Inferring Transients

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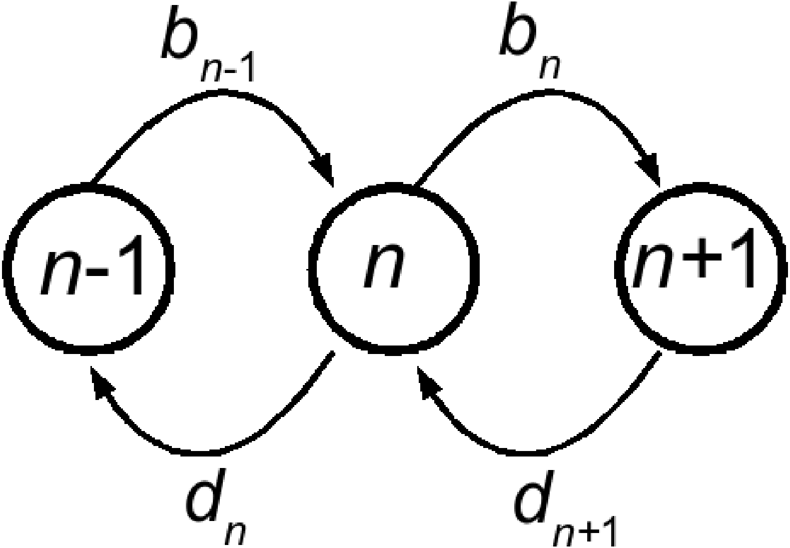
How do we answer

"is my system in a transient?"

Yes

(Provided $N > 0$)

Everything is transient?



Everything is transient?

- Any finite population is guaranteed to go extinct

Models vs reality

- Approximately stationary?

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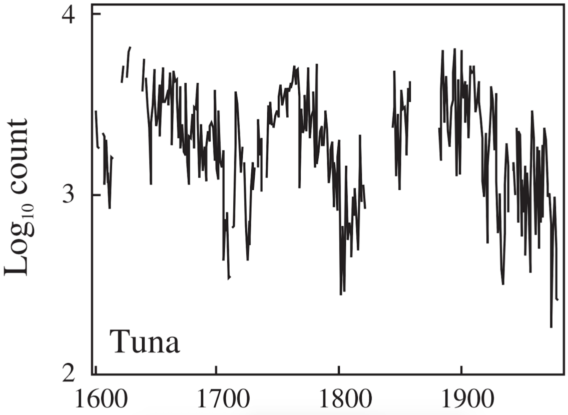
Models vs reality

- Approximately stationary?
- Deterministic model
- Continuous state model, environmental noise
- Quasi-stationary distribution

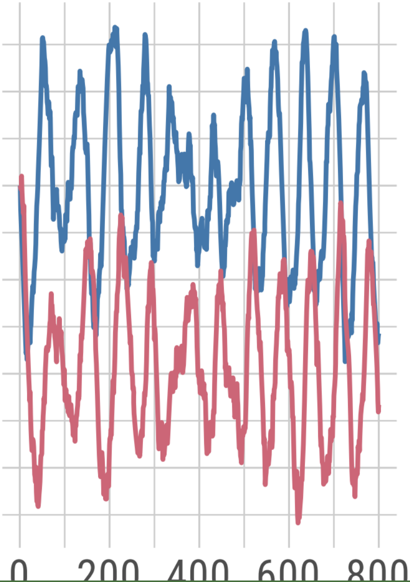
Identifying transient dynamics in actual data can be challenging!

Is this transient behavior?

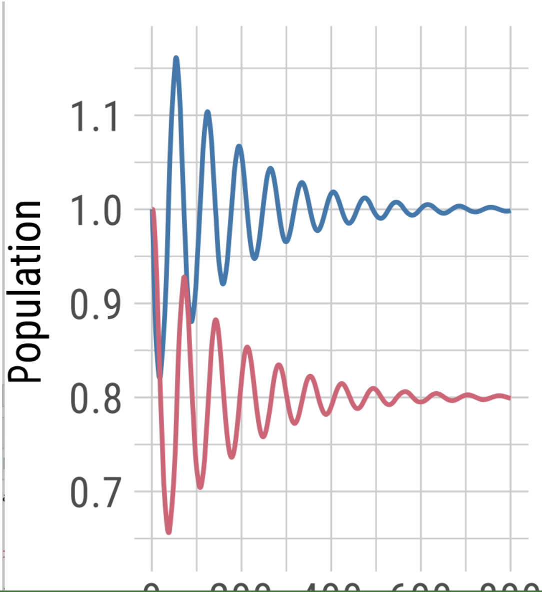
Eastern Atlantic Bluefin Tuna population dynamics, Bjørnstad et al. (2004).



Simulations from proposed model



Simulations from deterministic skeleton



Proposed model: Quasi-cycles

$$x_{t+1} = x_t + x_t r \left(1 - \frac{x_t}{K}\right) - b x_t y_t + \xi_{x,t}$$

$$y_{t+1} = y_t + c x_t y_t - d y_t + \xi_{y,t}$$

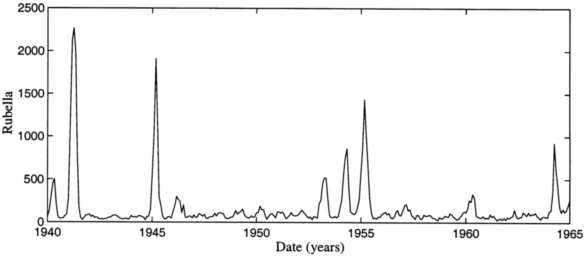
Nisbet & Gurney *Nature* (1976)

So what do quasi-cycles have to do with inferring transients?

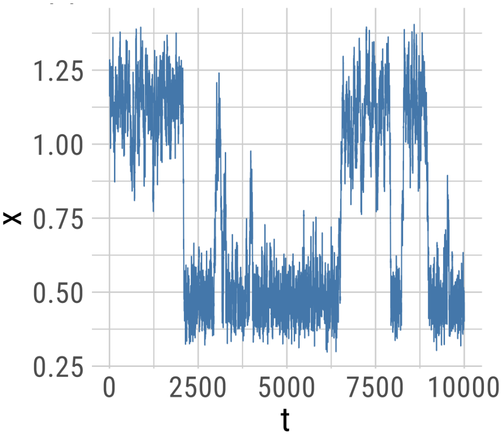
- Transient behavior in the deterministic skeleton reveals / drives the stationary behavior of the full stochastic model.

Is this a transient?

Prevalence of rubella in Copenhagen, Keeling et al 2001.



Model dynamics

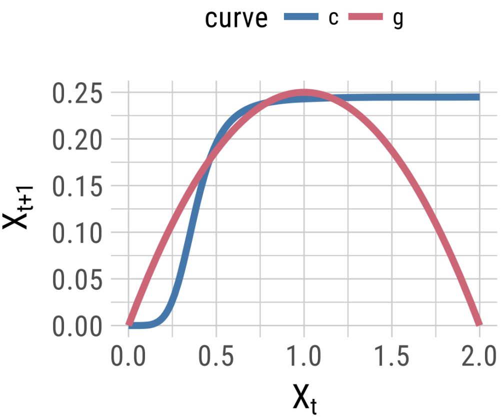


Model

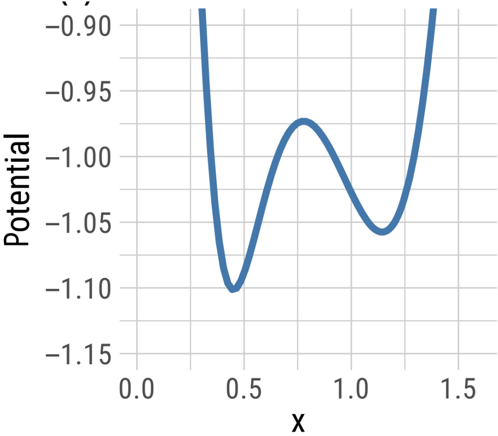
$$X_{t+1} = X_t + \underbrace{X_t r \left(1 - \frac{X_t}{K}\right)}_{\text{growth, } g(X_t)} - \underbrace{\frac{a X_t^Q}{X_t^Q + H^Q}}_{\text{consumption, } c(X_t)} + \xi_t,$$

May (1977) *Nature*

Stochastic Switching



Stochastic Switching



Implications for inferring transients

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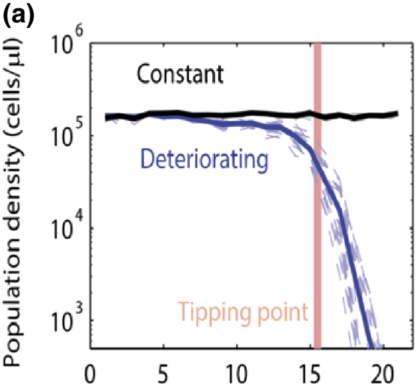
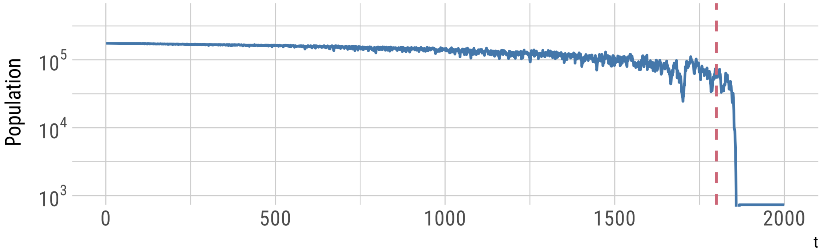
- Asymptotic behavior may tell us very little – not even what attractor we are in
- Short-term behavior may be more informative
- Stochastic transition across saddle point is rapid (large deviation theory)

Is this a transient?



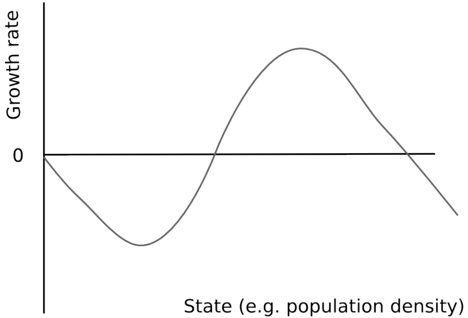
Model simulation

(c) Yeast population size

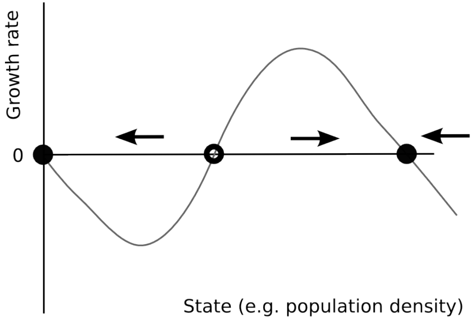


Vertical red dashed line indicates tipping point location

EWS – transient detection?



Critical slowing down



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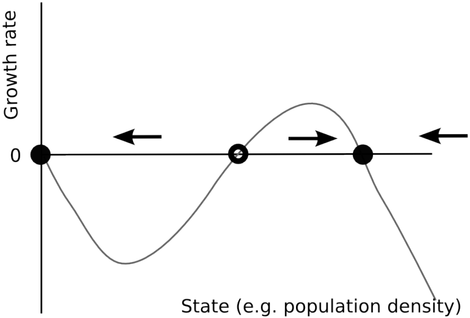

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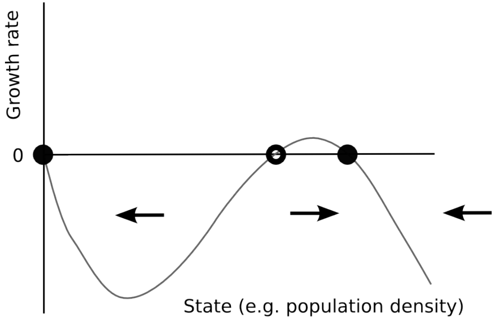

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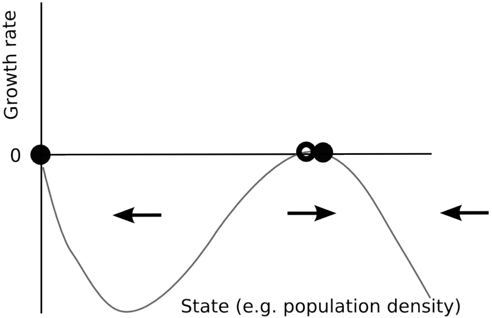

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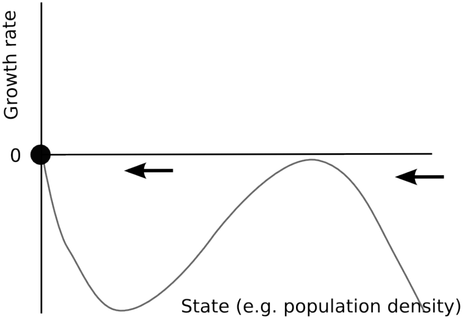

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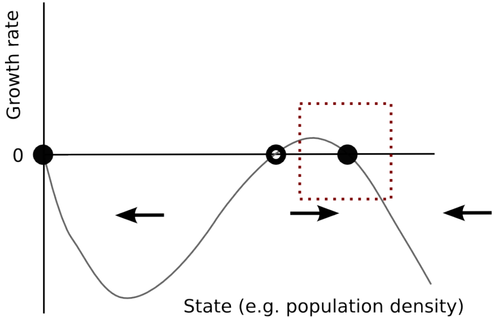
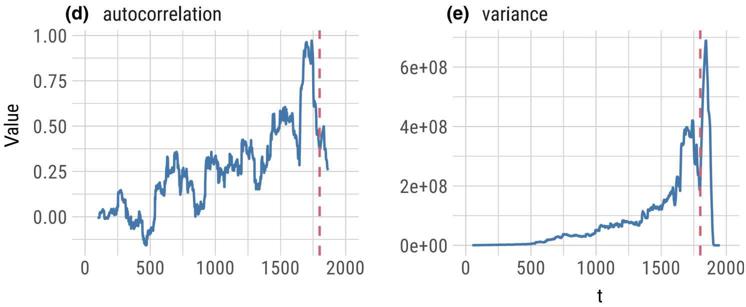


Signatures of CSD?



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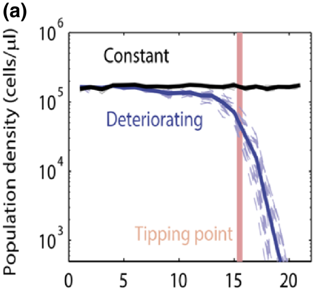
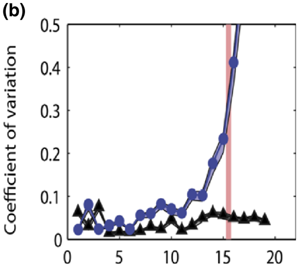
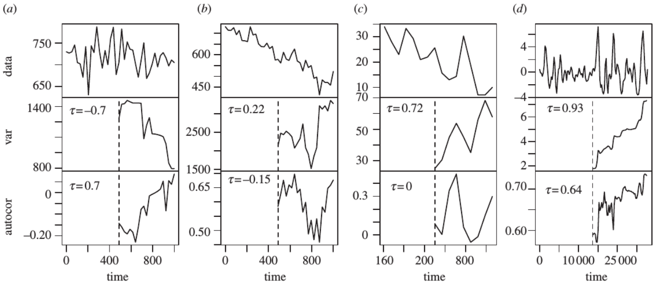


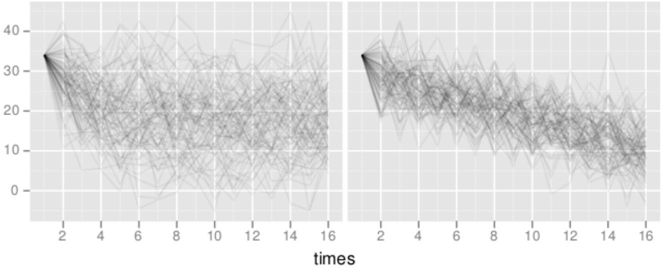


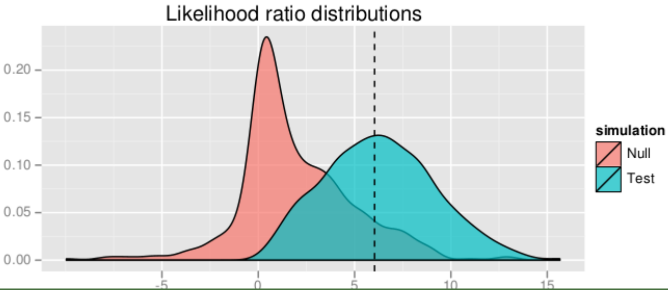
When will we actually observe critical slowing down?



Boettiger & Hastings (2012)

Statistical power for detection





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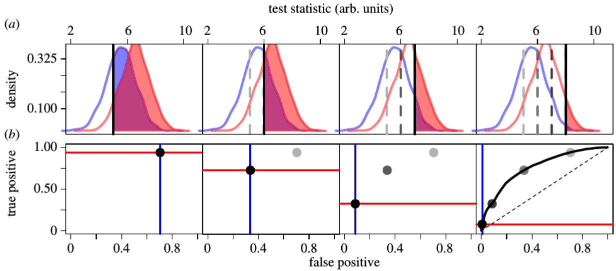

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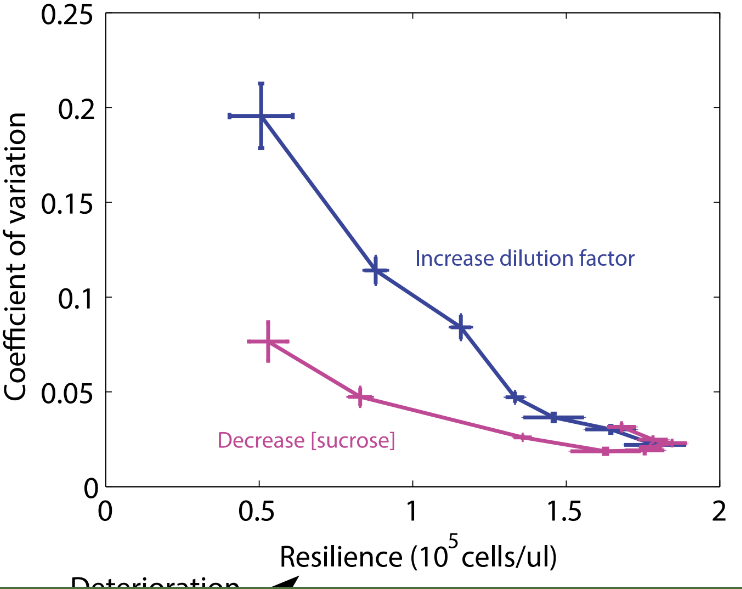
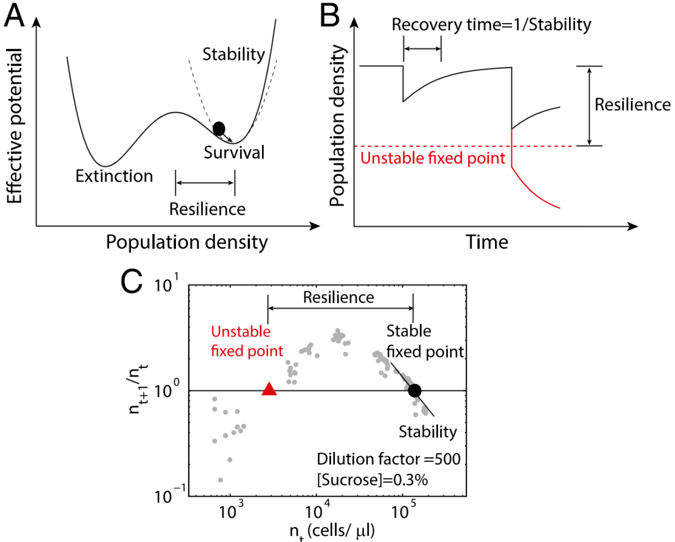


Different transient paths



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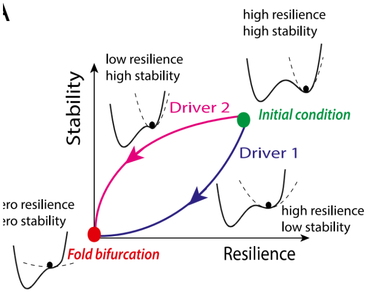
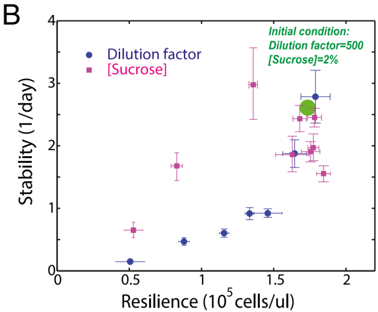
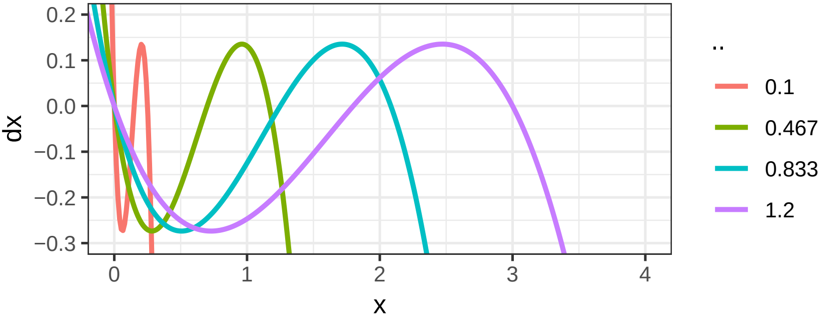




Critical speeding up?

$$dX_t = X_t \frac{r}{\beta}\left(\frac{X_t}{\beta A} - 1\right)\left(1 - \frac{X_t}{\beta C}\right)dt + \sigma dB_t$$

- Carrying capacity C, Allee threshold A



Titus et al (2019)

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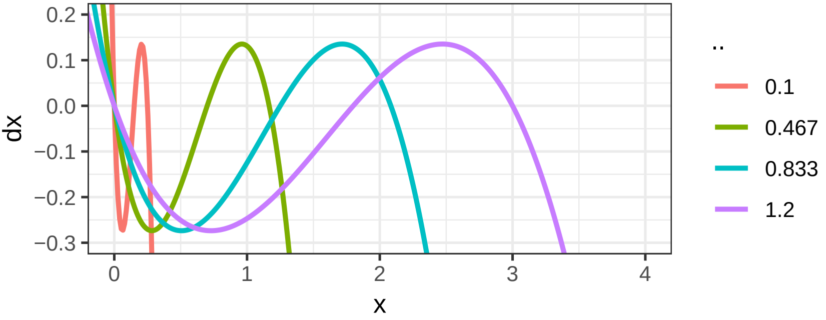
- Carrying capacity C, Allee threshold A
- available territory β



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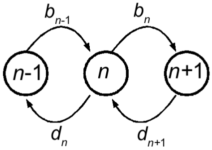
- (Ironically we often assume ergodicity in computing EWS)
- Endogonize parameter change as a state variable, can recover stationary oscillator dynamics

Transient signatures in fluctuation dynamics

Markov process



$\Longrightarrow n = \Omega\phi + \Omega^{1/2}\xi \Longrightarrow$

Linear Noise Approximation

Fundamental Equations

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \alpha_{1,0}(\phi) + \alpha''_{1,0}(\phi)\sigma^2 \tag{1}$$

$$\frac{\mathrm{d}\sigma^2}{\mathrm{d}t} = 2\alpha'_{1,0}(\phi)\sigma^2 + \alpha_{2,0}(\phi) \tag{2}$$

$$\alpha_{1,0}(\phi) = b(\phi) - d(\phi), \quad \alpha_{2,0} = b(\phi) + d(\phi)$$

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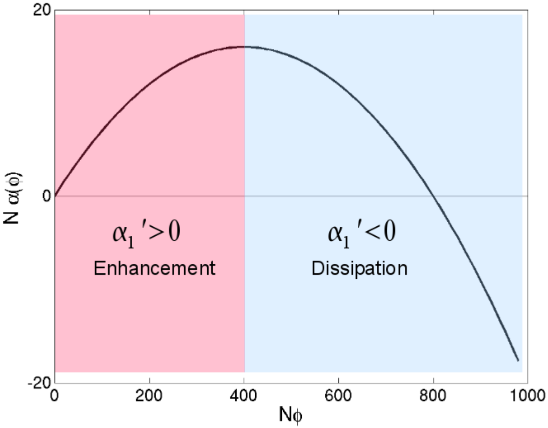

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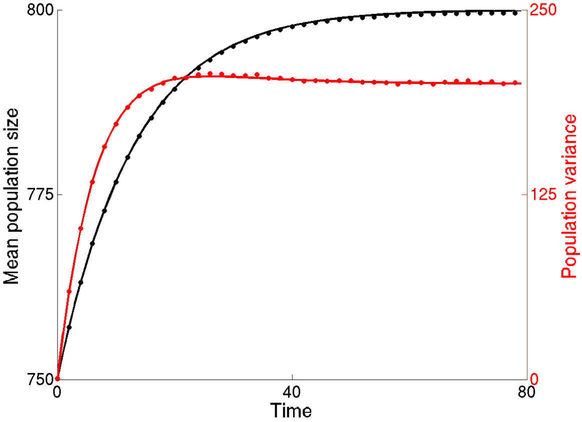
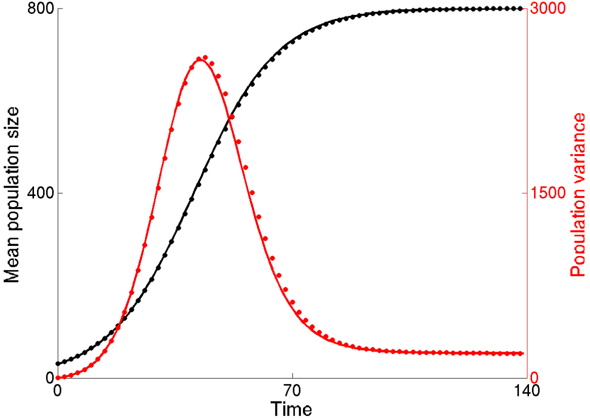


Fluctuation Enhancement

With an initial condition starting deep in the enhancement regime, fluctuations grow exponentially. At $N = 400$, dissipation takes over and fluctuations return to the same equilibrium as before.



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- Testing if system meets *assumptions* of the model may matter more than statistical inferences conditional on the model(s)
- Stochasticity matters – can change how transients behave, or even change our definition of transient behavior.
- Stochasticity + transient dynamics can be a rich source of information in a complex and nonlinear world