Forecasting Models with R

Section 3: Moving Averages and Exponential Smoothing Methods

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Moving Averages and Exponential Smoothing Methods Overview

- **Moving averages (MA) and exponential smoothing (ETS) methods** are used for flattening time series data.
- In moving averages (MA) previous data have equal weighted influence on current forecast.
- And in exponential moving averages (EMA) or exponentially weighted moving averages (EWMA) they have exponentially decaying influence the older they become.
- Exponential smoothing methods are a special case of the last. Their notation is ETS (error, trend, seasonality) and each of them can be either none (N), additive (A), additive damped (Ad), multiplicative (M), multiplicative damped (Md).

Simple Moving Average

- **Simple moving average (SMA)** consists of forecasted value equal to the arithmetic mean of previous period's data.
- **SMA (k)**

$$\hat{y}_t = SMA = \frac{1}{k} * \sum_{t=0}^{k} y_{t-k-1}$$

Brown's Simple Exponential Smoothing

- **Brown's simple exponential smoothing (SES)** is appropriate for forecasting data with no trend or seasonal patterns. (Robert G. Brown. "Exponential Smoothing for Predicting Demand". *Arthur D. Little Inc*. 1956)
- ***ETS(A,N,N) : error = additive, trend = none, seasonality = none.***

$$\widehat{y}_t = l_t$$
$$l_t = \alpha * y_{t-1} + (1-\alpha) * \widehat{y}_{t-1}$$
$$initial\ l_t = y_{t-1}$$
$$0 \le \alpha \le 1$$

Holt's Linear Trend Method

- **Holt's linear trend method** is appropriate for forecasting data with linear trend pattern (Charles C. Holt. "Forecasting Trends and Seasonal by Exponentially Weighted Averages". *Office of Naval Research Memorandum*. 1957.)
- ***ETS(A,A,N) : error = additive, trend = additive, seasonality = none.***

$$\hat{y}_t = l_t + b_t$$

$$l_t = \alpha * y_t + (1 - \alpha) * (l_{t-1} + b_{t-1})$$
$$initial\ l_t = y_{t-1}$$
$$0 \le \alpha \le 1$$

$$b_t = \beta * (l_t - l_{t-1}) + (1 - \beta) * b_{t-1}$$
$$initial\ b_t = y_{t-1} - y_{t-2}$$
$$0 \le \beta \le 1$$

Exponential Trend Method

- **Exponential trend method** is appropriate for forecasting data with exponential trend pattern.
- ***ETS(A,M,N) : error = additive, trend = multiplicative, seasonality = none.***

$$\hat{y}_t = l_t * b_t$$

$$l_t = \alpha * y_t + (1-\alpha) * (l_{t-1} + b_{t-1})$$
$$initial\ l_t = y_{t-1}$$
$$0 \le \alpha \le 1$$

$$b_t = \beta * \frac{l_t}{l_{t-1}} + (1-\beta) * b_{t-1}$$
$$initial\ b_t = y_{t-1}/y_{t-2}$$
$$0 \le \beta \le 1$$

Gardner's Additive Damped Trend Method

- **Gardner's Additive damped trend method** is appropriate for forecasting data with damped linear trend pattern. (Everette S. Gardner, Jr. and Eddie McKenzie. "Forecasting Trends in Time Series". *Management Science*. 1985.)
- ***ETS(A,Ad,N) : error = additive, trend = additive damped, seasonality = none.***

$$\hat{y}_t = l_t + \delta * b_t$$
$$0 \leq \delta \leq 1$$

$$l_t = \alpha * y_t + (1 - \alpha) * (l_{t-1} + \delta * b_{t-1})$$
$$initial\ l_t = y_{t-1}$$
$$0 \leq \alpha \leq 1$$

$$b_t = \beta * (l_t - l_{t-1}) + (1 - \beta) * \delta * b_{t-1}$$
$$initial\ b_t = y_{t-1} - y_{t-2}$$
$$0 \leq \beta \leq 1$$

Taylor's Multiplicative Damped Trend Method

- **Taylor's Multiplicative damped trend method** is appropriate for forecasting data with damped or restrained exponential trend pattern. (James W. Taylor. "Exponential Smoothing with a Dampen Multiplicative Trend". *International Journal of Forecasting*. 2003.)
- ***ETS(A,Md,N) : error = additive, trend = multiplicative damped, seasonality = none.***

$$\hat{y}_t = l_t + b_t^{\delta}$$
$$0 \leq \delta \leq 1$$

$$l_t = \alpha * y_t + (1-\alpha) * l_{t-1} * b_{t-1}^{\delta}$$
$$initial\ l_t = y_{t-1}$$
$$0 \leq \alpha \leq 1$$

$$b_t = \beta * \frac{l_t}{l_{t-1}} + (1-\beta) * b_{t-1}^{\delta}$$
$$initial\ b_t = y_{t-1}/y_{t-2}$$
$$0 \leq \beta \leq 1$$

Holt-Winters Additive Method

- **Holt-Winters additive method** is appropriate for forecasting data with linear trend and additive seasonal patterns. (Peter R. Winters. "Forecasting Sales by Exponentially Weighted Moving Averages". *Management Science*. 1960)
- ***ETS(A,A,A) : error = additive, trend = additive, seasonality = additive.***

$$\hat{y}_t = l_t + b_t + s_t$$

$$l_t = \alpha * (y_t - s_{t-m}) + (1-\alpha) * (l_{t-1} + b_{t-1})$$
$$initial\ l_t = y_{t-1}$$
$$0 \le \alpha \le 1$$

$$b_t = \beta * (l_t - l_{t-1}) + (1-\beta) * b_{t-1}$$
$$initial\ b_t = y_{t-1} - y_{t-2}$$
$$0 \le \beta \le 1$$

$$s_t = \gamma * (y_t - l_{t-1} - b_{t-1}) + (1-\gamma) * s_{t-m}$$
$$initial\ s_t = y_t - initial\ l_t$$
$$0 \le \gamma \le 1$$

Holt-Winters Multiplicative Method

- **Holt-Winters multiplicative method** is appropriate for forecasting data with linear trend and multiplicative seasonal patterns. (Peter R. Winters. "Forecasting Sales by Exponentially Weighted Moving Averages". *Management Science*. 1960)
- ***ETS(A,A,M) : error = additive, trend = additive, seasonality = multiplicative.***

$$\hat{y}_t = (l_t + b_t) * s_t$$

$$l_t = \alpha * \frac{y_t}{s_{t-m}} + (1-\alpha) * (l_{t-1} + b_{t-1})$$
$$initial\ l_t = y_{t-1}$$
$$0 \le \alpha \le 1$$

$$b_t = \beta * (l_t - l_{t-1}) + (1-\beta) * b_{t-1}$$
$$initial\ b_t = y_{t-1} - y_{t-2}$$
$$0 \le \beta \le 1$$

$$s_t = \gamma * \frac{y_t}{(l_{t-1} + b_{t-1})} + (1-\gamma) * s_{t-m}$$
$$initial\ s_t = y_t / initial\ l_t$$
$$0 \le \gamma \le 1$$

Holt-Winters Damped Method

- **Holt-Winters damped method** is appropriate for forecasting data with damped or restrained linear trend and multiplicative seasonal patterns. (Peter R. Winters. "Forecasting Sales by Exponentially Weighted Moving Averages". *Management Science*. 1960)
- ***ETS(A,Ad,M) : error = additive, trend = additive damped, seasonality = multiplicative.***

$$\hat{y}_t = (l_t + \delta * b_t) * s_t$$
$$0 \leq \delta \leq 1$$

$$l_t = \alpha * \frac{y_t}{s_{t-m}} + (1-\alpha) * (l_{t-1} + \delta * b_{t-1})$$
$$initial\ l_t = y_{t-1}$$
$$0 \leq \alpha \leq 1$$

$$b_t = \beta * (l_t - l_{t-1}) + (1-\beta) * \delta * b_{t-1}$$
$$initial\ b_t = y_{t-1} - y_{t-2}$$
$$0 \leq \beta \leq 1$$

$$s_t = \gamma * \frac{y_t}{(l_{t-1} + \delta * b_{t-1})} + (1-\gamma) * s_{t-m}$$
$$initial\ s_t = y_t / initial\ l_t$$
$$0 \leq \gamma \leq 1$$

Forecasting Methods Accuracy

- **Forecasting methods accuracy** is evaluated on which one minimizes the residuals or forecasting errors based on scale-dependent and scale-independent measures.
- **Scale-independent:**
  - **Mean absolute scaled error (MASE)** (Rob J. Hyndman and Anne B. Koehler. "Another Look at Measures of Forecast Accuracy". *International Journal of Forecasting*. 2006.)

$$MASE = \frac{MAPE_m}{MAPE_r}$$