Roots and Maximums of Random Numbers (November 24, 2025)

Matt Parker, proprietor of the YouTube channel "Stand-up Maths," does a nice video explaining why the n^{th} root of a random number (from the unit interval) has the same distribution as the maximum of n independent such numbers. That is, if

$$\begin{aligned}
Y_1 &= \sqrt[n]{X} \\
Y_2 &= \max(X_1, X_2, \ldots, X_n)
\end{aligned}$$

where all the X's are drawn randomly from $[0, 1]$, then Y_1 and Y_2 have exactly the same probability distribution, which may or may not be counterintuitive.

Showing this is a nice exercise for undergraduate probability students as the tools of calculus give the result easily and elegantly. What we mean by "X is drawn randomly from $[0, 1]$" is that X has probability density function $f(x) = 1$ and cumulative distribution function $F(x) = x$ (for $x \in [0, 1]$). If we know that the pdf is the derivative of the cdf, and we know differential calculus, then

$$\begin{aligned}
f_{Y_1}(y) &= \frac{d}{dy} P\left(\sqrt[n]{X} \le y\right) \\
&= \frac{d}{dy} F(y^n) \\
&= f(y^n) n y^{n-1} \\
&= n y^{n-1}
\end{aligned}$$

Also,

$$\begin{aligned}
f_{Y_2}(y) &= \frac{d}{dy} P\left(\max(X_1, X_2, \ldots, X_n) \le y\right) \\
&= \frac{d}{dy} P(X_1 \le y) P(X_2 \le y) \cdots P(X_n \le y) \\
&= \frac{d}{dy} F(y)^n \\
&= n F(y)^{n-1} f(y) \\
&= n y^{n-1}
\end{aligned}$$