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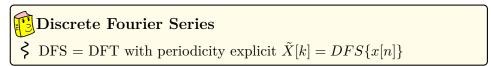
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1 Week 4 Module 3:

1.1 Part 2 - Advanced Fourier Analysise

1.1.1 Discrete Fourier Series DFS

TODO Discrete Fourier series



- The DFS maps an N-Periodic signal onto an N-Periodic sequence of Fourier coefficients
- \bullet The inverse DFS maps $n_{\rm periodic}$ sequence of Fourier coefficients a set onto an N-periodic signal
- DFS is an extension of the DFT for periodic sequencies
- A circular time-shift is an natural extension of a shift fo finite length signals.

Finite-length time shifts revisted

• The DFS helps us understand how to define time shifts for finite-lenght signals.

test

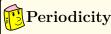
For an N-periodic sequence $\tilde{x}[n]$

$$\tilde{x}[n-M]$$
 is well-defined for all $M \in \mathbb{N}$
$$DFS\left\{\tilde{x}[n-M]\right\} = e^{-j\frac{2\pi}{N}Mk}\tilde{X}[k] \text{ delay factor}$$

$$IDFS\left\{e^{-j\frac{2\pi}{N}Mk}\tilde{X}[k]\right\} = \tilde{x}[n-M] \text{ delay factor}$$

For an N-length signal x[n]

$$\begin{split} &\tilde{x}[n-M] \text{ not well-defined for all } M \in \mathbb{N} \\ &build \ \tilde{x}[n] = x[n \ mod \ N] \Rightarrow \ \tilde{X}[k] = X[k] \\ &IDFT \left\{ \underbrace{e^{-j\frac{2\pi}{N}Mk}}_{} X[k] \ \right\} = IDFS \left\{ \underbrace{e^{-j\frac{2\pi}{N}Mk}}_{} \tilde{X}[k] \ \right\} = \tilde{x}[n-M] = x[(n-M) \ mod \ N] \end{split}$$



Shifts for finite-length signals are "naturally" circular

TODO Karplus-Strong revisted and DFS

Analysis Formula for a N-Periodic Signal in the frequency domain

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n]e^{-j\frac{2\pi}{N}nk}, \ k \in \mathbb{Z}$$

$$\tag{1}$$

X[k] Signal vector in the frequency domain

x[n] Signal vector in the (discrete) time domain

Reminder This is the inner Product in explicite form

Synthesis Formula for a N-Periodic Signal in the time domain

$$\tilde{x}[n] = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{X}[k] e^{j\frac{2\pi}{N}nk}, k \in \mathbb{Z}$$
 (2)

x[n] Signal vector in the (discrete) time domain

X[k] Signal vector in the frequency domain

 $\frac{1}{N}$ Normalisation coeficent

Reminder This is the inner Product in explicite fashion

1.1.2 The Discret-Time Fourier Transform (DTFT)

Overview Fourier Transform

- N-Point finite-length siganls: DFT
- N-Point periodic signals: DFS
- Infinite length (non periodic) signals: DTFT

Karplus Strong revisted and the DTFT

Plotting the DTFT

- Frequencies go from $-\pi$ to π
- Positive frequencies are on the right hand side of the x-axis
- Negative frequencies are on the left hand side of the x-axis
- Low frequencies are centered around 0
- High frequnecies will be on the extreme of the bound

1.1.3 Existence and properties of the DTFT

Formal Definition of the DTFT

- $x[n] \in \ell_2(\mathbb{Z})$, the space of square summable infinity sequences
- define the function of $\omega \in \mathbb{R}$

$$F(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$
, with $\omega = \frac{2\pi}{N}$ and $N \to \infty$

• inversion (when $F(\omega)$ exists):

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(e^{j\omega}); e^{j\omega n} d\omega, \text{ with } n \in \mathbb{Z}$$

Properties of the DTFT

linearity $DTFT\{\alpha x[n] + \beta y[n]\} = \alpha X(e^{j\omega}) + \beta Y(e^{j\omega})$

timeshift $DTFT\{x[n-M]\} = e^{-j\omega M} X(e^{j\omega})$

modulation $DTFT\{e^{-j\omega_0 M} x[n]\} = X(e^{j(\omega - \omega_0)})$

time reversal $DTFT\{x[-n]\} = X(e^{-jw})$

 $\text{conjugation} \qquad \qquad DTFT\{x^*[n]\} = X^*X(e^{-j\;\omega})$

Some particular cases

- if x[n] is symmetric, the DTFT is symmetric: $x[n] = x[-n] \iff X(e^{j\omega}) = X(e^{-j\omega})$
- if x[n] is real, the DTFT is Hemitian-symmetric: $x[n] = x^*[n] \iff X(e^{j\omega}) = X^*(e^{-j\omega})$
- if x[n] is real, the magnitude of th eDTFT is symmetric $x[n] \in \mathbb{R} \implies |X(e^{j\omega})| = |X(e^{-j\omega})|$
- if x[n] is real and symmetric, $X(e^{j\omega})$ is also real and symmetric

TODO The DTFT as a change of basis

1.1.4 TODO Sinusoidal Modulation

TODO Sinusoidal modulation

TODO Tuning a guitar

TODO Signal of the day: Tristan Chord

1.1.5 **TODO** Notes and Supplementary Material

TODO Relation Ship between transforms

TODO The fast fourier transform