Those crazy exponentials

Suppose the population of a bacteria culture t hours after it is activated is well modeled by the function

$$f(t) = 100e^{.04t}$$

You may recognize this as an example of an exponential growth model. In 20 hours, the size of the culture will be approximately

$$f(20) = 100e^{(.04)(20)} \approx 1113$$

Often, the growth constant, .04 is computed or estimated somehow. Suppose in doing this computation, you used one more place of decimal accuracy and obtained the model

$$g(t) = 100e^{.044t}$$

You would compute the size of the culture in 20 hours to be approximately

$$g(20) = 100e^{(.044)(20)} \approx 1205$$

Our answers are quite different even though our estimates of the growth constant only differed in the third decimal place.

Suppose I compute the growth constant to be r, and you compute it to be s, and let's say that $r < s$. Then my estimate of the size of the culture in t hours is $100e^{rt}$ and yours is $100e^{st}$. Their ratio is

$$\frac{100e^{rt}}{100e^{st}} = e^{(r-s)t} \tag{1}$$

Because my growth constant is smaller than yours, my estimate will be smaller than yours, but if our estimates are about the same, then the ratio in (1) should be about 1. However, the right hand side of (1) decays *exponentially*. If your growth constant is .01 more than mine, and we wish to estimate the size of the culture in 10 hours, this ratio will be about .9, which means that my estimate is 90% of yours–a significant difference. If we estimate the size of the culture in 70 hours (less than three days), your estimate will be more than twice as big as mine.