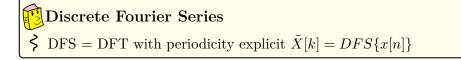
Contents

1	We	ek 4 Mo	dule 3: Part 2 - Advanced Fourier Analysise	1
	1.1	Discrete	e Fourier Series DFS	1
		1.1.1	Finite-length time shifts revisted	1
	1.2	The Dis	scret-Time Fourier Transform (DTFT)	3
		1.2.1	Overview Fourier Transform	3
		1.2.2	Karplus Strong revisted and the DTFT	3
		1.2.3	Formal Definition of the DTFT	3
		1.2.4	Properties of the DTFT	4
		1.2.5	Some particular cases	4
	1.3	TODO	Sinusoidal Modulation	4

1 Week 4 Module 3: Part 2 - Advanced Fourier Analysise

1.1 Discrete Fourier Series DFS



- The DFS maps an N-Periodic signal onto an N-Periodic sequence of Fourier coefficients
- \bullet The inverse DFS maps $n_{\rm periodic}$ sequence of Fourier coefficnets a set onto an N-periodic signal
- DFS is an extension of the DFT for periodic sequencies
- A circular time-shift is an natural extension of a shift fo finite length signals.

1.1.1 Finite-length time shifts revisted

• The DFS helps us understand how to define time shifts for finite-length signals.

test

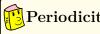
For an N-periodic sequence $\tilde{x}[n]$

$$\tilde{x}[n-M]$$
 is well-defined for all $M \in \mathbb{N}$
$$DFS\left\{\tilde{x}[n-M]\right\} = e^{-j\frac{2\pi}{N}Mk}\tilde{X}[k] \text{ delay factor}$$

$$IDFS\left\{e^{-j\frac{2\pi}{N}Mk}\tilde{X}[k]\right\} = \tilde{x}[n-M] \text{ delay factor}$$

For an N-length signal x[n]

 $\tilde{x}[n-M]$ not well-defined for all $M \in \mathbb{N}$ build $\tilde{x}[n] = x[n \mod N] \Rightarrow \tilde{X}[k] = X[k]$ $IDFT\left\{ e^{-j\frac{2\pi}{N}Mk}X[k] \right\} = IDFS\left\{ e^{-j\frac{2\pi}{N}Mk}\tilde{X}[k] \right\} = \tilde{x}[n-M] = x[(n-M) \bmod N]$



Periodicity
Shifts for finite-length signals are "naturally" circular

Analysis Formula for a N-Periodic Signal in the frequency domain

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n]e^{-j\frac{2\pi}{N}nk}, k \in \mathbb{Z}$$
(1)

X[k] Signal vector in the frequency domain

x[n] Signal vector in the (discrete) time domain

Reminder This is the inner Product in explicite form

Synthesis Formula for a N-Periodic Signal in the time domain

$$\tilde{x}[n] = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{X}[k] e^{j\frac{2\pi}{N}nk}, k \in \mathbb{Z}$$
 (2)

x[n] Signal vector in the (discrete) time domain

X[k] Signal vector in the frequency domain

 $\frac{1}{N}$ Normalisation coeficent

Reminder This is the inner Product in explicite fashion

1.2 The Discret-Time Fourier Transform (DTFT)

1.2.1 Overview Fourier Transform

- N-Point finite-length siganls: DFT
- N-Point periodic signals: DFS
- Infinite length (non periodic) signals: DTFT

1.2.2 Karplus Strong revisted and the DTFT

Plotting the DTFT

- Frequencies go from $-\pi$ to π
- Positive frequencies are on the right hand side of the x-axis
- Negative frequencies are on the left hand side of the x-axis
- Low frequencies are centered around 0
- High frequnecies will be on the extreme of the bound

1.2.3 Formal Definition of the DTFT

- $x[n] \in \ell_2(\mathbb{Z})$, the space of square summable infinity sequences
- define the function of $\omega \in \mathbb{R}$

$$F(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$
, with $\omega = \frac{2\pi}{N}$ and $N \to \infty$

• inversion (when $F(\omega)$ exists):

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(e^{j\omega}); e^{j\omega n} d\omega, \text{ with } n \in \mathbb{Z}$$

1.2.4 Properties of the DTFT

linearity
$$DTFT\{\alpha x[n] + \beta y[n]\} = \alpha X(e^{j\omega}) + \beta Y(e^{j\omega})$$

timeshift
$$DTFT\{x[n-M]\} = e^{-j\omega M} X(e^{j\omega})$$

modulation
$$DTFT\{e^{-j\omega_0 M} x[n]\} = X(e^{j(\omega - \omega_0)})$$

time reversal
$$DTFT\{x[-n]\} = X(e^{-jw})$$

conjugation
$$DTFT\{x^*[n]\} = X^*X(e^{-j\omega})$$

1.2.5 Some particular cases

- if x[n] is symmetric, the DTFT is symmetric: $x[n] = x[-n] \iff X(e^{j\omega}) = X(e^{-j\omega})$
- if x[n] is real, the DTFT is Hemitian-symmetric: $x[n]=x^*[n] \iff X(e^{j\omega})=X^*(e^{-j\omega})$
- if x[n] is real, the magnitude of th eDTFT is symmetric $x[n] \in \mathbb{R} \implies |X(e^{j\omega})| = |X(e^{-j\omega})|$
- if x[n] is real and symmetric, $X(e^{j\omega})$ is also real and symmetric

1.3 TODO Sinusoidal Modulation