No indicators for stochastic transitions but establishing baselines for early warning signals remains a general challenge

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In Boettiger & Hastings [1] we demonstrated that conditioning on observing a purely stochastic transition from one stable basin to another could generate time-series trajectories that could be mistaken for an early warning signal of a critical transition (such as might be due to a fold bifurcation; [2], when instead the shift is merely due to chance. While the goal was to highlight a potential danger in mining historical records for patterns showing sudden shifts when seeking to test early warning techniques, Drake [3] draws attention to a potentially more interesting consequence of our analysis. Drake argues that the bias observed could be used to forecast purely stochastic transitions – a task previously thought to be impossible (e.g. @Ditsleven2010, also Reference 4 for a brief overview of other examples). Unfortunately, we feel this interpretation too generous and must agree with the prevailing opinion that early warning signals for purely stochastic transitions do not exist. Drake demonstrates how the patterns presented in [1] show an indisputable difference between the time series that transition by chance and the much larger population of replicate time series driven by the identical mechanism that do not transition by chance. We show that this pattern in question is a consequence of any large deviation, and not caused by or indicative of any impending shift to an alternative stable state.

We here provide a numerical demonstration that the pattern in question for consideration of an early warning signal appears not only before purely stochastic transitions (as seen in Reference 1) but also in large deviations from a single stable point, in which no such transition is possible.

We replicate the analysis of Boettiger and Hastings [1] using time series replicates produced by an Ornstein-Uhlenbeck (OU) process: a stochastic differential equation in which there is only a single optimum whose strength is proportional to the displacement,

$$dX = -\alpha X dt + \sigma dBt$$

Instead of conditioning on trajectories that experience a large deviation, we condition on trajectories that experience a very large deviation. We then compute warning signals for each of these large deviation trajectories,

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and compare their distribution to that of the entire population of trajectories as shown in Figure 1. We note that the same bias is observed. This should help illustrate that observations we reported are driven by the large deviation preceding the transition, and are simply evidence of this fact and not of an impending transition per se. While trajectories already far from the origin are more likely to transition than ones close to the average, such events can be trivially identified by comparing their states to the historical average.

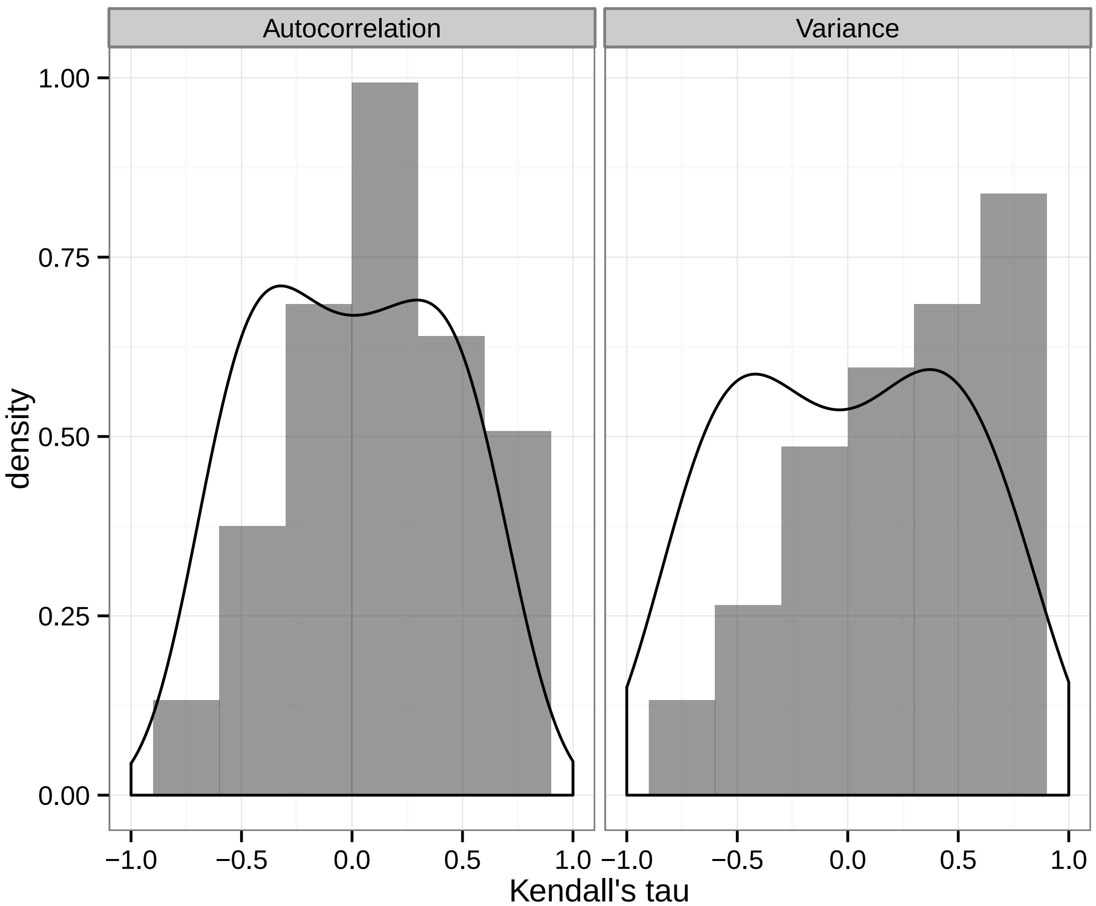

Figure 1: Figure 1. Histogram shows the frequency the correlation statistic τ observed for each warning signal (variance, autocorrelation coefficient) on the large deviation samples. Background distribution of all samples show by smooth line (kernel density estimate). More positive values of tau are supposed to indicate a rising indicator which can be a signal of an approaching transition [2]. $\alpha = 5$, $\sigma = 3.5$, $t \in (0, 10)$, 2000 replicates, 20,000 sample points each. Conditionally selected trajectories experiencing a deviation of at least -4, and analyzed the 1,500 data points prior to the threshold to determine a warning signal (following Reference 5). (link to code, null distribution data, conditional distribution data)

Observing the bias shown in Figure 1 depends on having a rapid enough sample frequency to capture the escape trajectory and a long enough trajectory for the statistic to demonstrate an increase over time. Since large deviations due to stochastic forces alone must be fast so must the accompanying warning signal and managemnt response, which will show up on the time scale of the perturbation. Of course fast relative to the system dynamics, may or may not be fast relative to the timescale of management; just as in the case of bifurcation-driven warning signals (see Reference 6).

One might consider this a corallary of the Prosecutor's Fallacy we originally presented, which demonstrated that examples of sudden transitions historically selected from the literature could be mistaken for positive evidence of early warning signals when they were in fact due to purely stochastic transitions. Here we have seen how any large deviation could be similarly misleading, whether or not it results in a stochastic transition to an alternative stable state. From a classical result of the large deviation theory one can gain considerable intuition about why these chance deviations show much higher variance and autocorrelation than expected from the stationary distribution of a stable point. Though large deviations are rare – the time we must wait to observe a deviation of size L in the system above scales as $\exp\left(L^2/\sigma^2\right)$ (the familiar Ahrennius relationship), when these deviations occur they occur very rapidly. The expected time for an exersion to a distant point L that does not again cross the stable point before reaching L scales as $\log(L/\sigma)$, just as a trajectory returing down the gradient of the attractor from L to the stable point (proofs in Reference 7 or Reference 8). While most trajectories in the stationary distribution take steps in each direction with equal probability, these large deviations moving rapidly to the boundary will consequently show the much greater autocorrelation, and in achieving a much greater deviation than typically observed, also show the spike in variance we observe. That such trajectories appear to be pulled in the direction of their escape rather than climbing away against a restorative force has led to confusion before. Reference 8 argues how this shows how a "punctuated equilibrium" pattern of statis followed by rapid change could arise entirely from small steps, and Reference 9 empirically demonstrates this phenomena in the trajectories of local population extinctions.

In conclusion, we heartily agree with the need for a decision-theoretic approach to early warning signal questions [10]. Central to a decision-theoretic approach is enumerating alternative scenarios that are possible given the observed data. We have highlighted how purley stochastic transitions and large deviations are such possiblities. The challenge of sufficient or unique early warning indicators is not limited to stochastic shifts, but includes the more typical critical transitions. For instance, rising variance or autocorrelation patterns typical of fold bifurcations can be observed in more benign bifurcations or smooth transitions [11]. Early warning signals may offer a promising technique that will one day allow us to avoid seemingly unpredictable catastrophes – but we must not lose sight of just how difficult are the challenges involved. A key step here and for early warning indicators more generally is to understand these other circumstances in which they can arise, that we may then develop ways to eliminate those possibilities. Though we may never be able to detect purely stochastic transitions, perhaps these approaches in this discussion may lead to more unique and sufficient indicators for true critical transitions.

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