Contents

1 Week 2 Module 2: Vector Spaces

Vector Space

Vector spaces build among others a common framework to work with the four classes of signals:

- Finite Length Signal
- Infinte Length Signal
- Periodic Signal
- Finite Support Signal

Finite length and periodic signal, i.e. the "practical signal processing" live in the $\mathbb{C}^N$ Space. To represent infinite length signals we need something more. We require sequneces to be square-summabe $\sum_{n=-\infty}^{\infty} |x[n]|^2$

$\mathbb{R}^2$, $\mathbb{R}^3$	Euclidean space, geomtery
$\mathbb{R}^n$, $\mathbb{C}^n$	Linear algebra
$\ell_2(\mathbb{Z})$	Square-Summable infinite sequences
$L_2([a,b])$	Square-integrable functions over an interval

1.1 Operationl Definitions

Inner Product

Measure of similarity between vectors

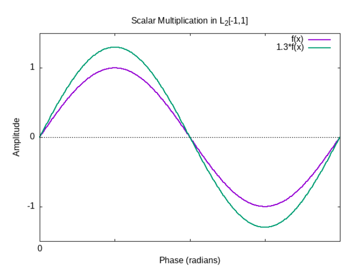
Inner Product	$\langle \mathbf{x}, \mathbf{y} \rangle := \sum_{n=0}^{N-1} x_n \, y_n$ A vector space with an inner product is called an **inner product space**
Inner Product in $\mathbb{R}^2$	$\langle \mathbf{x}, \mathbf{y} \rangle = x_0 y_0 + x_1 y_1 = \mathbf{x} + \mathbf{y} cos(\alpha)$
Inner Product in $\mathbb{L}_{[-1,1]}$	$\langle \mathbf{x}, \mathbf{y} \rangle = \int\limits_{-1}^{1} x(t) y(t) dt$
Norm of a Vector	$\mathbf{v} := \langle \mathbf{v}, \mathbf{v} \rangle = \|\|\mathbf{v}\|\|^2$ self inner product
Orthogonal	$\langle \mathbf{p}, \mathbf{q} \rangle = 0$ maximal different vectors inner product $= 0$
Distance	$d(x, y) = \mathbf{x} - \mathbf{y}_2$

1.2 Some Examples

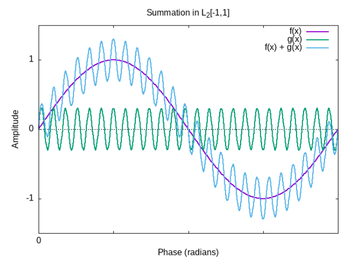
Not all vector spaces have got a graphical representation. The following table shows the graphical representation of vector spaces

graphical representation	no graphical respresentation
$\mathbb{R}^2$	$\mathbb{C}^N$ for N>1
$\mathbb{R}^3$	$\mathbb{R}^N$ for N>3
$\mathbb{L}_{[-1,1]}$	

Scalar Multiplication in $\mathbb{L}_2[-1, 1]$

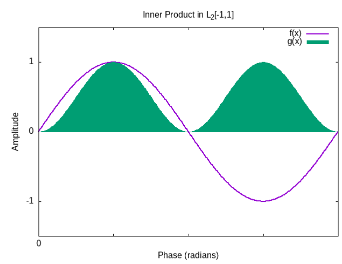


Summation of two Vectors in $\mathbb{L}_2[-1,1]$



Inner Product in $\mathbb{L}_2[-1,1]$ - The Norm:
with x = sin(π\,t)

$$\begin{aligned}\langle \mathbf{x}, \mathbf{x} \rangle &= ||\mathbf{x}||^2 \\ &= \int_{-1}^{1} sin^2(\pi) dt = 1\end{aligned}$$



1.3 Hilbert Space

A hilbert space is an **inner product space** which fulfills completeness.

1.4 Signal Spaces

Finite length signal live in $\mathbb{C}^N$

- all operations well defined and intuitive
- space of N-periodic signals sometimes indicated by $\tilde{\mathbb{C}}^N$

1.5 TODO Vecotor Bases

1.6 TODO Subspace Approximations

1.6.1 Least-Square Approximation

Consider a orthonormal basis for subspace S, called S of K

$$s-(k)_{k=0,1..,k-1} \text{ orthonormal basis for S}$$

orthonormal projection is defined as follows:

$$\hat{x} = \sum_{k=0}^{k-1} \langle s^{(k)}, x \rangle s^{(k)}$$

- orthogonal projection has minimum-norm error:

$$arg\ min||x - y|| = \hat{x}$$

- :

$$\langle x - \hat{x}, \hat{x} \rangle = 0$$