Week 12 Homework

Due Apr 5 at 11:59pm **Points** 17 **Questions** 17
Available Mar 27 at 8am - Apr 5 at 11:59pm 10 days **Time Limit** None

This quiz was locked Apr 5 at 11:59pm.

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	12,175 minutes	11 out of 17

Score for this quiz: **11** out of 17
Submitted Apr 5 at 11:59pm
This attempt took 12,175 minutes.

Question 1

0 / 1 pts

(Lesson 8.1: Introduction to Input Modeling.) It's GIGO time! Let's consider an $M/M/1$ queueing system with Exp(λ) interarrivals and Exp(μ) FIFO services at a single server. You may recall from some class (either this one or stochastic processes) that the steady-state expected cycle time (i.e., the time that the customer is in the system, including wait + service) is $w = 1/(\mu - \lambda)$.

If you were to try this out in Arena, let's say with $\mathtt{EXPO}(10 = 1/\lambda)$ interarrivals and $\mathtt{EXPO}(8 = 1/\mu)$ services (note the notation change between my usual "Exp" and Arena's " $\mathtt{EXPO}$ "), then we'd get $w = 1/(0.125 - 0.1) = 40$. Go ahead, see for yourself in Arena, but make sure that you run the system for 100,000 or so customers so that you can be sure that you're in steady-state!

Finally, here's the GIGO question, which will show what can happen when you mis-model a component of your process: What is the (approximate) steady-state expected cycle time if you have i.i.d. `UNIF(5, 15)` interarrivals instead of $\mathtt{EXPO}(10 = 1/\lambda)$ interarrivals? Note that both interarrival distributions have the same mean (10), but that doesn't necessarily imply that they'll have the same expected cycle times. Hint: You may want to use Arena as described above.

ou Answered

a. about 1

b. about 10

orrect Answer

c. about 23

d. about 40

e. about 62

(c). The`UNIF`case has waaaay smaller tails than the **`EXPO`**, so it's reasonable to assume that the cycle times will tend to be lower for the **`UNIF`** case. In fact, after 100,000 customers in Arena, I got an average time of 23.5. Thus, (c) is the right answer.

Question 2

1 / 1 pts

(Lesson 8.2: Identifying Distributions.) Let's play Name That Distribution!

The number of times a "3" comes up in 10 dice tosses.

a. Bernoulli

Correct!

b. Binomial

c. Geometric

d. Negative Binomial

e. Pareto

(b).

Question 3

1 / 1 pts

(Lesson 8.2: Identifying Distributions.) Name That Distribution!

The number of dice tosses until a 3 comes up.

a. Bernoulli

b. Binomial

Correct!

c. Geometric

d. Negative Binomial

e. Pareto

(c).

Question 4

0 / 1 pts

(Lesson 8.2: Identifying Distributions.) Name That Distribution!

The number of dice tosses until a 3 comes up for the 4th time.

a. Bernoulli

ou Answered

b. Binomial

c. Geometric

orrect Answer

d. Negative Binomial

e. Pareto

(d).

Question 5

1 / 1 pts

(Lesson 8.2: Identifying Distributions.) Name That Distribution!

IQs

a. Uniform

Correct!

b. Normal

c. Exponential

d. Weibull

e. Pareto

(b).

Question 6

1 / 1 pts

(Lesson 8.2: Identifying Distributions.) Name That Distribution!

Cases in which you have limited information, e.g., you only know the min, max, and "most likely" values that a random variable can take.

a. Bernoulli

b. Poisson

Correct!

c. Triangular

d. Weibull

e. Pareto

(c).

Question 7

0 / 1 pts

(Lesson 8.3: Unbiased Point Estimation.) Find the sample variance of $-3, -2, -1, 0, 1, 2, 3$

a. 0

b. 1

ou Answered

c. 4

orrect Answer

d. 14/3

e. 6

$$S^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X})^2 = \frac{1}{n-1}\left(\sum_{i=1}^n X_i^2 - n\bar{X}^2\right) = \frac{1}{6}(28 - 7(0)) = 14/3.$$

So the answer is (d).

Question 8

1 / 1 pts

(Lesson 8.3: Unbiased Point Estimation.) If $X_1, \ldots, X_{10}$ are i.i.d. Pois(6), what is the expected value of the sample variance S^2?

- a. 1/6
- b. 1/36
- c. 6 — Correct!
- d. 36
- e. 60

S^2 is always unbiased for the variance of X_i. Thus, we have $E[S^2] = Var(X_i) = \lambda = 6$. So the answer is (c).

Question 9

0 / 1 pts

(Lesson 8.4: Mean Squared Error.) Suppose that estimator A has bias = 3 and variance = 12, while estimator B has bias -2 and variance = 14. Which estimator (A or B) has the lower mean squared error?

ou Answered

a. A

orrect Answer

b. B

MSE = Bias² + Var, so

$MSE(\mathrm{A}) = 9 + 12 = 21 \quad \text{and} \quad MSE(\mathrm{B}) = 4 + 14 = 18.$

Thus, B has lower MSE.

Question 10

1 / 1 pts

(Lessons 8.5 and 8.6: Maximum Likelihood Estimators.) If $X_1 = 2$, $X_2 = -2$, and $X_3 = 0$ are i.i.d. realizations from a Nor(μ, σ^2) distribution, what is the value of the maximum likelihood estimate for the variance σ^2?

a. 0

b. 1

Correct!

c. 8/3

d. 4

e. None of the above

We know from a class example that

$$\widehat{\sigma^2} = \frac{n-1}{n} S^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - 0)^2 = 8/3.$$

Thus, the answer is (c).

Question 11

0 / 1 pts

(Lessons 8.5 and 8.6: Maximum Likelihood Estimators.) Suppose we observe the Pois(λ) realizations $X_1 = 5, X_2 = 9$ and $X_3 = 1$. What is the maximum likelihood estimate of λ?

a. 0

orrect Answer

b. 5

c. 25

ou Answered

d. 1/5

e. 1/25

I don't remember if we derived the MLE of λ in class or not, but here's how anyway. The likelihood function is

$$L(\lambda) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!} = \frac{e^{-n\lambda}\lambda^{\sum_i x_i}}{\prod_i (x_i!)}.$$

Thus,

$$\ln(L(\lambda)) = -n\lambda + \sum_{i=1}^{n} x_i \ln(\lambda) - \ln\left(\prod_{i=1}^{n}(x_i!)\right).$$

This implies that

$$\frac{d}{d\lambda}\ln(L(\lambda)) = -n + \frac{\sum_{i=1}^{n} x_i}{\lambda}.$$

Setting the derivative to 0 and solving yields $\hat{\lambda} = \bar{x} = 5$. Duh! What a surprise! So the answer is (b).

Question 12

1 / 1 pts

(Lesson 8.7: Invariance Property of MLEs) Suppose that we have a number of observations from a Pois(λ) distribution, and it turns out that the MLE for λ is $\hat{\lambda} = 5$. What's the maximum likelihood estimate of $\Pr(X = 3)$?

Correct!

- a. 0.1404
- b. 5
- c. 25
- d. 1/5
- e. 1/25

By invariance,

$$\widehat{\Pr}(X = k) \;=\; \frac{e^{-\hat{\lambda}}\hat{\lambda}^k}{k!},$$

so that we have

$$\widehat{\Pr}(X = 3) \;=\; \frac{e^{-\bar{x}}\bar{x}^k}{k!} \;=\; \frac{e^{-5}5^3}{3!} \;=\; 0.1404.$$

Thus, the answer is (a).

Question 13

1 / 1 pts

(Lesson 8.8: Method of Moments.) BONUS: Suppose that we observe $X_1 = 5$, $X_2 = 9$, and $X_3 = 1$. What's the method of moments estimate of $E[X^2]$?

- a. 5
- b. 25

Correct!

- c. 35.7
- d. 1/5
- e. 1/25

The MOM estimator for $E[X^2]$ is $\frac{1}{n}\sum_{i=1}^{n} x_i^2 = (25+81+1)/3 = 107/3$. Thus, the answer is (c).

Question 14

0 / 1 pts

(Lesson 8.9: Goodness-of-Fit Tests.) Suppose we're conducting a χ^2 goodness-of-fit test with Type I error rate $\alpha = 0.01$ to determine whether or not 100 i.i.d. observations are from a lognormal distribution with unknown parameters μ and σ^2. If we divide the observations into 5 equal-probability intervals and we observe a g-o-f statistic of $\chi_0^2 = 11.2$, will we ACCEPT (i.e., fail to reject) or REJECT the null hypothesis of lognormality?

ou Answered

- Accept

orrect Answer

- Reject

Note that the χ^2 test has $\nu = n - s - 1 = 5 - 2 - 1 = 2$ degrees of freedom. Then

$\chi_0^2 \;=\; 11.2;\; \chi^2_{0.01,2} \;=\; 9.21,$
so we REJECT even though we're being conservative with such a small α!

Question 15

1 / 1 pts

(Lessons 8.10 and 8.11: χ^2 Goodness-of-Fit Test.) This problem has a really long description, but the question itself will be very short! Be patient!

The number of defects in printed circuit boards is hypothesized to follow a Geometric(p) distribution. A random sample of $n = 70$ printed boards has been collected, and the number of defects observed. Here are the results.

Number of Defects	Observed Frequency
1	34
2	18
3	2
4	9
5	7

It turns out that the MLE of p for the Geom(p) is $\hat{p} = 1/\bar{X}$. (See the following proof if you don't believe me!)

Proof: The likelihood function is

$$L(p) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} (1-p)^{x_i - 1} p = (1-p)^{\sum_{i=1}^{n} x_i - n} p^n.$$

Thus,

$$\ln(L(p)) = \left(\sum_{i=1}^{n} x_i - n\right) \ln(1-p) + n \ln(p),$$

and so,

$\frac{d\ln(L(p))}{dp} = \frac{-\sum_{i=1}^{n} x_i + n}{1-p} + \frac{n}{p} = 0$

Solving for p gives the MLE $\hat{p} = \frac{1}{\bar{X}}$.

So in this particular case, we have:

$$\bar{X} = \frac{1(34)+2(18)+3(2)+4(9)+5(7)}{70} = 2.1,$$

and thus $\hat{p} = 0.476.$

Anyhow, we are interested in performing a χ^2 goodness-of-fit test to test the Geometric hypothesis. To this end, let's get the test statistic, χ_0^2? To do so, let's make a little table, assuming that $\hat{p} = 0.476$ is correct. Note that the expected number of observations having a certain value x is $E_x = n\,\Pr(X = x) = n(1-\hat{p})^{x-1}\hat{p}$. Also note that I've combined the entries in the last row (≥ 5) so the probabilities add up to one.

x	$P(X=x)$	E_x	O_x
1	0.4762	33.33	34
2	0.2494	17.46	18
3	0.1307	9.15	2
4	0.0684	4.79	9
≥ 5	0.0752	5.27	7
	1.0000	70	70

Technically speaking, we really ought to combine the last two cells, since $E_4 = 4.79 < 5$. Let's do so to get the following new-and-improved table.

x	$P(X=x)$	E_x	O_x
1	0.4762	33.33	34
2	0.2494	17.46	18
3	0.1307	9.15	2
≥ 4	0.1436	10.06	16
	1.0000	70	70

Thus, the test statistic is

$$\chi_0^2 = \sum_{x=1}^{4} \frac{(E_x - O_x)^2}{E_x}$$
$$= \frac{(33.33 - 34)^2}{33.33} + \frac{(17.46 - 18)^2}{17.46} + \frac{(9.15 - 2)^2}{9.15} + \frac{(10.06 - 16)^2}{10.06}$$
$$= 9.12.$$

Now, let's use our old friend $\alpha = 0.05$ in our test. Let $k = 4$ denote the number of cells (that we ultimately ended up with) and let $s = 1$ denote the number of parameters we had to estimate. Then we compare against $\chi^2_{0.05,k-s-1} = \chi^2_{0.05,2} = 5.99$. So after all this time, here's my question: Do we ACCEPT (i.e., fail to reject) or REJECT the Geometric hypothesis?

a. Accept

Correct!

b. Reject

Since $\chi_0^2 > \chi^2_{0.05,k-s-1}$, we REJECT. That wasn't so bad, was it?

Question 16

1 / 1 pts

(Lesson 8.12: Kolmogorov--Smirnov Test.) Consider the PRN's $U_1 = 0.1, U_2 = 0.9, \text{and } U_3 = 0.2$. Use Kolmogorov-Smirnov with $\alpha = 0.05$ to test to see if these numbers are indeed uniform. Do we ACCEPT (i.e., fail to reject) or REJECT uniformity?

Correct!

a. Accept

b. Reject

Need to make a little table, where the $R_{(i)}$ denote the ordered PRN's.

i	1	2	3
$R_{(i)}$	0.1	0.2	0.9
$\frac{i}{n} - R_{(i)}$	0.233	0.467	0.1
$R_{(i)} - \frac{i-1}{n}$	0.1	-	0.233

Thus, $D^+ = \max_i\{\frac{i}{n} - R_{(i)}\} = 0.467$, and $D^- = \max_i\{R_{(i)} - \frac{i-1}{n}\} = 0.233$, and so finally the K-S test stat is $D = \max(D^+, D^-) = 0.467$.

The necessary quantile is $D_{\alpha,n} = D_{0.05,3} = 0.708$.

Since $D < D_{\alpha,n}$, we ACCEPT (i.e., fail to reject) uniformity (though it's kind of a joke since it's only based on 3 observations)

Question 17

1 / 1 pts

(Lesson 8.14: Arena Input Analyzer.) You don't have to turn anything in for this, but I'd simply like you to play around with the Arena Input Analyzer. So, did you look at the Input Analyzer? (You should answer YES.)

Correct!

- a. Yes
- b. No

Quiz Score: **11** out of 17