Week 8 Homework

Due Mar 8 at 11:59pm **Points** 12 **Questions** 12
Available after Feb 28 at 8am **Time Limit** None

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	10,509 minutes	11 out of 12

Score for this quiz: **11** out of 12
Submitted Mar 8 at 11:59pm
This attempt took 10,509 minutes.

Question 1

1 / 1 pts

(Lesson 6.1: Introduction to Uniform Random Numbers.) TRUE or FALSE? It's actually a good thing for you to be able to reproduce a sequence of PRNs, should you so desire.

Correct!

- (selected) True
- False

TRUE. [You'd better to be able to reproduce your work if your boss doesn't believe you!]

Question 2

1 / 1 pts

(Lesson 6.2: Some Lousy Generators.) Consider von Neumann's mid-square PRN method, and suppose that $X_0 = 6632$. What is $R_3 = X_3/10000$? [Note: For purposes of this problem, treat all X_i's as if they were 8 digits, e.g., treat 123456 as if it were 00123456.]

a. 9834

b. 0.9834

c. 50055625

d. 556

Correct!

e. 0.0556

Following the notes, $X_0 = 6632$, so that $X_0^2 = 43983424$, so that $X_1 = 9834$. Similarly, $X_2 = 7075$ and $X_3 = 0556$, so that $R_3 = 0.0556$. Thus, (e) is the correct answer.

Question 3

1 / 1 pts

(Lesson 6.3: Linear Congruential Generators.) YES or NO? Does $X_i = (X_{i-1} + 12)\mathbf{mod}(13)$ have full period?

Correct!

True

False

YES! [It trivially cycles through $0, 1, 2, \cdots, 11$.]

Question 4

1 / 1 pts

(Lesson 6.3: Linear Congruential Generators. Problem 7.1 from Law 2015). Consider the generator

$X_i = (5X_{i-1} + 3)\mathbf{mod}(16)$.

Starting from $X_0 = 7$, find X_{500}.

- a. 0
- b. 6
- c. 7
- Correct! d. 11
- e. 38

If $X_0 = 7$, then $X_1 = 38\mathbf{mod}(16) = 6$, $X_2 = 1$, $X_3 = 8$, ..., $X_{16} = 7$. So the cycle repeats itself every 16 iterations. This implies that $X_{32} = 7 = X_{48} = \cdots = X_{496}$, and so $X_{500} = X_{496+4} = X_4 = 11$. Thus, the answer is (d).

Question 5

1 / 1 pts

(Lesson 6.3: Linear Congruential Generators.) Which uniform generator was recommended in class, at least as a ``desert island'' generator?

- a. $X_i = 16807X_{i-1}\mathbf{mod}(2^{31})$
- Correct! b. $X_i = 16807X_{i-1}\mathbf{mod}(2^{31} - 1)$

c. $X_i = 16807(X_{i-1} - 1)\mathbf{mod}(2^{31})$

d. $X_i = 16807(X_{i-1} - 1)\mathbf{mod}(2^{31} - 1)$

(b).

Question 6

1 / 1 pts

(Lesson 6.4: Tausworthe Generators.) Suppose that a Tausworthe generator gave you the series of bits 1010101. If you use all 7 bits, what Unif(0,1) random number would that translate to?

a. 0.3825

b. 0.5

Correct!

c. 0.6641

d. 0.9826

(c). Using the usual base-2 notation, we have

$$\frac{1010101_2}{2^7} = \frac{85}{128} = 0.6641$$

Question 7

1 / 1 pts

(Lesson 6.5: Generalizations of LCGs.) TRUE or FALSE? There are some great PRN generators out there with incredible cycle lengths $\approx 2^{191}$ and even 2^{19937}!

Correct!

- [x] True
- [ ] False

TRUE! Amazing!

Question 8

1 / 1 pts

(Lesson 6.6: Choosing a Generator --- Theory.) Which of the following statements about the RANDU generator is true?

- [ ] a. Something just ain't right about that boy.
- [ ] b. The generator is given by $X_i = 65539 X_{i-1} \,\mathbf{mod}(2^{31})$
- [ ] c. The PRNs appear at first glance to be uniform, but funny things happen when you look at the plots of the PRNs in multiple dimensions.
- [ ] d. The PRNs are distributed on just 15 hyperplanes.

Correct!

- [x] e. All of the above.

(e).

Question 9

1 / 1 pts

(Lesson 6.7: Statistical Considerations - Intro.) Suppose the guy on trial is actually guilty but you incorrectly acquit him. So you've incorrectly accepted the null hypothesis of innocence. What type of error have you just made - Type I or Type II?

a. Type I

Correct!

b. Type II

(b). You've incorrectly accepted the null hypothesis of innocence.

Question 10

1 / 1 pts

(Lesson 6.8: Goodness-of-Fit Tests.) Suppose we observe 1000 PRNs to obtain the following data.

interval	$[0.00, 0.25)$	$[0.25, 0.50)$	$[0.50, 0.75)$	$[0.75, 1.0]$
number observed	240	255	243	262

Conduct a χ^2 goodness-of-fit test to see if these numbers are approximately Unif(0,1). Use level of significance $\alpha = 0.05$.
Here are some table entries that you may need: $\chi^2_{0.05,3} = 7.81$, $\chi^2_{0.05,4} = 9.49$, and $\chi^2_{0.05,5} = 11.1$. ACCEPT or REJECT?

Correct!

a. Accept

b. Reject

The expected number of observations per equal-probability cell is $E_i = n/k = 1000/4 = 250$. This gives us the following augmented table.

interval i	$[0.00, 0.25)$	$[0.25, 0.50)$	$[0.50, 0.75)$	$[0.75, 1.0]$
number observed O_i	240	255	243	262
expected number E_i	250	250	250	250

Thus, the χ^2 goodness-of-fit statistic is

$$\chi_0^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} = \frac{100+25+49+144}{250} = 1.272$$

Meanwhile, the appropriate quantile $\chi^2_{\alpha,k-1} = \chi^2_{0.05,3} = 7.81$. Since $\chi_0^2 < \chi_\alpha^2$, we ACCEPT H_0. In other words, we are willing to assume that the numbers are approximately Unif(0,1).

Question 11

1 / 1 pts

(Lesson 6.9: Runs Tests for Independence.) Consider the following $n = 30$ PRNs. (Read from left to right, and then down.)

0.79	0.68	0.46	0.69	0.90	0.93	0.99	0.86	0.33	0.22
0.60	0.18	0.59	0.38	0.69	0.76	0.91	0.62	0.22	0.19
0.11	0.45	0.72	0.88	0.65	0.55	0.31	0.27	0.46	0.89

Let's conduct a runs *up and down* test to test H_0: the U_i's are independent with level α = 0.05. ACCEPT or REJECT?

a. Accept

Correct!

b. Reject

Solution: Let's associate $+$ and $-$ for up and down, respectively. Then the 30 PRNs translate to

$$-- \; ++++ \; --- \; + \; - \; + \; - \; +++ \; ---- \; +++ \; ---- \; ++$$

so that we have $A = 12$ runs up and down.

Recall that

$$A \sim \text{Nor}\left(\frac{2n-1}{3}, \frac{16n-29}{90}\right) \sim \text{Nor}\,(19.67,\, 5.01)\,.$$

so that the test stat is

$$Z_0 = \frac{A - \mathsf{E}[A]}{\sqrt{\mathsf{Var}(A)}} = \frac{12 - 19.67}{\sqrt{5.01}} = -3.43.$$

Meanwhile, the appropriate quantile is $z_{\alpha/2} = 1.96$.

Since $Z_0 > z_{\alpha/2}$, we REJECT H_0. In other words, these fellas probably ain't indep.
$\square$

Question 12

0 / 1 pts

(Lesson 6.9: Runs Tests for Independence.) Suppose that U_1, U_2, U_3 are i.i.d. Unif(0,1). Let's denote the number of runs up-and-down by X. Find the EXACT distribution of X.

a. X ~ Unif(0,1)

ou Answered

b. X~ Norm(0,1)

c. $\Pr(X=0) = 0.2, \Pr(X=1) = 0.3, \Pr(X=2) = 0.3, \Pr(X=3) = 0.2$

d. $\Pr(X=1) = 0.5, \Pr(X=2) = 0.5$

orrect Answer

e. $\Pr(X=1) = 1/3,\; \Pr(X=2) = 2/3$

Solution: We write out every possible permutation of the order of the three U_i's, each of which will have equal probability of occurring.

event	+/−	number of runs
$U_1 < U_2 < U_3$	$++$	1
$U_1 < U_3 < U_2$	$+-$	2
$U_2 < U_1 < U_3$	$-+$	2
$U_2 < U_3 < U_1$	$+-$	2
$U_3 < U_1 < U_2$	$-+$	2
$U_3 < U_2 < U_1$	$--$	1

Hence, in the case of three PRNs, there is 1 run with probability 1/3 and two runs w.p. 2/3. Thus, the answer is (e). $\square$

Quiz Score: **11** out of 12