This quiz has been regraded; your score was not affected.

Week 4 Homework

Due Feb 9 at 11:59pm **Points** 14 **Questions** 14
Available after Jan 31 at 8am **Time Limit** None

Attempt History

	Attempt	Time	Score	Regraded
LATEST	Attempt 1	9,929 minutes	13 out of 14	13 out of 14

Score for this quiz: **13** out of 14
Submitted Feb 8 at 1:31pm
This attempt took 9,929 minutes.

Question 1

1 / 1 pts

(Lesson 3.1: Solving a Differential Equation.) Suppose that $f(x) = e^{2x}$. We know that if h is small, then

$f'(x) \approx \frac{f(x+h)-f(x)}{h}$.

Using this expression with $h = 0.01$, find an approximate value for $f'(1)$.

a. 1

b. 2.72

c. 7.38

Correct! d. 14.93

We have

$f'(x) \approx \frac{f(x+h)-f(x)}{h} = \frac{e^{2(x+h)}-e^{2x}}{h}$

So using h = 0.01, we have

$f'(1) \approx \frac{e^{2.02}-e^{2}}{0.01} = 14.93$

Thus, the answer is (d).

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So using h = 0.01, we have

$f'(1) \approx \frac{e^{2.02}-e^{2}}{0.01} = 14.93$

Thus, the answer is (d).

Question 2

1 / 1 pts

(Lesson 3.1: Solving a Differential Equation.) Suppose that $f(x) = e^{2x}$. What is the actual value of $f'(1)$?

a. 1

b. $e \approx 2.72$

c. $e^2 \approx 7.39$

Correct!

d. $2e^2 \approx 14.78$

$f'(x) = 2e^{2x}$, so that $f'(1) = 2e^2$, and thus the answer is (d).

e. 14.93

$f'(x) = 2e^{2x}$, so that $f'(1) = 2e^2$,
and thus the answer is (d).

Question 3

1 / 1 pts

(Lesson 3.1: Solving a Differential Equation.) Consider the differential equation $f'(x) = (x+1)f(x)$ with $f(0) = 1$. What is the exact formula for $f(x)$?

a. $f(x) = e^x$

b. $f(x) = e^{2x}$

Correct!

c. $f(x) = \exp\left\{\frac{x^2}{2} + x\right\}$

This takes a little work. The good news is that you can actually get the true answer using the technique of separation of variables. We have

$$\frac{f'(x)}{f(x)} = x + 1,$$

so that

$$\int \frac{f'(x)}{f(x)} dx = \int x + 1 \; dx$$

Which implies

$$\ln(f(x)) = \frac{x^2}{2} + x + C,$$

so that $f(x) = Ke^{\frac{x^2}{2}+x}$, where C and K are arbitrary constants. Setting $f(0) = 1$ implies that $K = 1$, so that the exact answer is , the answer is $f(x) = e^{\frac{x^2}{2}+x}$, i.e., choice (c).

d. $f(x) = \exp\{x^2 + 2x\}$

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$$\frac{f'(x)}{f(x)} = x + 1,$$

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Question 4

1 / 1 pts

(Lesson 3.1: Solving Differential Equations.) Consider the differential equation $f'(x) = (x+1)f(x)$ with $f(0) = 1$. Solve for $f(0.20)$ using Euler's approximation method with increment $h = 0.01$ for $x \in [0, 0.20]$.

- a. $f(0.20) \approx 0.0$
- b. $f(0.20) \approx 1.0$
- Correct! c. $f(0.20) \approx 1.24$

By previous question, the true answer is the answer is

$f(x) = e^{\frac{x^2}{2}+x}$.

But our job is to use Euler to come up with an iterative approximation, so here it goes. As usual, we start with
$f(x+h) = f(x) + h\,f'(x) = f(x) + h(x+1)\,f(x) = f(x)[1 + h(x+1)]$,
from which we obtain the following table.

x	Euler approx	true $f(x)$
0.00	1.0000	1.0000
0.01	1.0100	1.0101
0.02	1.0202	1.0204
0.03	1.0306	1.0309
0.04	1.0412	1.0416
0.05	1.0521	1.0526
0.06	1.0631	1.0637
0.07	1.0744	1.0751
0.08	1.0859	1.0868
0.09	1.0976	1.0986
0.10	1.1096	1.01107
$\vdots$	$\vdots$	$\vdots$
0.19	1.2287	1.2313
0.20	1.2433	1.2461

Wow, what a good match! In any case, the answer is (c).

d. $f(0.20) \approx 2.49$

By previous question, the true answer is the answer is $f(x) = e^{\frac{x^2}{2}+x}$.

But our job is to use Euler to come up with an iterative approximation, so here it goes. As usual, we start with $f(x+h) = f(x) + h\,f'(x) = f(x) + h(x+1)\,f(x) = f(x)[1+h(x+1)]$, from which we obtain the following table.

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0.02	1.0202	1.0204
0.03	1.0306	1.0309
0.04	1.0412	1.0416
0.05	1.0521	1.0526
0.06	1.0631	1.0637
0.07	1.0744	1.0751
0.08	1.0859	1.0868
0.09	1.0976	1.0986
0.10	1.1096	1.01107
$\vdots$	$\vdots$	$\vdots$
0.19	1.2287	1.2313
0.20	1.2433	1.2461

Wow, what a good match! In any case, the answer is (c).

Question 5

1 / 1 pts

(Lesson 3.2: Monte Carlo Integration.) Suppose that we want to use Monte Carlo integration to approximate $I = \int_1^3 \frac{1}{1+x}\,dx$. If $U_1, U_2, \ldots, U_n$ are i.i.d. Unif(0,1)'s, what's a good approximation $\bar{I}_n$ for I?

Correct!

a. $\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+U_i}$

In the notation of the lesson, the general approximation we've been using is

$$\begin{aligned}
\bar{I}_n &= \frac{b-a}{n}\sum_{i=1}^{n} g(a+(b-a)U_i) \\
&= \frac{3-1}{n}\sum_{i=1}^{n} g(1+(3-1)U_i) \\
&= \frac{2}{n}\sum_{i=1}^{n} g(1+2U_i) \\
&= \frac{2}{n}\sum_{i=1}^{n} \frac{1}{1+(1+2U_i)} \\
&= \frac{1}{n}\sum_{i=1}^{n} \frac{1}{1+U_i}
\end{aligned}$$

so that the answer has simplified very nicely to (a).

b. $\frac{2}{n}\sum_{i=1}^{n}\frac{1}{1+U_i}$

c. $\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+2U_i}$

d. $\frac{2}{n}\sum_{i=1}^{n}\frac{1}{1+2U_i}$

e. $\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+3U_i}$

In the notation of the lesson, the general approximation we've been using is

$$\begin{aligned}\bar{I}_n &= \frac{b-a}{n}\sum_{i=1}^{n} g(a+(b-a)U_i) \\ &= \frac{3-1}{n}\sum_{i=1}^{n} g(1+(3-1)U_i) \\ &= \frac{2}{n}\sum_{i=1}^{n} g(1+2U_i) \\ &= \frac{2}{n}\sum_{i=1}^{n} \frac{1}{1+(1+2U_i)} \\ &= \frac{1}{n}\sum_{i=1}^{n} \frac{1}{1+U_i}\end{aligned}$$

so that the answer has simplified very nicely to (a).

Question 6

1 / 1 pts

(Lesson 3.2: Monte Carlo Integration.) Again suppose that we want to use Monte Carlo integration to approximate $I = \int_1^3 \frac{1}{1+x}\,dx$. You may have recently discovered that the MC estimator is of the form

$\bar{I}_n = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{1+U_i}$.

Estimate the integral I by calculating $\bar{I}_n$ with the following 4 uniforms:

$U_1 = 0.3 \qquad U_2 = 0.9 \qquad U_3 = 0.2 \qquad U_4 = 0.7$

- a. 0
- b. 0.2
- c. 0.321

Correct!

d. 0.679

$\bar{I}_4 = \frac{1}{4}\sum_{i=1}^{n}\frac{1}{1+U_i} = \mathbf{0.679}$, so the answer is (d).

e. 0.8

$\bar{I}_4 = \frac{1}{4}\sum_{i=1}^{n}\frac{1}{1+U_i} = \mathbf{0.679}$, so the answer is (d).

Question 7

1 / 1 pts

(Lesson 3.2: Monte Carlo Integration.) Yet again suppose that we want to use Monte Carlo integration to approximate $I = \int_1^3 \frac{1}{1+x}\,dx$. What is the *exact* value of I?

a. 0.197

Correct!

b. 0.693

$I = \ln(1+x)|_1^3 = \ln(4) - \ln(2) = \mathbf{0.693.}$ Thus, the answer is (b).

c. 1.386

d. 2.773

$I = \ln(1+x)|_1^3 = \ln(4) - \ln(2) = \mathbf{0.693.}$ Thus, the answer is (b).

Question 8

1 / 1 pts

(Lesson 3.3: Making Some π.) Inscribe a circle in a unit square and toss $n = 1000$ random darts at the square. Suppose that 760 of those darts land in the circle. Using the technology developed in class, what is the resulting estimate for π?

a. π

b. 4.0 (UGA answer)

c. 3.2

Correct!

d. 3.04

The estimate $\hat{\pi}_n = 4 \times (\text{proportion in circle}) = 4(760/1000) = 3.04$ Thus, the answer is (d).

e. 3.12

The estimate $\hat{\pi}_n = 4 \times (\text{proportion in circle}) = 4(760/1000) = 3.04$ Thus, the answer is (d).

Question 9

1 / 1 pts

(Lesson 3.3: Making Some π.) BONUS: Now suppose that we can somehow toss n random darts into a unit *cube*. Further, suppose that we've inscribed a *sphere* with radius 1/2 inside the cube. Let $\hat{p}_n$ be the proportion of the n darts that actually fall within the sphere. Give a Monte Carlo scheme to estimate π.

a. $\hat{\pi}_n = 2\hat{p}_n$

b. $\hat{\pi}_n = \frac{4}{3}\hat{p}_n$

c. $\hat{\pi}_n = 4\hat{p}_n$

Correct!

d. $\hat{\pi}_n = 6\hat{p}_n$

The probability that a dart falls inside the sphere is the volume of the sphere divided by the volume of the unit cube, i.e., $\frac{4}{3}\pi r^3 = \pi/6$. Thus, for large n, we have $\hat{p}_n \approx \pi/6$, so that $\hat{\pi}_n = 6\hat{p}_n$ should do the trick. Therefore, the answer is (d).

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Question 10

1 / 1 pts

(Lesson 3.4: Single-Server Queue.) Consider a single-server Q with *LIFO* (*last*-in-first-out) services. Suppose that three customers show up at times 5, 6, and 8, and that they all have service times of 4. When does customer 2 leave the system?

a. 3

b. 9

c. 13

Correct!

d. 17

Let's make a version of our usual table.

i	A_i	T_i	W_i^Q	S_i	D_i
1	5	5	0	4	9
2	6	13	7	4	**17**
3	8	9	1	4	13

Thus, the answer is (d).

e. 19

Let's make a version of our usual table.

i	A_i	T_i	W_i^Q	S_i	D_i
1	5	5	0	4	9
2	6	13	7	4	**17**
3	8	9	1	4	13

Thus, the answer is (d).

Question 11 Original Score: 1 / 1 pts Regraded Score: 1 / 1 pts

This question has been regraded.

(Lesson 3.5: (s, S) Inventory Model.) Consider our numerical example from the lesson. What would the third day's total revenues have been if we had used a (4,10) policy instead of a (3,10)?

a. -22

b. -13

Correct!

c. 44

Day i	*begin stock*	D_i	I_i	Z_i	*sales rev*	*order cost*	*hold cost*	*penalty cost*	*TOTAL rev*
1	10	5	5	0	50	0	-5	0	45
2	5	2	3	7	20	$-(2+4(7))$	-3	0	-13
3	10	8	2	8	80	$-(2+4(8))$	-2	0	**44**

Thus, the answer is (c).

d. 45

e. 80

Day i	*begin stock*	D_i	I_i	Z_i	*sales rev*	*order cost*	*hold cost*	*penalty cost*	*TOTAL rev*
1	10	5	5	0	50	0	-5	0	45
2	5	2	3	7	20	$-(2+4(7))$	-3	0	-13
3	10	8	2	8	80	$-(2+4(8))$	-2	0	**44**

Thus, the answer is (c).

Question 12

1 / 1 pts

(Lesson 3.6: Simulating Random Variables.) If U is a Unif(0,1) random number, what is the distribution of $-0.5\ell n(U)$?

a. Who knows?

Correct!

b. Exp(2)

By the Inverse Transform Theorem, we know that $-\frac{1}{\lambda}\ell n(1-U) \sim \text{Exp}(\lambda)$. But since U and $1-U$ are both Unif(0,1) (why?), we also have

$-\frac{1}{\lambda}\ell n(U) \ \sim \ \text{Exp}(\lambda).$

In particular,

$-0.5\ell n(U) \ \sim \ \text{Exp}(2),$

so that the answer is (b).

c. Exp(1/2)

d. Exp(-2)

e. Exp$(-1/2)$

By the Inverse Transform Theorem, we know that $-\frac{1}{\lambda}\ell n(1-U) \sim \text{Exp}(\lambda)$. But since U and $1-U$ are both Unif(0,1) (why?), we also have

$-\frac{1}{\lambda}\ell n(U) \ \sim \ \text{Exp}(\lambda).$

In particular,

$-0.5\ell n(U) \ \sim \ \text{Exp}(2),$

so that the answer is (b).

Question 13

0 / 1 pts

(Lesson 3.6: Simulating Random Variables.) If U_1 and U_2 are i.i.d. Unif(0,1) random variables, what is the distribution of $U_1 + U_2$? Hints: (i) I may have mentioned this in class at some point; (ii) You may be able to reason this out by looking at the distribution of the sum of two dice tosses; or (iii) You can use something like Excel to simulate $U_1 + U_2$ many times and make a histogram of the results.

a. Unif(0,2)

ou Answered

b. Normal

c. Exponential

orrect Answer

d. Triangular

By any of the hints, you get a Triangular(0,1,2) distribution, i.e., answer (d).

Question 14

1 / 1 pts

(Lesson 3.7: Spreadsheet Simulation.) I stole this problem from the Banks, Carson, Nelson and Nicol text (5th edition). Expenses for Joey's college attendance next year are as follows (in $):

Tuition = 8400
Dormitory = 5400
Meals $\sim$ Unif(900,1350)
Entertainment $\sim$ Unif(600,1200)
Transportation $\sim$ Unif(200,600)
Books $\sim$ Unif(400,800)

Here are the income streams the student has for next year:
Scholarship = 3000
Parents = 4000
Waiting Tables $\sim$ Unif(3000,5000)
Library Job $\sim$ Unif(2000,3000)

Use Monte Carlo simulation to estimate the expected value of the loan that will be needed to enable Joey to go to college next year.

a. $2500

b. $3250

Correct!

c. $3325

An easy spreadsheet simulation (or an almost-as-easy exact analytical calculation) reveals that the expected loan amount is $3325, or answer (c).

If you don't believe me, here's some Matlab code (if you happen to have Matlab)...

```
m = 1000000; % reps
Income = 7000 + unifrnd(3000,5000,[1 m]) + unifrnd(2000,3000,[1 m]);
Expenses = 13800 + unifrnd(900,1350,[1 m]) + unifrnd(600,1200,[1 m])
+ unifrnd(200,600,[1 m]) + unifrnd(400,800,[1 m]);
Totals = Income - Expenses;
hist(Totals,100)
mean(Totals)
var(Totals)
```

d. $3450

e. $4000

An easy spreadsheet simulation (or an almost-as-easy exact analytical calculation) reveals that the expected loan amount is $3325, or answer (c).

If you don't believe me, here's some Matlab code (if you happen to have Matlab)...

```
m = 1000000; % reps
Income = 7000 + unifrnd(3000,5000,[1 m]) + unifrnd(2000,3000,[1 m]);
Expenses = 13800 + unifrnd(900,1350,[1 m]) + unifrnd(600,1200,[1 m])
+ unifrnd(200,600,[1 m]) + unifrnd(400,800,[1 m]);
Totals = Income - Expenses;
hist(Totals,100)
mean(Totals)
var(Totals)
```

Quiz Score: **13** out of 14