Week 9 Homework

Due Mar 15 at 11:59pm **Points** 18 **Questions** 18
Available after Mar 6 at 8am **Time Limit** None

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	7,764 minutes	15 out of 18

Score for this quiz: **15** out of 18
Submitted Mar 15 at 11:59pm
This attempt took 7,764 minutes.

Question 1 1 / 1 pts

(Lesson 7.1: Introduction to Random Variate Generation.) Unif(0,1) PRNs can be used to generate which of the following random entities?

a. Exp(λ) random variates

b. Nor(0,1) random variates

c. Triangular random variates

d. Bern(p) random variates

e. Nonhomogeneous Poisson processes

Correct!

f. All of the above --- and just about anything else!

(f).

Question 2

1 / 1 pts

(Lesson 7.2: Inverse Transform Theorem --- Intro.) If X is an Exp(λ) random variable with c.d.f. $F(x) = 1 - e^{-\lambda x}$, what's the distribution of the random variable $1 - e^{-\lambda X}$?

Correct!

- a. Unif(0,1)
- b. Nor(0,1)
- c. Triangular
- d. Exp(λ)
- e. None of the above

Note that $1 - e^{-\lambda X} = F(X) \sim \textbf{Unif}(0, 1)$, where the last step follows by the Inverse Transform Theorem. Thus, the correct answer is (a).

Question 3

1 / 1 pts

(Lesson 7.2: Inverse Transform Theorem --- Intro.) If U is a Unif(0,1) random variable, what's the distribution of $-\frac{1}{\lambda}\ln(U)$?

- a. Unif(0,1)
- b. Nor(0,1)
- c. Triangular

Correct!

- d. Exp(λ)

e. None of the above

Since U and $1-U$ are both Unif(0,1) (by symmetry), we have $-\frac{1}{\lambda}\ln(U) \;\sim\; -\frac{1}{\lambda}\ln(1-U) \;\sim\; \text{Exp}(\lambda),$
where the last step follows from Lesson 2's Inverse Transform Theorem example. Thus, the answer is (d).

Question 4

1 / 1 pts

(Lesson 7.2: Inverse Transform Theorem --- Intro.) Suppose that $U_1, U_2, \ldots, U_{5000}$ are i.i.d. Unif(0,1) random variables. Using Excel (or your favorite programming language), simulate $X_i = -\ln(U_i) \text{ for } i = 1, 2, \ldots, 5000$. Draw a histogram of the 5000 numbers. What p.d.f. does the histogram look like?

- a. Uniform
- b. Normal
- c. Triangular
- d. Exponential (Correct!)
- e. Bernoulli

By the Inverse Transform Theorem, all of the X_i's are Exp($\lambda = 1$). Since we have a histogram of 5000 of these, it really ought to look like an exponential p.d.f., $f(x) = e^{-x}$, $x \geq 0$. Thus, the answer is (d).

Question 5

1 / 1 pts

(Lesson 7.3: Inverse Transform --- Continuous Examples.) Suppose the c.d.f. of X is $F(x) = x^3/8$, $0 \le x \le 2$. Develop a generator for X and demonstrate with $U = 0.54$.

- a. $X = U^3/8 = 0.0197$
- b. $X = 8U^3 = 1.260$

Correct!

- c. $X = 2U^{1/3} = 1.629$
- d. $X = 4U^{1/3} = 3.257$
- e. $X = 8U^{1/3} = 6.515$

Set $F(X) = U \sim \text{Unif}(0,1)$. Then $U = X^3/8$, and so $X = 2U^{1/3}$. Plugging in $U = 0.54$, we get $X = 1.629$. Thus, the correct answer is (c)

Question 6

1 / 1 pts

(Lesson 7.3: Inverse Transform --- Continuous Examples.) If X is a Nor(0,1) random variate, and $\Phi(x)$ is the Nor(0,1) c.d.f., what is the distribution of $\Phi(X)$?

Correct!

- a. Uniform
- b. Normal
- c. Triangular

d. Exponential

e. Bernoulli

By the Inverse Transform Theorem, $\Phi(X) \sim \text{Unif}(0,1)$; so the answer is (a).

Question 7

1 / 1 pts

(Lesson 7.3: Inverse Transform --- Continuous Examples.) If U is a Unif(0,1) random variate, and $\Phi(x)$ is the Nor(0,1) c.d.f., what is the distribution of $2\Phi^{-1}(U)+3$?

a. Unif(0,1)

b. Unif(3,2)

c. Nor(0,1)

d. Nor(3,2)

Correct!

e. Nor(3,4)

By the Inverse Transform Theorem,
$2\Phi^{-1}(U)+3 \ \sim \ 2\,\text{Nor}(0,1)+3 \ \sim \ \text{Nor}(3,4)$

Thus, (e) is the correct answer.

Question 8

1 / 1 pts

(Lesson 7.4: Inverse Transform --- Discrete Examples.) How would you simulate the sum of two 6-sided dice tosses? (Note that $\lceil \cdot \rceil$ is the round-up function; and all of the U's denote PRNs.)

a. $12U$

b. $\lceil 12U \rceil$

c. $6U_1 + 6U_2$

Correct!

d. $\lceil 6U_1 \rceil + \lceil 6U_2 \rceil$

e. None of the above

Choice (a) just gives a random real number between 0 and 12. (b) gives a discrete uniform random integer from 1,2,...,12. (c) gives a continuous triangular distribution. Meanwhile, recall that we learned in class that $\lceil 6U \rceil$ is a 6-sided die toss. Thus, since (d) is simply the sum of two of these tosses, it is the correct answer.

Question 9

1 / 1 pts

(Lesson 7.4: Inverse Transform --- Discrete Examples.) If U is Unif(0,1), how can we simulate a Geom(0.6) random variate?

a. $\lceil \ln(U)/\ln(0.4) \rceil$

b. $\lceil \ln(1-U)/\ln(0.4) \rceil$

c. $\lceil \ln(U)/\ln(0.6) \rceil$

d. $\lceil \ln(1-U)/\ln(0.6) \rceil$

Correct!

e. Both (a) and (b)

f. Both (c) and (d)

From the notes, we have
$\left\lceil \frac{\ln(U)}{\ln(1-p)} \right\rceil \sim \left\lceil \frac{\ln(1-U)}{\ln(1-p))} \right\rceil \sim \text{Geom}(p).$
So the answer is (e).

Question 10

1 / 1 pts

(Lesson 7.5: Inverse Transform --- Empirical Distributions.) BONUS: Consider four observations from some unknown distribution, $X_1 = 1.5$, $X_2 = -3.7$, $X_3 = 2.7$, and $X_4 = 0.6$. What is the fourth order statistic, which we denoted by $X_{(4)}$ in class?

a. 1.5

b. -3.7

Correct!

c. 2.7

d. 0.6

$X_{(4)}$ merely means the largest of the sample of 4 observations. Thus, (c) is the correct answer.

Question 11

1 / 1 pts

(Lesson 7.6: Convolution.) Suppose that U and V are PRNs. Let $X = U + V$. Simulate this 5000 times, and draw a histogram of the 5000 numbers. What p.d.f. does the histogram look like?

a. Uniform

b. Normal

Correct!

c. Triangular

d. Exponential

e. Bernoulli

By the lesson notes, we know that the 5000 Xi's are all Triangular(0,1,2). Since we have a histogram of 5000 of these, it really ought to look like a triangular p.d.f. Thus, the answer is (c).

Question 12

0 / 1 pts

(Lesson 7.6: Convolution.) Suppose that $U_1, U_2, \ldots, U_{24}$ are i.i.d. PRNs. What is the approximate distribution of $X = 5 + 3\sum_{i=1}^{24} U_i$?

ou Answered

a. Uniform

b. Nor(0,1)

c. Nor(5,1)

d. Nor(12,2)

e. Nor(41,6)

orrect Answer

f. Nor(41,18)

By the lesson notes regarding the "desert island" normal generator, we know that

$\sum_{i=1}^{24} U_i \approx \text{Nor}(n/2, n/12) \sim \text{Nor}(12, 2)$

Thus,

$X = 5 + 3\sum_{i=1}^{24} U_i \approx 5 + 3\text{Nor}(12, 2) \sim 5 + \text{Nor}(36, 18) \sim \text{Nor}(41, 18).$

Thus, (f) is the correct answer.

Question 13

0 / 1 pts

(Lesson 7.6: Convolution.) If are PRNs, what's the distribution of $-2\ln(U_1^2(1-U_2)^2U_3^2)$?

a. Exp(1/2)

b. Exp(4)

c. $\text{Erlang}_2(1/2)$

orrect Answer

d. $\text{Erlang}_3(1/4)$

ou Answered

e. $\text{Erlang}_3(2)$

Since U and $1 - U$ are both Unif(0,1), we have

$$-2\ln(U_1^2(1-U_2)^2U_3^2) \;=\; -4\ln(U_1(1-U_2)U_3) \;\sim\; -4\ln(U_1U_2U_3) \;\sim\; \textbf{Erlang}_3(1/4).$$

Thus, the answer is (d).

Question 14

1 / 1 pts

(Lesson 7.7: Acceptance-Rejection --- Intro.) In general, the majorizing function $t(x)$ is itself a p.d.f. $f(x)$.

- True
- Correct! False

Since $t(x) \geq f(x)$, we have

$$\int_{\mathbb{R}} t(x)\,dx \;\geq\; \int_{\mathbb{R}} f(x)\,dx \;=\; 1.$$

Thus, the majorizing function generally integrates to a number greater than 1, and so it cannot be a legitimate p.d.f.

Question 15

1 / 1 pts

(Lesson 7.8: Acceptance-Rejection --- Proof.) BONUS: Which of the following are true?

a. The closer the majorizing function $t(x)$ is to the true p.d.f. $f(x)$, the more efficient the A-R algorithm is.

b. $h(y) = t(y) / \int_{\mathbb{R}} t(x)\, dx$ is itself a p.d.f.

c. Random Variates from $h(y)$ should be "easy" to generate.

d. Dave is the best teacher ever!

Correct!

e. All of the above.

(e). [And I hope we agree that (d) is really, really true!]

Question 16

1 / 1 pts

(Lesson 7.9: Acceptance-Rejection --- Continuous Examples.) Suppose that X is a continuous RV with p.d.f. $f(x) = 30x^4(1-x)$, for $0 < x < 1$. What's a good method that you can use to generate a realization of X?

a. Inversion

b. Convolution

c. Box-Muller

Correct!

d. Acceptance-Rejection

e. Composition

(d).

Question 17

0 / 1 pts

(Lesson 7.9: Acceptance-Rejection --- Continuous Examples.) Consider the constant $c = \int_{\mathbb{R}} t(x)\, dx = 5$. On average, how many iterations (trials) will the A-R algorithm require?

ou Answered

- a. 1/5

orrect Answer

- b. 5
- c. 10
- d. 25
- e. None of the above

The number of trials required is Geom($1/c$), which has expected value $c = 5$. Therefore, (b) is our guy.

Question 18

1 / 1 pts

(Lesson 7.10: Acceptance-Rejection --- Poisson Distribution.) Suppose that $U_1 = 0.65$, $U_2 = 0.45, U_3 = 0.82, U_4 = 0.09$, and $U_5 = 0.26$. Use our acceptance-rejection technique from class to generate $N \sim \text{Pois}(\lambda = 3.7)$. (You may not need to use all of the uniforms.)

a. N=0

b. N=1

c. N=2

Correct!

d. N=3

Define $p_n \equiv \prod_{i=1}^{n+1} U_i$. We'll stop as soon as $p_n < e^{-3.7} = 0.0247$. Let's make the following convenient table.

n	U_{n+1}	p_n	Stop?
0	0.65	0.65	nope
1	0.45	0.2925	nope
2	0.82	0.2399	nope
3	0.09	0.0216	yup

So we take N = 3, and the answer is (d).

e. N=4

Define $p_n \equiv \prod_{i=1}^{n+1} U_i$. We'll stop as soon as $p_n < e^{-3.7} = 0.0247$. Let's make the following convenient table.

n	U_{n+1}	p_n	Stop?
0	0.65	0.65	nope
1	0.45	0.2925	nope
2	0.82	0.2399	nope
3	0.09	0.0216	yup

So we take N = 3, and the answer is (d).

Quiz Score: **15** out of 18