5. (a) Let $\sum u_n(x)$ be a series of functions continuous in any interval (a,b), then prove that $\int_a^b \sum u_{\wedge}(x)\,dx = \sum \int_a^b u_n(x)\,dx$. **8**

(b) Define pointwise and uniform convergence of sequence of functions and show that the series : **8**

$$\frac{2x}{1+x^2}+\frac{4x^3}{1+x^2}+\frac{8x^7}{1+x^6}+\ldots\ldots$$

is uniformly convergent in $-1<x<1$.

Unit IV

6. (a) State and prove Taylor's theorem. **8**

(b) Prove that $f(x,y)=|xy|^{\frac{1}{2}}$ is not differentiable at $(0,0)$. **8**

7. (a) A linear transformation on a $f \cdot d \cdot$ vector space X is one and one if and only if range of T is X. **8**

S.C.No.—M/22/21703104

M. Sc. EXAMINATION, 2022

(First Semester)

(Batch 2021)

MATHEMATICS

21MTH-104

Real Analysis

Time : 3 Hours *Maximum Marks :* 80

Note : Attempt *Five* questions in all. All questions carry equal marks.

Unit I

1. (a) Define Riemann-Stieltjes Integral.

(b) Give an example of rectifiable curve.

(c) Show that Lebesgue integral is generalization of Riemann integral.

(d) State Weierstrass M-test.

(e) State Implicit function theorem.

(f) Give an example of a linear transformation and a non-linear transformation.

(g) Define outer measure. Prove that $[0,1]$ is uncountable.

(h) Show that the outer measure of Cantor set is zero. **2×8=16**

Unit II

2. (a) If f and α' are Riemann integrable on $[a, b]$, then show that $f \in (R(\alpha)$ and $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$. **8**

(b) Evaluate : **8**

(i) $\int_1^4 (x-[x])dx^2$

(ii) $\int_0^3 \sqrt{x^3}dx^3$.

3. (a) If P* is a refinement of P, then show that $L(f,P,\alpha) \leq L(f,P^*,\alpha)$ and $U(f,P,\alpha) \geq U(f,P^*,\alpha)$. **8**

(b) If $f \in R(\alpha)$ on $[a,b]$ then $|f|$ is also Riemann-Stieltjes integrable on $[a,b]$ and $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| d\alpha$. **8**

Unit III

4. (a) Consider the series : **8**

$$\sum_{n=1}^{\infty}\left[\frac{nx}{1+n^2x^2} - \frac{(n-1)x}{1+(n-1)^2x^2}\right], \quad 0 \leq x \leq 1$$

Show that the series is integrable term by term although 0 is a point of non-uniform convergence.

(b) Test the series $\sum_1^{\infty}\frac{\sin nx}{n^p}$ for uniform convergence in any finite interval, $p > 0$. **8**

(b) If $U = \frac{x^2+y^2+z^2}{x}$, $V = \frac{x^2+y^2+z^2}{y}$,

$W = \frac{x^2+y^2+z^2}{z}$, compute $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. **8**

Unit V

8. (a) Define outer measure of a set. Show that outer measure of an interval is its length. **8**

(b) Prove that union and intersection of two measurable sets is measurable. **8**

9. (a) Show the existence of a non-measurable set. **8**

(b) Let $\{E_i\}$ be an infinite decreasing sequence of measurable sets, then prove that : **8**

$$m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n} mE_n .$$

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