Q3

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$$argmin||X - \hat{X}||_F^2 where \hat{X} = \sum_r^R \lambda_r A_r^{(1)} \circ A_r^{(2)} ... \circ A_r^{(d)}$$

Assuming a third order matrix, the CP decomposition can be rewritten by factor matrices in matrix form:

$$\hat{X}_{(1)} \approx A^{(1)} \Lambda (A^{(3)} \odot A^{(2)})^T$$
$$\hat{X}_{(2)} \approx A^{(2)} \Lambda (A^{(3)} \odot A^{(1)})^T$$
$$\hat{X}_{(3)} \approx A^{(3)} \Lambda (A^{(2)} \odot A^{(1)})^T$$

Where $\odot$ is Khatri Rao product and $X_{(1)}$ denotes the first mode of X and X and $A^{(i)} \forall i \in [d$ are known.
 Since The Khatri-Rao product is the matching columnwise Kroenecker product the above Khatri-Rao products can be rewritten as vecotor Kroenecker products:

$$\hat{X}_{(1)} \approx A^{(1)} \Lambda (a_1^{(3)} \otimes a_1^{(2)} ... a_k^{(3)} \otimes a_k^{(2)})^T$$
$$\hat{X}_{(2)} \approx A^{(3)} \Lambda (a_1^{(3)} \otimes a_1^{(1)} ... a_k^{(3)} \otimes a_k^{(1)})^T$$
$$\hat{X}_{(3)} \approx A^{(3)} \Lambda (a_1^{(2)} \otimes a_1^{(1)} ... a_k^{(2)} \otimes a_k^{(1)})^T$$

$$argmin||X_1 - \hat{X}_1||_F^2 where \hat{X}_1 = A^{(1)} \Lambda (a_1^{(3)} \otimes a_1^{(2)} ... a_k^{(3)} \otimes a_k^{(2)})^T$$