(b) Two persons A and B throw a coin, the one who throws head first wins the toss. The game starts with A. Find their respective chances of winning.

(c) If X is a continuous r.v. with density function :

$$f(x) = \begin{cases} ax; & 0 < x \le 1 \\ a; & 1 \le x \le 2 \\ 0; & \text{otherwise} \end{cases}$$

Determine the constant a and $P(X \le 1.5)$.

4. (a) The joint distribution of X and Y is given by :

$$f(x, y) = 4xye^{-(x^2+y^2)}; x \ge 0, y \ge 0$$

Find marginal density functions of X and Y; and the conditional density function of X given Y = y.

(b) If $f(x) = \frac{1}{a}\left[1 - \frac{|x-b|}{a}\right]; |x-b| < a$, find E(X), E($X^2$) and variance of X.

S.C.No.—M/22/18703105

M. Sc. EXAMINATION, 2022

(First Semester)

(Batch 2020/2019/2018)

MATHEMATICS

18MTH-105

Mathematical Statistics

Time : 3 Hours *Maximum Marks :* 80

Note : Attempt *Five* questions in all. All questions carry equal marks.

1. (a) Two fair coins are tossed once and observation is recorded. Write the set of exhaustive cases. Find the probability of observing exactly one head.

(b) If the p.d.f. of a continuous random variable X is given by :

$$f(x) = \begin{cases} C\left(4x - 2x^2\right), & 0 < x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

find the value of C and P(X $\le$ 1).

(c) Compute Variance (X) when X represents the outcome when we roll a fair dice.

(d) Find moment generating for the distribution :

$$f(x) = pq^{x-1};\ x = 1, 2, 3, \ldots\ldots..$$

where p is probability of success in one trial and q is probability of failure in one trial.

(e) Find M.G.F. and mean for the distribution :

$$f(x) = \begin{cases} \theta e^{-\theta x}, & x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

(f) Find mean deviation about mean for uniform distribution.

(g) Define sufficiency and efficiency in estimation theory.

(h) What do you mean by critical region ?

2. (a) State and prove addition theorem of probability for arbitrary events A_1, A_2, A_3.

(b) State and prove Bayes' theorem on probability.

(c) A bag-I contains 4 white and 6 black balls while another bag-II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags, and it is found to be black. Find the probability that it was drawn from bag-I.

3. (a) What do you mean by sample space and discrete random variable ? A r.v. X has the following probability function :

X	-2	-1	0	1	2	3
$f(x)$	k	0.3	$2k$	0.2	0.1	$3k$

Find the value of k; and P(X $\le$ 0); distribution functions of X.

5. (a) Define Poisson distribution. Find its M.G.F. and cumulant generating function. Also show that all cumulants are equal for Poisson's distribution.

(b) If X is a Poisson variate such that P(X = 1) = P(X = 2), then find P(X = 4). Also obtain first three moments about mean for Binomial distribution.

6. (a) Prove that moments of odd order are zero for a normal distribution and even order moments are given by :

$$\mu_{2n} = 1.3.5 \ldots\ldots (2n-1)\sigma^{2n}.$$

(b) Find points of inflexion of a Normal Curve.

7. (a) Find mean and mean deviation about mean of exponential distribution.

(b) State and prove central limit theorem.

8. (a) Discuss unbiasedness, consistency and two types of errors along with one example for each.

(b) Write short notes on the following :

Level of significance, Consistency, Null and Alternate Hypothesis.

9. (a) Ten individuals were chosen at random from a normal population and their heights were found to be in inches as 63, 63, 66, 67, 68, 69, 70, 70, 71 and 71. Test the hypothesis that the mean height of the population is 66 inches. Also find the 95% confidence limits for the true population mean μ. (Given $t_{9(0.05)}$ = 2.26).

(b) 15000 random numbers were taken from some logarithmic table and the following frequencies of each digit were obtained :

Digit	**Frequency**
0	1493
1	1441
2	1461
3	1552
4	1494
5	1454
6	1613
7	1491
8	1482
9	1519

Use Chi-square test to assess the correctness of the hypothesis that each digit had an equal chance of being chosen. (Given Chi-square at 5% level for 9 d.f. is = 16.919).