No indicators for stochastic transitions but establishing baselines for early warning signals remains a general challenge

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In Boettiger & Hastings [1] we demonstrated that conditioning on observing a purely stochastic transition from one stable basin to another could generate time-series trajectories that could be mistaken for an early warning signal of a critical transition (such as might be due to a fold bifurcation; [2], when instead the shift is merely due to chance. While the goal was to highlight a potential danger in mining historical records for patterns showing sudden shifts when seeking to test early warning techniques, Drake [3] draws attention to a potentially more interesting consequence of our analysis. Drake argues that the bias observed could be used to forecast purely stochastic transitions – a task previously thought to be impossible [4]. Unfortunately, we feel this interpretation too generous and must agree with the prevailing opinion that early warning signals for purely stochastic transitions do not exist. Drake demonstrates how the patterns presented in [1] show an indisputable difference between the time series that transition by chance and the much larger population of replicate time series driven by the identical mechanism that do not transition by chance. We argue that the appropriate null distribution is the subset of trajectories that had experienced an equally large deviation. Both numerical and analytical work show that the apparent signal to which Drake refers is an artifact of a large deviation, rather than than the probability of an impending transition.

What is meant by having found evidence of an early warning signal? We can only define this relative to some null distribution or baseline indicating what we might expect when no warning signal is present. This question has so far received insufficient attention, empirical and theoretical, throughout the recent literature on the subject [5]. Drake's analysis presents the choice of baseline or null distribution as the population of all replicates experiencing the identical dynamics. We argue that the appropriate null distribution in this case would rather be only the subset of those trajectories that had experienced an equally large deviation (with or without experiencing a transition), rather than the entire population. After all, our goal is to detect approaching transitions before they happen, not merely detect large deviations after they happen.

A numerical example shows that a similar pattern to that observed [1] can be seen in the large deviations of

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a purely stable system. This suggests that the pattern is not due to the curvature or any other feature of the chance transition, but only due to the conditional selection being biased to such chance transitions. We also discuss some earlier analytic work which helps explain the patterns observed.

We replicate the analysis of Boettiger and Hastings [1] using time series replicates produced by an Ornstein-Uhlenbeck (OU) process: a stochastic differential equation in which there is only a single optimum whose strength is proportional to the displacement,

$$dX = -\alpha X dt + \sigma dBt$$

Instead of conditioning on trajectories that experience a large deviation, we condition on trajectories that experience a very large deviation. We then compute warning signals for each of these large deviation trajectories, and compare their distribution to that of the entire population of trajectories as shown in Figure 1. We note that the same bias is observed. This should help illustrate that observations we reported are driven by the large deviation preceding the transition, and are simply evidence of this fact and not of an impending transition per se. While trajectories already far from the origin are more likely to transition than ones close to the average, such events can be trivially identified by comparing their states to the historical average.

Observing the bias shown in Figure 1 depends on having a rapid enough sample frequency to capture the escape trajectory and a long enough trajectory for the statistic to demonstrate an increase over time. Since large deviations due to stochastic forces alone must be fast so must the accompanying warning signal and managemnt response, which will show up on the time scale of the perturbation. Of course fast relative to the system dynamics, may or may not be fast relative to the timescale of management; just as in the case of bifurcation-driven warning signals, see Reference 7.

The realization that the trajectories of large deviations have a pattern that looks like an early warning signal can be better understood in light of early work on large deviations. Reference 1 observes that any trajectory that does manage to escape by chance does so quickly. This rapid dash to the boundary results in an escape path that is highly autocorrelated and marked by a rapid increase in variance as it departs from the mean. It turns out that we can be more precise about the speed of this trajectory: the expected transit time from the vicinity of the stable point to some distant deviation L can be shown to scale as $\log(L/\sigma)$. This result is rather remarkable considering it is the very same scaling to go against the gradient, uphill, as the expected time to *return* downhill from that distant point L to the neighborhood of the stable point. For comparison, the waiting time to observe such a trajectory scales as $\exp(L^2/\sigma^2)$, such that we must wait a long time before observing such a path. These results follow in the limit of $L \gg \epsilon$ for a region ϵ around the stable point and for small noise $\sigma \ll L$, and a careful walk through of the calculation can be found in Reference 8 for the OU process as considered here, which follows the derivation of Reference 9. This result has been rediscovered in other ecological and evolutionary contexts, e.g. Reference 10, and Drake and Griffen in fact provide an impressive empirical demonstration of the phenomenon

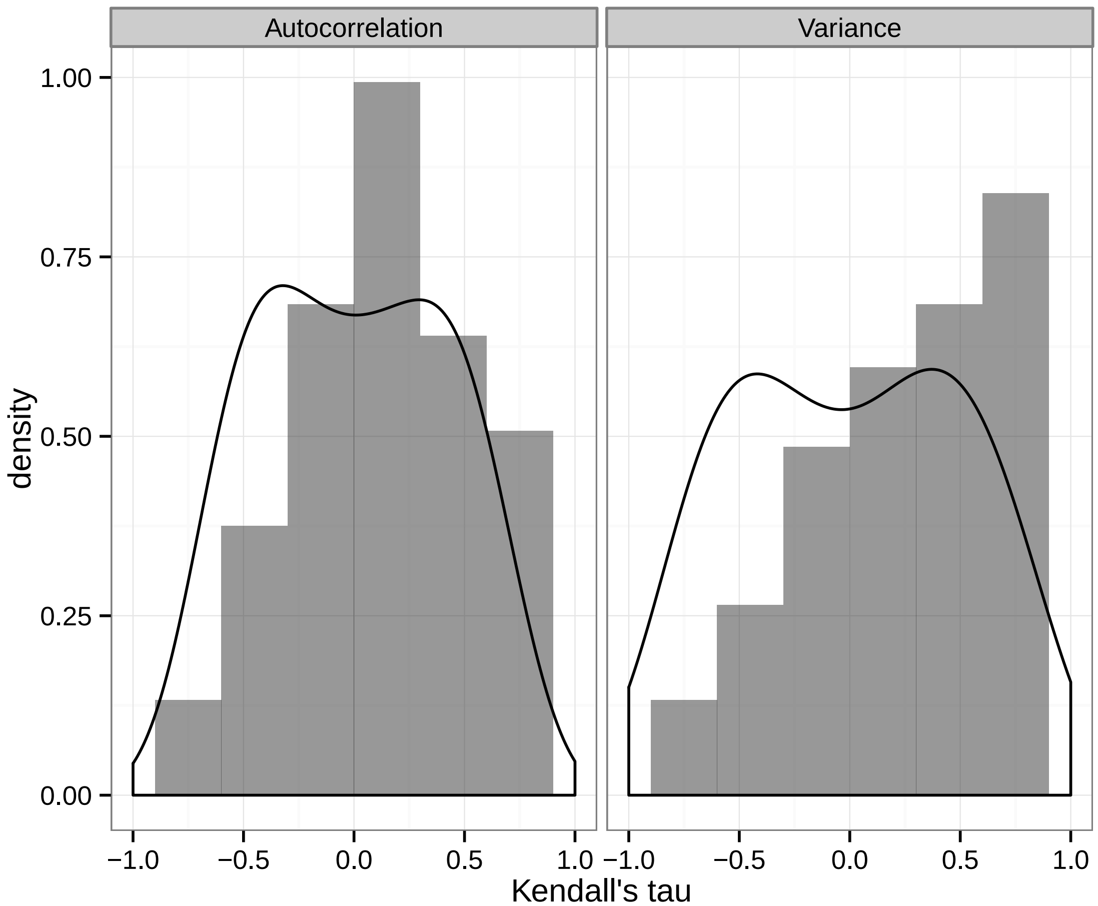

Figure 1: Figure 1. Histogram shows the frequency the correlation statistic τ observed for each warning signal (variance, autocorrelation coefficient) on the large deviation samples. Background distribution of all samples show by smooth line (kernel density estimate). More positive values of tau are supposed to indicate a rising indicator which can be a signal of an approaching transition [2]. $\alpha = 5$, $\sigma = 3.5$, $t \in (0, 10)$, 2000 replicates, 20,000 sample points each. Conditionally selected trajectories experiencing a deviation of at least -4, and analyzed the 1,500 data points prior to the threshold to determine a warning signal (following Reference 6). (link to code, null distribution data, conditional distribution data)

in recovery and extinction patterns in Reference 11. It is a direct consequence of this rapid trajectory that the resulting pattern shows a sudden rise in variance and autocorrelation as the random walk follows its trajectory to the large deviation. More precise statements for the expected variance and autocorrelation along such a trajectory could be solved for along these same lines (or more directly through the large deviation theory, see Reference 12) but are beyond the scope of this comment.

Both the numerical and analytic lines of evidence suggest that pattern Drake alludes arises from the large deviation, rather than from some signal that the system approaches a stochastic transition per se. If we are to consider this a warning signal of stochastic transitions, it is only in this weak sense in which the signal can be seen in any large deviation rather than being sufficient evidence of the chance transition. Relative to the background of all trajectories obeying the same dynamics, this has positive predictive value. Relative only to other large deviations, it does not. We heartily agree with the need for a decision-theoretic approach to early warning signal questions [13]. The challenge of sufficient or unique early warning indicators is not limited to stochastic shifts, but includes the more typical critical transitions. For instance, rising variance or autocorrelation patterns typical of fold bifurcations can be observed in more benign bifurcations or smooth transitions [14]. Early warning signals may offer a promising technique that will one day allow us to avoid seemingly unpredictable catastrophes – but we must not lose sight of just how difficult are the challenges involved. A key step here and for early warning indicators more generally is to understand these other circumstances in which they can arise, that we may then develop ways to eliminate those possibilities. Though we may never be able to detect purely stochastic transitions, perhaps these approaches in this discussion may lead to more unique and sufficient indicators for true critical transitions.

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