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# Secure Distributed Poker using MPC

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# Abstract



# Resumé



# Acknowledgments

*Christian Bobach,  
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# Chapter 1

## Introduction

In this thesis I have made a practical study of the application of a Multi Party Computation(MPC) protocol. To show what can be done by a MPC protocol and how it can be used a poker game has been developed as a proof of concept.

It is easy to think of how one could be cheated when playing an online game of poker. It is hard for me as a player to know if the dealer and one of the other players has an agrangement such that the dealer always deals better cards to that player such this player wins in the long run. The idea by using a MPC protocol here is to guarantee that the cards are dealt fairly. Such that the player of online poker can trust the protocol and know that the cards are guaranteed to be dealt fairly.

To ensure that the card are dealt fairly I will use a MPC protocol to take care of the shuffling of the cards. In this study I will use a two party computation(2PC) protocol called *DUPLO* which will be introduced in chapter 2. In this thesis a two party heads up poker game will be studied. The study is a showcase of the possibilities of MPC protocols and what can be achieved by them. It should be possible to easy extend the work done in this thesis to work in cases with more that only two parties using a another MPC protocol designed for that purpose. It should also be equally easy to extend the game to work with more players.

The hope is to show that a MPC protocol can be used to guarantee that the cards are deal fairly without any big cost in terms of time delay for the players.

For the inplemetation of the poker game I have studied various fields both in computer science and other fields. I have read up on different types of poker games to figure out which one was best suited for a two party setting. I have studied the underlying MPC protocol to understand how it works and to ensure that it for fills the right properties needed for an application as a poker game. I have studied different permutation algorithms and implemented them to compare them and see what effects they have on the underlying protocol.

TODO: introduce chapter on DUPLO

TODO: introduce chapter on shuffle algorithms

In chapter 3 on shuffle algorithms I introduce the different algorithms studied during the project. I argue for the ideas behind the algorithm and why they work in the application of a poker game. Some optimizations that can be done to the algorithm to reduce their size are perposed. At last the algorithms will be compared on a teoretical level to see different benefits.

TODO: introduce chapter on Poker Implementation

TODO: introduce chapter on conclusion and proposals of futher studies

In the next section the variant of poker chosen for this study will be introduced and others will be mentioned to give an idea of their differences.

## 1.1 The Poker Game

A poker game is a card game played in various rounds where the player draw cards and place bets. The bets are won according to a predefined list where the card constellation with the lowest probability wins. There exists many different variants of poker but only one will be chosen. The variant chosen to use in this thesis is known as 'five card draw' poker. In this study the game will be played between two parties. In this variant of poker five cards are dealt to each player in the first round. After this the first betting round occurs. Then a swap round occurs where the players have the possibility to chose how many cards to change to try to improve their hand. Then a last betting round is performed before the cards is revealed and a winner is declared.

Five card draw poker is played with a deck of 52. This poses some requirements for our shuffling algorithms. Since there are 52 cards in the deck this yields  $52!$  different permutations. We require a shuffle algorithm that can produce exact these permutations to represent all the possible shuffles of the card deck. Because only the first 20 card of the deck is needed per game it is enough for the algorithm to produce a complete shuffle of these card and not the remaining 32 cards. This implies that the algorithm used to shuffle the cards only needs to produce

$$\frac{52!}{(52 - 20)!}$$

different permutations. Since each player is dealt five card and at most can chane all these cards in the swap round. This yealds 10 card per player and therefore 20 in total.

Other variants of poker require a different amount of cards per game. One example could be if the came included tree players instead of two, then 30 cards of the complete deck would be needed. An other example could be the Texas Hold'em variant which is played by dealing two cards to each player and placing tree cards face upwards on the table. These cards are the used as a part of each of the players hand. After this a betting round is performed. This is continued

by another card dealt facing upwards on the table. This is done twice before the final revelation phase where the winner is found. If the game involves two players then 4 card is dealt to the players and 5 to the table resulting in a total of 9 cards used. This implies an algorithm producing

$$\frac{52!}{(52 - 9)!}$$

different permutations of the card deck is needed. This is also known as an  $m$  out of  $n$  permutation.

From here on when talking about a poker game the five card draw poker will be the reference otherwise it will be specified. This is especially interesting when looking for optimizations on the shuffle algorithms which will be introduced in chapter 3 and when they are compared. When coming to chapter 4 this will have effect when the cards are dealt. Both in terms of the amount of data sent and the time used by the protocol.



## Chapter 2

# DUPLO

In this chapter I will introduce the *DUPLO* framework introduced in [A2] and why this was chosen to handle the communication and security of the poker game. I will explain how the structure of the *DUPLO* protocol works and describe what the different framework calls handle. I will go over the security details of the protocol to illustrate how this is guaranteed. Lastly I will introduce the *Frigate* compiler which is shipped with the *DUPLO* framework to generate circuits for the evaluation.

As it can be read in [A2] the *DUPLO* framework is among the latest papers where the efficiency of a two party computation (2PC) protocol using garbled circuit in a malicious setting is studied. In the paper *DUPLO* is claimed to reach the protocol performs better than any existing protocol. Their idea came from the fact that the two extreme variants of cut and chose protocols did not perform well in each end of the spectrum when it comes to the size of the circuits. As they came up with a new approach to the way cut and chose 2PC protocols could be done in the malicious setting. The idea is to garble subcomponents of the circuit and get a optimum somewhere in between the two extremes; garbling of complete circuits or garbling on gate level, cut and chose. The aim was to show that the gate level cut and chose added an overhead when soldering these together again when the circuits for evaluation is built. At the same time to show when the number of subcomponents goes up there is a performance gain compared to whole gate cut and chose because of the amortized benefits.

As seen in section 7 on performance in [A2] it is clearly that the experiments done on real life circuits yields an optimal cut and chose strategy which differs from the earlier known possibilities. The gain in terms of running time increases as the size of the circuits get bigger. Which shows that the *DUPLO* protocol scales significantly better than the rest.

As this is the best performing 2PC protocol at the moment combined with the fact that it is developed at Aarhus University such that the people with knowledge of the protocol is in the same building was the main factors for using *DUPLO*. The fact that *DUPLO* supported the possibility of single and distinct wire openings to a desired party helped the decision.

In the next section I will introduce the different functions in the framework and what they achieve.

## 2.1 The *DUPLO* Framework

The *DUPLO*2PC framework was chosen to use during the experiment of implementing a poker game. *DUPLO* consists of two parties, a *Constructor* and an *Evaluator* with different roles during the protocol. The *Constructor* generates the garbled circuits and sends them to the *Evaluator*. The *Evaluator* verifies a number of these circuits. If these pass the *Evaluator* trusts that the remaining circuits are valid. Then these remaining circuits are used during evaluation.

The overall construction of the framework consists of different functions to call to allow for the right communication between the two parties. It is specified in which order these functions should be called to ensure that the right information is at the parties at the right time. At the same time the split up of the functions allows for local computations to be done between these framework calls.

To run the protocol and use the framework a *Constructor* and an *Evaluator* is created. First of all they read the circuit file specifying the functionality desired. In our case it is the shuffle algorithm which is introduced in chapter 3.

Once these are created they run the framework function calls in parallel. First the two parties connect to each other via the **Connect** call. In this case it is the *Constructor* hosting the service and then the *Evaluator* connects to this. When they are connected they each make a call to the **Setup** function to initialize the communication protocol which is the *XOR*-homomorphic commitment protocol. After this they start the preprocess phase of the components in the circuit by running the **PreprocessComponentType** function call. Which generates a garbled representation of the shuffle algorithm.

Then the **PrepareComponents** function is called to produce input and output authentication such that the inputs and outputs can be transferred securely between the two parties.

After this the composed *Circuit* is constructed by calling the **Build** function. This constructs the complete circuit which is to be evaluated later by the call to **Evaluate**. This ensures that the function components specified by the composed circuit file is soldered together in the right way. Such that the output wires from one subfunction is fed to the right input wire on another subfunction. This is also done using the *XOR*-homomorphic commitments. When the next call is made to **Evaluate** the input to the circuit is given and the circuit is evaluated on this. Such that a garbled output is generated. When this is done a call to **DecodeKeys** can be made and the output of the circuit can be learned.

The evaluation of circuits in the *DUPLO* protocol allows for openings of outwires to both parties or only one. This will allow us to only reveal some cards to one player and other cards to the other. The split up of **Evaluate** and **DecodeKey** functions allows for opening of output wires in different rounds which helps us achieve good round complexity when doing our implementation.

**TODO:** Evt. introduction to; 2PC, GC, *XOR*-HC. Fordele ulemper



## 2.2 Security

In this section I will introduce the security of the protocol to show that players playing a game of poker with an implementation using the *DUPLO* framework will have *oblivioness*, implying that the oponent can not learn more than supposed to. The players will also be ensured *correctness* of the protocol, meaning that if garbled evaluation is done it gives the right output. *DUPLO* also ensures *authenticity* because it is not possible for a player to do evaluation of the functionality on other input than the party garbling the circuit.

The proof of security for the protocol is done using the Universal Composition (UC) framework. This is an easy digested abstract protocol proof technique which allows for sequential predefined interaction between parties using actions and reactions. It has a modular approach to functionality proofs, when one functionality has been proved it can be used as a steppingstone for the next proof. In *DUPLO* they use the hybrid model with ideal functionalities  $\mathcal{F}_{HCOM}$  and  $\mathcal{F}_{OT}$ . Where the  $\mathcal{F}_{HCOM}$  functionality is for the *XOR*-homomorphic commitment scheme used by the protocol, and  $\mathcal{F}_{OT}$  for the one out of two oblivious transfer. These functions are then used to prove *correctness*, *obliviousness* and *authenticity* of the protocol.

**TODO: Protocol overall security proof appendix a**

In the section on protocol details in [A2] appendix A they describe and analyse the protocol. Here structuring the main protocol and going into details on how *correctness*, *obliviousness* and *authenticity* is guaranteed through the different protocol function calls. The proof ends up being rather complex as the main structure consists of 8 subfunctions, which each is a combination of further subcircuits. All of these functions are guaranteed to satisfy the properties. During the analysis of these functions they end up with lemmas proving correctness of; soldering and evaluation of subcircuits. They end up with lemmas proving robustness of; the key authenticator buckets, evaluation of key authenticators, input of constructor, input of evaluator, evaluation of subcircuits and output of evaluator. This culminates in the theorem proving robustness of the protocol, showing that if the *constructor* is corrupt and the *evaluator* is honest and the protocol does not abort, then the protocol completes holding the before mentioned property, except with negligible probability. As known when using MPC protocols where half or more of the parties are corrupt we can not guarantee termination.

**TODO: Protocol security analysis appendix b**

In appendix B they prove the fact that the protocol is secure against a corrupt *constructor* or *evaluator*. Since it is a 2PC we may assume that one of the parties is honest as the parties trust in themselves. When proving in the UC framework it is worth to remember that a poly-time simulator  $\mathcal{S}$  should be presented. For the case of a corrupted *constructor*,  $\mathbf{G}$  for generator, and a honest *Evaluator*,  $\mathbf{E}$ .  $\mathcal{S}$  plays the role of  $\mathbf{E}$  in the protocol, but is not given access to the inputs of  $\mathbf{E}$ ,  $x_{\mathbf{E}}$ . Instead  $\mathcal{S}$  has access to an oracle  $\mathcal{O}_{x_{\mathbf{E}}}(\cdot)$  containing  $x_{\mathbf{E}}$ .  $\mathcal{S}$  might contact  $\mathcal{O}_{\mathbf{E}}$  giving input  $x_{\mathbf{G}}$  and in return learn  $y_{\mathbf{G}}$  as if the evaluation of the functionality was done with  $x_{\mathbf{E}}$  and  $x_{\mathbf{G}}$  as input.

To show that the protocol is secure in this setting we need to show that a *constructor* running the protocol can not distinguish between talking to **E** or **S**. **TODO: END PROOF**

**TODO: What is needed of the protocol, active security, single wire opening, duplo**

**TODO: Preprocessing of duplo, serversetting interesting to study in stead of application**

## 2.3 Frigate the *DUPLO*Circuit Compiler

In this section I will introduce how the new version of the *Frigate* circuit compiler workes.

First of all when installing the compiler some special versions of libarys are required, which are not the latest. Folowing the instructions in the installation guide and some amount of internet search I was able to get it up and running. During the thesis I have been using hardware running Ubuntu 16.04 LTS or higher. Where the standard version of *flex* and *bison* is higher that supported by *DUPLO*. This requires that the right versions are installed and kept back such that these are not updated later. But as the compilation of circuits are done once and prior to the compilation of the actual *DUPLO*implemetation this is not a complete deal breaker.

**TODO: Describe the frigate programming language**

The *Frigate* compiler came shipped with the documentation for the first version and was not updated when extended to fit the *DUPLO*framework. This should not be expected since the functionality of the compilers input language was not changed. But this resulted in some time consuming trial and error since it was not well written and specified. It was a long time since I last had worked with circuits in such a way as was the case for generating big circuits with a circuit compiler. This resulted in some hard earned experience on small exapmles.

Some of the most important and different things to take into account when programming for circuit generation is the possibility to do wire access. First of all it is possible to specify which and how many wires should represent a value by using `y=x{index:size}`. This is the way long bit input strings are translated into higher level representations of smaler instances. This gave some occasions for frustrations because is it not possible to acces vires based on variable inputs which cannot be pre determined by *Frigate*. This makes perfect sence since the compiler can not know which wires should be used as the representation. A second point to remember when working with *Frigate* is that it do not allow for more than one level functions. Which gives some restrictions in programming compared to many other languages. This restriction makes it harder to create small functions with one single specific focus that could be called when the desired functionality was required. This did not give possibility recursive functions. But as the circuits generated is a static representation this

is a restriction which cannot easily be handled, since the size of the circuit can not vary based on the inputs. Another small oddity is that *Frigate* only allows for assignments in the main method through function calls.

Otherwise the programming language used by *Frigate* resembles the well known *C* language. The compiler requires you to specify how many parties the functionality is used by, by the call to `#parties n`. At the same time it requires that the size of input is specified for each of the parties using `input i size_i` and `#output i size_o`. But otherwise it allowed for definitions of constants, types, structures and imports just like in *C* which allows for some easy readability.

When the desired functionality has been implemented using the *wir* described above used by *Frigate* the compiler can be used, from inside the compiled *DUPLO* framework, by calling:

```
./built/release/Frigate path/to/file.wir -dp
```

The `-db` flag ensures that the right *DUPLO* format is generated. This call to the compiler generates different files with different extensions. The file we are interested in is the file with the extension `.wir.GC_duplo` this is the one that the *DOPLO* framework can use as input file.

The *DOPLO* framework has a functionality that allowed for evaluation of these input files without setting up both parties. This gave the possibility to specify the inputs for the evaluation and learn the result in a first return cycle. This possibility allowed me to study the implementations of the programming language and how the algorithms behaved compared to expected while implementing them. First I tried to implement the *Fisher-Yates* algorithm, which is described in details in section 3.1 and can be seen as algorithm 1, in the `.wir` format. During the implementation I encountered some problems. Since I did not have any earlier experience working with the *Frigate* compiler I was not sure if the problem was in the implementation of the algorithm or in the compiler. At first the focus was on the implementation as the lack of experience working with *Frigate* easily could lead to implementation errors. After a lot of modularisation and debugging it was clear that something was wrong with the version of the *Frigate* compiler used to generate *DUPLO*-circuits. During the debugging process I created a framework that allowed me to test the different modules of the implementation one by one. It was then clear that something was off when using the modulo reduction in the *Fisher-Yates* algorithm. After different attempts to get it to work without any luck the focus shifted from being on the implementation to be on the compiler. After breaking the implementation down and testing the modulo operator `%`, as specified in the documentation, it was clear that some error was introduced during compilation. Therefore I started to take a deeper look at the compiler. Since the compiler was a modified version of the *Frigate* compiler the possibility was that a bug could have been introduced when adding the new *DUPLO* features. Therefore the old version of *Frigate* was installed to test if that implementation had the same bug. Then

$l$	$r$	0	NOR	$\neg x$	AND $y$	$\neg x$	$x$ AND $\neg y$	$\neg y$	XOR	NAND	AND	NXOR	$y$	If $x$ Then $y$	$x$	If $y$ Then $x$	OR	1
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	1
0	1	0	0	1	1	0	0	1	1	0	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1

Table 2.1: A table of the 16 different gate types that can be used in a circuit of the type used in duplo

a problem arose because the old version did not support the *DUPLO*circuit format. But because *DUPLO* also has supports for another circuit file format known as *bristol*, I wrote a parser that took the output from the old version of *Frigate* and translated that into the *bristol* format. This gave me two different formatted circuit files that *DUPLO* could use as input files. Based on these two formats is created a test framework to see if there was any differences on the output when ran on the same input. This showed that there was a difference in the results produced, especailly in the case when using the modular reduction. When taking a deeper look at the problem it came clear that not all gate types was created during compilation using the *DUPLO* version of the *Frigate* compiler.

This resulted in a fix of the new version of the *Frigate* compiler and a complete change in the representation format of the circuits. Now all gates in the *DUPLO*circuit format has two input and one putput wire by standart and are not explicitly written in the formatted file as earlier. The representation of gates chaged from a more human readable type like *XOR* to a truth table frendly type like 0110 for *XOR*. This change in representation ensured that all 16 gate types which can be seen in table ?? are now implemented. The representation of the two constant wires **0** and **1** is handled as special cases as all gates now has two input wires. For the case of **0** it is handled as an *XOR* gate with input wires with the same value. For the case of **1** it is now handled as the *NXOR* of input wires with the same value.

In this way by going into details and debugging my implementaion of the algorithm I have contributed to the *DUPLO*project by reporting my findings and allowed them to fix this problem before publishing their finding. This has helped to secure a stronger overall research product.

On the other side the compiler has not been updated to support modulo with a deviser that is not the power of 2. This is not mentioned anywhere in the documentation for *Frigate*. When I accounted the problem of the modulo operator I used a small amount of time to reserch if it was posible to implement this functionality easily. During the search I noticed that the implementation of modulo to some power of 2 is simple. While the research I did seemed to indicate that for valuse not a power of 2 are not trivilay implemented. Therefor this was leaft unfixed and my implementation uses a hack to overcome this problem which is stated in section 3.1.

## Chapter 3

# Shuffling Algorithms

In this chapter I will introduce the different shuffling algorithms studied during this project. I will introduce the ideas behind each algorithm studied and what makes it special. I will introduce why these were chosen. I will explain how they were optimized to fit better to the specific needs for a poker game. Lastly I will compare the algorithms to see the different benefits, and based on this choose which algorithm to use in the implementation.

The permutation algorithms studied are with the purpose of shuffling card decks. It is important to choose an algorithm that ensures that the correct amount of permutations is reached.

The first algorithm studied is the Fisher-Yates algorithm introduced in [B4]. It may also be known as Knuth shuffle which was introduced to computer science by R. Durstenfeld in [A1] as algorithm 235. This algorithm uses an in place permutation approach and gives a perfect uniform random permutation. This algorithm is introduced in section 3.1 and can be seen in pseudocode as algorithm 1.

The second algorithm proposed uses ideas from shuffling networks and [A3] as conditional swap combined with the well known *Bubble-sort* algorithm. The idea is simple and use conditional swaps gadgets which swaps two inputs based on some condition. This algorithm is introduced in section 3.2 and can be seen in pseudocode as algorithm 2. This algorithm yields a perfect uniform permutation.

These shuffle algorithms is optimized to fit to the poker setting introduced in the section 1.1 on poker in chapter 1. This is done such that it only shuffles the required cards and not the whole deck.

The implementations of these algorithms will be introduced in section 3.3 where the choices made will be discussed. At last in section 3.4 a comparison of the algorithms is done. Here I chose which algorithm to use in the implementation of the poker game and benchmark upon. In this section other type of shuffling networks called Bitonic shuffle network will be introduced and discussed shortly. No implementation of such a shuffle network was done.

---

**Algorithm 1 *Fisher-Yates***

---

*deck* is initialized to hold  $n$  cards  $c$ .

*seed* is initialized to hold  $n$  random  $r$  values where  $r_i \in [i, n]$  for  $i \in [1, n]$ .

---

```
1: function SWAP(card1, card2)
2:    $tmp = card1$ 
3:    $card1 = card2$ 
4:    $card2 = tmp$ 
5: end function
6:
7: function SHUFFLE(deck, seeds)
8:   for  $i=1$  to  $n$  do
9:      $r = seeds[i]$ 
10:    SWAP( $deck[i]$ ,  $deck[r]$ )
11:   end for
12: end function
```

---

### 3.1 Fisher-Yates

The *Fisher-Yates* algorithm can be seen in algorithm 1. It is a well known in place permutation algorithm that given two arrays as input; one that contains the values that should be shuffled, here denoted *deck*, and another holding the values specifying how the first array should be shuffled, here denoted *seed*. These swap values from *seed* indicate where each of the original values should go in the swap. When the algorithm runs through the first array which is supposed to be permuted it swaps the value at a given index with the value specified by the swap value of the second array. Think of the input to be shuffled as a card deck then you take the top card of the deck and swap it with another card at a position defined by the swap value.

This implies that the algorithm takes two inputs of the same size where the one is holding the values to be permuted, *deck* with  $n$  values  $card_i$ , for  $i = 0, \dots, n$ . The other holding the values for which the different  $card_i$  in the *deck* is to be swapped, *seed* with  $n$  values  $seed_i$ . If the swap values  $seed_i$  from the *seed* are not given in the correct interval the probability for the different permutations is not equally likely. Therefore it is important that the  $seed_i$  values are chosen accordingly to the algorithm. The algorithm states that  $seed_i$  is chosen from an interval starting with its own index  $i$  to the size  $n$  of the *deck*. This gives exactly the number of permutations required as  $card_1$  has exactly  $n$  possible places to go.  $card_2$  has  $n - 1$  possible places and so forth until the algorithm reaches  $card_n$  which has no other place to go. Since  $seed_i \in [i, n]$  we have  $n!$  because  $i$  runs from 1 to  $n$  which should be the case as described in section 1.1.

If the  $seed_i$  values contained in *seed* is not chosen for the right interval but instead all is chosen from 1 to  $n$  we would end up having a skew on the probability of the different permutations. As  $card_i$  in this case has  $n$  possible places to go, this yields  $n^n$  distinct permutations. This introduces an error into the algorithm as there should only be  $n!$  and as  $n^n$  is not divisible by  $n!$  for

Seeds:	1	51	14	20	10	37	9	33	37		
Deck:	1	2	3	4	5	6	7	8	9	...	52
	1	2	3	4	5	6	7	8	9	...	52
	1	51	3	4	5	6	7	8	9	...	52
	1	51	14	4	5	6	7	8	9	...	52
	1	51	14	20	5	6	7	8	9	...	52
	1	51	14	20	10	6	7	8	9	...	52
	1	51	14	20	10	37	7	8	9	...	52
	1	51	14	20	10	37	9	8	7	...	52
	1	51	14	20	10	37	9	33	7	...	52
	1	51	14	20	10	37	9	33	6	...	52
Result:	1	51	14	20	10	37	9	33	6		

Figure 3.1: Fisher-Yates algorithm in action: In this figure a 9 out of 52 shuffle has been completed to illustrate how the algorithm works. First 1 is swapped with 1. Then 2 is swapped with 51. 3 with 14. 4 with 20 and so on until the first 9 numbers have completed a full permutation. Resulting in 1, 51, 14, 20, 10, 37, 9, 33, 6.

$n > 2$ . This results in a non uniform probability of the different permutations. The same is the problem if  $seed_i$  is not chosen from  $[i, n]$  but instead  $]i, n]$  such that the own index is not in the interval. By introducing this error to the algorithm the empty shuffle is not possible. In other words it is not possible to get the same output as the input. Which does not give the desired uniform distribution of permutations.

In case of the poker game we need  $52!$  permutations. If all  $seed_i$  is chosen from  $[i; 52]$  we would get  $52^{52}$  possible permutations. As described  $52^{52}$  is not divisible by  $52!$  since  $52 > 2$ . If  $seed_i$  instead is chosen from  $]i; 52]$  we get  $(52 - 1)!$  permutations which is neither divisible by  $52!$ .

As described in section 1.1 no more than a permutation on the first 20 cards is needed. Which means that we only need the  $\frac{52!}{32!}$  specific permutations out of the total of  $52!$  different permutations. Doing a  $m$  out of  $n$  permutation using the *Fisher-Yates* algorithm is straight forward. Instead of running through  $n$  swaps indicated by the size of  $seed$  it is enough to run through  $m$  swaps. In our case resulting in the input  $seed$  only need to have size 20 and therefore the for-loop seen in algorithm 1 in the shuffle function needs to have fewer iterations. Those giving us a full permutation on the first  $m$  indexes of  $deck$ .

In figure 3.1 it is possible to see the *Fisher-Yates* shuffle in action. Here the first 9 cards of a sorted deck is shuffled according to the given seed. Running the algorithm on these inputs gives the 9 first cards 1, 51, 14, 20, 10, 37, 9, 33, 6 as output. It is interesting to notice that 37 in the seed twice. Since the algorithm

permute the input *deck* the value 37 will not be in the output twice. We see that 6 is swapped in the second time the seed 37 is used. This is because the first time 6 and 37 was swapped. This illustrate that it is possible for a *card* to be swapped multiple times.

## 3.2 Shuffle Networks

Shuffling networks or permutation networks has a lot of resemblance to sorting networks. The idea behind this type of networks is that they consist of a number of input wires and equally many output wires. These wires go through the entire network. On these wires a swap gadget is placed. This gadget is constructed such that if a condition is satisfied the input on the two wires are swapped. By placing these swap gadgets correctly on the input wires it is possible to get a complete uniform random permutation of the input on the output wires. The swap gadgets are created according to [A3] as figure 3.

Applying such a shuffle network in the setting of a poker game is simple. The input to the shuffle algorithm is the *deck* that we want to shuffle and the output is the shuffled *deck*. The more interesting part is how to place the swap gadgets to ensure that the right number of possible permutations is satisfied. There are many different shuffle algorithms that can be implemented using shuffle networks. The one I have looked into and implemented builds on ideas from [A3] where they introduces the conditional swap gadget. The algorithm is a combination of the well known *bubble-sort* algorithm and the conditional swap.

In the next section I will introduce the conditional swap algorithm, which can be seen as algorithm 2.

**Conditional Swap:** The conditional swap algorithm takes two inputs; the first input is an array, denoted *deck* of  $n$  cards  $card_i$  for  $i = 1, \dots, n$ , and the second an array *seed* of size  $l = \frac{n^2}{2}$  bits  $b_j$  where  $j = 1, \dots, l$ . The algorithm creates  $n - 1$  layers of conditional swap gadgets. The first layer contains  $n - 1$  conditional swap gadgets. The second  $n - 2$  and so on until the last layer consisting of one gate. Each layer is constructed such that a swap gadget is placed on two adjacent input wires. Each of these gates overlap with one of the input wires at the adjacent swap gadget. This is illustrated in figure 3.2. The layers are stacked in such a way that the first input wire is only represented in the first layer. Thereby is the first value on the first output wires determined by the first layer of swap gadgets. Resulting in the first input  $card_1$  has  $n$  places to go. The second layer determines which output  $card_2$  will have and so on. Continuing this way until reaching the last layer where the two last outputs  $card_{n-1}$  and  $card_n$  will be determined. This gives us a shuffle algorithm with a perfect shuffle and  $n!$  different permutations as desired.

If each layer of the swap gadgets are not decreasing by one on the amount of swap gadgets this algorithm suffers the problem of producing  $n^n$  permutations. Which is not divisible by the desired  $n!$  permutations. This resulting in a skew



---

**Algorithm 2** *Conditional swap*

---

*deck* is initialized to hold  $n$  cards  $c$ .

*seed* is initialized to hold  $\frac{n^2}{2}$  random *bit* values where  $bit_i \in [0, 1]$  for  $i \in [1, \frac{n^2}{2}]$ .

---

```
1: function CONDITIONALSWAP(bit, card1, card2)
2:   if bit equal 1 then
3:     tmp = card1
4:     card1 = card2
5:     card2 = tmp
6:   end if
7: end function
8:
9: function SHUFFLE(deck, seeds)
10:  index = 0
11:  for i=1 to n do
12:    for j=n-1 to i do
13:      index = index + 1
14:      bit = seeds[index]
15:      CONDITIONALSWAP(bit, deck[j], deck[j + 1])
16:    end for
17:  end for
18: end function
```

---

of the probability on the different permutations such that the propability of each permutation is no longer uniform.

Again some optimization can be done to the algorithm since we only need a  $m$  out of  $n$  permutation. This can be done by letting the outer loop of algorithm 2 run for  $m$  iterations instead of  $n$ . This yields  $n$  possible values for  $card_1$ ,  $n - 1$  possible values for  $card_2$  and so one until  $n - m$  values for  $card_{n-m}$ . This is exactly the amount of permutation we require for our optimized algorithm as this gives us  $\frac{n!}{(n-m)!}$ . Which is enough for our poker implementation as described in section 1.1

In figure 3.2 a run of algorithm 2 can be seen Here a 9 out of 52 variant is used. It can be seen that the inputs *deck* is sorted and holds the values to be shuffled and *seed* which are binary and indicates if two values should be swapped. The first 52 bits of the *seed* decides if the first *card* values should be shuffled. Which is not the case in this run. Then the next 51 bist from *seed* indicate that 51 should be swapped all the way accross to the wire repecting the second out card. This implies that all cards 51 passed on its way will now be on the right adjacent wire to where it was prior to the swap. That is why the third output *card* with value 14 starts at wire index 15 and output wire four with value 20 starts at wire index 21. So the algorithm continiues until it outputs the first 9 cards shuffled as 1, 51, 14, 20, 10, 37, 9, 33, 6.

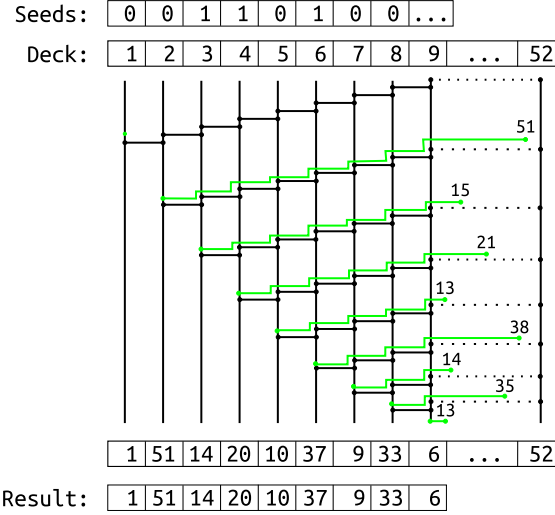


Figure 3.2: Conditional swap algorithm in action: In this figure a 9 out of 52 shuffle has been completed to illustrate how the algorithm works. Each bit in the *seed* indicate if a gate should be swapped. Since the size of *seed* is so big I have tried to illustrate which wire each value is located at before moved in a layer resulting in 1, 51, 14, 20, 10, 37, 9, 33, 6.

### 3.3 Implementation

Because the *DUPLO* protocol was chosen as the MPC protocol to use in this study the first hurdle was to generate the circuits that should handle the shuffling of the cards. Since circuits become rather complex when trying to implement functions it was important to have a compiler that could translate the algorithms into the desired circuit representation. Luckily the *DUPLO* project came shipped with a compiler for generating circuits with the right format. Therefore before starting to implement the shuffle algorithms I had to read up on the documentation for the compiler, which can be found by following the descriptions in appendix A.3. The documentation was from the first version of the *Frigate* compiler which was extended during the *DUPLO* project generate the desired *DUPLO* format. An introduction to the *Frigate* compiler created in the *DUPLO* project can be found in section 2.3 and the codebase is introduced in appendix A.3. Because it was a long time since I last had worked with circuits and the documentation of the incorporated functionality of *Frigate* I used some time getting up to speed. Much of this time was by trial and error which gave me some hard earned experience through small examples.

Before going into details on the implementations a link to the sourcecode can be found in appendix A.4.1. In the implementation of the two shuffle algorithms there are five different functions that were implemented as different modules that could be used. Both shuffle algorithms *Fisher-Yates* and *Conditional-swap* make calls to the function `initDeck`. Which initializes a hardcoded representation of the card deck which is to be shuffled when evaluating the generated circuit.

Gate Type	Non-optimized	Optimized	Difference(%)
Gates	145491	57364	60
Free Gates	97753	39001	60
Non-free Gates	47739	18363	62

Table 3.1: *Conditional-swap*: Comparison of the non-optimized and optimized versions of the algorithm. The comparison is done on the amount of each gate type in the compiled circuit.

This is done by a for-loop inserting the values of the deck on the right wires. In the implementation 6 bit variables is used for the representation of each card, as 6 bits allows for 64 different representations, because  $2^6 = 64$ . Which is enough to represent each unique card in the deck. It is important to notice that the variable indexing the start position of every card needs a representation with at least 9 bits to hold the correct value, since  $\lceil \log_2(52 \cdot 6) \rceil = 9$ . If only 6 bits were used for indexing only wires up to position 63 could be assigned and not all 312, as required since  $52 \cdot 6 = 312$ . Therefore 9 bit is used for this variable.

Then looking into the rest of the structure of the *Conditional-swap* algorithm we see that the first function used is the **xorSeed**. This function handles the *XOR* of the *seed* received from the two parties. This is a straightforward implementation using the build-in *XOR* function `^`. After this the call to **initDeck** is done such that the result for the two functions can be fed into the shuffle algorithm. Such that the last function used in the *Conditional-swap* algorithm is the **shuffleDeck** function. This is the function handling the actual shuffling. This is implemented using two for-loops; one for constructing the layers in the network, and another for generating the swap gadgets in each respective layer. Following the structure of algorithm 2 we ensure that we are ending up with an algorithm producing the right amount of permutations according to the discussion made in section 3.2. Using the optimizations proposed in the same sections results in a clear reduction in the number of gates, as seen in table 3.1. Most important we see a 62% reduction in the non-free gates on the optimized version.

In case of the *Fisher-Yates* algorithm the things stack up a bit differently. The first function in this case is the **correctSeed**. The function takes the *seed* from the two parties and correct them as described in section 3.1. At first each of the inputs are splitted up into representations of 6 bits such that they each hold 52 values,  $seed_{C_i}$  for the *Constructors* and  $seed_{E_i}$  for the *Evaluator*. Then the new input representations are added such that  $seed_{C_1}$  is added with  $seed_{E_1}$ . This ensures that the new value  $seed_i$  is at most  $2 \cdot (2^6 - 1)$  because of the representation. Since the addition of two 6 bit values can not be guaranteed to fit inside another 6 bit value a representation with more bits is used to store the resulting value. The idea was to use a modulo reduction on  $seed_i$  to guarantee that it was inside the interval described in section 3.1. Since the modulo reduction implemented in *Frigate* only supports divisors that is a

Gate Type	Non-optimized	Optimized	Difference(%)
Gates	99150	57790	42
Free Gates	61806	37433	39
Non-free Gates	37344	22357	40

Table 3.2: *Fisher-Yates*: Comparison of the non-optimized and optimized versions of the algorithm. The comparison is done on the amount of each gate type in the compiled circuit.

power of 2 the modulo reduction in *Frigate* can not be used, because  $seed_i$  is not guaranteed to have this property. I used a lot of time to figure out that the *Frigate* modulo operator only worked on powers of 2. This implementation detail was not specified anywhere in the documentation. The first idea was to fix the problem by implementing a modulo function that I could use instead. I therfor put some research time into this problem, but it is to be rather complex to achive. Therefor another solution was chosen to overcome the problem. Since the input  $seed_{C_i}$  and  $seed_{E_i}$  to the functionality is assumed to be in the right intervalls the solution was to subtract the boundray of the interval  $I_u = 52 - i$  from  $seed_i$  if this exceets  $I_u$ . Doing it this way it yealds a resulting value  $seed_i$  in the right intervall. This is do to the fact that  $seed_{C_i}$  and  $seed_{E_i}$  can at most be  $I_u$ . This ensures that  $seed_i$  is at most  $2 \cdot I_u$ . Then  $seed_i - I_u$  is guaranteed to be at most  $I_u$ . In the implemetation this was done by introducing an *if* statment checking if  $seed_i$  exceeded  $I_u$ . It is nothworthy to mention that all values have had an unsigned representation until now. But since the comparison of to values needs a signed representation as stated by the documentation  $seed_i$  was converted. This corrections to the implemetations now ensures that the randomness given to **shuffleDeck** has the right form. But only if  $seed_{C_i}$  and  $seed_{E_i}$  are inside the right intervall  $[0; I_u]$ .

The second function used in the implementation is the same as in the case for the *Conditional-swap*algorithm where the **initDeck** function is called to initialize the representation of the *deck* which is to be shuffled. The last function called is the **shuffleDeck** function which is different from the one from the *Conditional-swap*algorithm. This function consist of an outer for-loop that runs through the cards  $card_i$  of the deck. Because circuits are a static representation as disscused earlier in section 2.3 it is not possible to assign a wire value based on a variable input. Therefor this for-loop generates layers of conditional swap gadgets. Resulting in  $52 - i$  swap gadgets in each layer, for  $i = 0, \dots, 51$ . This is represented by the inner for-loop. In this way a composed gadget is generated for each  $card_i$  such that the  $card_i$  can be swapped with any other  $card_j$ , where  $j = i, \dots, 51$ . This composed gadget is a composition of  $52 - j$  desitinc swap gadgets. Such that the desired propability is reached as described in section 1.1. Both an optimized and non-optimized version of the algorithm was implemented to see how big the gain of the optimization was. This can be seen in table 3.2, where we see that the optimization result in a 40% decresae in the number of non-free *XOR* gates.

### 3.4 Comparison

In this section I will try to compare the two algorithms on their internal structure. I will compare the algorithms based upon their gate composition and based on that choose which one to continue with in the implementation of the poker game. When comparing the algorithms I use the optimized versions as it would be one of these that will be used because of their gain in the number of non-free *XOR*-gates.

The first we will look at is the input to the **shuffleDeck** functions. The both take the *deck* as input, which in both cases is generated by the function **initDeck**. This does therefore not yield any difference to the algorithms. Then when looking at the *seed* it is clear that there are some differences. Both in terms of representation and in size. First looking at the representation of *seed*, in *Fisher-Yatesseed<sub>FY</sub>* and *Conditional-swapseed<sub>CS</sub>*. The *seed<sub>FY</sub>* is a representation of 20 values *seed<sub>FY<sub>i</sub></sub>* in the interval  $[0; 52 - i]$ , for  $i = 1, \dots, 52$ . Where *seed<sub>CS</sub>* does not have any abstract representation and therefore *seed<sub>CS<sub>i</sub></sub>* has the binary representation  $[0; 1]$ . The difference in the representations is one reason why we see a difference in the size of the *seed<sub>FY</sub>* and *seed<sub>CS</sub>* in terms of bits. Where the size of *seed<sub>FY</sub>* is 112 since 6 bits are used for the representation of the 20 seed values. The size of *seed<sub>FY</sub>* is 830 because one bit is needed per swap gadget, which is  $\sum_{i=52-20}^{51} i$ . As we see it is also the way the algorithm uses the *seed* that affects the size. Where the *Fisher-Yates* algorithm constructs composed gadgets consisting of multiple swap gadgets the *Conditional-swap* algorithm only constructs swap gadgets. Even though the algorithms have different approaches to generate the swap gadgets they end up generating the same amount of swap gadgets. Because *Fisher-Yates* generates 19 composed gadgets *CG<sub>i</sub>* consisting of  $52 - i$  swap gadgets, for  $i = 1, \dots, 19$ . Yielding the same number of swap gadgets as in *Conditional-swap*.

If we then instead turn our attention to the way the algorithms handle the *seed* before they are fed to **shuffleDeck** we see some big differences. Because both algorithms take *seed<sub>C</sub>* and *seed<sub>E</sub>* as inputs from *Constructor* and *Evaluator* the algorithms need to generate one single *seed* that can be used by **shuffleDeck**. This adds an overhead to the algorithms. This is not much for the *Conditional-swap* algorithm since it can use the *XOR* function because it uses one bit of randomness at a time. As described in section 3.1 this is not the case for *Fisher-Yates* since it uses 6 bits of randomness *seed<sub>i</sub>* at a time. Because there is the restriction on the *seed<sub>C</sub>* and *seed<sub>E</sub>* and the *seed* input to **shuffleDeck** in *Fisher-Yates* the overhead added is more. The split-up of *seed<sub>C</sub>* and *seed<sub>E</sub>* into *seed<sub>C<sub>i</sub></sub>* and *seed<sub>E<sub>i</sub></sub>* does not add any overhead, but the addition of these does. Also the check to test if *seed<sub>i</sub>* is greater than  $I_u$  and the subtraction adds an overhead to the overall circuit. The differences can be seen in table 3.3 where it is clear that **xorSeed** adds significantly less overhead to the circuit compared to **correctedSeed**.

We see that the **xorSeed** has two non-free gates, which is strange since it only does *XOR* as should be free. This is because of the bit constants **0** and **1** which are implemented using *AND* or *NAND* with both inputs coming

Gate Type	<b>correctSeed</b>	<b>xorSeed</b>	Difference(%)
Gates	6220	1663	73
Free Gates	4347	1661	62
Non-free Gates	1873	2	100

Table 3.3: Comparison of the overhead added to the algorithms by handling the *seed*'s. The comparison is done on the amount of each gate type in the compiled circuit.

Gate Types	<i>Fisher-Yates</i>	<i>Conditional-swap</i>	Difference(%)
Gates	59790	57364	4
Free Gates	37433	39001	-4
Non-free Gates	22357	18363	18

Table 3.4: Compariason of the *Fisher-Yates* and *Conditional-swap* algorithms after compilation to *DUPLO* circuits. The comparison is done on the amount of each gate type.

from the same wire. Therefore will every circuit generated with the compiler have these two non-free *XOR* gates.

When looking in to the overall composition of *Fisher-Yates* and *Conditional-swap* we get the results as seen in table 3.4. We see that *Conditional-swap* is overall 4% bigger in terms of the amount of gates than *Fisher-Yates*. On the more important comparison is that *Conditional-swap* has 18% less non-free *XOR* gates. Therefore is the *Conditional-swap* algorithm the one that will be used in the implementation of the poker game.

As a note to the comparison it should be mentioned that some of the variables used in **correctedSeed** are bigger in terms of bit size than needed. But since this is only a fraction of the 1873 non-free gates seen in table 3.3 this would not change that *Conditional-swap* has less non-free gates than *Fisher-Yates*.

At last I will give a short comment on another possible shuffle algorithm that could have been used. This other types of sorting networks that the one studied in section 3.2. In [A3] they also uses a algorithm known as the *Bitonic* merge sort algorithm. Such an algorithm is constructed of what is known as *half - cleansers*. These *half - cleansers* are constructed such that the input is guaranteed to have one peak  $p$ ,  $i_1 \leq \dots \leq p \geq \dots \geq i_n$ . Then the output is half sorted such that the highest values are in one of the two halves. Resulting in a algorithm that can sort any input. This guarantees that one input  $seed_i$  can every other place. If the conditional swap gadget is used then it can be used as a shuffle algorithm instead of a sorting algorithm. This type of sorting network generates a circuit of size  $O(n \cdot \log(n))$  which is better then what the *Conditional-swap* and *Fisher-Yates* algorithms can aquire which is  $O(n^2)$ . But as argued earlier the *Conditional-swap* and *Fisher-Yates* algorithms are easily optimized.

TODO: Bitonic algorithm, optimization? can maybe be done by flipping such a sorting network and removing half cleansers

As a result of this we see that for some card games it can be better to use another algorithm than the ones studied in this thesis since it may outperform them. We now know that a *Bitonic* algorithm would produce a smaller circuit than *Fisher-Yates* and *Conditional-swap* but since the two algorithms are easy to optimize such that they produce a relatively small circuit. It is not clear that a *Bitonic* algorithm will produce a circuit that is smaller. This implies that there will be a cross over at some point where it is better to shuffle a complete deck than only parts of it as is done in our case. This can even be the case when only a part of the deck is needed.





## Chapter 4

# Poker Implemetation

The main idea of this chapter is to introduce the different stages during the implementation. Here I will introduce the different problems I have encountered and how I have chosen to overcome those.

TODO: Introduce different sections

### 4.1 The Poker Game

As described in section 2.1 where the structure of the framework was introduced. This structure is what every implemetation using the *DUPLO* protocol needs to have. In the next part I will introduce how this was used to implement the poker game in this project.

TODO: The pokser setting

Before starting on the implemetation it is important to decide which setting of the poker game should be studdied. Two different settings was proposed and discussed, these can be seen in figure 4.1. The first is one where the *Constructor* and *Evaluatr* act as players of the poker game themselves. They will play against each other and each decide on which and how many cards should be

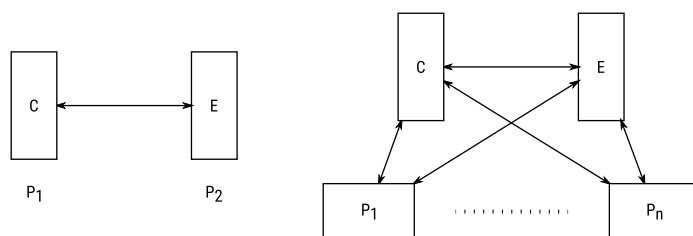


Figure 4.1: The two different settings discussed to implement. The one on the left where the *Constructor* and the *Evaluator* also are the players and the one on the right whr they act as servers where players connect and then locally construct output based on output from the servers.

changed. The other setting is where the *Constructor* and *Evaluator* act as servers where players connect to. These players will act as clients connecting to both the *Constructor* and *Evaluator*. Here the *Constructor* and *Evaluator* will run the protocol as described, but use inputs from the clients. The first one is the one chosen mainly because of the simplicity and timelimit of the project. The other setting has some interesting features which I will mention here. The second setting resembles what is used in real world online poker today. Today a player connects to a server which handles the shuffling and dealing of cards. Then instead of connecting to one server which is not completely trusted the player connects to two servers which runs a secret share of the *DUPLO* protocol. Then the player will receive a share from each party and reconstruct the computed output. This setting distributes the trust issue with the setting used to day. It could be the example that one of the servers in the *DUPLO* setting is a State authorized server in which the player then would put its trust. Then game makers is then required to use the protocol against this server to be allowed to sell games in that country. It will allow for a distributed instead of a single point trust issue. In the second setting there are no limits to how many players could connect to the servers to play a game of poker. In this way the *2PC* is not a restriction for the amount of players. This setting requires a way of handling authenticity between the players, the *Constructor* and *Evaluator*. This is needed to ensure that none of the players sends different inputs to the *Constructor* and *Evaluator*, and to ensure that the *Constructor* and *Evaluator* does not send different outputs to the players. This is not a simple task to overcome, therefore the first setting was chosen.

The way *DUPLO* is constructed it allows for preprocessing of the circuits, such that the right information is at the right party before evaluation. The overall protocol can be categorized in 3 different main phases. The first phase is the *Preprocess* phase which consists of the framework calls from section 2.1; **Connect**, **Setup**, **PreprocessComponentType**, **PrepareComponents** and **Build**. The next phase is the *Evaluation* phase which calls the **Evaluate** framework function. The last phase is the *Online* phase which consists of the **DecodeKeys** calls. Building the protocol this way allows for an intense *Preprocess* phase where a lot of data is processed and communications is done. This allows for a faster *Evaluation* phase and results in a small overhead when running the *Online* phase.

I will now describe the implementation of the poker game. First of all we need to both implement a *Constructor* and an *Evaluator*. These run the protocol in parallel with the same framework calls, but hold different information during the phases. First we read the composed circuits in to both parties such that they know which functionality is to be computed. Then the first interaction between the two parties is when the **Connect** call is done. On the *Constructor* site this generates the server functionality which the *Evaluator* connects to with this call. **TODO: Missing setup???** Then a call to **PreprocessComponentType** is done on each of the subcircuits. **TODO: What does preprocessComponentType???** . After this a call to **PrepareComponents** is done, such that **TODO: What does**

prepareComponets???

. At last the circuit are generated by the call to **Build**. **TODO: What deos build???**

. As can be seen no special input is given to any of the parties during the phase except of the circuit representing the functionality to be computed. Therefor this is called the *Preprocess* pase. As this can be done without futher input.

The next phase is the *Evaluate* phase, where the evaluation of the circuit that has been build. But before the call to **Evaluate** can be done each of the parties need to generate their inputs to the shuffle algorithms. Since the *deck* is harcoded into the circuit it is sufficient for them to provide the *seed*. This is done by generating 16 bytes of random data and using this as a seed for a pseudo random generator producing the 830 bit randomness used for each *deck*. Then this is used as input to the **Evaluate** function.

The last phase is the *Online* phase it is here the calls to **DecodeKeys** is done. This phase is run for each game played. The amount of games possible is specified in the number of simuntainiusly shuffled decks. In this phase the *Constructor* and *Evaluator* must first agree on which outputwires should be opened to which party. Therefor the first the do is to generate an array containing the wire indexes to be opened to the *Constructor* and then those to the *Evaluator*. These are then opened and displayed to the players. For the first hand the *Constructor* is always dealt the first five cards from the deck, the *Evaluator* five cards starting from index 10. This is the most optimal way to deal the cards as this requires the least calls to **DecodeKeys**. This does not change the propability of the cards dealt. As known for real world card games one or two cards are normaly dealt at a time, this is propably done because the shuffle algorithm doeas not have a uniform distribution. But has a skew such that sequences of cards are more likely to repeat in the next game. This is not the case for our shuffle algorithms, since the start with the same initial *deck* and has a uniform propability distribution on the output.

Then a input interface is displayed to the players to allow them to choose which cards to change. This is done using a terminal interface where the user writes; 0 if no cards needs to changes, 1 if the first card should be changed, 2 for the second and so on, if multiple cards is to be changes this is done by seperating the card indexes with a comma , like 4, 5. To allow for a new hand to be dealt with the specified cards changed the parties first sends the amount of cards thay want to change, after this the indexes of the cards are sent. Now each party knows which wires should be opened, therefor the old index of output wires is updated to hold the new index wires. Then the second and final hand is opened an displayed to the parties by the second call to **DecodeKeys** The cards with indexes from 5 to 9 is reserved for the *Constructor*'s second hand and the cards 15 to 19 to the *Evaluator*'s. By opening the last hand in this way adds an overhead since one card may be openend in both the first and second hand. But this allows for less communication before the last revilation round in the game.

The last round of each game is the revelation phase of the oponents hand. This is done without any communicatin other than a last call to **DecodeKeys**. The input to this last call is done by switching the inputs for the *Constructor*

and *Evaluator* as they already know which wires was opened in the last hand to the oponent. By opening the oponents card this way we ensure that the oponent does not learn the cards which were discarded.

At last when all the decks are played the statistics are written to the log files. This is the timings and amount of data sent by the different calls. This data is used in section 4.2 to discuss the efficiency of the *DUPLO* protocol in the setting of a poker game.

A small note on the leak of information when communication indexes for **DecodeKeys**. The oponent always knows how many and which cards are changed because the parties need to agree on which wires to open, but this is not any different from real life poker. Therefore this leak of information should not be considered a security problem.

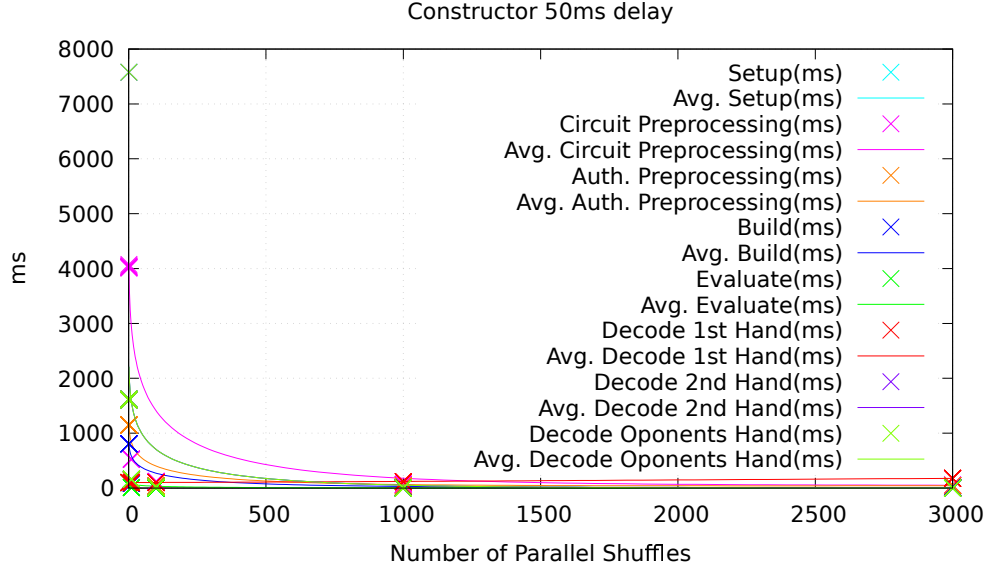
## 4.2 Benchmarking

**TODO: Implementation interaction, flags**

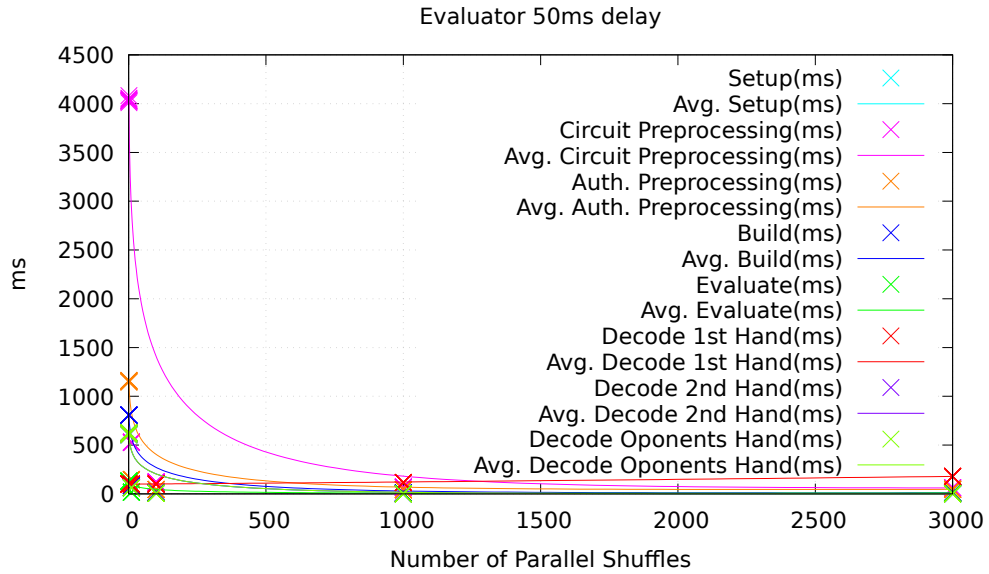
First a shuffle algorithm is compiled which is described how to do in appendix A.3. We use the *Conditional-swap* circuit implemented in section 3.3 as this has the least non-free *XOR* gates as seen in section 3.4. When this is in place

**TODO: Bash scripts**

**TODO: Removed values, ref logged values**

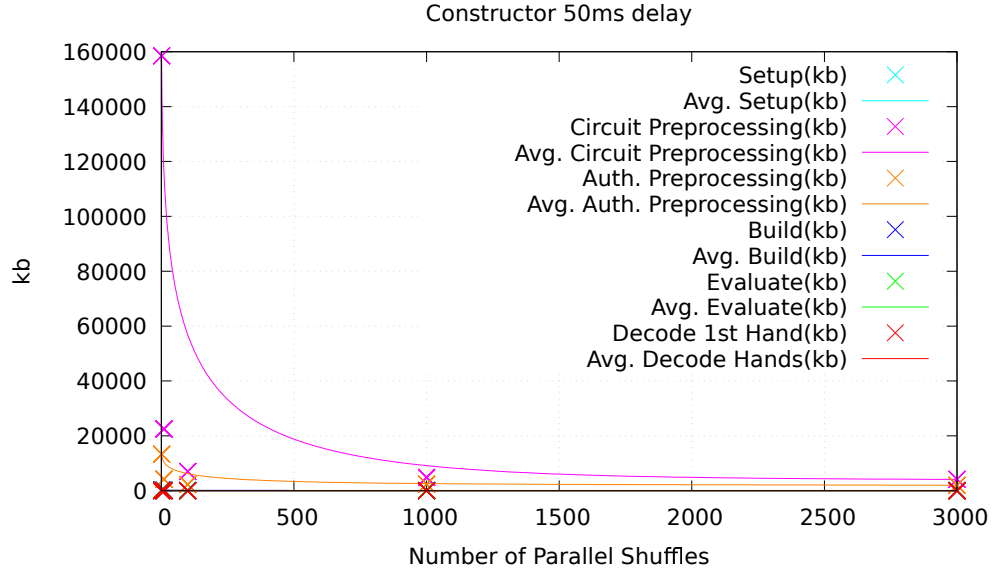


(a) Constructor: 50ms simulated network latency. Showing the time used in different phases per deck shuffled.

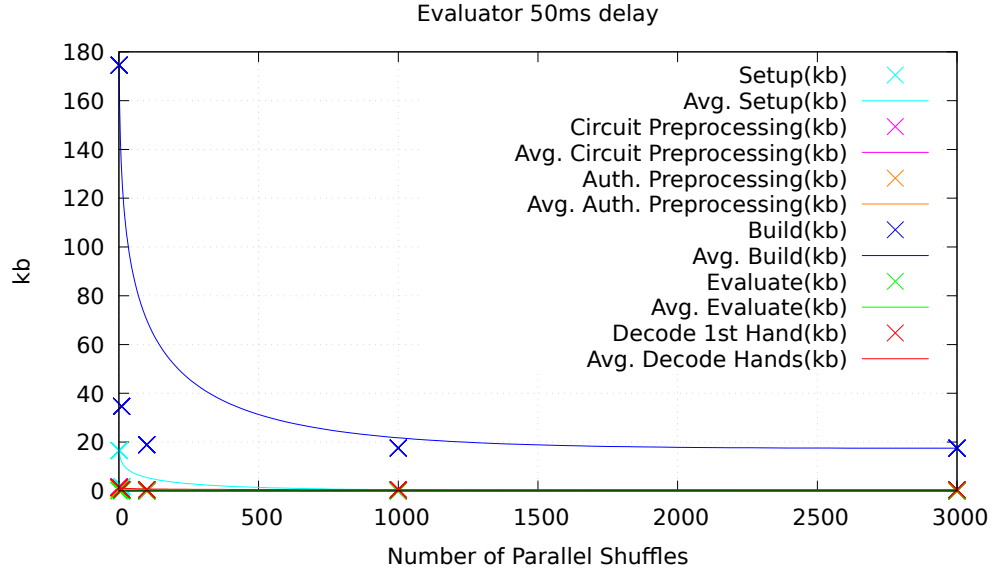


(b) Evaluator: 50ms simulated network latency. Showing the time used in different phases per deck shuffled.

Figure 4.2: Comparison of the Constructor and Evaluator on which phases requires the most time.



(a) Constructor:  $kb$  sent in different phases.



(b) Evaluator:  $kb$  sent in different phases.

Figure 4.3: Comparison of Constructor and Evaluator on which phases most  $kb$  are sent to the other party.

Phase	Delay(ms)	Number of Parallel Shuffles				
		1	10	100	1000	3000
Setup	50	2210.37	162.11	17.63	2.86	2.17
	0	1431.11	114.23	12.66	2.36	2.00
	-	779.29	47.88	4.97	0.50	0.17
Circuit preprocess	50	4033.93	527.35	119.99	67.52	51.36
	0	1564.52	201.07	50.75	29.75	19.50
	-	2469.41	326.28	69.24	37.77	31.86
Auth preprocess	50	1150.64	143.22	39.92	37.93	42.54
	0	93.10	31.95	20.02	22.90	33.57
	-	1057.54	121.27	19.90	17.03	8.97
Build	50	807.18	86.06	15.78	8.42	10.82
	0	22.84	6.79	6.27	6.46	9.02
	-	784.34	79.27	9.51	1.96	1.8
Evaluate	50	100.82	10.22	1.18	0.30	0.30
	0	0.69	0.24	0.24	0.23	0.24
	-	100.13	9.98	0.94	0.07	0.06
Decode keys 1st hand	50	100.37	100.39	100.91	115.62	177.92
	0	0.20	0.13	0.37	10.20	77.15
	-	100.17	100.26	100.54	105.42	100.77
Decode keys 2nd hand	50	100.37	10.23	1.24	0.50	0.86
	0	0.13	0.08	0.90	0.25	1.30
	-	100.24	10.15	0.34	0.25	-0.44
Decode Opponents hand	50	100.35	10.22	1.23	0.48	0.57
	0	0.11	0.08	0.09	0.23	0.91
	-	100.24	10.14	1.14	0.25	-0.34

(a) Constructor: comparison of the different phases and concurrent shuffles with and without delay.

Phase	Delay(ms)	Number of Parallel Shuffles				
		1	10	100	1000	3000
Setup	50	620.25	61.89	6.15	0.62	0.21
	0	117.58	11.36	1.05	0.09	0.03
	-	502.67	50.53	5.01	0.53	0.18
Circuit preprocess	50	4037.57	528.96	120.39	68.30	60.86
	0	1574.70	202.97	51.89	30.79	29.63
	-	2462.87	325.99	68.50	37.51	31.23
Auth preprocess	50	1154.60	146.40	44.90	42.32	46.33
	0	94.98	34.70	24.73	27.37	37.65
	-	1059.62	111.70	20.17	14.95	8.68
Build	50	807.23	86.12	15.83	8.50	10.88
	0	22.88	6.82	6.33	6.59	9.14
	-	784.35	79.30	9.50	1.91	1.74
Evaluate	50	129.49	17.91	5.43	5.44	9.58
	0	15.71	5.13	2.92	2.29	7.96
	-	113.78	12.78	2.51	3.15	1.62
Decode keys 1st hand	50	100.60	100.63	101.17	115.91	179.12
	0	0.29	0.19	0.43	15.01	125.68
	-	100.31	100.44	100.26	100.90	53.44
Decode keys 2nd hand	50	100.59	10.23	1.47	22.08	128.10
	0	0.18	0.08	0.22	14.31	124.84
	-	100.41	10.15	1.25	7.77	3.26
Decode Opponents hand	50	100.57	10.23	12.70	21.81	122.89
	0	0.17	0.07	0.22	14.16	123.14
	-	100.40	10.16	12.48	7.65	-0.25

(b) Evaluator: comparison of the different phases and number of concurrent shuffles with and without delay.

Table 4.1: Comparison of the time consumption of the Constructor and Evaluator.

Phase	Number of Parallel Shuffles				
	1	10	100	1000	3000
Setup	12.70	1.27	0.12	0.01	0.00
Circuit Preprocess	158470.00	22518.10	7023.94	4837.94	4127.91
Auth. Preprocess	13329.90	4170.08	2249.69	2437.49	1986.51
Build	327.71	128.49	107.24	105.12	104.96
Evaluate	122.84	122.84	122.84	122.48	122.84
Decode keys	5.94	2.38	2.02	1.99	1.99

(a) Constructor:  $kb$  sent per shuffled deck in different phases.

Phase	Number of Parallel Shuffles				
	1	10	100	1000	3000
Setup	16.55	1.65	0.17	0.02	0.01
Circuit Preprocess	1.59	0.16	0.02	0.00	0.00
Auth. Preprocess	1.06	0.11	0.01	0.00	0.00
Build	174.55	34.67	18.95	17.55	17.49
Evaluate	0.11	0.10	0.10	0.10	0.10
Decode keys	1.44	0.58	0.49	0.48	0.48

(b) Evaluator:  $kb$  sent per shuffled deck in different phases.

Table 4.2: Comparison of the Constructor and Evaluator on the amount of data sent in the different phases.



## Chapter 5

# Conclusion

TODO: Conclusion bla ...



# Appendix A

## Codebase

In this appendix references will be presented to the different codebase used during the thesis. An url to the repositories on github will be presented together with a short description of where the most interesting parts for this project can be found.

### A.1 Hardware

For compilations of the circuits with the *Frigate* compiler the following setup has been used:

OS:           Ubuntu 16.10/17.04  
Processor: Intel i5-4210U CPU @ 1.70GHz  
Cores:        2  
Threads:      4  
RAM:          12 GB

I did not encounter any problems or slow compilations of circuits using this setup.

For the testing of the poker game that setup was not sufficient as it could not handle more than 500 simultaneous shuffles. Therefore another setup was used:

OS:           Ubuntu 16.04 LTS  
Processor: Intel i7-3770K CPU @ 3.50GHz  
Cores:        4  
Threads:      8  
RAMS:         32 GB

This setup allowed for testing up to 3000 simultaneous shuffles, which is the highest done in the testing phase. When going up to 4000 this setup ran out of memory on the *Evaluator* site of the execution.

## A.2 DUPLO

The *DUPLO* repository at GitHub can be found here <sup>1</sup>.

The documentation on the site is clear and illustrates clearly how it is compiled such it can be tested. No documentation is presented for how interacting with the framework can be done for new implementations. The most interesting part for the sake of this project is located in the `src` folder. Here the `CMakeLists.txt` file is located which specifies how the project is compiled, this is overwritten when compiling the poker implementation. The folder `src/dublo-mains` is where the actual implementations of the *Constructor* and *Evaluator* can be found. Here the implementations of the poker *Constructor* and *Evaluator* will be placed.

For a easy setup of duplo a docker instance is created and can be found here <sup>2</sup>. This can be started in docker version `17.05.0-ce` with the command;

```
docker run -it --network:host cbobach/duplo
```

The `--network:host` flag is not secure but is the fast easy way to let the container running the *Constructor* expose the port on wich the container running the *Evaluator* needs to connect. When running two instances of these docker containers the *Constructor* and *Evaluator* is runned using one for these commands for the default setting:

```
./build/release/DuploConstructor  
./build/release/DuploEvaluator
```

## A.3 Frigate

The *Frigate* repository on GitHub is a subrepository to *DUPLO* and can be found here <sup>3</sup>.

The documentation of how *Frigate* is installed with the special versions of some of the libraries used is specified in the documentation of *DUPLO*, the link can be found in appendix A.2. It is also here the documentation of how to compile *DUPLO* circuit formats are done.

To find the documentation of the `.wir` file format a look should be taken at the link above. Here the specifications are of how wire acces is done for example. It is here all functionalities that are implemented in the language is listed and how they are used. This documentation is narrow at some places. It does for example not specify that the modulo operator `%` does only work on powers of 2.

Using the docker image from docker <sup>4</sup> and running the following command, in docker version `17.05.0-ce`;

---

<sup>1</sup><https://github.com/AarhusCrypto/DUPLO>

<sup>2</sup><https://hub.docker.com/r/cbobach/duplo/>

<sup>3</sup><https://github.com/AarhusCrypto/DUPLO/tree/master/frigate>

<sup>4</sup><https://hub.docker.com/r/cbobach/duplo/>

```
docker run -it -v host/dir:container/dir cbobach/duplo
```

will start a container where it is possible to compile a `.wir` file using the container. For it to work the `.wir` files has to be located in the `host/dir` then the following command can be run to compile the functionality:

```
./build/release/Frigate container/dir/functionality.wir -dp
```

The `-db` flag ensures that the *DUPLO* file format is generated. The *DUPLO* generate file will have the extension `.wir.GC_duplo` this can then be feeded to the *DUPLO* framework using

```
./build/release/DuploConstructor -f container/dir/functionality.wir.GC_duplo  
./build/release/DuploEvaluator -f container/dir/functionality.wir.GC_duplo
```

Then the new functionality will be runned in the default *DUPLO* environment.

## A.4 Poker

In this section the GitHub repositories to the different phases will be linked. A short description to where the interesting parts are will be presented.

### A.4.1 Circuit implementation

The different circuit `.wir` files can be found in the Github repository here <sup>5</sup>.

Here the implementations of the shuffle algorithms are present as `fisher_yates_shuffle.wir` and `conditional_swap_shuffle*.wir`. Multiple versions of the `conditional_swap_shuffle*.wir` file are present with different values for `*`. This is to allow for multiple sequential hands to be played, these files are then used when testing the *DUPLO* framework to show its capabilities.

Only one version of `fisher_yates_shuffle.wir` is located in the repository since this is a slower algorithm in this setting as discussed in section 3.4.

The files `init_deck.wir`, `xor_seed.wir` and `corrected_seed.wir` are all modules that are called by the shuffle algorithms. The `init_deck.wir` file is used by both algorithms and hardwires the card values to their respective wires. The `corrected_seed.wir` file is used by the *Fisher-Yates* algorithm to ensure that the seed feeded to the shuffle algorithm is in the correct intervals as explained in section 3.1. The `xor_seed.wir` file is used by the *Conditional-swap* algorithm to generate the seed used by the shuffle.

It is also here that the parser used to debug the *Frigate* compiler is located and is found as `parse.py`. The other python script found in `count-gate-types.py` is the one used to compare the amount of gates types for the compiled circuits.

---

<sup>5</sup>[https://github.com/cbobach/speciale\\_circuit](https://github.com/cbobach/speciale_circuit)

### A.4.2 DUPLO implemetation

The poker repository for the implementation using the duplo framework can be found here <sup>6</sup>.

Here the `CMakeLists.txt` file is the one used to overwrite the original file found in the *DUPLO* framework to allow for compilation of the poker *Constructor* and *Evaluator*. In the folder `duplo-mains` the implementations of these are located as `poker-const-main.cpp` for the *Constructor* and `poker-eval-main.cpp` for the *Evaluator*. In this folder their shared functionality is found in the `poker-mains.h`.

Back in the main dir of the repository the docker files are found for generating the docker instance of *DUPLO* used in appendix A.2 and A.3. This is the `Dockerfile.DUPLO` where as the `Dockerfile` is the one used for running the poker implementation. The `entry-point.sh` file is used to start the docker containers correct such that they can run in the background. The docker image can be found here <sup>7</sup>. The containers can be started using the following commands in docker:

```
docker run -d -p 2800:2800 cbobach/duplo-poker --profile const -i 0
docker run -d --network:host cbobach/duplo-poker --profile eval -i 0
```

The `-d` flag tells docker that the containers should run detached. The `-p` flag tells docker to connect the host port 2800 to the containers internal port 2800. `--network:host` is the easy way to let the container have access to the hosts network ports. These commands will play one hand of poker in the background, if more are required the `-f` flag can be used to specify which circuits should be used. To get the right timings the `-n` flag is required together with the `-f` flag. This flag needs to reflect the number of simultaneous shuffles in the circuit.

Using the `-it` flag in docker instead of the `-d` flag allow for interactive rounds of poker if the `-i` flag is set to 1 instead of 0.

### A.4.3 Test results

In this section a link to the repository on GitHub with all the generated statistics. Here all generated graphs and timings can be found. The repository can be found here <sup>8</sup>

TODO: Add log files to GitHub

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<sup>6</sup>[https://github.com/cbobach/speciale\\_implementation](https://github.com/cbobach/speciale_implementation)

<sup>7</sup><https://hub.docker.com/r/cbobach/duplo-poker/>

<sup>8</sup>[https://github.com/cbobach/speciale\\_thesis/tree/master/figurs](https://github.com/cbobach/speciale_thesis/tree/master/figurs)

# Primary Bibliography

- [A1] Richard Durstenfeld. Algorithm 235: Random permutation. <http://doi.acm.org/10.1145/364520.364540>, July 1964.
- [A2] Vladimir Kolesnikov, Jesper Buus Nielsen, Mike Rosulek, Ni Trieu, and Roberto Trifiletti. Duplo: Unifying cut-and-choose for garbled circuits. Cryptology ePrint Archive, Report 2017/344, 2017. <http://eprint.iacr.org/2017/344>.
- [A3] J. Katz Y. Huang, D. Evans. Private set intersection: Are garbled circuits better than custom protocols? <https://www.cs.virginia.edu/~evans/pubs/ndss2012/>, 2012.





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- [B4] F. Yates R.A. Fisher. Statistical tables for biological, agricultural and medical research, 6th ed. <http://hdl.handle.net/2440/10701>, 1963.