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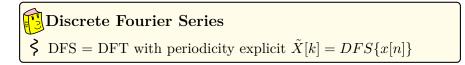
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1 Week 4 Module 3:

1.1 Part 2 - Advanced Fourier Analysise

1.1.1 Discrete Fourier Series DFS

1. **TODO** Discrete Fourier series



- The DFS maps an N-Periodic signal onto an N-Periodic sequence of Fourier coefficients
- \bullet The inverse DFS maps $n_{\rm periodic}$ sequence of Fourier coefficients a set onto an N-periodic signal
- DFS is an extension of the DFT for periodic sequencies
- A circular time-shift is an natural extension of a shift fo finite length signals.
- (a) Finite-length time shifts revisted
 - The DFS helps us understand how to define time shifts for finite-lenght signals.

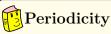
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For an N-periodic sequence $\tilde{x}[n]$

$$\begin{split} &\tilde{x}[n-M] \text{ is well-defined for all } M \in \mathbb{N} \\ &DFS\left\{\tilde{x}[n-M]\right\} = e^{-j\frac{2\pi}{N}Mk}\tilde{X}[k] \text{ delay factor} \\ &IDFS\left\{e^{-j\frac{2\pi}{N}Mk}\tilde{X}[k]\right\} = \tilde{x}[n-M] \text{ delay factor} \end{split}$$

For an N-length signal x[n]

 $\tilde{x}[n-M]$ not well-defined for all $M \in \mathbb{N}$ build $\tilde{x}[n] = x[n \mod N] \Rightarrow \tilde{X}[k] = X[k]$ $IDFT\left\{e^{-j\frac{2\pi}{N}Mk}X[k]\right\} = IDFS\left\{e^{-j\frac{2\pi}{N}Mk}\tilde{X}[k]\right\} = \tilde{x}[n-M] = x[(n-M) \mod N]$



₹ Shifts for finite-length signals are "naturally" circular

- 2. **TODO** Karplus-Strong revisted and DFS
 - (a) Analysis Formula for a N-Periodic Signal in the frequency domain

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n]e^{-j\frac{2\pi}{N}nk}, \ k \in \mathbb{Z}$$

$$\tag{1}$$

X[k] Signal vector in the frequency domain

x[n] Signal vector in the (discrete) time domain

Reminder This is the inner Product in explicite form

(b) Synthesis Formula for a N-Periodic Signal in the time domain

$$\tilde{x}[n] = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{X}[k] e^{j\frac{2\pi}{N}nk}, \ k \in \mathbb{Z}$$

$$(2)$$

x[n] Signal vector in the (discrete) time domain

X[k] Signal vector in the frequency domain

 $\frac{1}{N}$ Normalisation coeficent

Reminder This is the inner Product in explicite fashion

1.1.2 The Discret-Time Fourier Transform (DTFT)

- 1. Overview Fourier Transform
 - N-Point finite-length siganls: DFT
 - N-Point periodic signals: DFS
 - Infinite length (non periodic) signals: DTFT
- 2. Karplus Strong revisted and the DTFT
 - (a) Plotting the DTFT
 - Frequencies go from $-\pi$ to π
 - Positive frequencies are on the right hand side of the x-axis
 - Negative frequencies are on the left hand side of the x-axis
 - Low frequencies are centered around 0
 - High frequnecies will be on the extreme of the bound

1.1.3 Existence and properties of the DTFT

- 1. Formal Definition of the DTFT
 - $x[n] \in \ell_2(\mathbb{Z})$, the space of square summable infinity sequences
 - define the function of $\omega \in \mathbb{R}$

$$F(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$
, with $\omega = \frac{2\pi}{N}$ and $N \to \infty$

• inversion (when $F(\omega)$ exists):

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(e^{j\omega}); e^{j\omega n} d\omega, \text{ with } n \in \mathbb{Z}$$

2. Properties of the DTFT

$$\begin{split} & \operatorname{linearity} & DTFT\{\alpha x[n]+\beta y[n]\} = \alpha X(e^{j\omega})+\beta Y(e^{j\omega}) \\ & \operatorname{timeshift} & DTFT\{x[n-M]\} = e^{-j\omega M} \ X(e^{j\omega}) \\ & \operatorname{modulation} & DTFT\{e^{-j\omega_0 M} \ x[n]\} = X(e^{j\ (\omega-\omega_0)}) \end{split}$$

time reversal $DTFT\{x[-n]\} = X(e^{-jw})$ conjugation $DTFT\{x^*[n]\} = X^*X(e^{-j\;\omega})$

- 3. Some particular cases
 - if x[n] is symmetric, the DTFT is symmetric: $x[n]=x[-n] \iff X(e^{j\omega})=X(e^{-j\omega})$
 - if x[n] is real, the DTFT is Hemitian-symmetric: $x[n]=x^*[n] \iff X(e^{j\omega})=X^*(e^{-j\omega})$
 - if x[n] is real, the magnitude of th eDTFT is symmetric $x[n] \in \mathbb{R} \implies |X(e^{j\omega})| = |X(e^{-j\omega})|$
 - if x[n] is real and symmetric, $X(e^{j\omega})$ is also real and symmetric
- 4. **TODO** The DTFT as a change of basis

1.1.4 TODO Sinusoidal Modulation

- 1. **TODO** Sinusoidal modulation
- 2. **TODO** Tuning a guitar
- 3. **TODO** Signal of the day: Tristan Chord

1.1.5 **TODO** Notes and Supplementary Material

- 1. **TODO** Relation Ship between transforms
- 2. **TODO** The fast fourier transform