Contents

1 Week 4 Module 3:

1.1 Part 2 - Advanced Fourier Analysise

1.1.1 Discrete Fourier Series DFS

1. **TODO** Discrete Fourier series

   Discrete Fourier Series

   DFS = DFT with periodicity explicit $\tilde{X}[k] = DFS\{x[n]\}$

   - The DFS maps an N-Periodic signal onto an N-Periodic sequence of Fourier coeffients
   - The inverse DFS maps $\text{n}_{\text{periodic}}$ sequence of Fourier coeffienςts a set onto an N-periodic signal
   - DFS is an extension of the DFT for periodic sequencies
   - A circular time-shift is an natural extension of a shift fo finite length signals.

   (a) Finite-length time shifts revisted

      - The DFS helps us understand how to define time shifts for finite-lenght signals.

      test

For an N-periodic sequence $\tilde{x}[n]$

$\tilde{x}[n-M]$ is well-defined for all $M \in \mathbb{N}$

$DFS\left\{\tilde{x}[n-M]\right\} = \boxed{e^{-j\frac{2\pi}{N}Mk}}\tilde{X}[k]$ delay factor

$IDFS\left\{\boxed{e^{-j\frac{2\pi}{N}Mk}}\tilde{X}[k]\right\} = \tilde{x}[n-M]$ delay factor

For an N-length signal $x[n]$

$\tilde{x}[n-M]$ not well-defined for all $M \in \mathbb{N}$

$build\ \tilde{x}[n] = x[n\ mod\ N] \Rightarrow\ \tilde{X}[k] = X[k]$

$IDFT\left\{\boxed{e^{-j\frac{2\pi}{N}Mk}}X[k]\right\} = IDFS\left\{\boxed{e^{-j\frac{2\pi}{N}Mk}}\tilde{X}[k]\right\} = \tilde{x}[n-M] = x[(n-M)\ mod\ N$

Periodicity

Shifts for finite-length signals are "naturally" circular

2. **TODO** Karplus-Strong revisted and DFS

   (a) Analysis Formula for a N-Periodic Signal in the frequency domain

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n]e^{-j\frac{2\pi}{N}nk},\ k \in \mathbb{Z} \tag{1}$$

   $X[k]$ Signal vector in the frequency domain

   $x[n]$ Signal vector in the (discrete) time domain

   Reminder This is the inner Product in explicite form

   (b) Synthesis Formula for a N-Periodic Signal in the time domain

$$\tilde{x}[n] = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{X}[k]e^{j\frac{2\pi}{N}nk},\ k \in \mathbb{Z} \tag{2}$$

   $x[n]$ Signal vector in the (discrete) time domain

   $X[k]$ Signal vector in the frequency domain

   $\frac{1}{N}$ Normalisation coeficent

   Reminder This is the inner Product in explicite fashion

1.1.2 The Discret-Time Fourier Transform (DTFT)

1. Overview Fourier Transform

   - N-Point finite-length siganls: DFT
   - N-Point periodic signals: DFS
   - Infinite length (non periodic) signals: DTFT

2. Karplus Strong revisted and the DTFT

   (a) Plotting the DTFT
      - Frequencies go from $-\pi$ to π
      - Positive frequencies are on the right hand side of the x-axis
      - Negative frequencies are on the left hand side of the x-axis
      - Low frequencies are centered around 0
      - High frequnecies will be on the extreme of the bound

1.1.3 Existence and properties of the DTFT

1. Formal Definition of the DTFT

   - $x[n] \in \ell_2(\mathbb{Z})$, the space of square summable infinity sequneces
   - define the function of $\omega \in \mathbb{R}$

   $$\boxed{F(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}, \text{ with } \omega = \frac{2\pi}{N} \text{ and } N \to \infty}$$

   - inversion (when $F(\omega)$ exists):

   $$\boxed{x[n] = \frac{1}{2\,\pi} \int_{-\pi}^{\pi} F(e^{j\omega}); e^{j\,\omega\,n} \mathrm{d}\omega, \text{ with } n \in \mathbb{Z}}$$

2. Properties of the DTFT

linearity	$DTFT\{\alpha x[n]+\beta y[n]\} = \alpha X(e^{j\omega})+\beta Y(e^{j\omega})$
timeshift	$DTFT\{x[n-M]\} = e^{-j\omega M}\ X(e^{j\omega})$
modulation	$DTFT\{e^{-j\omega_0 M}\ x[n]\} = X(e^{j\,(\omega-\omega_0)})$

time reversal $DTFT\{x[-n]\} = X(e^{-jw})$

conjugation $DTFT\{x^*[n]\} = X^* X(e^{-j\ \omega})$

3. Some particular cases

   - if $x[n]$ is symmetric, the DTFT is symmetric: $x[n] = x[-n] \iff X(e^{j\omega}) = X(e^{-j\omega})$
   - if $x[n]$ is real, the DTFT is Hemitian-symmetric: $x[n] = x^*[n] \iff X(e^{j\omega}) = X^*(e^{-j\omega})$
   - if $x[n]$ is real, the magnitude of th eDTFT is symmetric $x[n] \in \mathbb{R} \implies |X(e^{j\omega})| = |X(e^{-j\omega})|$
   - if $x[n]$ is real and symmetric, $X(e^{j\omega})$ is also real and symmetric

4. **TODO** The DTFT as a change of basis

1.1.4 TODO Sinusoidal Modulation

1. **TODO** Sinusoidal modulation
2. **TODO** Tuning a guitar
3. **TODO** Signal of the day: Tristan Chord

1.1.5 TODO Notes and Supplementary Material

1. **TODO** Relation Ship between transforms
2. **TODO** The fast fourier transform