Are we learning to build better models at the cost of better decisions?

Carl Boettiger[a,1]

[a]*Department of Environmental Science, Policy, and Management, University of California, 130 Mulford Hall Berkeley, CA 94720-3114, USA*

Abstract

Model-based forecasts have been enabled by recent explosion of data and computational methods and spurred on by decision-makers appetite for forecasts in everything from elections to pandemic response. It is taken for granted that the model which makes the most accurate forecast, accounting for uncertainty, will also be the best model to inform decision-making. Using a classic example from fisheries management, I demonstrate that selecting the model that produces the most accurate and precise forecast can lead to decidedly worse outcomes.

Keywords: transients, optimal control, adaptive management, stochasticity, uncertainty, ecological management

Global change issues are complex and outcomes are difficult to predict, making approaches to uncertainty a central part of effective decision-making [@ @]. While some uncertainty is intrinsic to the underlying processes (stochasticity) or by limits to what variables we can observe and how accurately we measure them (measurement uncertainty), much uncertainty is the result of our imperfect knowledge of the processes involved, as expressed by the structure and parameters of mathematical models used to approximate those processes (model uncertainty). Unlike the other forms of uncertainty, model uncertainty can be reduced by gathering additional data, which may rule out certain models or parameter values as implausible[1]. When model parameters are estimated directly from available data, there is a risk that the best-fitting models may *overfit* to patterns arising from by chance from the stochasticity or measurement error which do not reflect the underlying process. Researchers have often favored simpler models which are less prone to over-fitting, sometimes through explicit penalties such as information criteria [@], though these can be misleading [@]. With the rapid expansion of available ecological and environmental data [@], it is increasingly possible to rely instead on comparisons between model predictions and data purposely excluded from model estimation (cross-validation), or better, comparisons between model predictions and data collected in the future (forecasting, J. S. Clark et al. (2001); Dietze et al. (2018)]). Over sufficiently long timescales, comparing model forecasts to future observations will select the models with greatest predictive accuracy. Because model-based forecasts frequently play an important role in decision-making, it is commonly assumed (e.g. Carl J. Walters 1981; J. S. Clark et al. 2001; Dietze et al. 2018) that this approach for addressing model uncertainty will also improve decision-making outcomes. Here, I illustrate that this need not be the case: it is quiet possible that any approach with improves the forecast accuracy of the model(s) over time can simultaneously lead to worse decision outcomes. This example illustrates that knowing when increased forecast accuracy will or will not also improve decision outcomes in general remains an open problem, as is the challenge of what to do about it.

Email address: `cboettig@berkeley.edu` (Carl Boettiger)

[1]While some authors distinguish between model uncertainty and parameter uncertainty, I will refer collectively to any uncertainty that arises from our imperfect knowledge of the system as "model uncertainty." Uncertainty in model structure can often be reflected by appropriate parameterization, just as models with different parameter values can be treated distinctly. Whereas stochasticity is inherent to the process (or rather, to our choice of state variables, Boettiger (2018)) and measurement uncertainty can only be reduced by more accurate tools for measurement, model uncertainty is reducible by learning from additional data.

I illustrate this problem using an example from fisheries management. Fisheries are a significant economic and conservation concern world wide and their management remains an important debate (e.g. Worm et al. 2006, 2009; Costello et al. 2016). Moreover, their management has been a proving grounds for theoretical and practical decision-making issues which are widely applicable in other areas of ecology and conservation (D. Ludwig, Hilborn, and Walters 1993; Lande, Engen, and Saether 1994), and one that has long wrestled with issues of uncertainty in the context of management decisions (e.g. C. W. Clark 1973; Reed 1979; Carl J. Walters 1981; Donald Ludwig and Walters 1982). While modern fisheries management frequently relies on complex models which may contain scores of parameters to reflect the specific age or stage structure of a specific fish stock (Database 2018; Ricard et al. 2011), I will rely on simple, well-studied models which permit greater intuition and generalization (Levins 1966). Consistent with such previous work (Schaefer 1954; C. W. Clark 1973; Reed 1979; Carl J. Walters 1981; Donald Ludwig and Walters 1982; Costello et al. 2016), let us consider the problem of determining the optimal harvest policy given a measurement of the current stock size.

Ecological Models

The sustainable harvest decision problem can be stated as follows: The fish stock is observed to be in state X_t at time t, and is then subjected to some harvest H_t before recruiting new fish, subject to stochastic environmental noise ξ_t, to bring the stock to $X_t + 1$,

$$X_{t+1} = f(X_t - H_t, \xi_t) \tag{1}$$

A manager seeks each year to select the harvest H_t which will maximize the sum of the utility derived from such a harvest and such a state, $U(X_t, H_t)$, over all time, subject to discount rate δ (C. W. Clark 1973):

$$\sum_{t=0}^{t=\infty} U(X_t, H_t)\delta^t \tag{2}$$

Further we imagine that the function f is not known precisely, and so we will rely on an evaluation of forecasting skill across a set of candidate models to determine which one to use to manage the fishery. Again for simplicity, we will restrict ourselves to two simple candidate models f_1 and f_2. Both share the same underlying structure of logistic recruitment (known as the Gordon-Schaefer model in fisheries context owing to groundbreaking work independently by Schaefer (1954) and Gordon and Press (1954)), differing only in their choice of certain parameters:

$$f_i(Y) = Y + r_i Y \left(1 - \frac{Y}{K_i}\right) * \xi_t(\sigma) \tag{3}$$

Where $\xi_t(\sigma)$ represents log-normal random noise with mean of unity and log-standard-deviation σ. Model 1 is given by $r_1 = 2$, $K_1 = 16$, $\sigma_1 = 0.05$, Model 2 by $r_2 = 0.5$, $K_2 = 10$, $\sigma = 0.075$ (in dimensionless units). Having both the larger growth rate and the larger carrying capacity, Model 1 is clearly the more optimistic of the two choices.

Selecting between Model 1 and Model 2 can thus be considered the simplest illustration of the model uncertainty problem. This is a subset of the more general problem of selecting model parameters, assuming a logistic growth, which itself is a subset of estimating the best structural from (e.g. Ricker, Beverton-Holt, etc). There is no need to consider these more complicated versions of the model uncertainty problem here, since they all inherit the same issue. Reducing the model selection problem to these two models simplifies the presentation and will aid intuition at no loss of generality.

The only additional assumption we will need is that the "true" model is not among the suite of models under consideration. Mathematical models are, at best, approximations of the underlying processes. Ecological processes are much too complex to ever be modeled exactly. For illustrative purposes, we will thus assume the "true" process to be given by Model 3, which is unknown to the decision-maker, but similar enough to at least one of the candidate models might be considered a reasonable approximation:

$$f_3(Y) = Y + r_3 Y^4 \left(1 - \frac{Y}{K_3}\right) \tag{4}$$

with $r_3 = 0.002$, $K_3 = 10$ and $\sigma_3 = 0.05$.

The task of deciding whether Model 1 or Model 2 would be the better choice for decision making is thus perhaps the simplest example of the much studied issue of model uncertainty that we can pose. As in any real world scenario, neither model is the true model, but nevertheless this model set contains a good enough approximation of the true model to make good decisions. However, any of the well-developed approaches for decision-making under model uncertainty will prefer Model 2 over Model 1, despite the fact that the optimal policy under Model 2 leads to much worse outcomes ecologically and economically.

Methods for Managing Under Model Uncertainty

A wide range of paradigms are available for approaching the issue of decision-making under uncertainty. These approaches can roughly be divided into two groups: the first group treats the issue of model uncertainty independently from the decision itself, while the second integrates the process of reducing model uncertainty into the process of decision making to maximize the value of some objective. There are a wide range of techniques within each, and it is also possible to blend approaches. The key distinction is that methods in the first group do not involve any direct consideration of the possible actions or the utility that may result from those actions in how they select models (statistical approaches such as information criteria e.g. Burnham and Anderson 1998); and in particular, forecasting evaluation J. S. Clark et al. (2001); Dietze et al. (2018)], while those in the second group require a more explicit statement of possible actions and the desired objectives (Decision theoretic approaches, for which Polasky et al. 2011 provides an excellent and accessible review). I illustrate how the most promising techniques from each of these groups would be applied to this simple problem, and demonstrate that it both cases they lead us away from the model that produces the most desirable decisions towards worse outcomes. In retrospect, it will become obvious that neither these nor any other widely applied methods will select the model that leads to the best outcomes from the set of models considered.

Forecasts and Proper Scores

Like many decision problems, the task of setting a sustainable harvest quota appears to hinge on having an accurate forecast: if we can predict to what size the fish stock will increase next year, $X_t + 1$, and we know the current stock, X_t, then we can safely harvest $X_{t+1} - X_t$. Overestimating or underestimating such recruitment will result in over-harvesting or under-harvesting, respectively. Selecting a model based on forecast skill is also justifiable on theoretical grounds, since it reduces the risk of over-fitting by comparing model predictions to later observations that were not used to estimate the model. Gneiting and Raftery (2007) provides a rigorous proof for "proper" scoring criteria, which have the desirable property which no model predicting the distribution of future outcomes, $Q(x)$ can achieve a better average score than the true model $P(x)$. That is, unlike likelihood or other goodness-of-fit scores, it is impossible to overfit when conditioning on a strictly proper score – since no model model can out-perform the true model. Not that strictly proper scoring rules score *probabilistic forecasts* and not just point predictions, favoring models which accurately reflect the uncertainty over those which under-estimate it. These features have made proper scoring rules for probabilistic forecasts a successful and popular approach for addressing model uncertainty in many other areas (Gneiting and Katzfuss 2014; Raftery 2016) and a promising tool for evaluation of ecological forecasts (Dietze et al. 2018).

I illustrate the process of model selection by strictly proper scoring rules using two scenarios. In Scenario A (passive observation) the fish stock is unharvested and allowed to recover towards carrying capacity (as simulated under our "true" model, Model 3) while comparing the observed stock size in each subsequent time step to the distribution predicted under Model 1 and Model 2 respectively [Fig 1]. The mean, μ

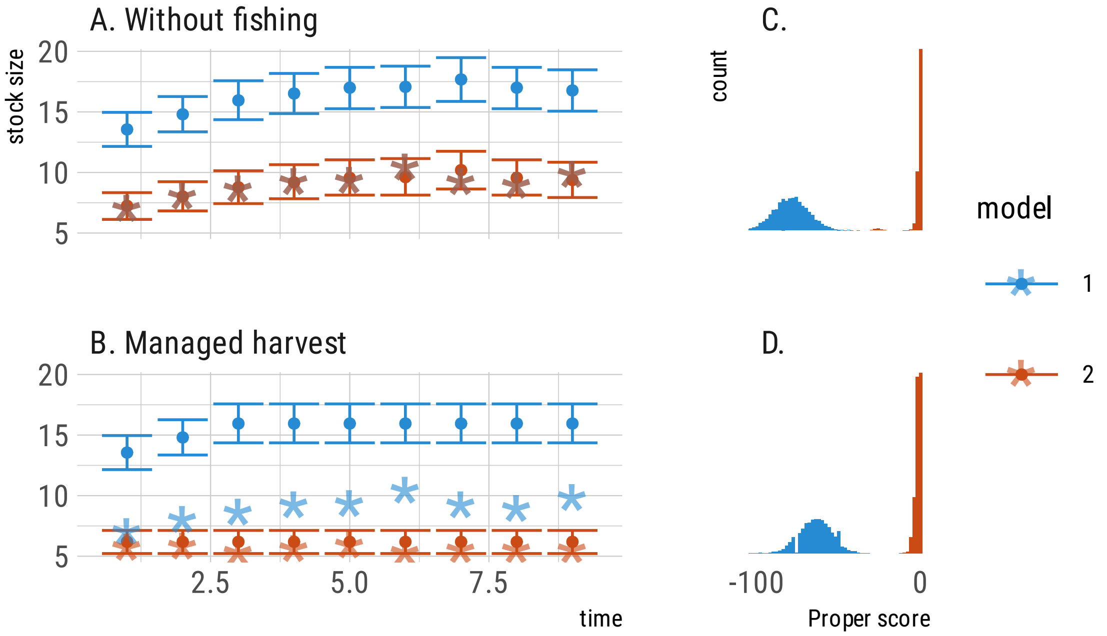

Figure 1: Forecast performance of each model. Panels A, B: Step ahead predictions of stock size under unfished (A) and fished (B) scenarios. Error bars indicating the 95% confidence intervals around each prediction, while stars denote the observed value in that year. Because the models make different decisions each year in the fished scenario, the observed stock size in year 2, 3, etc under the management of model 1 (blue stars) is different from that under model 2 (red stars). Panels C, D: corresponding distribution of proper scores across all predictions (100 replicates of 100 timesteps). Higher scores are better, confirming that model 2 makes the better forecasts.

and variance, σ of the forecast are compared against the true observation x using a Proper scoring rule of Gneiting and Raftery (2007),

$$-(\mu - x)^2/\sigma^2 - \log(\sigma) \tag{5}$$

for each prediction over 100 replicate simulations of 100 time steps each [Fig 1].

In Scenario B (actively harvest), I have first solved for the optimal management strategy for both Model 1 and Model 2 [Fig 1b]. For small noise and concave functions with linear reward structure this can be done analytically (see proof in Reed 1979), or solved more generally by stochastic dynamic programming (see review by Marescot et al. 2013, details in Appendix). Under this scenario, replicate simulations of the stock are harvested at each time step using the optimal quota dictated by either Model 1 and Model 2. The resulting stock sizes in the time-step following this harvest are once again compared to the probabilities predicted by each model using Eq (5). Model 2 unequivocally outperforms Model 1 in both scenarios of passive observation and active harvest.

Despite the clearly superior predictive accuracy of Model 2 in both scenarios, the outcomes from management under Model 2 are substantially worse. We can assess such outcomes in less abstract terms than forecasting skill, such as economic value (in dollars) or the ecological value (unharvested biomass). In our simple formulation of the decision problem, the "utility" the manager seeks to maximize is simply the economic value (net present value: the discounted sum of all profits from future harvests, Eq (2)) of harvested fish. This formulation ignores any utility provided by fish that are not harvested, beyond their contribution to future potential harvests. While it is possible to include such contributions directly in the utility function being optimized (e.g. Halpern et al. 2013), even without doing so, Model 1 maintains both a higher unharvested biomass and also leads to higher economic returns throughout [Fig 2].

Had we been able to include Model 3 in our forecast comparisons, it would equal or outperform the forecasting skill of both Model 1 and Model 2 (as guaranteed by the theorem of Gneiting and Raftery

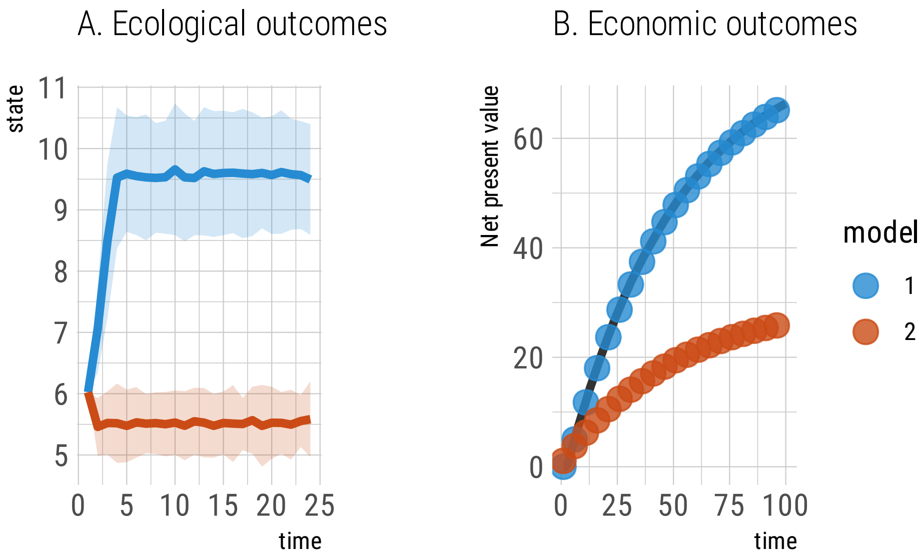

Figure 2: Ecological and economic performance of each forecast. Harvest quotas derived from Model 1 result in a significantly higher fish stock size than under Model 2 (panel A). Economic returns under Model 1 are also substantially higher (panel B)

(2007)), while also matching or out-performing their economic utility (as guaranteed by the theorem of Reed (1979)). In practice, we never have access to the generating model.

One obvious limitation in this comparison is that scenario B treats each model as fixed over the entire course of the simulation. Research has long emphasized the importance of learning and adaption in the face of new data whenever we are dealing with model uncertainty (Polasky et al. 2011). A related limitation is that in Scenario B, decisions were based either on assuming Model 1 is correct or assuming Model 2 is correct. A more robust approach (e.g. Carl J. Walters and Hilborn 1978) incorporates the uncertainty over models directly into the decision calculations by integrating over the probability each model M (or parameter value) is correct when calculating the utility; $U(x_t, a_t) = \int U(x_t, a_t|M)P(M)$. The probability assigned to each model can then be updated after each subsequent action in light of the resulting outcomes Smith and Walters (1981). While a the wide range of strategies for such iterative updating are known (Polasky et al. 2011); all will fail to select the higher-performing of the two simple models considered here.

Decision-Theoretic Approaches

Decision-theoretic approaches include scenario analysis, resilience thinking, optimal control and related methods (Polasky et al. 2011), which have a long history in ecology and particularly in fisheries management. One of the most influential of these is *adaptive management* (sensu Carl J. Walters and Hilborn 1978), in which a manager seeks to both reduce uncertainty over time while also achieving the best outcomes given current knowledge. While the term today is frequently used in a looser sense, adaptive management as originally developed can be considered an example of optimal control under model uncertainty (Carl J. Walters 1981; Smith and Walters 1981). In this approach, a manager assigns probabilities to each of the possible models under consideration, i.e. models 1 & 2. The expected utility of any action reflects both the intrinsic uncertainty over future states under either model (process uncertainty) and the manager's uncertainty over the choice of models (or parameter values). Unlike the forecast comparison, an adaptive management approach explicitly considers the utility function, Eqn (2), which it seeks to maximize by intergrating over model uncertainty.

In passive adaptive management, the manager chooses the action which maximizes that expected future utility. After observing the consequences of the action, the manager updates the probabilities each model is correct, typically by Bayes rule (e.g. Carl J. Walters 1981; Smith and Walters 1981; Donald Ludwig and Walters 1982). In active adaptive management, the key difference is that the manager may chose an

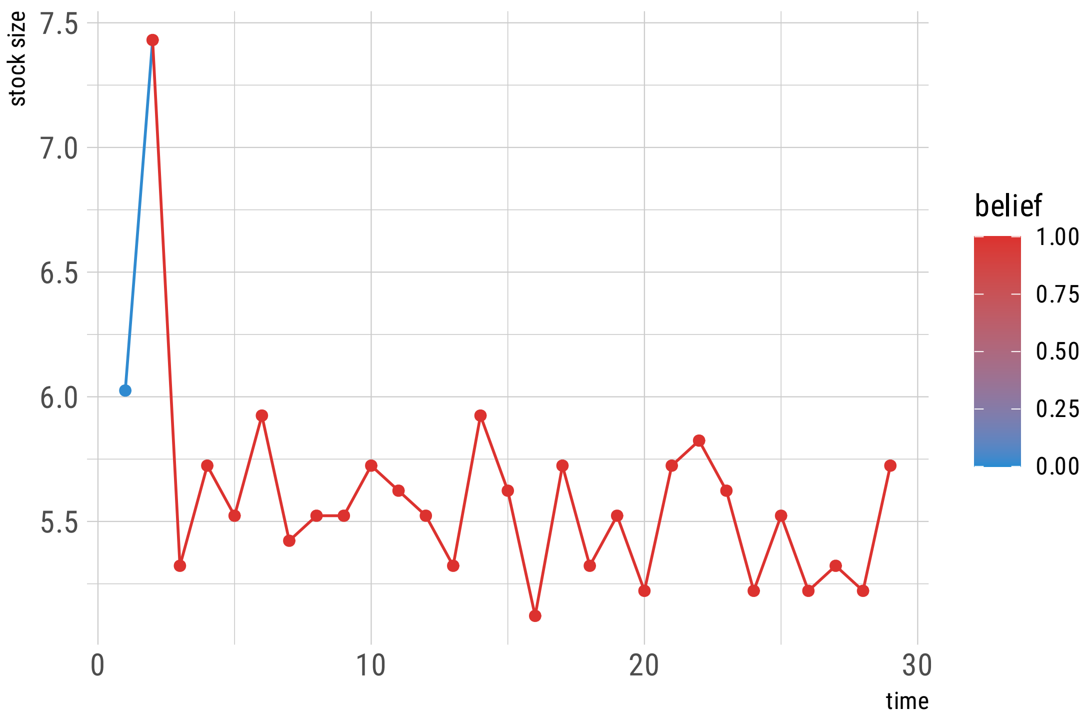

Figure 3: Adaptive management under model uncertainty, color indicates the belief that Model 2 is correct (red). Each model is assigned an initial prior belief. To ensure that any issues with passive exploration are not the cause of preferring Model 2, the initial belief in Model 2 is set to 1%. Within a single iteration of adaptive management, the belief over models is updated to near certainty in Model 2, resulting in higher harvests and lower stock sizes similar to managing under Model 2 alone, Fig 2.

action which does not maximize the utility *given the current model uncertainty*, but instead, takes an action whose consequences will more quickly discriminate between the models and thus more rapidly reduce the uncertainty. After observing the outcome of this exploratory step, model probabilities are updated as before. Under some scenarios the cost of this 'exploration' step can be made up later by actions based on the more precise knowledge gained as a result ('exploitation'). Alternative approaches such as scenario analysis can be considered approximations of this approach. A scenario analysis does not rely on quantitative estimates of model probabilities or expected utility, but still consider possible outcomes over a suite of models (scenarios), select actions to minimize the risk or or other suitable objective, and update or eliminate scenarios deemed implausible over time (Polasky et al. 2011).

Each of these approaches swiftly decides that model 1 in our example is implausible, and settles for making decisions driven almost exclusively by model 2. I illustrate this using a passive adaptive management strategy following classic examples for parameter (Donald Ludwig and Walters 1982) or structural (Smith and Walters 1981) model uncertainty. To demonstrate that the behavior is not driven by failure to explore sufficiently, (which might be addressed by an active adaptive management), I will assign initial probability that model 2 is true at 1%. After a single iteration of learning, these probabilities are completely reversed, with the manager deciding that model 2 is almost certainly correct [Fig 4]. As before, this results in a management practice with much worse ecological and economic outcomes than would have been realized by a manager who stubbornly clung to model 1 without updating [Fig 2, achieving a net present value over 100 time steps that only 31% that expected under management using Model 1 alone]. If instead of considering only these two models we consider a whole suite of Gordon-Schaefer-type models with varying r and K parameters, learning is slightly slower but no less counter-productive: mean stock size is 46 and net present value is 20 of what would be achieved under Model 1 (Appendix, Fig S1).

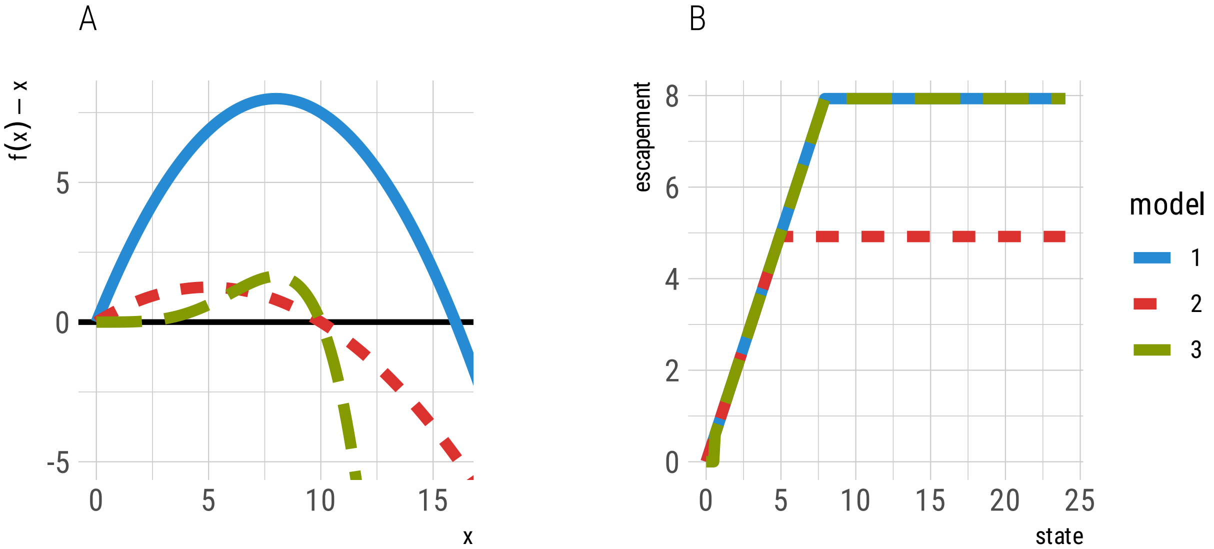

Figure 4: Panel A: Population growth curves of each model. The positive equilibrium of each model occurs where the curve crosses the horizontal axis. Note that while Model 2 is a better approximation to the truth (Model 3), Model 1 better approximates the stock size which leads to maximum growth.

Discussion

Given this simplified version of model uncertainty in which one of the two models leads to effectively optimal ecological and economic outcomes, current approaches invariably choose the other. The reason for this may appear obvious once we compare both curves to that of the underlying model, Model 3. Plotting the growth rate functions of each model, [Fig 4a], it is hardly surprising that no method which would prefer the closely overlapping Model 2 to the no-where-close Model 1 as the better approximation of Model 3. Nevertheless, decisions based on model 1 are nearly indistinguishable from those based on the true model [Fig 4b], while Model 2 leads to over-harvesting. So why do our best approaches fail such a simple test case? It is tempting to dismiss this example as having little relevance to real-world decision problems on a variety of grounds: (1) this problem is an artifact of overly-simplified models, (2) this problem is an artifact of an unrealistic optimal control solution, (3) the problem arises from having started with different objectives, (4) the problem is already solved. I address these in turn.

Is this result an artifact of the simple models considered here? A more realistic approach would consider uncertainty over model parameters, as well as models with different structural forms, such as Beverton-Holt (Beverton and Holt 1957), Ricker (Ricker 1954), or Shepherd (Shepherd and Cushing 1980). Such complexity would not be sufficient to resolve the paradox of model uncertainty illustrated here. Simplicity facilitates intuition: Model 1 performs best because it correctly approximates the stock size which achieves maximum growth rate under the true model. In contrast, each of those widely used recruitment relationships are symmetric, and their best-fitting parameter estimates will always have a maximum growth rate at a lower stock size. It is easy to point out that even this wider array of parameter and model values is too simple, since fisheries models in practice are much more complicated still (indeed, too complicated for the formal integration over model uncertainty considered here). It is indeed true that either the forecasting or adaptive management example would work as expected if the set of candidate models included the true model, or at least a functional form that could better approximate the peak of the true model while also fitting better than model 2. But we never have the true model in real systems. If the candidate models are only caricatures, so too is the “true” model. The candidate models considered here are no doubt much closer approximations to model 3 (all being single species, unstructured in age, stage, time or space, etc) than any model used in practice is to reality. Morever, a more accurate model is not required to reach optimal outcomes – Model 1 already provides this by correctly approximating the key feature in this problem, even while it is wrong about much else. How do proceed when our best methods fail in such a simple case?

Is this result an artifact of optimal control? The analysis has relied on optimal control theory to determine the optimal decision associated with each model or distribution of models. The resulting policies are mathematically optimal under the assumptions given Marescot et al. (2013). In most realistic decision problems, model complexity currently precludes the calculation of an optimal control solution at all ("the curse of dimensionality," e.g. Polasky et al. 2011; Marescot et al. 2013). Under the assumptions of any of the models considered here, the optimal control solution leads to what is known as a "constant escapement" policy, which seeks to bring biomass to the target escapement level, B_{MSY} as quickly as possible, $H_t = \min(X_t - B_{MSY}, 0)$. If the assumptions are not met, a constant-escapement policy may not be optimal, as in the case when observations of the stock X_t are not perfect (Memarzadeh and Boettiger 2019). While constant escapement is used as a basis of management in some real world cases such as salmon fisheries, most marine fisheries are based on a target of "constant mortality," (F_{MSY}). Notably, a constant mortality policy, implemented as $H_t = F_{MSY} X_t$, would not perform better under Model 1 than Model 2. Constant mortality is optimal only under deterministic growth at equilibrium (where the policy is the same under constant escapement). Does that mean the result is an artifact of assuming constant escapement rather than constant mortality as the basis for management? No, it does not. It is just as easy to construct a model under which the constant mortality matches the constant mortality of the true model while being a poor predictor. Moreover, using a non-optimal policy would always beg the question of whether the poor outcomes were merely the result of using a non-optimal policy rather than a consequence of model uncertainty. Because the space of possible actions is smaller than that of possible models, we should expect most decision problems to have their own "Model 1," a model that would lead to correct actions while being a poor predictor of future outcomes. The trouble is that we have no theory which would not reject such a model.

Is this result merely an artifact of differing objectives? Gneiting and Raftery (2007)'s proper scoring rules are designed to select models which make the best forecast, and Model 2 really is the best choice for that task. However, the adaptive management approach explicitly seeks to maximize expected utility, integrating over the uncertainty in the model, and yet it evolves away from Model 1 which would achieve that goal and settles on Model 2. Note that this issue is not resolved by updating the learning step to reflect only the immediate reward – it is in fact easy to see that so-called greedy algorithms would take the largest harvest possible and never reach the sustainable yield achieved by lower harvest rates of Model 1. To select Model 1 over alternatives like Model 2 which make better predictions but lead to worse outcomes requires a more long-term approach that forgoes iterative updating of probabilities as new data becomes available. Note that this is the opposite recommendation of *adaptive* management (Carl J. Walters and Hilborn 1978) or *iterative* forecasting (Dietze et al. 2018).

Perhaps Model 1 is a unicorn, a mythically simple model that gives optimal results but has no analogue in the real world. This is not an easy assertion to prove either way. Future observations consistently fall outside the 95% confidence intervals predicted under Model 1. Unfortunately, such innaccurate predictions are only too common in the real world. Have any of those led to effective policy despite the innaccuracy? Have any been replaced with more accurate models that have also driven worse decisions? That is more difficult to answer. What this example demonstrates is that if such unicorn models exist, our current paradigm will surely filter them out eventually, to be replaced with worse outcomes.

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