# Inferring Transients

Carl Boettiger
Dept of Environmental Science, Policy, & Management
University of California, Berkeley
@cboettig

29 March 2019

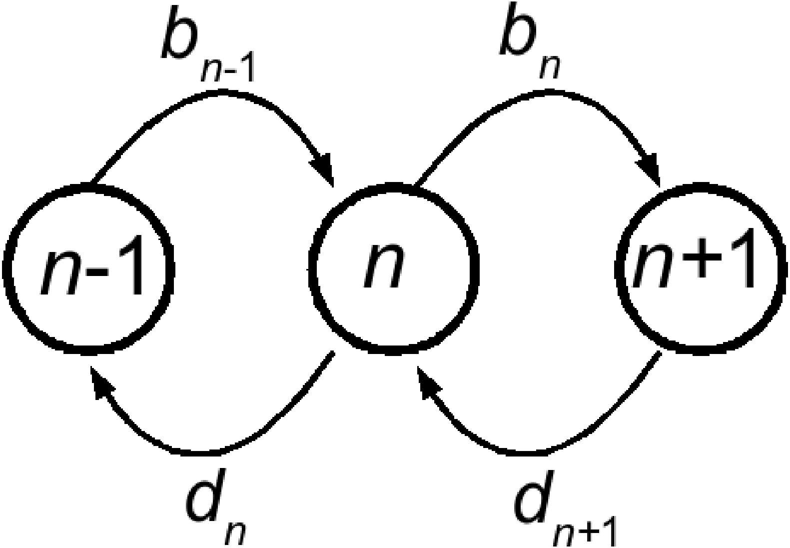
# How do we answer

"is my system in a transient?"

Yes

(Provided $N > 0$)

# Everything is transient?



# Everything is transient?

- Any finite population is guaranteed to go extinct

# Models vs reality

- Approximately stationary?

# Models vs reality

- Approximately stationary?
- Deterministic model

# Models vs reality

- Approximately stationary?
- Deterministic model
- Continuous state model, environmental noise

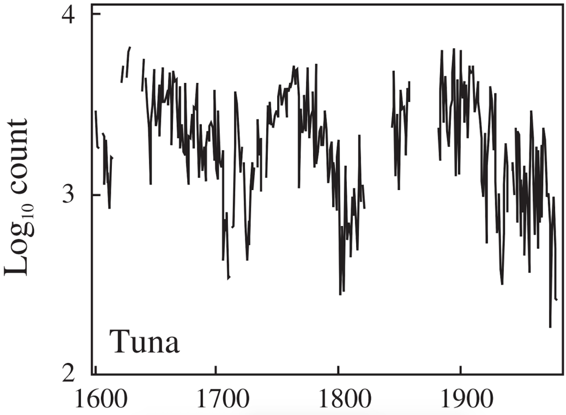
# Models vs reality

- Approximately stationary?
- Deterministic model
- Continuous state model, environmental noise
- Quasi-stationary distribution

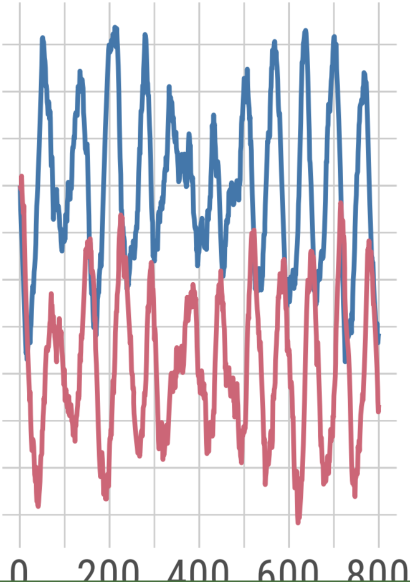
Identifying transient dynamics in actual data can be challenging!

# Is this transient behavior?

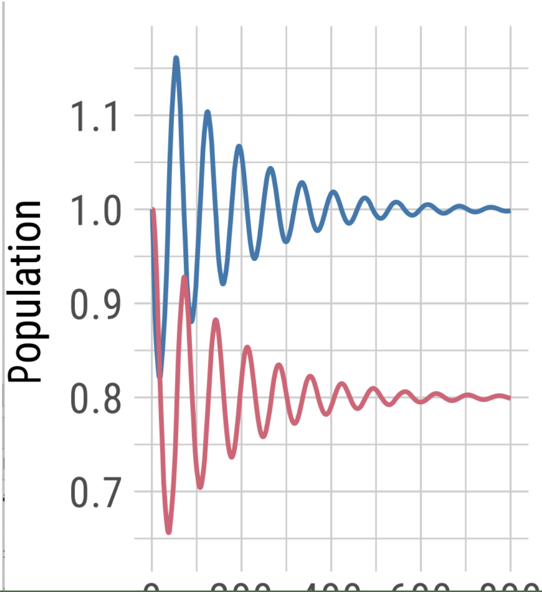
Eastern Atlantic Bluefin Tuna population dynamics, Bjørnstad et al. (2004).



# Simulations from proposed model



# Simulations from deterministic skeleton



# Proposed model: Quasi-cycles

$$x_{t+1} = x_t + x_t r \left(1 - \frac{x_t}{K}\right) - b x_t y_t + \xi_{x,t}$$

$$y_{t+1} = y_t + c x_t y_t - d y_t + \xi_{y,t}$$

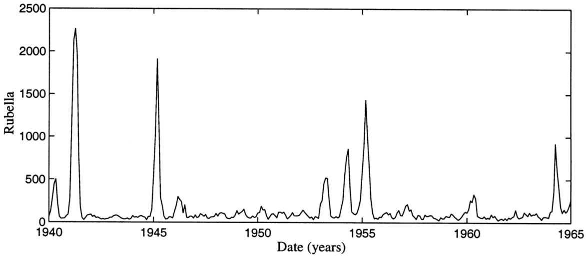
Nisbet & Gurney *Nature* (1976)

# So what do quasi-cycles have to do with inferring transients?

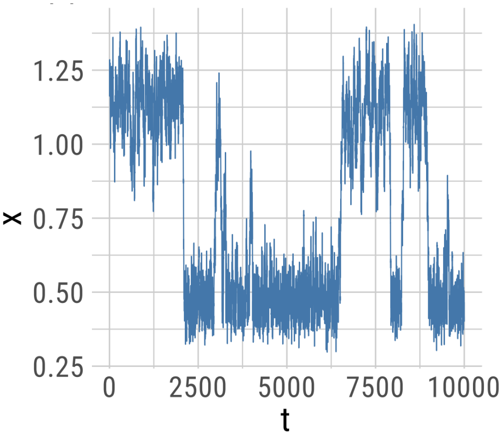
- Transient behavior in the deterministic skeleton reveals / drives the stationary behavior of the full stochastic model.

# Is this a transient?

Prevalence of rubella in Copenhagen, Keeling et al 2001.



# Model dynamics

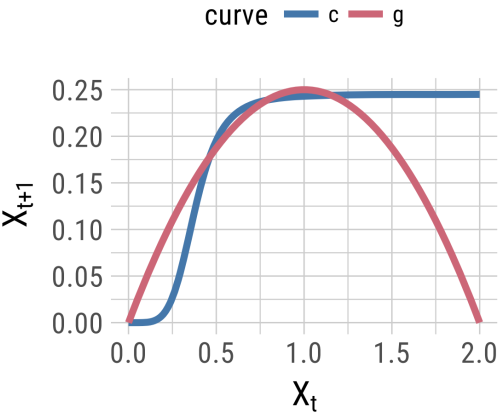


# Model

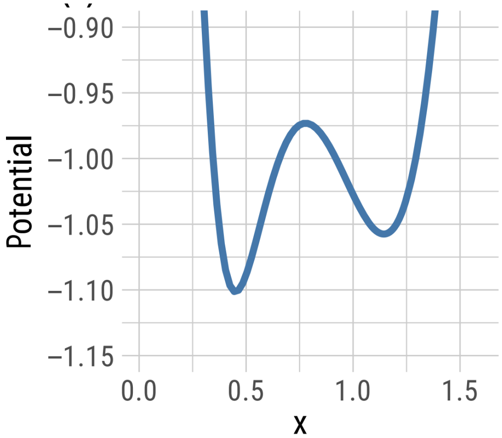
$$X_{t+1} = X_t + \underbrace{X_t r \left(1 - \frac{X_t}{K}\right)}_{\text{growth, } g(X_t)} - \underbrace{\frac{a X_t^Q}{X_t^Q + H^Q}}_{\text{consumption, } c(X_t)} + \xi_t,$$

May (1977) *Nature*

# Stochastic Switching



# Stochastic Switching



# Implications for inferring transients

- Are these transitions transient dynamics?

# Implications for inferring transients

- Are these transitions transient dynamics?
- Asymptotic behavior may tell us very little – not even what attractor we are in

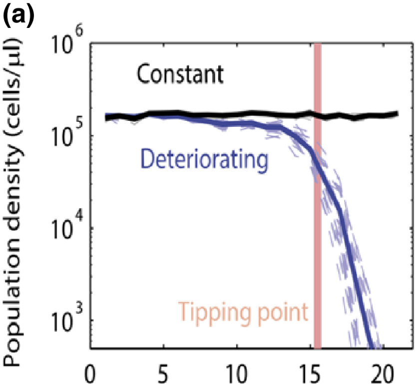
# Implications for inferring transients

- Are these transitions transient dynamics?
- Asymptotic behavior may tell us very little – not even what attractor we are in
- Short-term behavior may be more informative

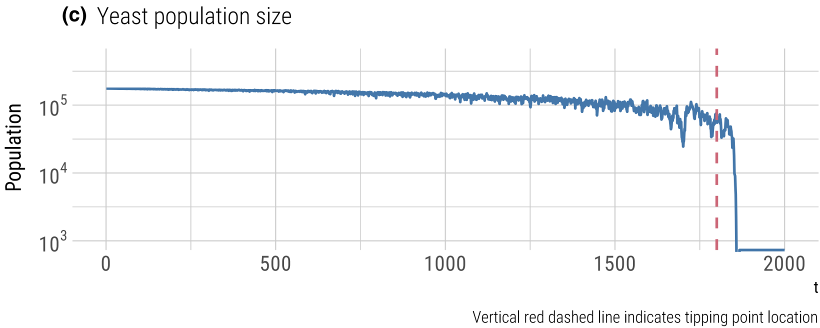
# Implications for inferring transients

- Are these transitions transient dynamics?
- Asymptotic behavior may tell us very little – not even what attractor we are in
- Short-term behavior may be more informative
- Stochastic transition across saddle point is rapid (large deviation theory)

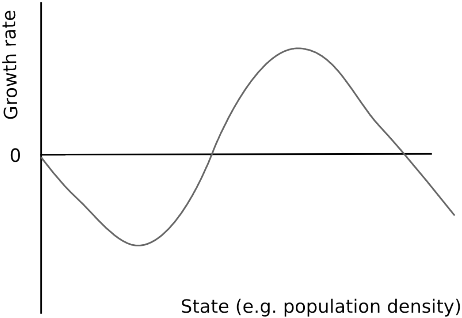
# Is this a transient?



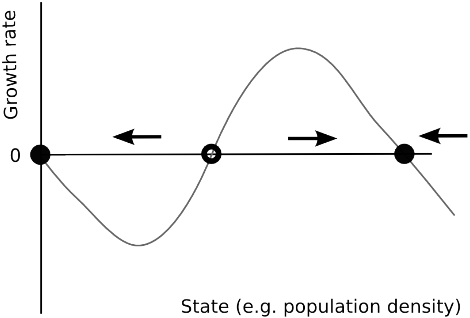
# Model simulation



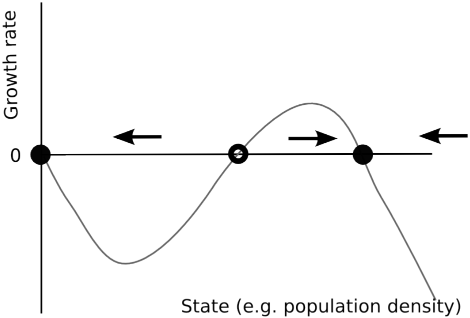
# EWS – transient detection?



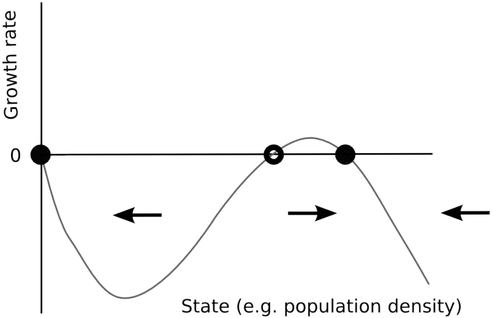
# Critical slowing down



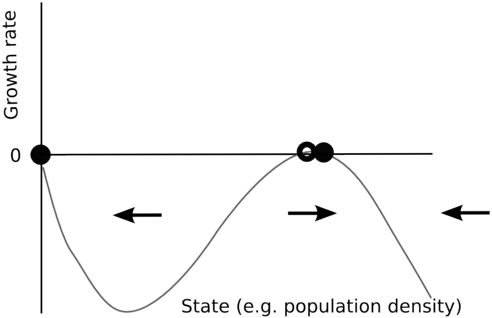
# Critical slowing down



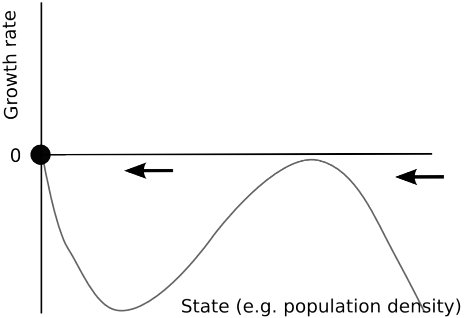
# Critical slowing down



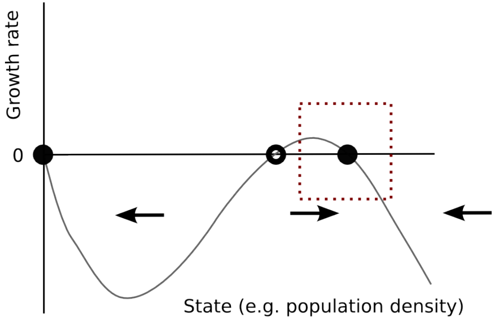
# Critical slowing down



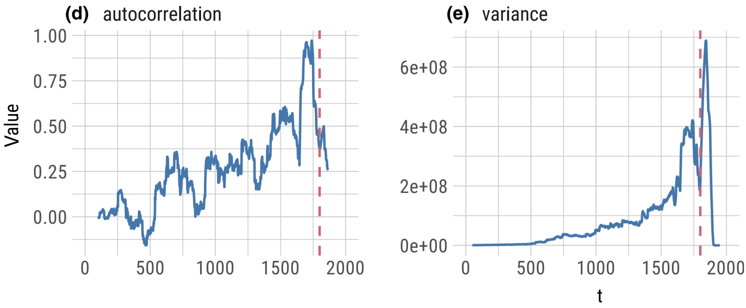
# Critical slowing down



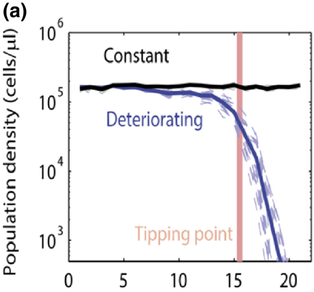
# Critical slowing down

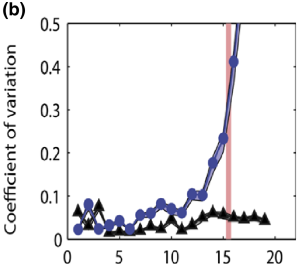


# Signatures of CSD?

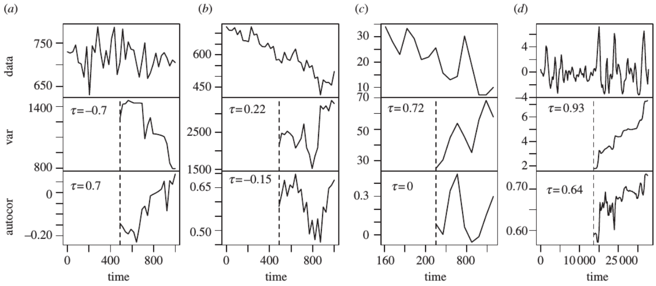


# EWS – transient detection?



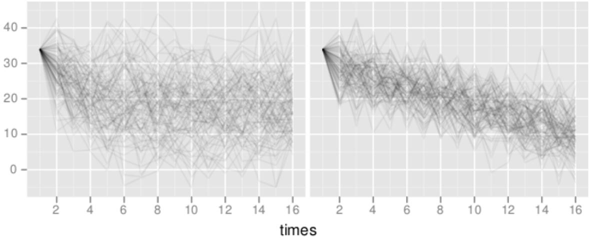



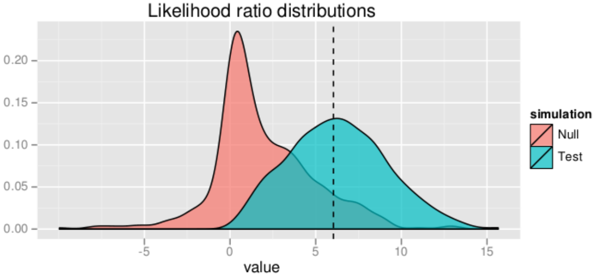
# When will we actually observe critical slowing down?



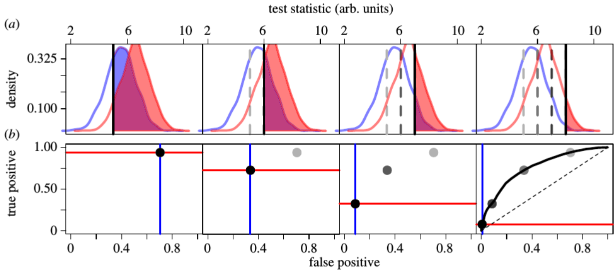

Boettiger & Hastings (2012)

# Statistical power for detection



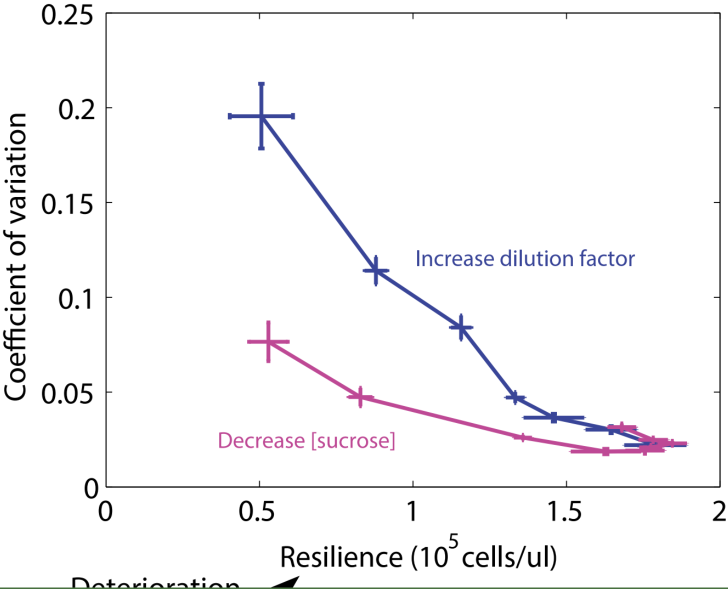



# Statistical power for detection

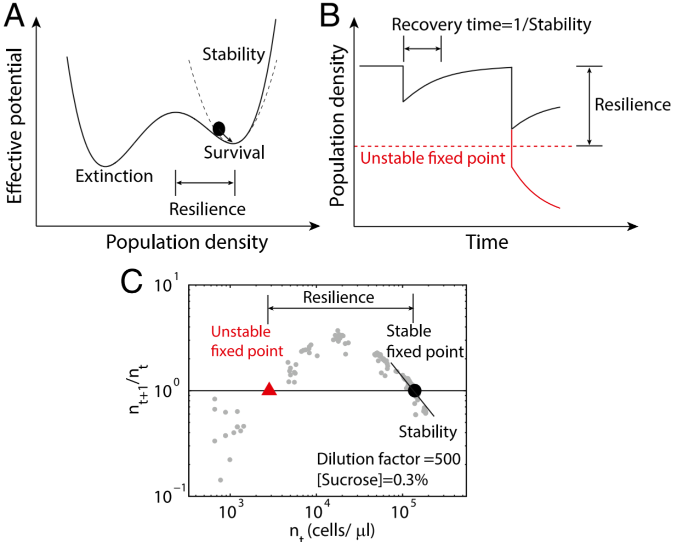



Boettiger & Hastings (2012)

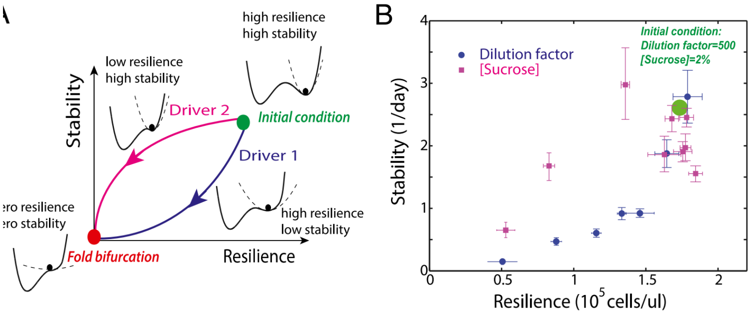
# EWS – transient detection?



# Different transient paths



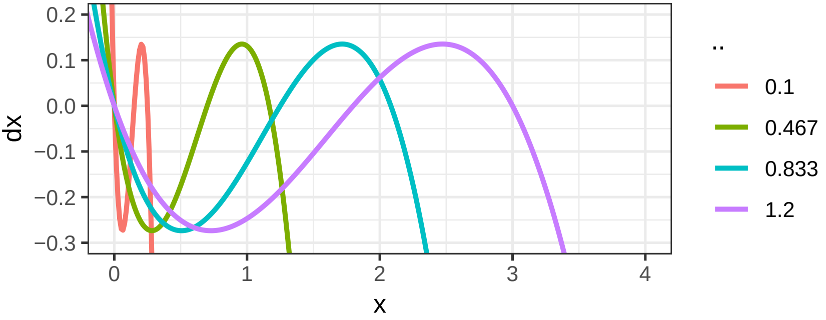
# Different transient paths



# Critical speeding up?

$$dX_t = X_t \frac{r}{\beta} \left( \frac{X_t}{\beta A} - 1 \right) \left( 1 - \frac{X_t}{\beta C} \right) dt + \sigma dB_t$$

- Carrying capacity $C$, Allee threshold $A$

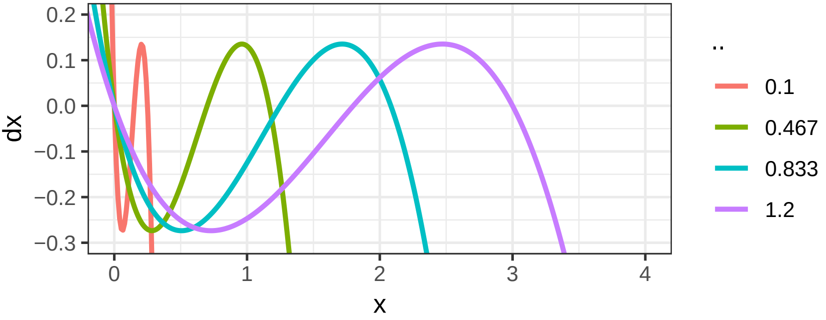



Titus et al (2019)

# Critical speeding up?

$$dX_t = X_t \frac{r}{\beta}\left(\frac{X_t}{\beta A} - 1\right)\left(1 - \frac{X_t}{\beta C}\right) dt + \sigma dB_t$$

- Carrying capacity $C$, Allee threshold $A$
- available territory $\beta$




Titus et al (2019)

# Transients & bifurcations

- External driver: slowly changing parameter, drives transient behavior

# Transients & bifurcations

- External driver: slowly changing parameter, drives transient behavior
- System that appears approx stationary experiences dramatic change *due to this transient shift*

# Transients & bifurcations

- External driver: slowly changing parameter, drives transient behavior
- System that appears approx stationary experiences dramatic change *due to this transient shift*
- System dynamics, e.g. bifurcation point, suggest a mechanism to detect this slow transient

# Transients & bifurcations

- External driver: slowly changing parameter, drives transient behavior
- System that appears approx stationary experiences dramatic change *due to this transient shift*
- System dynamics, e.g. bifurcation point, suggest a mechanism to detect this slow transient
- As before, transient detection is predicated on knowing the right model structure / mechanisms

# Transients & bifurcations

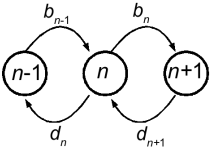
- External driver: slowly changing parameter, drives transient behavior
- System that appears approx stationary experiences dramatic change *due to this transient shift*
- System dynamics, e.g. bifurcation point, suggest a mechanism to detect this slow transient
- As before, transient detection is predicated on knowing the right model structure / mechanisms
- (Ironically we often assume ergodicity in computing EWS)

# Transients & bifurcations

- External driver: slowly changing parameter, drives transient behavior
- System that appears approx stationary experiences dramatic change *due to this transient shift*
- System dynamics, e.g. bifurcation point, suggest a mechanism to detect this slow transient
- As before, transient detection is predicated on knowing the right model structure / mechanisms
- (Ironically we often assume ergodicity in computing EWS)
- Endogonize parameter change as a state variable, can recover stationary oscillator dynamics

# Transient signatures in fluctuation dynamics

Markov process




$\Longrightarrow n = \Omega\phi + \Omega^{1/2}\xi \Longrightarrow$

Linear Noise Approximation

## Fundamental Equations

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \alpha_{1,0}(\phi) + \alpha''_{1,0}(\phi)\sigma^2 \tag{1}$$

$$\frac{\mathrm{d}\sigma^2}{\mathrm{d}t} = 2\alpha'_{1,0}(\phi)\sigma^2 + \alpha_{2,0}(\phi) \tag{2}$$

$$\alpha_{1,0}(\phi) = b(\phi) - d(\phi), \quad \alpha_{2,0} = b(\phi) + d(\phi)$$

# Distinct Fluctuation Regimes

# Distinct Fluctuation Regimes

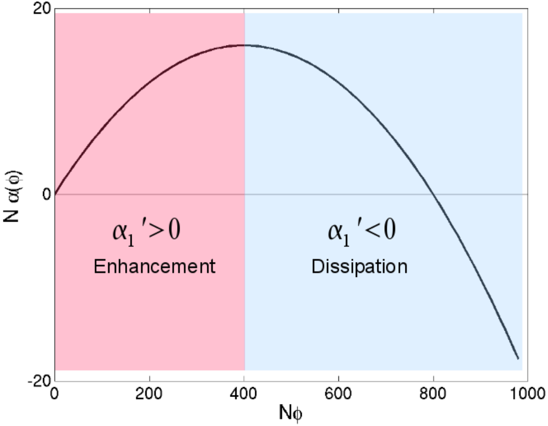
$$\frac{\mathrm{d}n}{\mathrm{d}t} = \underbrace{c\frac{n}{N}\left(1 - \frac{n}{N}\right)}_{b_n} - \underbrace{e\frac{n}{N}}_{d_n}$$

$$\frac{\mathrm{d}\sigma^2}{\mathrm{d}t} = 2\alpha'_{1,0}(\phi)\sigma^2 + \alpha_{2,0}(\phi)$$

# Distinct Fluctuation Regimes

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \underbrace{c\frac{n}{N}\left(1-\frac{n}{N}\right)}_{b_n} - \underbrace{e\frac{n}{N}}_{d_n}$$

$$\frac{\mathrm{d}\sigma^2}{\mathrm{d}t} = 2\alpha'_{1,0}(\phi)\sigma^2 + \alpha_{2,0}(\phi)$$



# Near Equilibrium: Fluctuation Dissipation Regime

In the dissipation regime, fluctuations exponentially relax to the equilibrium level

# Near Equilibrium: Fluctuation Dissipation Regime

In the dissipation regime, fluctuations exponentially relax to the equilibrium level

$$\hat{\sigma}^2 = \frac{b(n)+d(n)}{2[d'(n)-b'(n)]}$$

# Near Equilibrium: Fluctuation Dissipation Regime

In the dissipation regime, fluctuations exponentially relax to the equilibrium level

$$\hat{\sigma}^2 = \frac{b(n)+d(n)}{2[d'(n)-b'(n)]}$$

- $N = 1000$, $e = 0.2$, $c = 1$

# Near Equilibrium: Fluctuation Dissipation Regime

In the dissipation regime, fluctuations exponentially relax to the equilibrium level

$$\hat{\sigma}^2 = \frac{b(n)+d(n)}{2[d'(n)-b'(n)]}$$

- $N = 1000$, $e = 0.2$, $c = 1$
- $\hat{n} = N\left[1 - \frac{e}{c}\right] = 800$

# Near Equilibrium: Fluctuation Dissipation Regime

In the dissipation regime, fluctuations exponentially relax to the equilibrium level

$$\hat{\sigma}^2 = \frac{b(n)+d(n)}{2[d'(n)-b'(n)]}$$

- $N = 1000$, $e = 0.2$, $c = 1$
- $\hat{n} = N\left[1 - \frac{e}{c}\right] = 800$
- $\hat{\sigma}^2 = N\frac{e}{c} = 200$

# Near Equilibrium: Fluctuation Dissipation Regime

In the dissipation regime, fluctuations exponentially relax to the equilibrium level

$$\hat{\sigma}^2 = \frac{b(n)+d(n)}{2[d'(n)-b'(n)]}$$

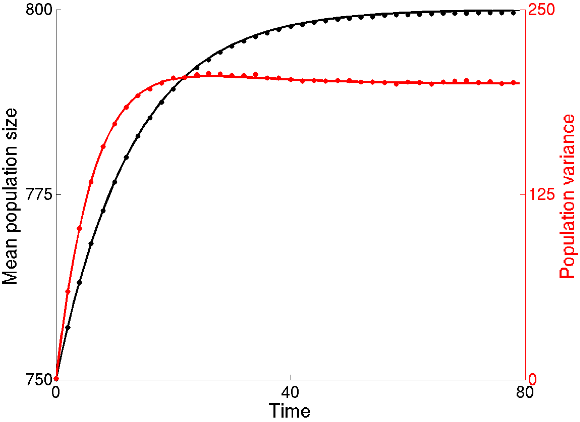
- $N = 1000$, $e = 0.2$, $c = 1$
- $\hat{n} = N\left[1 - \frac{e}{c}\right] = 800$
- $\hat{\sigma}^2 = N\frac{e}{c} = 200$
- Dots are simulation averages, lines are theoretical prediction

# Near Equilibrium: Fluctuation Dissipation Regime

In the dissipation regime, fluctuations exponentially relax to the equilibrium level

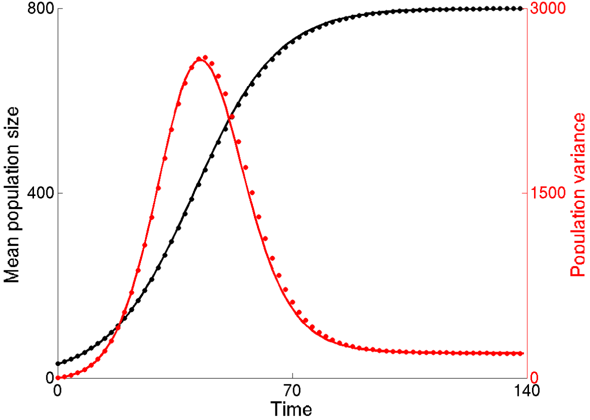
$$\hat{\sigma}^2 = \frac{b(n)+d(n)}{2[d'(n)-b'(n)]}$$

- $N = 1000$, $e = 0.2$, $c = 1$
- $\hat{n} = N\left[1 - \frac{e}{c}\right] = 800$
- $\hat{\sigma}^2 = N\frac{e}{c} = 200$
- Dots are simulation averages, lines are theoretical prediction



# Fluctuation Enhancement

With an initial condition starting deep in the enhancement regime, fluctuations grow exponentially. At $N = 400$, dissipation takes over and fluctuations return to the same equilibrium as before.



# Conclusions

- Inference of transients depends on the model

# Conclusions

- Inference of transients depends on the model
- Testing if system meets *assumptions* of the model may matter more than statistical inferences conditional on the model(s)

# Conclusions

- Inference of transients depends on the model
- Testing if system meets *assumptions* of the model may matter more than statistical inferences conditional on the model(s)
- Stochasticity matters – can change how transients behave, or even change our definition of transient behavior.

# Conclusions

- Inference of transients depends on the model
- Testing if system meets *assumptions* of the model may matter more than statistical inferences conditional on the model(s)
- Stochasticity matters – can change how transients behave, or even change our definition of transient behavior.
- Stochasticity + transient dynamics can be a rich source of information in a complex and nonlinear world