1

Density estimation: Psychological experiments. (50 points)

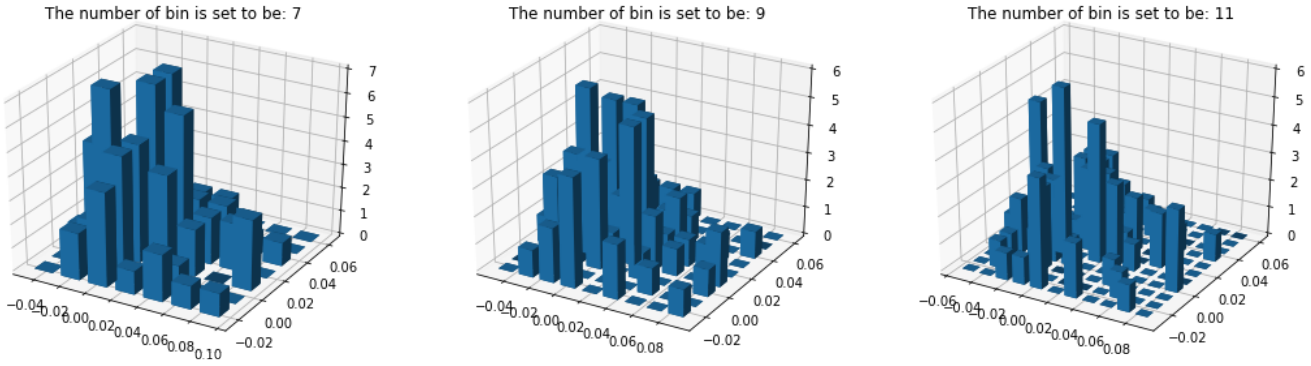
The data set n90pol.csv contains information on 90 university students who participated in a psychological experiment designed to look for relationships between the size of different regions of the brain and political views. The variables amygdala and acc indicate the volume of two particular brain regions known to be involved in emotions and decision-making, the amygdala and the anterior cingulate cortex; more exactly, these are residuals from the predicted volume, after adjusting for height, sex, and similar body-type variables. The variable orientation gives the students' locations on a five-point scale from 1 (very conservative) to 5 (very liberal).

1. Form 2-dimensional histogram for the pairs of variables (amygdala, acc). Decide on a suitable number of bins so you can see the shape of the distribution clearly.

   Base on the Silverman's rule of thumb for deciding the optimal bandwidth h for variable "amygdala":
   $h = 1.06\hat{\sigma}m^{-1/5} = 0.01397$

   For varibale "amygdala", its range is (-0.0676, 0.0812). So we can calculate the number of bins is:
   number of bin $= \frac{0.0812+abs(-0.0676)}{0.01397} \approx 11$

   Thus I decided to use 11 as the number of bins. Besides I also tried bin=7 and bin=9. Actually I think 7 and 9 are also OK.



(a) Bin = 7 (b) Bin = 9 (c) Bin = 11

Figure 1: Different numbers of bins

2. Now implement kernel-density-estimation (KDE) to estimate the 2-dimensional with a two-dimensional density function of (amygdala, acc). Use a simple multi-dimensional Gaussian kernel, for $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$, where x_1 and x_2 are the two dimensions respectively

$$K(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_1^2 + x_2^2)}{2}}.$$

Recall in this case, the kernel density estimator (KDE) for a density is given by

$$p(x) = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{h} K\left(\frac{x^i - x}{h}\right)$$

where x^i are two-dimensional vectors, $h > 0$ is the kernel bandwidth. Set an appropriate h so you can see the shape of the distribution clearly. Plot of contour plot (like the ones in slides) for your estimated density.

Again, I choose h according the Silverman's rule. However this time I used h=0.00876 according to the variable "acc". Below is the shape of distribution of the whole dataset

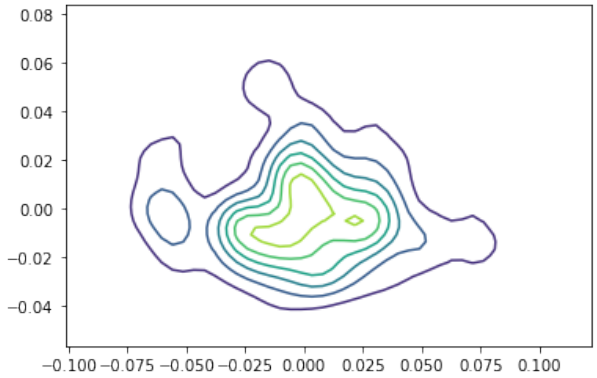
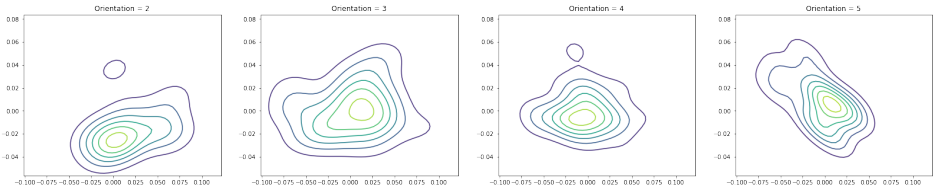


Figure 2: The contour of shape of the distribution for the all dataset



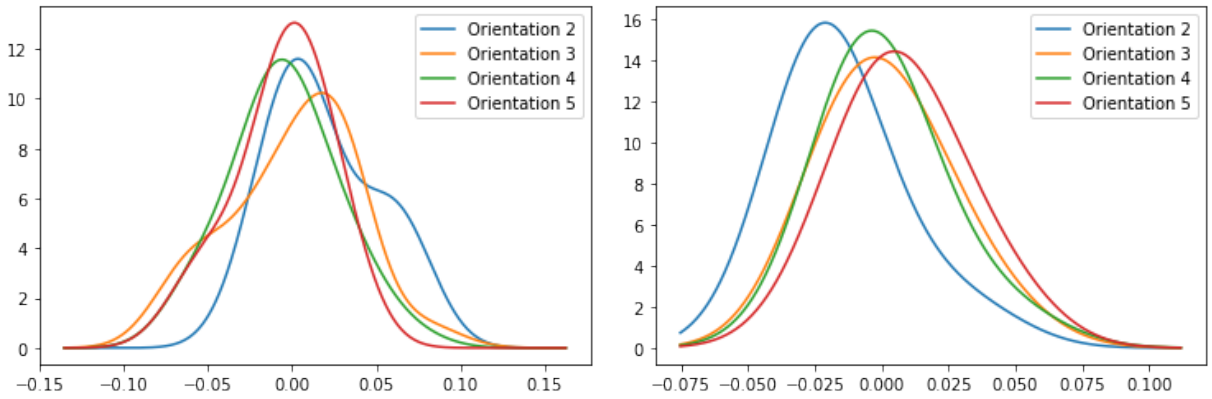
(a) Orientation =2 (b) Orientation =3 (c) Orientation =4 (d) Orientation =5

Figure 3: Different orientations

I also made contour plot for different orientations. The plots are shown above.

3. Plot the condition distribution of the volume of the amygdala as a function of political orientation: $p(\text{amygdala}|\text{orientation} = a)$, $a = 1, \ldots, 5$. Do the same for the volume of the acc. Plot $p(\text{acc}|\text{orientation} = a)$, $a = 1, \ldots, 5$. You may either use histogram or KDE to achieve the goal.

   I used KDE method to get the conditional probability for each variable.



(a) amygdala (b) acc

Figure 4: Different orientations

2

Implementing EM algorithm for MNIST dataset. (50 points)

Implement the EM algorithm for fitting a Gaussian mixture model for the MNIST dataset. We reduce the dataset to be only two cases, of digits "2" and "6" only. Thus, you will fit GMM with $C = 2$. Use the data file data.mat or data.dat on Canvas. True label of the data are also provided in label.mat and label.dat

The matrix images is of size 784-by-1990, i.e., there are totally 1990 images, and each column of the matrix corresponds to one image of size 28-by-28 pixels (the image is vectorized; the original image can be recovered, e.g., using MATLAB code, reshape(images(:,1),28, 28).

1. Select from data one raw image of "2" and "6" and visualize them, respectively.

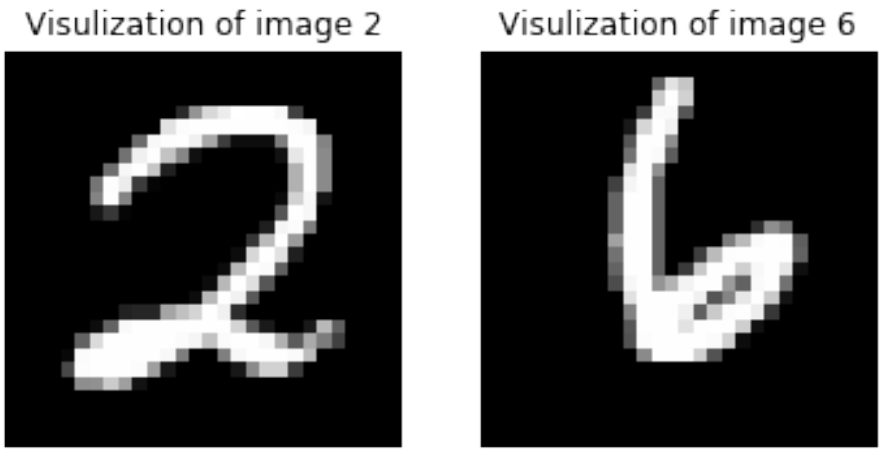


Figure 5: Visulization of image 2 and 6

2. Use random Gaussian vector with zero mean as initial means, and identity matrix as initial covariance matrix for the clusters. Please plot the log-likelihood function versus the number of iterations to show your algorithm is converging.

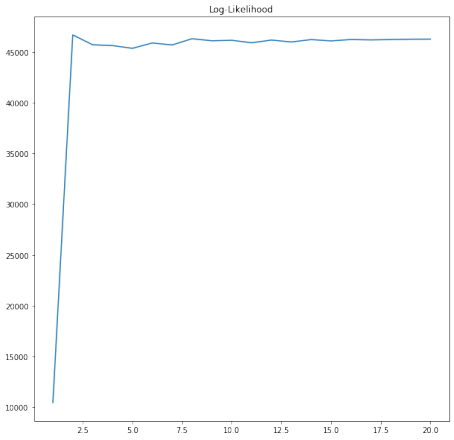


Figure 6: Log likelihood vs Iteration

3. Report the finally fitting GMM model when EM terminates: the weights for each component, the mean vectors (please reformat the vectors into 28-by-28 images and show these images in your submission). Ideally, you should be able to see these means corresponds to "average" images. No need to report the covariance matrices.

   The weights for each component are: 0.51708543 and 0.48291457.

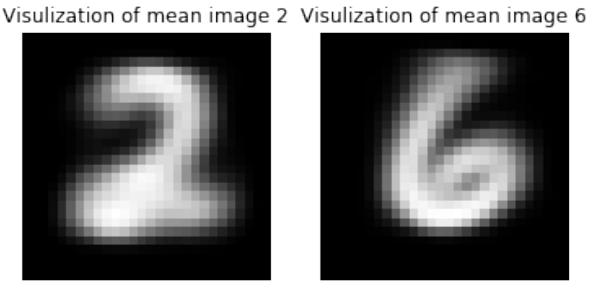


Figure 7: Iamges of mean vectors

4. (Optional). Use the p_{ic} to infer the labels of the images, and compare with the true labels. Report the miss classification rate for digits "2" and "6" respectively. Perform K-means clustering with $K = 2$. Find out the miss classification rate for digits "2" and "6" respectively, and compare with GMM. Which one achieves the better performance?