Contents

1 Week 1 Module 1: Basics of Digital Signal Processing

1.1 Introduction to digital signal processing

1.1.1 Signal

- Description of the evoultion of a physical phenomenon

phenomenon	signal
weather	temperature
sound	pressure
sound	magnetic deviation
light intensity	gray level on paper

1.1.2 Processing

- **Analysis:** Understanding the information carried by the signal
- **Synthesis:** Creating a signal to contain the given information

1.1.3 Digital

- Discrete Time
  - Splice up time into a series of descrete instance without loosing information
  - Harry Nyquist and Claude Shannon state with the Sampling Theorem that continous time representation and discrete time representation are equivalent.
  - The Sampling Theorem: Under appropriate "slowness" conditions for x(t) we have

$$x(t) = \sum_{n=-\infty}^{\infty} x[n]\ sinc(\frac{t - nT_s}{T_s}) \tag{1}$$

  - The conditiion under which the Sampling Theorem holds was given by Fourier and it's Fourier Analysis.
  - The fouriere transform will give us a quantitive measure how fast a signal moves
- Discrete Amplitude
  - Through discretisation of ampltitudes only a set of predefined values are possible.
  - The set of levels is countable i.e. we can always map the level of a sample to an integer. If our data is represented by integer it becomes complete abstract and general which has very importand consequences in the following three domains:
    * **Storage** special devices for recoding needed
    * **Processing** General purpose microprocessor is sufficient
    * **Transmission** Reproduction of the original signal and therefore eliminating nois is easy

1.1.4 From Analog to Digital Signal Processing

- Analog asks for $f_{(t)} =?$
- Digital represents data as a sequence of numbers (scaled with a factor of 1000)

  -12 -12 -12 -11 -11 -12 -12 -11 -11 -10

-10 -10 -9 -10 -10 -9 -9 -9 -9 -9

-8 -8 -7 -7 -8 -8 -8 -7 -7 -7

1.2 Discrete-Time Signals

1.2.1 Basic Definitions

- Sequence: defined as complex-valued function
- Discrete-Time Signal: a sequnece of complex numbers
  - one dimension (for now)
  - notation: x[n]
  - two-sides sequencies: x: $\mathbb{Z} \rightarrow \mathbb{C}$
  - n is *a-dimensional* "time", sets an order on the sequence of samples
  - analysis: periodic measurement
  - synthesis: stream of generated samples, reproduce a physical phenomenon

1.2.2 Basic Signals

[width=5cm, xticks=5]-5, 5-1.2, 1.2 [linecolor=blue, xmin=-5, xmax=-1]0 [linecolor=blue]0 1 [linecolor=blue, xmin=1, xmax=5]0

Impulse

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \tag{2}$$

[width=5cm, xticks=5]-5, 5-1.2, 1.2 [linecolor=blue, xmin=-5, xmax=-1]0 [linecolor=blue, xmin=0, xmax=5]1

Unit Step

$$\mu[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{3}$$

[width=5cm, xticks=5]-5, 5-1.2, 1.2 [linecolor=blue, xmin=-5, xmax=-1]0 [linecolor=blue, xmin=0, xmax=5]0.7 x exp 1 mul

Exponential Decay

$$x[n] = a^n \times \mu[n],\ a \in \mathbb{C},\ |a| < 1 \tag{4}$$

[width=5cm, xticks=5]-15, 15-1.2, 1.2 [linecolor=blue]5 3.14 mul x mul 3.14 add cos

Real value sinus

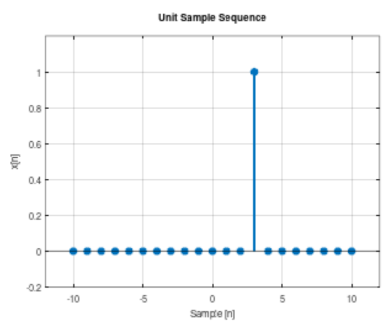
$$x[n] = cos(\omega_0 n + \Phi) \tag{5}$$

1.2.3 Octave Algorithm for some basic Signals

Unit Impulse

```
function [x,n] = impseq(n0,n1,n2)
% Generates x(n) = delta(n-n0); n1 <= n0 <= n2
% ----------------------------------------------
% [x,n] = impseq(n0,n1,n2)
%
  n = [n1:n2]; x = [(n-n0) == 0];
end
```

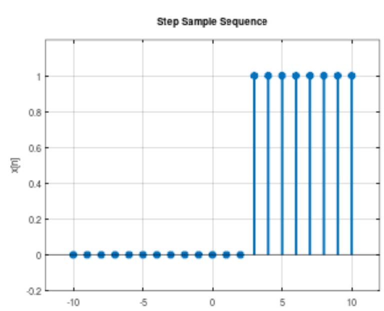
$$x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



Unit Step

```
function [x,n] = stepseq(n0,n1,n2)
% Generates x(n) = delta(n-n0); n1 <= n0 <= n2
% ----------------------------------------------
% [x,n] = stepseq(n0,n1,n2)
%
  n = [n1:n2]; x = [(n-n0) >= 0];
end
```

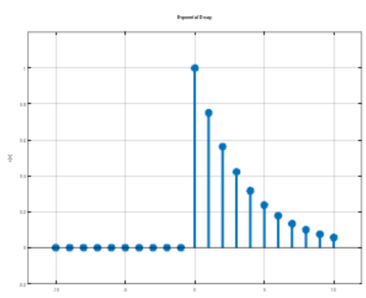
$$x[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



Real-valued exponential Sequence

```
function [x,n] = expseq(n1,n2,a)
% Generates x(n) = a^n
% ---------------------------------------------
% [x,n] = expseq(n1,n2,A,omega,phi)
%
  n = [n1:n2];
  for (i = 1 : length(n))
    if (n(i) >= 0)
      x(i) = (a).^n(i);
    else
      x(i) = 0;
    end
  end
end
```

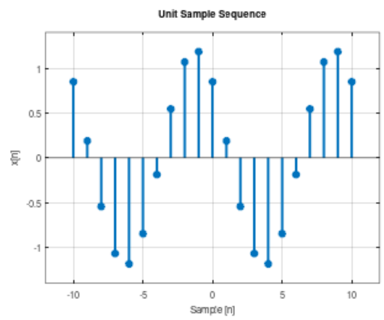
$$x[n] = a^n, \forall n \; a \in \mathbb{R}$$



Sinusoidal Sequence

```
function [x,n] = cosseq(n1,n2,A, omega, phi)
% Generates x(n) = A*cos(2*pi*omega*n + phi); n1 <= n2
% ---------------------------------------------
% [x,n] = cosseq(n1,n2,A,omega,phi)
%
  n = [n1:n2]; x = A*cos(2*pi*omega*n + phi);
end
```

$x[n] = A\,cos(\omega_0 n + \Phi)$



1.2.4 Classes of Discrete-Time signals

1. Finite-Length
   - indicate notation: $x[n]$, $n = 0.1.2.....N-1$
   - vector notation: $x = [x_0, x_1, ...x_{N-1}]^T$
   - practical entities, good for numerical packages (e.g. numpy)
2. Infinte-Length
   - sequence notation: $x[n]$, n $\in \mathbb{Z}$
   - abstraction, good for theorems
3. Periodic
   - N-periodic sequence: $\tilde{x}[n] = \tilde{x}[n + kN]$, n,k,N $\in \mathbb{Z}$
   - same information as in finite-length of length N
   - natural bridge between finite and infinite length
4. Finite-Support Finite-support sequence

$$\overline{x}[n] = \begin{cases} x[n] & if 0 \leq n < N, n \in \mathbb{Z} \\ 0 & otherwise \end{cases} \tag{6}$$

   - same information as in finite-length of length N
   - another bridge between finite and infinite lengths
5. Elementary Operations

Scaling

$$y[n] = ax[n] \rightarrow \begin{cases} a > 0 & amplification \\ a < 0 & attenuation \end{cases} \tag{7}$$

Sum

$$y[n] = x[n] + z[n] \tag{8}$$

Product

$$y[n] = x[n] * z[n] \tag{9}$$

Shift

$$y[n] = x[n-k] \rightarrow \begin{cases} k > 0 & deleay \\ k < 0 & anticipate \end{cases} \tag{10}$$

Integration

$$y[n] = \sum_{k=-\infty}^{n} x[k] \tag{11}$$

Differentation

$$y[n] = x[n] - x[n-1] \tag{12}$$

[logo=,couleur=yellow!10,barre=snake,arrondi=0.1]Relation Operator and Signals

- The unit step can be optained by applying the integration operator to the discrete time pulse.
- The unit impulse can be optained by applying the differentation operator to the unit step.

1.2.5 Energy and Power

Energy Many sequencies have an infinity amount of energy e.g. the unit step u[n],

$$E_x = ||x||_2^2 = \sum_{k=-\infty}^{\infty} |x[n]|^2 \tag{13}$$

Power To describe the energetic properties of the sequencies we use the concept of power

$$P_x = ||x||_2^2 = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 \tag{14}$$

Many signals have infi

1.3 Basic signal processing

1.3.1 How a PC plays discrete-time sounds

1. The discrete-time sinusoid

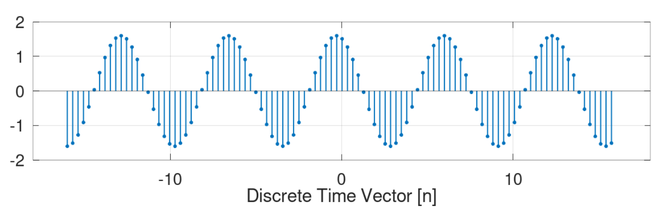
$$x[n] = sin(\omega_0\, t + \Theta)$$

```
N=33                           # Vector lenght
n=-(N-1)/2:pi/10:(N-1)/2; # Discrete Time Vector
omega0 = pi/10;
theta = pi/2

f = 1.6*sin(omega0+n + theta); # The sinusoid

# Do not open the graphic window in org
figure( 1, "visible", "off");

stem(n,f, "filled", "linewidth", 2, "markersize", 6);
axis([-(N-1+4)/2 (N-1+4)/2 -2 2])
set(gca, "fontsize", 24);
grid on ;
xlabel("Discrete Time Vector [n]");
print -dpng "-S1400,350" ./image/sin.png;
# Org-Mode specific output
ans = "./image/sin.png";
```



2. Digital vs physical frequency

   - Discrete Time:
     - Periodicity: how many samples before the pattern repeats (M)
     - n: no physical dimension

- Physical World:
  - Periodicity: hoq many seconds before the pattern repeats
  - frequency measured in Hz
- Soundcard T_s System Clock
  - A sound card takes ever T_s an new sample from the discrete-time sequence.
  - periodicity of M samples $\rightarrow$ periodicity of $M\ T_s$ seconds
  - real world frequency

$$f = \frac{1}{M\ T_s} Hz \tag{15}$$

- Example
  - usually we choose F_s the number of samples per seconds
  - $T_s = 1/F_s$

$$\begin{aligned} F_s &= 48000 \text{e.g. a typical value} \\ T_s &= 20.8\mu\ s \\ f &= 440Hz \text{ , with M = 110} \end{aligned}$$

1.3.2 The Karplus Strong Algorithm

1. The Moving Average

   - simple average (2 point average)

$$m = \frac{a+b}{2} \tag{16}$$

   - moving average: take a "local" average

$$y[n] = \frac{x[n] + x[n-1]}{2} \tag{17}$$

   - Average a sinusoid

$$\begin{aligned} x[n] &= cos(\omega\ n) \\ y[n] &= \frac{cos(\omega\ n) - cos(\omega\ (n-1)}{2} \\ y[n] &= cos(\omega\ n + \theta) \end{aligned}$$

[couleur=yellow!10, arrondi = 0.1, logo=, ombre=true]Linear Transformation Applying a linear transformation to a sinusoidal input results in a sinusoidal output of the same frequency with a phase shift.

2. Reversing the loop

$$y[n] = x[n] + \alpha\, y[n-1] \rightarrow \text{ The Karplus Strong Algorithm} \tag{18}$$

- **Zero Initial Conditions:**
  - set a start time (usually $\mathrm{n}_0 = 0$)
  - assume input and output are zero for all time before N_0

1.4 Digital Frequency

[logo=,couleur=yellow!10,barre=snake]Digital Frequency

$$\begin{aligned} \sin\Big(n\big(\omega + 2k\pi\big)\Big) &= \sin\big(n\omega + \phi\big), \text{ k in } \mathbb{Z} \\ &= e^{i(\phi + n*2\pi\omega)} \end{aligned} \tag{19}$$

[logo=,couleur=yellow!10, barre=snake]Complex Exponential

$$\omega = \frac{M}{N} \times 2 \times \pi \tag{20}$$

1.5 The Reproduction Formula

[logo=, couleur=yellow!10, barre=snake, arrondi=0.1]Reproduction Formula

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \tag{21}$$

Any signal can be expressed as a linear combination of wighted and shifted pulses.