic energy of a rigid body rotating about a fixed point.

4. (a) Derive the Lagrange's equation for a holonomic dynamical system.

(b) Two uniform rods AB, BC of masses m_1, m_2 and lengths $2a$, $2b$ are smoothly hinged at B and initially they lie at rest on a smooth table and in a straight line. AB receives a blow of impulse I at A perpendiculate to AB. Construct the equations of motion of the system just after impact.

5. (a) Derive Hamilton canonical equation in polar coordinates.

(b) Discuss the physical significance of Conservation of Energy.

9. (a) Show that a family of right circular cones with a common axis and vertex is a possible family of equipotential surfaces. Hence find the potential function.

(b) Obtain the surface density in terms of surface harmonics.

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