# Week 13 Homework

**Due** Apr 12 at 11:59pm **Points** 9 **Questions** 9
**Available** Apr 3 at 8am - Apr 12 at 11:59pm 10 days **Time Limit** None

This quiz was locked Apr 12 at 11:59pm.

## Attempt History

| | Attempt | Time | Score |
|---|---|---|---|
| LATEST | Attempt 1 | 2,209 minutes | 7 out of 9 |

Score for this quiz: **7** out of 9
Submitted Apr 12 at 11:59pm
This attempt took 2,209 minutes.

### Question 1

**1 / 1 pts**

(Lesson 9.1: Introduction to Output Analysis.) Which of the following problems might best be characterized by a finite-horizon simulation?

a. Simulating long-term hurricane patterns

b. Simulating a manufacturing cell 24/7/365

Correct!

c. Simulating the operations of a bank from 9:00 a.m. until 5:00 p.m.

d. Simulating the steady-state distribution of a Markov chain

(c). [All of the other answers involve long-run, steady-state simulations.]

## Question 2

1 / 1 pts

(Lesson 9.1: Introduction to Output Analysis.) Let's run a simulation whose output is a sequence of daily inventory levels for a particular product. Which of the following statements is true?

a. The consecutive daily inventory levels are independent.

b. The consecutive daily inventory levels are uncorrelated.

c. The consecutive daily inventory levels are normally distributed.

Correct!

d. The consecutive daily inventory levels may not be identically distributed.

Just (d). Consecutive inventory levels are rarely i.i.d. normal

## Question 3

1 / 1 pts

(Lesson 9.2: A Mathematical Interlude.) BONUS: Suppose that $X_1, X_2, \ldots$ is a stationary (steady-state) stochastic process with covariance function $R_k \equiv Cov(X_1, X_{1+k})$, for $k = 0, 1, \ldots$. We know from class that the variance of the sample mean can be represented as $Var(\bar{X}_n) = \frac{1}{n}\left[R_0 + 2\sum_{k=1}^{n-1}\left(1 - \frac{k}{n}\right)R_k\right]$.

We also know from class that for a simple AR(1) process, we have $R_k = \phi^k$, $k = 0, 1, 2, \ldots$ Compute $Var(\bar{X}_n)$ for an AR(1) process with $n = 3$ and $\phi = 0.8$.

a. -0.831

b. -0.5

c. 0

d. 0.5

Correct!

e. 0.831

$$\begin{aligned} Var(\bar{X}_n) &= \frac{1}{3}\left[R_0 + 2\sum_{k=1}^{2}\left(1-\frac{k}{3}\right)R_k\right] \\ &= \frac{1}{3}\left[R_0 + 2\left(1-\frac{1}{3}\right)R_1 + 2\left(1-\frac{2}{3}\right)R_2\right] \\ &= \frac{1}{3}\left[\phi^0 + \frac{4}{3}\phi^1 + \frac{2}{3}\phi^2\right] \\ &= \frac{1}{3}\left[1 + \frac{4}{3}(0.8) + \frac{2}{3}(0.64)\right] \;=\; 0.831. \end{aligned}$$

Thus, the answer is (e).

## Question 4

**1 / 1 pts**

(Lesson 9.2: A Mathematical Interlude.) TRUE or FALSE? Using the notation of the previous question,

$$\lim_{n\to\infty} nVar(\bar{X}_n) \;=\; R_0 + 2\sum_{k=1}^{\infty} R_k \;=\; \sum_{k=-\infty}^{\infty} R_k.$$

Correct!

True

False

Just let the $n$'s get big (though you have to be a little non-rigorous , which I'll allow in this class since I like you.)

## Question 5

1 / 1 pts

(Lesson 9.3: Finite-Horizon Analysis.) Suppose we want to estimate the expected average waiting time for the first $m = 100$ customers at a bank. We make $r = 4$ independent replications of the system, each initialized empty and idle and consisting of 100 waiting times. The resulting replicate means are:

| $i$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $Z_i$ | 5.2 | 4.3 | 3.1 | 4.2 |

Find a 90% confidence interval for the mean average waiting time for the first 100 customers.

a. [4.2,4.3]

Correct!

b. [3.188,5.212]

c. 4.2

d. $3.5 \pm 2$

e. $4.2 \pm 2$

The sample mean and sample variance of the 4 replicate means are easily calculated as $\bar{Z}_5 = 4.2$ and $S_Z^2 = 0.74$. For level $\alpha = 0.10$, we have $t_{0.05,3} = 2.353$, and so the CI is:

$$\mu \in \bar{Z} \pm t_{\alpha/2,r-1}\sqrt{\frac{S_Z^2}{r}} = 4.2 \pm (2.353)\sqrt{\frac{0.74}{4}} = 4.2 \pm 1.012 = [3.188, 5.212].$$

Thus, the answer is (b).

## Question 6

**0 / 1 pts**

(Lesson 9.6: Steady-State Analysis.) Consider a particular data set of 100,000 stationary waiting times obtained from a large queueing system. Suppose your goal is to get a confidence interval for the unknown mean. Would you rather use (a) 50 batches of 2000 observations or (b) 10000 batches of 10 observations each?

orrect Answer

- (a)

ou Answered

- (b)

Recall that the method of batch means requires a very large batch size; and, further, the notes sort of recommend at least 30 batches, (but not necessarily more at the cost of smaller-than-needed batch size). Thus, (a) is the way to go!

## Question 7

**1 / 1 pts**

(Lesson 9.6: Steady-State Analysis.) Suppose $[0, 4]$ is a 95% nonoverlapping batch means confidence interval for the mean $\mu$ based on

20 batches of size 500. Now the boss has decided that she wants a 90% CI based on those same 20 batches of size 500. What is it?

- a. [0,4]
- b. [-1,5]

Correct!

- c. [0.348, 3.652]
- d. [0.948, 3.052]
- e. [-1,5]

$$[0,4] \;=\; \bar{X} \pm t_{\alpha/2,b-1}\sqrt{\widehat{V}_B/n}.$$

This implies that $\bar{X} = 2$ and the half-length is $t_{0.025,19}\sqrt{\widehat{V}_B/n} = 2$. Thus, the new 90% confidence interval is

$$\begin{aligned}\text{new CI} &= \bar{X} \pm t_{0.05,19}\sqrt{\widehat{V}_B/n} \\ &= 2 \pm \frac{t_{0.05,19}}{t_{0.025,19}} t_{0.025,19}\sqrt{\widehat{V}_B/n} \\ &= 2 \pm \frac{1.729}{2.093} \times 2 \\ &= 2 \pm 1.652 \;=\; [0.348,\ 3.652].\end{aligned}$$

Thus, the answer is (c).

## Question 8

1 / 1 pts

(Lesson 9.7: Properties of Batch Means.) (You can do this problem without watching Lesson 9.7. You can do it!)

Consider the output analysis method of non overlapping batch means. Assuming that you have a sufficiently large batch size, it can be shown that when the number of batches $b$ is even, the expected width of the 90%

two-sided confidence interval for $\mu$ is proportional to

$$\frac{t_{0.05,b-1}}{\sqrt{b-1}} \frac{\left(\frac{b-1}{2}\right)\left(\frac{b-3}{2}\right)\cdots\frac{1}{2}}{\left(\frac{b-2}{2}\right)!}.$$

Using the above equation, determine which of the following values of $b$ gives the smallest expected width.

a. b=2

b. b=4

Correct!

c. b=6

d. 2b or not 2b, that is a question.

e. Do b do b do. See www.youtube.com/watch?v=hlSbSKNk9f0 at time 2:23.

Let $h(b)$ denote the value of the above expression as a function of $b$. Then easy calculations reveal that $h(b) = 3.157$, $h(4) = 1.019$, and $h(6) = 0.845$. So the answer is $b = 6$, that is, (c)

## Question 9

**0 / 1 pts**

(Lesson 9.8: Other Steady-State Methods.) Consider the following observations:

54 70 75 62

If we choose a batch size of 3, calculate all of the overlapping batch means for me.

a. 65.25

b. 62.0, 68.5

orrect Answer

c. 66.3, 69.0

ou Answered

d. 65.25 $\pm$ 3

e. None of the above

$\bar{X}^o_{1,3} = \frac{1}{3}\sum_{i=1}^{3} X_i = 66.3$ and $\bar{X}^o_{2,3} = \frac{1}{3}\sum_{i=2}^{4} X_i = 69.0$.
Thus, the answer is (c).

Quiz Score: **7** out of 9