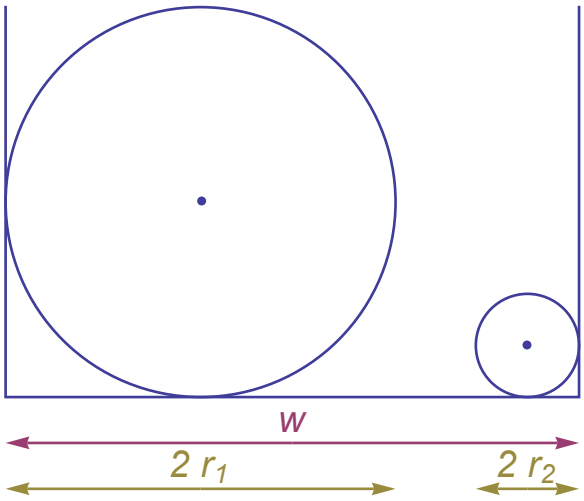
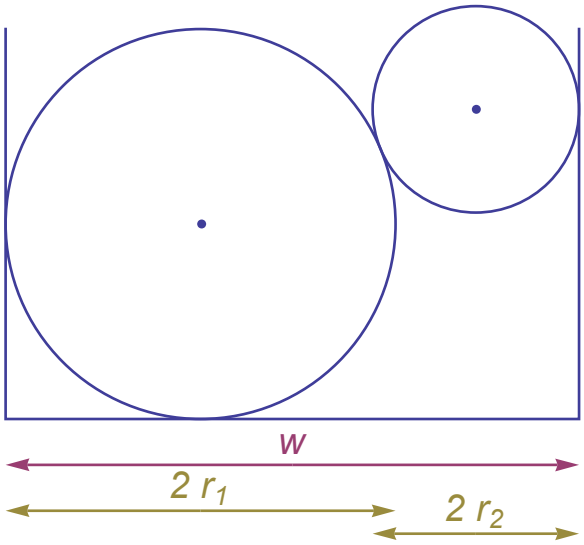
Packing Two Cylinders (August 22, 2024)

Suppose two cylinders of radii r_1 and r_2 are to be packed into a box of width w. What is the depth d of the shallowest possible box? If $w \geq 2(r_1+r_2)$, then both cylinders can sit on the bottom of the box, and the minimum depth is the larger of the two radii.



However, if $w < 2(r_1 + r_2)$, one of the circles will be leaning on the other and not make contact with the bottom of the box.



The way to compute the depth of the box is made clear by adding a right triangle whose hypotenuse connects the circle centers. Letting $s = r_1 + r_2$, the width of the box is $w = x + s$, so $x = w - s$. The hypotenuse of the triangle is s, so by the Pythagorean Theorem, $y = \sqrt{s^2 - (w-s)^2} = \sqrt{w(2s-w)}$. The depth of the box is $s + y$, and so our solution is

$$d = s + \sqrt{w(2s - w)}$$