Week 1 Homework

Due Jan 19 at 11:59pm **Points** 13 **Questions** 13
Available after Jan 10 at 8am **Time Limit** None

Instructions

Please answer all the questions below.

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	126 minutes	11 out of 13

Score for this quiz: **11** out of 13
Submitted Jan 15 at 12:35pm
This attempt took 126 minutes.

Question 1

1 / 1 pts

(Lesson 1.3: Deterministic Model.) Suppose you throw a rock off a cliff having height h_0 = 1000 feet. You're a strong bloke, so the initial downward velocity is v_0 = -100 feet/sec (slightly under 70 miles/hr). Further, in this neck of the woods, it turns out there is no friction in the atmosphere - amazing! Now you remember from your Baby Physics class that the height after time t is

$$h(t) = h_0 + v_0 t - 16t^2$$

When does the rock hit the ground?

a. -11.625 sec

b. 2 sec

Correct!

c. 5.375 sec

Set

$$0 \;=\; h(t) \;=\; 1000 - 100t - 16t^2,$$

and solve for t. Quadratics are easy:

$$t \;=\; \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \;=\; \frac{100 \pm \sqrt{100^2 + 4(16)(1000)}}{2(-16)} \;=\; \frac{-100 \pm 272.0}{32} \;=\; 5.375,$$

which we take as the answer since the negative answer doesn't make practical sense.

d. 11.625 sec

e. 10 sec

Set

$$0 \;=\; h(t) \;=\; 1000 - 100t - 16t^2,$$

and solve for t. Quadratics are easy:

$$t \;=\; \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \;=\; \frac{100 \pm \sqrt{100^2 + 4(16)(1000)}}{2(-16)} \;=\; \frac{-100 \pm 272.0}{32} \;=\; 5.375,$$

which we take as the answer since the negative answer doesn't make practical sense.

Question 2

0 / 1 pts

(Lesson 1.3: Stochastic Model.) Consider a single-server queueing system where the times between customer arrivals are independent, identically distributed Exp(λ = 2/hr) random variables; and the service times are i.i.d. Exp(μ = 3/hr). Unfortunately, if a potential arriving customer sees that the server is occupied, he gets mad and leaves the system. Thus, the system can have either 0 or 1 customer in it at any time. This is what's known as an M/M/1/1 queue. If $P(t)$ denotes the probability that a customer is being served at time t, trust me that it can be shown that

$$P(t) \;=\; \tfrac{\lambda}{\lambda+\mu} + \left[P(0) - \tfrac{\lambda}{\lambda+\mu}\right] e^{-(\lambda+\mu)t}.$$

If the system is empty at time 0, i.e., $P(0) = 0$, what is the probability that there will be no people in the system at time 1 hr?

a. 1

b. 2/3

ou Answered

c. 0.397

orrect Answer

d. 0.603

At time $t = 1$, we have

$$\begin{aligned} 1 - P(t) &= 1 - \frac{\lambda}{\lambda+\mu} - \left[P(0) - \frac{\lambda}{\lambda+\mu}\right] e^{-(\lambda+\mu)t} \\ &= \frac{\mu}{\lambda+\mu} - \left[P(0) - \frac{\lambda}{\lambda+\mu}\right] e^{-(\lambda+\mu)t} \\ &= \frac{3}{2+3} - \left[0 - \frac{2}{5}\right] e^{-(5)(1)} \\ &= 0.603 \end{aligned}$$

Question 3

1 / 1 pts

(Lesson 1.4: History.) Harry Markowitz (one of the big wheels in simulation language development) won his Nobel Prize for portfolio theory in 1990, though the work that earned him the award was conducted much earlier in the 1950s. Who won the 1990 Prize with him? You are allowed to look this one up.

Correct!

a. Merton Miller and William Sharpe

for accomplishments in related (but slightly different) subject areas.

b. Henry Kissinger

c. Albert Einstein

d. Subrahmanyan Chandrasekhar

(a) for accomplishments in related (but slightly different) subject areas.

Question 4

1 / 1 pts

(Lesson 1.5: Applications.) Which of the following situations might be good candidates to use simulation? (There may be more than one correct answer.)

a. We put $5000 into a savings account paying 2% continuously compounded interest per year, and we are interested in determining the account's value in 5 years.

Correct!

b. We are interested in investing one half of our portfolio in fixed-interest U.S. bonds and the remaining half in a stock market equity index. We have some information concerning the distribution of stock market returns, but we do not really know what will happen in the market with certainty.

Correct!

- [x] c. We have a new strategy for baseball batting orders, and we would like to know if this strategy beats other commonly used batting orders (e.g., a fast guy bats first, a big, strong guy bats fourth, etc.). We have information on the performance of the various team members, but there's a lot of randomness in baseball.

- [ ] d. We have an assembly station in which "customers" (for instance, parts to be manufactured) arrive every 5 minutes exactly and are processed in precisely 4 minutes by a single server. We would like to know how many parts the server can produce in a hour.

Correct!

- [x] e. Consider an assembly station in which parts arrive randomly, with independent exponential interarrival times. There is a single server who can process the parts in a random amount of time that is normally distributed. Moreover, the server takes random breaks every once in a while. We would like to know how big any line is likely to get.

Correct!

- [x] f. Suppose we are interested in determining the number of doctors needed on Friday night at a local emergency room. We need to insure that 90% of patients get treatment within one hour.

(a) and (d) do not require simulation, since we can easily "solve" those models with a simple equation or two. (b), (c), (e), and (f) will likely require simulation.

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Question 5

1 / 1 pts

(Lessons 1.6 and 1.7: Baby Examples.) The planet Glubnor has 50-day years.

Suppose there are 2 Glubnorians in the room. What's the probability that they'll have the same birthday?

a. $1/(49 \cdot 50)$

Correct!

b. 1/50

b. Let's call the two guys A and B. Whatever A's birthday is, the probability that B matches it is 1/50.

Let's try it another way. The total number of ways that two people can have birthdays is 50 × 50 = 2500. The total number of ways that they can have two *different* birthdays is 50 × 49 = 2450. Thus,

$$\mathsf{P}(\text{match}) = 1 - \mathsf{P}(\text{no matches}) = 1 - \frac{2450}{2500} = 1/50.$$

c. 1/25

d. 2/49

b. Let's call the two guys A and B. Whatever A's birthday is, the probability that B matches it is 1/50.

Let's try it another way. The total number of ways that two people can have birthdays is 50 × 50 = 2500. The total number of ways that they can have two *different* birthdays is 50 × 49 = 2450. Thus,

$$\mathsf{P}(\text{match}) = 1 - \mathsf{P}(\text{no matches}) = 1 - \frac{2450}{2500} = 1/50.$$

Question 6

1 / 1 pts

(Lessons 1.6 and 1.7: Baby Examples.) The planet Glubnor has 50-day years.

Now suppose there are 3 Glubnorians in the room. (They're big, so the room is getting crowded.) What's the probability that at least two of them have the same birthday?

a. 1/50

b. 2/50

c. 1/(49 · 50)

Correct!

d. 0.0592

d. I admit that this involves a teensy bit of probability (that you will eventually review in Module 2), but it should be easy enough. Mimicking the previous question, we have

$$1 - \mathsf{P}(\textsf{no matches}) = 1 - \frac{50(49)(48)}{50^3} = 0.0592.$$

d. I admit that this involves a teensy bit of probability (that you will eventually review in Module 2), but it should be easy enough. Mimicking the previous question, we have

$$1 - \mathsf{P}(\textsf{no matches}) = 1 - \frac{50(49)(48)}{50^3} = 0.0592.$$

Question 7

1 / 1 pts

(Lessons 1.6 and 1.7: Baby Examples.) Inscribe a circle in a unit square and toss $n = 500$ random darts at the square.

Suppose that 380 of those darts land in the circle. Using the technology developed in this lesson, what is the resulting estimate for π?

a. -3.14

b. 2.82

Correct!

c. 3.04

(c), since the estimate $\hat{\pi} = 4$ × (proportion in circle).

d. 3.14

e. 3.82

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Question 8

1 / 1 pts

(Lessons 1.6 and 1.7: Baby Examples.) Again inscribe a circle in a unit square and toss n random darts at the square.

What would our estimate be if we let $n \to \infty$ and we applied the same ratio strategy to estimate π?

Correct!

a. π

by the Law of Large Numbers.

b. $\pi/2$

c. 3.04

d. 3.14

e. 2π

(a). by the Law of Large Numbers.

Question 9

0 / 1 pts

(Lessons 1.6 and 1.7: Baby Examples.) Suppose customers arrive at a single-server ice cream parlor times 3, 6, 15, and 17. Further suppose that it takes the server 7, 9, 6, and 8 minutes, respectively, to serve the four customers. When does customer 4 leave the shoppe?

a. 18

ou Answered

b. 25

orrect Answer

c. 33

d. 45

(c).

Here is the sequence of relevant events

customer	arrive time	start service	service time	depart time
1	3	3	7	10
2	6	10	9	19
3	15	19	6	25
4	17	25	8	**33**

Question 10

1 / 1 pts

(Lesson 1.8: Generating Randomness.) Suppose we are using the (awful) pseudo-random number generator

$$X_i = (5X_{i-1} + 1)\text{mod}(8),$$

with starting value ("seed") $X_0 = 1$. Find the second PRN, $U_2 = X_2/m = X_2/8$.

a. 0

b. 1/8

Correct!

c. 7/8

We have

$$X_1 = (5X_0 + 1)\text{mod}(8) = 6\text{ mod}(8) = 6$$
$$\text{and then}$$
$$X_2 = (5X_1 + 1)\text{mod}(8) = 31\text{ mod}(8) = 7$$
$$\text{So } U_2 = X_2/8 = 7/8$$

d. 3

(c).

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$$X_1 = (5X_0 + 1)\text{mod}(8) = 6\ \text{mod}(8) = 6$$

and then

$$X_2 = (5X_1 + 1)\text{mod}(8) = 31\ \text{mod}(8) = 7$$

So $U_2 = X_2/8 = 7/8$

Question 11

1 / 1 pts

(Lesson 1.8: Generating Randomness.) Suppose we are using the "decent" pseudo-random number generator

$$X_i = 16807X_{i-1}\text{mod}(2^{31} - 1),$$

with seed X_0 = 12345678. Find the resulting integer X_1. Feel free to use something like Excel if you need to.

a. 352515241

b. 16808

Correct!

c. 1335380034

This is actually not quite so easy as it may seem, since you have to be a little careful not to lose significant digits. We'll learn more about this in Module 6. In any case,

$X_1 = 16807(12345678)\text{mod}(2^{31} - 1) = 207493810146\ \text{mod}(2^{31} - 1) = 1335380034,$

where I multiplied the big numbers and took the mod with the help of Excel.

d. 12345679

(c).

This is actually not quite so easy as it may seem, since you have to be a little careful not to lose significant digits. We'll learn more about this in Module 6. In any case,
$X_1 = 16807(12345678)\text{mod}(2^{31}-1) = 207493810146 \text{ mod}(2^{31}-1) = 1335380034,$
where I multiplied the big numbers and took the mod with the help of Excel.

Question 12

1 / 1 pts

(Lesson 1.8: Generating Randomness.) Suppose that we generate a pseudo-random number U = 0.128. Use this to generate an Exponential $(\lambda = 1/3)$ random variate.

a. -6.17

Correct!

b. 6.17

From the lesson notes, we have
$X = -(1/\lambda)\ell n(U) = -3\ell n(0.128) = 6.17.$
So the answer is (b).Note: It turns out that
$X = -(1/\lambda)\ell n(1-U) = -3\ell n(0.872) = 0.411.$
would also have been an acceptable answer. Can you see why?

c. -0.685

d. 0.685

(b).

From the lesson notes, we have
$X = -(1/\lambda)\ell n(U) = -3\ell n(0.128) = 6.17.$
So the answer is (b).Note: It turns out that
$X = -(1/\lambda)\ell n(1-U) = -3\ell n(0.872) = 0.411.$
would also have been an acceptable answer. Can you see why?

Question 13

1 / 1 pts

(Lesson 1.9: Output Analysis.) BONUS: Which scenarios are most apt for a steady-state analysis? (More than one answer may be right.)

- [ ] a. We simulate a bank from noon till 1:00 pm.

Correct!

- [x] b. We investigate a production line that runs 24/7.

- [ ] c. We are interested in seeing what our portfolio is likely to be 3 months from now.

Correct!

- [x] d. We try to estimate the unemployment rate 30 years from now.

(b) and (d), which deal with long-term phenomena.

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Quiz Score: **11** out of 13