Week 2 Homework

Due Jan 26 at 11:59pm **Points** 19 **Questions** 19
Available after Jan 17 at 8am **Time Limit** None

Instructions

Please answer all the questions below.

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	4,541 minutes	16 out of 19

Score for this quiz: **16** out of 19
Submitted Jan 25 at 1:57pm
This attempt took 4,541 minutes.

Question 1

1 / 1 pts

(Lesson 2.1: Derivatives.) BONUS: If $f(x) = \ell n(2x-3)$, find the derivative $f'(x)$.

a. $2x$

b. $\frac{1}{2}\ell n(2x-3)$

Correct!

c. $2/(2x-3)$

This follows by the chain rule,
$f'(x) = [\ell n(2x-3)]' = \frac{(2x-3)'}{2x-3} = \frac{2}{2x-2}$

d. $x/2$

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$f'(x) = [\ell n(2x-3)]' = \frac{(2x-3)'}{2x-3} = \frac{2}{2x-2}$

Question 2

0 / 1 pts

(Lesson 2.1: Derivatives.) BONUS: If $f(x) = \cos(1/x)$, find the derivative $f'(x)$.

a. $\cos(1/x^2)$

b. $\sin(1/x^2)$

ou Answered

c. $-\frac{1}{x^2}\sin(1/x)$

orrect Answer

d. $\frac{1}{x^2}\sin(1/x)$

By the chain rule,
$[\cos(1/x)]' = -\sin(1/x)[1/x]' = \frac{1}{x^2}\sin(1/x)$

Question 3

1 / 1 pts

(Lesson 2.2: Finding Zeroes.) BONUS: Suppose that $f(x) = e^{4x} - 4e^{2x} + 4$. Use any method you want to find a zero of $f(x)$, i.e., x such that $f(x) = 0$.

a. $x = 0$

b. $x = 1$

c. $x = \ell n(2) = 0.693$

Correct!

d. $x = \frac{1}{2}\ell n(2) = 0.347$

This doesn't take too much work. Namely, set
$0 = f(x) = e^{4x} - 4e^{2x} + 4 = (e^{2x} - 2)^2$.
This is the same as $e^{2x} = 2$, or $x = \frac{1}{2}\ell n(2) = 0.347$

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$0 = f(x) = e^{4x} - 4e^{2x} + 4 = (e^{2x} - 2)^2$.
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Question 4

1 / 1 pts

(Lesson 2.3: Integration.) BONUS: Find $\int_0^1 (2x+1)^2 dx$.

a. 1/2

b. 7/2

c. 7/3

Correct!

d. 13/3

We have
$\int_0^1 (2x+1)^2 dx = \frac{(2x+1)^3}{6}\Big|_0^1 = \frac{27}{6} - \frac{1}{6} = 13/3$

We have

$\int_0^1 (2x+1)^2\,dx = \frac{(2x+1)^3}{6}\Big|_0^1 = \frac{27}{6} - \frac{1}{6} = 13/3$

Question 5

1 / 1 pts

(Lesson 2.3: Integration.) BONUS: Find $\int_1^2 e^{2x}\,dx$.

a. 1

b. $e^2 - e$

Correct!

c. 23.6

We have

$\int_1^2 e^{2x} = \frac{1}{2}e^{2x}\Big|_1^2 = 23.60$

d. 46.2

We have

$\int_1^2 e^{2x} = \frac{1}{2}e^{2x}\Big|_1^2 = 23.60$

Question 6

1 / 1 pts

(Lesson 2.3: Integration.) BONUS: Find

$\lim_{x\to 0} \frac{\sin(x)-x}{x}$.

a. 1

Correct!

b. 0

If we let $f(x) = \sin(x) - x$ and $g(x) = x$, then
$\lim_{x \to 0} f(x) = 0$ and
$\lim_{x \to 0} g(x) = 0$, so that
$$\lim_{x \to 0} \frac{f(x)}{g(x)}$$
seems to get us into a 0/0 issue. Thus, we'll need to employ L'Hôspital's Rule:
$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f'(x)}{g'(x)} = \lim_{x \to 0} \frac{\cos(x) - 1}{1} = \frac{0}{1} = 0.$$

c. ∞

d. undetermined

If we let $f(x) = \sin(x) - x$ and $g(x) = x$, then
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Question 7

1 / 1 pts

(Lesson 2.4: Numerical Integration.) BONUS: Find the approximate value of the integral $\int_0^2 (x-1)^2 dx$ using the lesson's form of the Riemann sum with $f(x) = (x-1)^2, a = 0, b = 2$, and $n = 4$.

a. -2

b. 1/3

Correct!

c. 3/4

We have

$\int_0^2 (x-1)^2 dx \approx \frac{b-a}{n} \sum_{i=1}^{n} f(a + \frac{(b-a)i}{n}) = \frac{2}{4} \sum_{i=1}^{4} (\frac{2i}{4} - 1)^2 = 3/4$

Well, this is sort of close to the true integral of 2/3. Of course, we could've done even better if n had been bigger or if we had used the midpoint of each interval instead of the right endpoint.

d. 3

We have

$\int_0^2 (x-1)^2 dx \approx \frac{b-a}{n} \sum_{i=1}^{n} f(a + \frac{(b-a)i}{n}) = \frac{2}{4} \sum_{i=1}^{4} (\frac{2i}{4} - 1)^2 = 3/4$

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Question 8

1 / 1 pts

(Lesson 2.5: Probability Basics.) If $P(A) = P(B) = P(C) = 0.6$ and $A, B,$ and C are independent, find the probability that exactly one of $A, B,$ and C occurs.

a. 0.144

Correct!

b. 0.288

$$\begin{aligned} \mathsf{P}(\textbf{exactly one}) &= \mathsf{P}(A\cap\bar{B}\cap\bar{C})+\mathsf{P}(\bar{A}\cap B\cap\bar{C})+\mathsf{P}(\bar{A}\cap\bar{B}\cap C) \\ &= \mathsf{P}(A)\mathsf{P}(\bar{B})\mathsf{P}(\bar{C})+\mathsf{P}(\bar{A})\mathsf{P}(B)\mathsf{P}(\bar{C})+\mathsf{P}(\bar{A})\mathsf{P}(\bar{B})\mathsf{P}(C) \\ & \textbf{(by independence)} \\ &= (0.6)(0.4)(0.4)+(0.4)(0.6)(0.4)+(0.4)(0.4)(0.6) = 0.288. \end{aligned}$$

You could also have used a binomial distribution argument to solve this problem,
i.e.,

$$\mathsf{P}(\text{exactly one}) \;=\; \binom{3}{1}(0.6)^1(0.4)^2 = 0.288$$

c. 0.576

d. 0.6

e. I'm from The University Of Georgia. Is the answer -3?

$$\begin{aligned} \mathsf{P}(\textbf{exactly one}) &= \mathsf{P}(A\cap\bar{B}\cap\bar{C})+\mathsf{P}(\bar{A}\cap B\cap\bar{C})+\mathsf{P}(\bar{A}\cap\bar{B}\cap C) \\ &= \mathsf{P}(A)\mathsf{P}(\bar{B})\mathsf{P}(\bar{C})+\mathsf{P}(\bar{A})\mathsf{P}(B)\mathsf{P}(\bar{C})+\mathsf{P}(\bar{A})\mathsf{P}(\bar{B})\mathsf{P}(C) \\ & \textbf{(by independence)} \\ &= (0.6)(0.4)(0.4)+(0.4)(0.6)(0.4)+(0.4)(0.4)(0.6) = 0.288. \end{aligned}$$

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$$\mathsf{P}(\text{exactly one}) \;=\; \binom{3}{1}(0.6)^1(0.4)^2 = 0.288$$

Question 9

1 / 1 pts

(Lesson 2.5: Probability Basics.) Toss 3 dice. What's the probability that a "4" will come up exactly twice?

Correct!

a. 5/72

Write out every possible outcome explicitly, or use the following binomial argument: Let X denote the number of times a "4" comes up. Clearly,
$X \sim \mathrm{Bin}(3, \frac{1}{6})$. Thus $\mathrm{P}(X = 2) = \binom{3}{2}(\frac{1}{6})^2(\frac{5}{6})^{3-2} = \frac{5}{72}$.

b. 1/2

c. 13/16

d. 1/8

Write out every possible outcome explicitly, or use the following binomial argument: Let X denote the number of times a "4" comes up. Clearly,
$X \sim \mathrm{Bin}(3, \frac{1}{6})$. Thus $\mathrm{P}(X = 2) = \binom{3}{2}(\frac{1}{6})^2(\frac{5}{6})^{3-2} = \frac{5}{72}$.

Question 10

1 / 1 pts

(Lesson 2.6: Simulating Random Variables.) BONUS: Suppose U and V are independent Uniform(0,1) random variables. (You can simulate these using the RAND() function in Excel, for instance.) Consider the nasty-looking random variable

$$Z \; = \; \sqrt{-2\ell n(U)}\cos(2\pi V),$$

where the cosine calculation is carried out in radians (not degrees). Go ahead and calculate Z. . . don't be afraid. Now, repeat this task 1000 times (easy to do in Excel) and make a histogram of the 1000 Z's. What distribution does this look like?

Correct!

a. Normal

This is the Box-Muller method to generate normal random variables. We'll learn much more about this later on.

b. Unif(0,1)

c. Exponential

d. Weibull

This is the Box-Muller method to generate normal random variables. We'll learn much more about this later on.

Here's some example Matlab code that works well. . .

```
%Matlab code
clear all;close all;clc;
z_vec=zeros(1000,1);
for i = 1:1000
u = rand;
v = rand;
z_vec(i)= sqrt(-2*log(u))*cos(2*pi*v);
end
nbins = 30;
bins = linspace(-5,5,nbins);
histogram(z_vec,bins)
```

Of course, you can do this easily in R, Excel, Python etc.

Question 11

1 / 1 pts

(Lesson 2.7: Great Expectations.) Suppose that X is a discrete random variable having $X=-1$ with probability 0.2, and $X=3$ with probability 0.8. Find $\mathrm{E}[X]$.

a. -1

b. 3

c. 1

Correct!

d. 2.2

$\mathrm{E}[X] = \sum_x xf(x) = (-1)(0.2) + (3)(0.8) = 2.2$ So the answer is (d).

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Question 12

1 / 1 pts

(Lesson 2.7: Great Expectations.) Suppose that X is a discrete random variable having $X = -1$ with probability 0.2, and $X = 3$ with probability 0.8. Find $\mathrm{Var}[X]$.

a. -1

b. 1

Correct!

c. 2.56

In addition to the above work,

$$\mathrm{E}[X^2] = \sum_x x^2 f(x) = ((-1)^2)(0.2) + (3^2)(0.8) = 7.4,$$

so that we have $\mathrm{Var}(X) = \mathrm{E}[X^2] - (\mathrm{E}[X])^2 = 2.56$. So the answer is (c).

d. 5.12

In addition to the above work,

$$\mathrm{E}[X^2] \;=\; \sum_x x^2 f(x) \;=\; ((-1)^2)(0.2) + (3^2)(0.8) \;=\; 7.4,$$

so that we have $\mathrm{Var}(X) = \mathrm{E}[X^2] - (\mathrm{E}[X])^2 = 2.56$. So the answer is (c).

Question 13

1 / 1 pts

(Lesson 2.7: Great Expectations.) Suppose that X is a discrete random variable having $X = -1$ with probability 0.2, and $X = 3$ with probability 0.8. Find $\mathrm{E}[3 - \frac{1}{X}]$.

- a. 3
- b. ∞
- c. -2
- d. 44/15

Correct!

Finally, by LOTUS,

$$\mathrm{E}[1/X] \;=\; \sum_x (1/x) f(x) \;=\; 0.2/(-1) + 0.8/3 \;=\; 1/15,$$

so that $\mathrm{E}[3 - \frac{1}{X}] = 3 - \mathrm{E}[\frac{1}{X}] = \frac{44}{15}$. So the answer is (d).

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$$\mathrm{E}[1/X] \;=\; \sum_x (1/x) f(x) \;=\; 0.2/(-1) + 0.8/3 \;=\; 1/15,$$

so that $\mathrm{E}[3 - \frac{1}{X}] = 3 - \mathrm{E}[\frac{1}{X}] = \frac{44}{15}$. So the answer is (d).

Question 14

1 / 1 pts

(Lesson 2.7: Great Expectations.) Suppose X is a continuous random variable with p.d.f. $f(x) = 4x^3$ for $0 \le x \le 1$. Find $\mathrm{E}[1/X^2]$.

- a. 2/3
- b. 1
- c. 3/2
- d. 2

Correct!

By LOTUS,

$$\mathrm{E}[1/X^2] \;=\; \int_{\mathbb{R}} (1/x^2) f(x)\,dx \;=\; \int_0^1 4x\,dx \;=\; 2.$$

By LOTUS,

$$\mathrm{E}[1/X^2] \;=\; \int_{\mathbb{R}} (1/x^2) f(x)\,dx \;=\; \int_0^1 4x\,dx \;=\; 2.$$

Question 15

1 / 1 pts

(Lesson 2.8: Functions of a Random Variable.) Suppose X is the result of a 5-sided die toss having sides numbered $-2, -1, 0, 1, 2$. Find the probability mass function of $Y = X^2$.

- a. $\mathrm{P}(Y = 1) = \mathrm{P}(Y = 4) = 1/2$
- b. $\mathrm{P}(Y = 1) = \mathrm{P}(Y = 2) = 1/2$

Correct!

c. $P(Y=0) = \frac{1}{5}$, $and\ P(Y=1) = P(Y=4) = \frac{2}{5}$

This follows because

$$P(Y=0) = P(X^2=0) = P(X=0) = 1/5,$$
$$P(Y=1) = P(X^2=1) = P(X=-1)+P(X=1) = 2/5,$$
$$\text{and}$$
$$P(Y=4) = P(X^2=4) = P(X=-2)+P(X=2) = 2/5.$$

No other possible values for $Y = X^2$.

d. $P(Y=-2) = P(Y=-1) = P(Y=0) = P(Y=1) = P(Y=2) = 1/5$

This follows because

$$P(Y=0) = P(X^2=0) = P(X=0) = 1/5,$$
$$P(Y=1) = P(X^2=1) = P(X=-1)+P(X=1) = 2/5,$$
$$\text{and}$$
$$P(Y=4) = P(X^2=4) = P(X=-2)+P(X=2) = 2/5.$$

No other possible values for $Y = X^2$.

Question 16

0 / 1 pts

(Lesson 2.8: Functions of a Random Variable.) Suppose X is a continuous random variable with p.d.f. $f(x) = 2x$ for $0 < x < 1$. Find the p.d.f. $g(y)$ of $Y = X^2$. (This may be easier than you think.)

orrect Answer

a. $g(y) = 1$, for $0 < y < 1$

b. $g(y) = y$, for $0 < x < 1$

c. $g(y) = y^2$, for $-1 < y < 1$

ou Answered

d. $g(y) = x^2$, for $0 < y < 1$

Note that the c.d.f. of X is $F(x) = x^2$ (you can do this in your head). So by the Inverse Transform Theorem, we immediately have that $F(X) = X^2 = Y$ is Unif(0,1), with the p.d.f. $g(y) = 1$.

Question 17

1 / 1 pts

(Lesson 2.9: Jointly Distributed RVs.) Suppose that $f(x, y) = 6x$ for $0 \le x \le y \le 1$. Find $\mathrm{P}(X < 1/2 \text{ and } Y < 1/2)$.

- a. 1
- b. 1/2
- c. 1/4
- d. 1/8

Correct!

$$\begin{aligned} \mathrm{P}(X < 1/2 \text{ and } Y < 1/2) &= \int_0^{1/2} \int_0^y f(x, y)\, dx\, dy \\ &= \int_0^{1/2} \int_0^y 6x\, dx\, dy \\ &= 1/8. \end{aligned}$$

So the answer is (d).

$$\begin{aligned} \mathrm{P}(X < 1/2 \text{ and } Y < 1/2) &= \int_0^{1/2} \int_0^y f(x, y)\, dx\, dy \\ &= \int_0^{1/2} \int_0^y 6x\, dx\, dy \\ &= 1/8. \end{aligned}$$

So the answer is (d).

Question 18

0 / 1 pts

(Lesson 2.9: Jointly Distributed RVs.) Suppose that $f(x, y) = 6x$ for $0 \leq x \leq y \leq 1$. Find the marginal p.d.f. $f_X(x)$ of X.

orrect Answer

a. $6x(1-x)$, for $0 \leq x \leq 1$

ou Answered

b. $6x$, for $0 \leq x \leq 1$

c. $6y$, for $0 \leq x \leq 1$

d. $6x(1-y)$, for $0 \leq x \leq 1$

$f_X(x) = \int_{-\infty}^{\infty} f(x, y)\, dy = \int_x^1 6x\, dy = 6x(1-x)$, for $0 \leq x \leq 1$. So the answer is (a).

Question 19

1 / 1 pts

(Lesson 2.9: Jointly Distributed RVs.) YES or NO? Suppose X and Y have joint p.d.f. $f(x, y) = cxy/(1 + x^2 + y^2)$ for $0 < x < 1$, $0 < y < 1$, and whatever constant c makes the nasty mess integrate to 1. Are X and Y independent?

a. Yes

Correct!

b. No

NO! The lesson has a theorem that says that X,Y are independent if and only if you can write $f(x, y) = a(x)b(y)$ with no funny limits for some functions $a(x)$ and $b(y)$. Can't do such a factorization, so X and Y ain't indep.

NO! The lesson has a theorem that says that X,Y are independent if and only if you can write $f(x, y) = a(x)b(y)$ with no funny limits for some functions $a(x)$ and $b(y)$. Can't do such a factorization, so X and Y ain't indep.

Quiz Score: **16** out of 19