LLE

April 23, 2015

1 Exercice

The goal of this exercice is to that the optimal weigths W_{ij} minimizing the cost function $\mathcal{E}$ are independant with respect to the following symmetries: scaling, translating and rotating.

1.1 Scaling $\alpha\vec{X}_i$

$\sum_i |\alpha\vec{X}_i - \sum_j W_{ij}\alpha\vec{X}_j|^2$
$= \sum_i |\alpha\vec{X}_i - \alpha\sum_j W_{ij}\vec{X}_j|^2$
$= \sum_i |\alpha(\vec{X}_i - \sum_j W_{ij}\vec{X}_j)|^2$
$= \sum_i |\alpha|^2|\vec{X}_i - \sum_j W_{ij}\vec{X}_j|^2$
$= \alpha^2\sum_i |\vec{X}_i - \sum_j W_{ij}\vec{X}_j|$
$= \alpha^2\mathcal{E}(W)$ As $\alpha \in \mathbb{R}^+ \setminus 0$

As α. So the cost function is invariant for this symmetry.

1.2 Translation $\vec{X}_i + \vec{v}$

$\sum_i |\vec{X}_i + \vec{v} - \sum_j W_{ij}(\vec{X}_j + \vec{v})|^2$
$= \sum_i |\vec{X}_i(1 - \sum_j W_{ij}\vec{X}_j) + \vec{v}(1 - \sum_j W_{ij}\vec{v})|^2$

1.3 Rotation $U.\vec{X}_i$

$\sum_i |U.\vec{X}_i - \sum_j W_{ij}U.\vec{X}_j|^2$

2 Exercice

2.1

2.2

2.3