Quantum Mechanical Approach to Neutrino Oscillation

By

Sarvesh B	ID: 2020B5A71839G
Varun Venkatasubramanian	ID: 2020B5PS1839G

Department of Physics

Professor : Prasanta Kumar Das

01 May 2022

Contents

Abstract

The possibility of neutrino oscillations was first anticipated in response to the well-known solar neutrino problem, after the Homestake experiment conducted by Raymond Davis in 1960. Up till then, neutrino flavors had not yet been discovered and the oscillation theory presented involved a neutrino-antineutrino oscillation as a plausible explanation. In this paper, the specific case of two neutrino mixing has been explored, and a standard derivation of the oscillation probabilities is presented. This view is quite simplistic, even under the quantum mechanical regime. However, it can give one pretty interesting insight into the properties and behavior of neutrinos. Lastly, the future implications of neutrinos experiments and further work and possible study into the field of neutrino oscillations has been briefed upon.

Chapter 1

1 Introduction

1.1 Particle Physics Background

The Standard Model explains the composition of matter and how it holds together. It is based on two essential concepts: all matter is made up of particles, and these particles interact with one another by exchanging particles linked with the fundamental forces. Many of the subatomic particles that would eventually compose what particle physicists refer to as the Standard Model were discovered in accelerators by the mid-1960s.The interest in them grew from how it was learned through the intersection of physics and astronomy, that subatomic particles were crucial to the understanding of the formation of the universe.

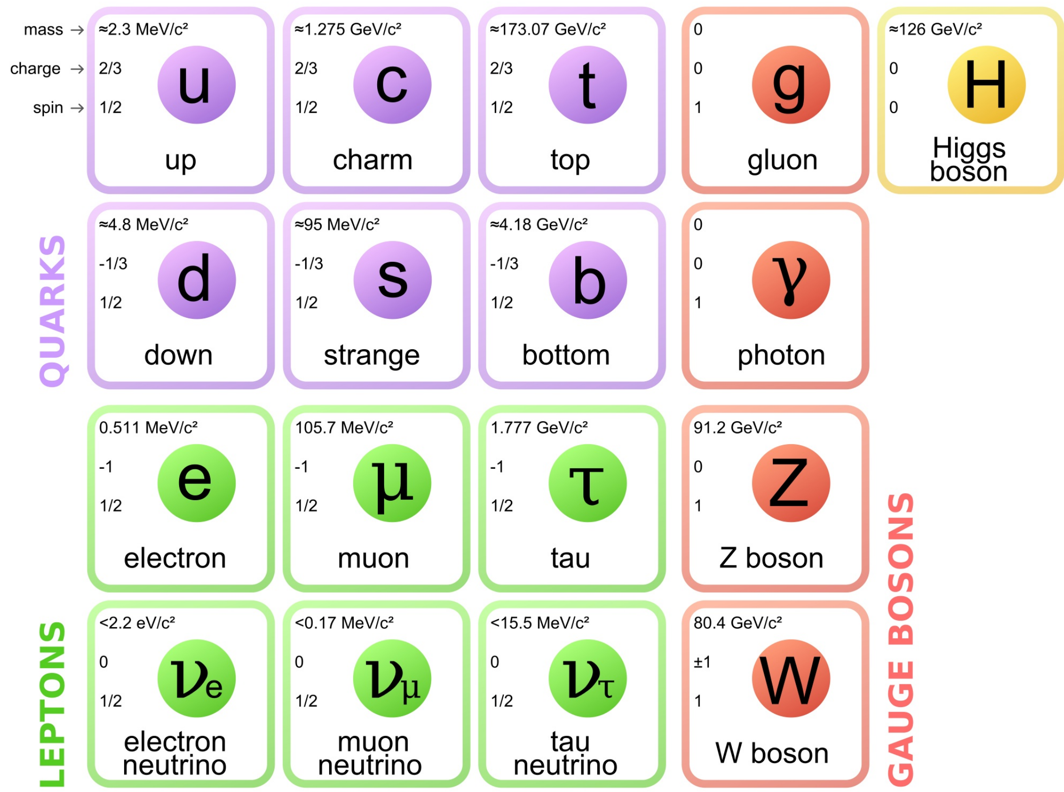

Figure 1: The above shows the Standard Model of particles which includes the four

forces - bosons - as well as the quarks and lepton elementary particles .

By the time the Higgs Boson was formally discovered in 2012, the Standard Model framework had been fully established. It was made up of three generations and a collection of bosons or forces, as indicated in figure 1, including the weak force (w and z bosons), strong force (gluon), electromagnetic force (photon), and the Higgs Boson. The quarks, the particles that make up neutrons and protons, are the model's initial category of particles. The neutrino is one of three types of leptons, which do not interact with the strong force and of which the neutrino is one. Each kind or flavor corresponds to the charged counterparts of the electron, muon, and tau varieties.

An important classification that I will remark upon here, is that neutrinos are not only leptons, but also a particle classification known as a fermion. This label is given to particles that are spin - $\frac{1}{2}$ particles. It means that they a constrained by the Pauli Exclusion Principle.To date, there are three flavors of neutrinos known to particle physicists that in fact correspond to the three generations of leptons within the Standard Model - electron,muon, and tau. The important characteristic that sets these particles apart from other Standard Model particles is that they only interact with the weak force and gravity. Thus, despite the fact that they are a highly studied topic in particle physics, this idea of not interacting much with the matter around them makes these leptons the least understood of the Standard Model particles.

1.2 The Neutrino : A Formal Description

Weak interactions are of two types. The first is the charged-current interaction, involving W-bosons as the particle of force propagation and the other is called the neutral current interaction for which the Z-bosons propagate the force. In the charged-current interactions, a lepton such as an electron, muon or a tau can absorb a W+ boson, and produce the corresponding neutrino. This neutrino would be either an electron neutrino, muon neutrino or tau neutrino respectively. Here, the type or ‘flavor’ of neutrino corresponds to the type of lepton that produces it. Apart from their inherent ‘flavor’ property, neutrinos are considered to be chargeless particles of extremely low mass that travel at near light speeds.

Now, in addition to the 3 ‘flavor’ eigenstates of neutrinos, they are also known to have 3 different mass eigenstates. Neutrinos having different mass eigenstates are represented as vi (i= 1,2,3 in the increasing order of their masses). Moreover, there is no one to one correspondence that can be drawn between the ‘flavor’ and ‘mass’ eigenstates of neutrinos. This means that each flavor eigenstate of a neutrino can be written as a superposition of its mass eigenstates and vice versa.

Generally Speaking,

$$|V_\alpha\rangle = \sum_j U^*_{\alpha j} |V_\mathrm{j}\rangle \qquad (\alpha = e, \mu, \tau) \tag{1.2.1}$$

and by inverting the above expression we get,

$$|V_j\rangle = \sum_j U_{\alpha j} |V_\alpha\rangle \tag{1.2.2}$$

In the above expression, U is the unitary matrix known as the Pontecorvo-Maki-Nakagawa-Sakata Matrix, or the PMNS matrix. This is the Lepton mixing matrix or the neutrino mixing matrix that contains the information of the mismatch of quantum states of neutrinos when they propagate freely in space and when they participate in weak interactions. In general, when neutrinos travel in free space, they show a definite mass eigenstate and their flavor state is in a superposition of the flavor eigenstates, whereas, when they participate in weak interactions, they have a definite flavor eigenstate and their mass state is in a superposition of mass eigenstates. The mass eigenstates of neutrinos also form a complete, orthonormal eigenbasis whose eigenvalues diagonalize the free particle hamiltonian for the neutrino.Since U is a unitary matrix, it can consequently be shown that the flavor eigenstates also form a complete, orthonormal set. U is therefore a non-diagonal matrix.

$$U^\dagger U = UU^\dagger = \mathbb{1} \quad \Longleftrightarrow \quad \delta_{\alpha\beta} = \sum_j U^*_{\alpha j} U_{\beta j} \quad , \quad \delta_{jk} = \sum_j U^*_{\alpha j} U_{\alpha k} \tag{1.2.3}$$

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

The following is the representation of the neutrino flavor vector as a product of the PMNS matrix and the neutrino mass vector. The parameterization of the PMNS matrix by mixing angles is shown in the as shown below.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{\mathrm{CP}}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{\mathrm{CP}}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{\mathrm{CP}}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{\mathrm{CP}}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{\mathrm{CP}}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{\mathrm{CP}}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{\mathrm{CP}}} & c_{23} c_{13} \end{bmatrix}.$$

Chapter 2

2 The Neutrino Oscillation

2.1 Discovery

Even before the first experiments that detected the effects of neutrino oscillations and before the existence of different neutrino 'flavors' were known, Bruno Pontecorvo in 1957-1958 suggested the idea of neutrino masses, mixing and probable oscillations. He believed that, analogous to the leptons and hadrons, neutrinos too could show oscillations similar to the K0-antiK0 system. However, he assumed that the oscillation would be produced between neutrinos and anti-neutrinos, since different neutrino flavors had not yet been hypothesized or discovered.

In the year 1960, the astrophysicists Raymond Davis Jr. and John N. Bahcall in the Homestake experiment, were the ones to first detect the effects of neutrino oscillations. The Homestake experiment was performed to collect and count neutrinos emitted as a result of the nuclear fusion taking place in the sun. The results of this experiment gave rise to the 'solar neutrino problem', which concerned a large inconsistency between the theoretically predicted flux of solar neutrinos and the experimentally obtained value of the solar neutrino flux. The experiment using a chlorine based detector found the solar neutrino flux to be in 2/3rds deficit of the theoretically predicted values. This deficit was subsequently confirmed by other experiments using radiochemical and water Cherenkov detectors, however, neutrino oscillations was not identified as the source of this disparity until the Sudbury Neutrino Observatory in Canada provided clear evidence of flavor change occurring in neutrinos in 2001. The complete theories of neutrino mixing and neutrino oscillations in vacuum were developed through the 1970s and '80s. Neutrino oscillations were finally discovered experimentally in the year 2015 by Arthur B and Takaaki Kajita, who received the Nobel prize in 2015 for the same!

2.2 Vacuum Oscillation Probability

THE STANDARD DERIVATION:

In section 1.2, a formal description of the neutrino and its flavors was proposed. Following from such a description, it is already known that the massive eigenstates of the neutrino form complete, orthonormal eigenstates for the neutrino's free particle hamiltonian.

$$\hat{\mathcal{H}}_0|\nu_j\rangle = E_j|\nu_j\rangle \tag{2.2.1}$$

where Ej is the jth energy eigenstate of the neutrino,and

$$\langle\nu_j|\nu_k\rangle = \delta_{jk} \tag{2.2.2}$$

The orthonormality of the mass eigenstates, and unitarity of the PMNS matrix suggests that the flavor eigenstates are also orthonormal as follows:

$$\langle\nu_\alpha|\nu_\beta\rangle = \delta_{\alpha\beta} \tag{2.2.3}$$

The properties of neutrinos when extended from section 1.2 lead us to understand that the near light speed neutrinos would display ultra-relativistic behavior. Under this regime, the E_j can be Taylor expanded as:

$$E_j = \sqrt{\overrightarrow{p}^2 + m_j^2} \simeq E + \frac{m_j^2}{E} \tag{2.2.4}$$

The above expression is of first order in m_j^2 (The mass energy term 'E' being neglected).

The above formalism conveys the fact that as a particle traveling freely through vacuum, the neutrino has a definite momentum, and hence a definite energy E. Hence the state of propagation of neutrinos for example from the Sun to Earth through a vacuum medium is in one of its massive eigenstates, or in other words, in a superposition of its flavor eigenstates.

Now, in order to approach the problem of neutrino oscillation in vacuum, consider the evolution of a neutrino as it travels through space, or in other words, the time evolution of the neutrino's quantum mechanical state.

This is described as per the Schrodinger's equation as:

$$i\frac{d}{dt}|V_j(t)\rangle = \hat{\mathcal{H}}|V_j(t)\rangle \tag{2.2.5}$$

The above equation has a plane wave solution as:
(for a time independent Hamiltonian)

$$|V_j(t)\rangle = e^{-iE_jt}|V_j\rangle \tag{2.2.6}$$

Now, consider the probability amplitude for transition of the neutrino from a flavor eigenstate of alpha to beta at a time 't':

$$A_{\psi_\alpha \to \psi_\beta} \equiv \langle \psi_\alpha | \psi_\beta \rangle \tag{2.2.7}$$

The probability for this corresponding transition at time 't' can be given as:

$$P_{\psi_\alpha \to \psi_\beta} = |A_{\psi_\alpha \to \psi_\beta}|^2 \tag{2.2.8}$$

$$A_{\nu_\alpha \to \nu_\beta}(t) \equiv \langle \nu_\beta | \nu_\alpha(t) \rangle = \sum_j U^*_{\alpha j} U_{\beta j} e^{-iE_jt} \tag{2.2.9}$$

$$P_{\nu_\alpha \to \nu_\beta}(t) = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i(E_j - E_k)t} \tag{2.2.10}$$

The above results are based on the assumption that different mass eigenstates have the same momentum $|p|$ and the energy can be approximated for the ultra-relativistic behavior of neutrinos. Also, under the assumption that distance L- approximately equal to the time of travel 't'.

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{\frac{-i\Delta m^2_{jk} L}{2E}} \tag{2.2.11}$$

(Δm^2_{jk} is the mass squared difference between m_j and m_k)
Here, the value of the indices j are proportional to the mass of the neutrino. The above probability for $P_{\nu_\alpha \to \nu_\beta}(L, E)$ is called the transition probability when $\alpha \neq \beta$ while survival probability when $\alpha = \beta$.The above derivation of vacuum oscillation probability is correct under the set of assumptions made for such a description and calculation.

2.3 Two Neutrino Mixing

The flavor oscillation probability obtained in section 2.2 is clearly slightly ambiguous, especially with regards to the actual elements and composition of the PMNS matrix U. In order to concretize this expression, we can start off by considering the mixing of simply 2 flavor states of the neutrino. This mixing could either simply be between two of the three flavor eigenstates of the neutrino, or linear combinations of these flavor eigenstates (eg: $\nu_e, c_\mu \nu_\mu + c_\tau \nu_\tau$) under the condition that the two states satisfy the normalization conditions. These approximations are feasible as experiments are not as receptive to the three neutrino mixing case.

Furthermore, this approximation has an edge in the sense of it being parameterized by fewer variables, allowing an easier interpretation of its results and measurement data.

In the case of this 2 neutrino mixing, the neutrino fields can be rephased, and the phase component of each of the three unitary matrices of U can thus be removed. Hence the PMNS matrix U can be parameterized by a single mixing angle θ and reduced to a 2x2 rotation matrix in θ as:

$$U = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

Equation (2.2.11) can now be simplified by substituting the above matrix for U appropriately, and we obtain:

$$P_{\nu_\alpha \to \nu_\beta} = \frac{1}{2} Sin^2(2\theta)[1 - cos(\frac{\Delta m^2 L}{2E})] \qquad (\alpha \neq \beta) \tag{2.3.1}$$

From the above equation, it can be noted that the oscillation probability is asymmetric under the mixing angle transformation $\theta \leftrightarrow \frac{\pi}{2} - \theta$, which in general is not true. The implication that oscillation probability is degenerate with respect to the mixing angle only holds for this specific case of vacuum oscillation probability for two neutrino mixing and under the specific underlying assumptions. In general, the oscillation probability turns out to be asymmetric with respect to the mixing angle. Moreover, we have disregarded in our derivation of the oscillation probability, the factor of incoherence of neutrinos. This further restricts the application of the above formula. However, as a general case, this formula can be accepted for the given underlying assumptions, and even analysis of this simple formula yields some remarkable insight as to the nature of neutrino oscillation experiments.

Chapter 3

3 Impact on Future Physics

3.1 Importance of the Mass-Squared Difference

We have seen why neutrino oscillation are required in the development of physics in terms of Quantum mechanics. The oscillation had previously been discovered after all,yet we are still caring about it till today.To answer this there is a term in the derivation of probability amplitude called $\triangle m^2_{ij}$ (The mass square difference).The parameter may seem to just simply calculations but in fact this parameter is a vital to understanding how neutrinos function in our universe. Therefore physicists who perform research in neutrino physics give importance to the $\triangle m^2_{ij}$ as well as the mixing angle θ and the CP phase δ_{CP}. Returning to the three neutrino mixing defined in the PMNS matrix at the beginning of this study to briefly explain why this is so crucial. Now, the three masses 1, 2, and 3 are connected by two separate mass squared differences by the difference $\triangle m^2_{12}$ and $\triangle m^2_{23}$ as seen in the figure 2 below.

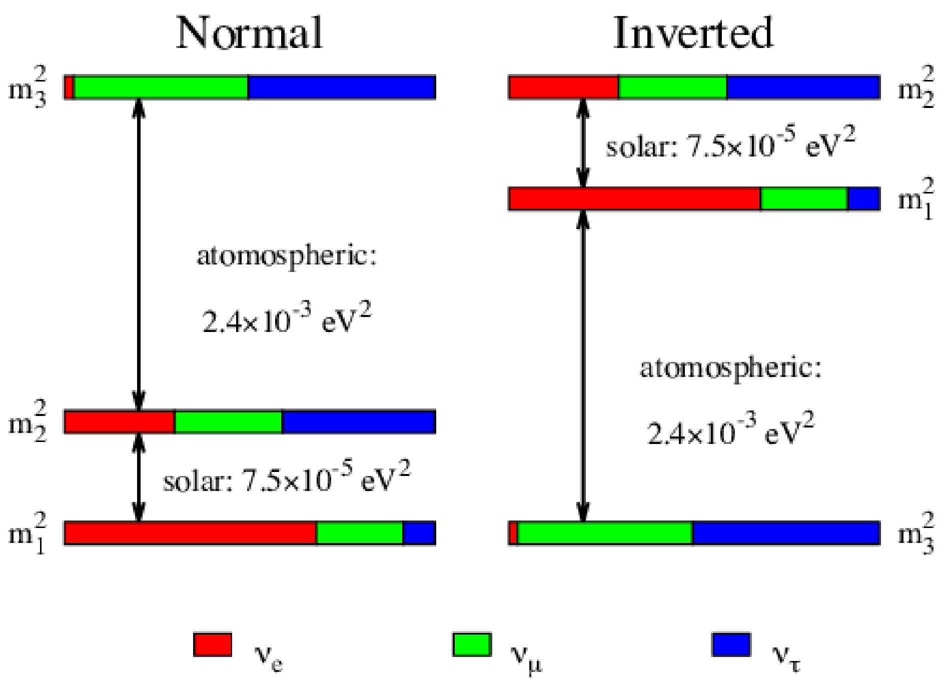

Figure 2: The above shows the two proposed neutrino mass hierarchies where the normal indicates a larger m_3 while the inverted case assumes the opposite - m_3 as the smallest.

Since the masses themselves are super-positions of the three flavors(i.e they mix) these differences change depending on whether the actual mass eigenstate of m_1 or m_3 is larger in size relative to the other two. Now, We define what are known as the two hierarchies under consideration in which the ordering of the mass-squared differences can be based which are referred to as normal or inverted. This idea of probing parameter space to determine which hierarchy is true is one of the important questions that is currently circulating within the physics community, as it will aid in the further characterization of neutrinos. While it has been investigated further in recent experiments, there is another concept that neutrino oscillations are used to investigate: the concept of a sterile neutrino.

3.2 Sterile Neutrinos

The sterile neutrino is a fourth potential neutrino that is currently being investigated by the neutrino physics community. This modification was first proposed to the neutrino physics community to explain experimental oddities, particularly in the oscillation process. These neutrinos behave differently from their known three flavor counterparts because, while they would participate in oscillations, in this "sterile" phase they would only interact with gravity - not the weak force. This means it can be seen as an extra phase in the overall oscillation process, but because they only interact with gravity, studying them is considerably more difficult.

While still a theoretical particle, it is the subject of an increasing number of experiments, ranging from Fermi National Accelerator Laboratory's current short Baseline (SBN) program and the future long Baseline experiment DUNE, to small reactor experiments like KamLAND, all of which will help to further constrain the parameter space with which one could detect a sterile neutrino.

3.3 Extension: Neutrinos and the Universe

Neutrinos, like other fermions, have two spin orientations: right-handed and left-handed. This is similar to the discussion of spin - $\frac{1}{2}$ particles in every quantum course. To date, only left-handed neutrinos have been discovered in neutrino experiments. To account for this, we state that a Dirac neutrino differs from a Majorana neutrino. The Higgs mechanism gives Dirac neutrinos their mass, which essentially means that their mass must be close to that of other particles. The strength of neutrino interactions with the Higgs Boson is too weak when compared to those with the top quark, therefore this is not widely accepted.

Majorana neutrinos have the property of being their own antiparticle, and they follow what is known as the seesaw mechanism, which explains how heavy right-handed neutrinos in the early stages of the cosmos must have decayed into the lighter left-handed neutrinos seen in tests. Furthermore, the decaying process would yield left-handed neutrinos and right-handed antineutrinos, which we know decay at somewhat different rates based on prior observations. This could explain why, despite the fact that there were equal amounts of matter and antimatter at the moment of the Big Bang, they didn't annihilate each other.

It's astounding that we're studying the physics of subatomic particles now, given the history of the cosmos. It's also incredible that a particle whose physics is based on something that every undergraduate physics major learns - the quantum mechanical formalism - can be studied easily by both the theoretical and experimental communities, and that its properties can aid our understanding of the nature of the universe.

Chapter 4

4 Future experiments

Neutrino experiments are notoriously challenging, and every result must be confirmed by a second independent experiment. Many experiments are being planned to test various neutrino mass and mixing ranges in the future. We'll go through a few of them.

1. Super-Kamiokande: Super-Kamiokande is now in operation and has already delivered significant results. For both atmospheric and solar neutrinos, more high-statistics results are eagerly anticipated. This is expected to shed light on issues such as :

   (a) Seasonal variations in solar neutrino flux, which are indicative of the vacuum oscillation solution,

   (b) The day–night effect of solar neutrinos, which will allow a distinction between the large and small angle MSW(Mikheyev-Smirnov-Wolfenstein) solutions,

   (c) Spectral shape deformation of solar neutrinos as a normalisation independent check on neutrino oscillations, and so on. It will also function as a neutrino telescope in the event of a supernova.

2. The Sudbury Neutrino Observatory (SNO) will use 1 kton of D_2O surrounded by 7.3 ktons of conventional water in this experiment. Neutrinos will be detected via

   (a) CC disintegration ($\nu_e + d \longrightarrow e^- + p + p$) via electron Cerenkov radiation,

   (b) NC disintegration via calorimetric detection of emission on neutron capture, and

   (c) CC and NC scattering ($\nu + e^- \longrightarrow \nu + e^-$) via Cerenkov radiation. Only contribute to the CC reactions, but all three flavours of sequential neutrinos contribute equally to the NC reactions. The estimated count rates per year for these reactions, according to the typical solar model, are 9,750, 2,800, and 1,100, respectively. At SNO, the neutrino energy threshold should be at 5 MeV.

3. Borexino: Using 100 tonnes of ultrapure liquid scintillator, this experiment at the Gran Sasso laboratory will seek solar neutrinos. It will use the recoil electron in $\nu - e^{-}$ scattering to detect a neutrino event with a threshold electron energy as low as 0.25 MeV. It possesses real-time sensitivity to Be neutrinos from the sun (the 0.86 MeV line, which accounts for nearly 90% of the emission). This is especially important because
(a) Present experimental data suggest that Be neutrinos are virtually completely suppressed, and
(b) All previous experiments sensitive to these neutrinos were radiochemical in origin. The SSM predicts 50 occurrences per day at Borexino based on flux predictions. This experiment's scintillation-based detection does not provide directional information.

4. Homestake iodine experiment: This is the new setup that will replace the chlorine experiment at Homestake, and it uses the same basic radiochemical principle with the reaction $\nu_e + I^{127} \longrightarrow Xe^{127} + e^{-}$. With a half-life of 36.4 days, Xe^{127} decays by electron capture. The first plan is to install a 100-ton detector, which will later be upgraded to a 1k ton detector.

5. Long baseline experiments: The atmospheric neutrino experiments seem to indicate the occurrence of $\nu_\mu \longleftrightarrow \nu_\tau$ oscillations with a mass splitting around $10^{-2} - 10^{-3} eV^2$. Since accelerators provide ν_μ beams of higher energy, in order to probe this favoured mass difference range using these neutrinos one must have long baselines of several hundreds of kilometres. A number of such experiments are in the planning stages:

(a) K2K: Among the long baseline accelerator experiments the one that is in themost advanced stage is K2K using a neutrino beam originating from KEK and detected at super-Kamiokande – a baseline of 250 km. The neutrinos will beproduced by delivering a 12 GeV proton beam on target producing a meanbeam energy of 1.4 GeV.

The ν_e contamination level is predicted to be around 1%. A toroidal magnetic field focuses positively charged particles at the target, resulting in a fourteen-fold increase in neutrino flux. By comparing with a nearby detector, the experiment will look for an excess of ν_e well as a spectrum distortion. $\nu_\mu \longleftrightarrow \nu_\tau$ Oscillations can only be sought for in the disappearance mode due to the beam's limited energy. Future plans call for employing 50 GeV protons to create a higher-energy neutrino beam, the $\nu_\mu \longleftrightarrow \nu_\tau$ oscillation will be detected by looking for the appearance of τS in the final state.

(b) CERN to Gran Sasso:Neutrino Oscillation Experiment (NOE) which plans to use 6.7 ktons of scintillating fibre. They want to look for $\nu_\mu \longleftrightarrow \nu_\tau$ oscillation by looking for τ decays and $\nu_\mu \longleftrightarrow \nu_e$ oscillations by measuring any excess of electrons. The relative intensities of the CC and NC signals will be used to quantify the disappearance of ν_μ due to oscillations. The OPERA experiment, which is based on a 1 kton Emulsion Cloud Chamber, is the third experiment under consideration. This is a hybrid device that alternates between emulsion plates and free space, and the setup is intended to be sensitive to $\triangle m^2 = 10^{-3} eV^2$.

(c) MINOS: This is a long baseline experiment being planned with a neutrino beamfrom Fermilab directed to the SOUDAN setup in Minnesota. The distance fromsource to detector will be 730 km and the average neutrino energy is expectedto be 10 GeV. The experiment will look for $\nu_\mu \longleftrightarrow \nu_\tau$ oscillations.

6. Reactor experiments: Nuclear reactors are copious sources of and have been a standard source for neutrino experiments since the fifties. Several experiments to look for oscillations are planned for the future using reactor antineutrinos.

   (a) Palo Verde: This experiment will use scintillation detectors at distances of 750,888, and 889 metres from the reactor to search for oscillations in the disappearance mode. The relevant reaction is $\nu_e + p \longrightarrow e^+ + n$ where the e^+ produces annihilation photons while the neutron is identified by the delayed photon emitted on absorption. The expected sensitivity of this experiment is to $\triangle m^2 > 1.3 * 10^{-3} eV^2$ and $\sin^2 2\theta > 0.1$

   (b) KAMLAND: Reactor antineutrinos with an average energy of 3 MeV will be used in this experiment. It will search for oscillations in the disappearance mode, with a baseline of 100 kilometres.

7. ICARUS: At the Gran Sasso facility, a 600 tonne liquid argon detector is also being installed. It has a 4–5 MeV neutrino energy threshold, hence it will only detect B neutrinos from the sun. It will detect neutrinos by detecting (a) scattered electrons in scattering, which occurs via both CC and NC interactions, and (b) the de-excitation of K produced in the reaction $^{40}Ar \longrightarrow ^{40} K^* + e^-$. The ground state of K has $J^\pi = 4^-$ hence the de-excitation occurs through the emission of several gammas.The angular distributions (a) is directed along the direction, while (b) is more isotropic and multiplicity (b) has many gammas identify occurrences of type (a) and (b). In the future, it may be able to detect solar neutrinos as well as quantify the atmospheric neutrino ratio $\frac{\nu_\mu + \overline{\nu_\mu}}{\nu_e + \overline{\nu_e}}$.

   There are plans for a 5 kton detector in the future (Future) along similar lines. Another method is to add 5% CD (deuterated methane) and watch for the CC reaction $\nu_e + d \longrightarrow e^- + p + p$.The electron will be isolated, while the two protons at the vertex will create severe ionisation. The facility might also be used as a baseline detector for CERN's NGS (Neutrino to Gran Sasso) beam.

Chapter 5

5 Conclusion

In this paper, we have discussed in rigor one of the many ways in which neutrino oscillations can be modeled based on the properties of neutrinos. The standard derivation of the vacuum oscillation case has been discussed and the result obtained in this case is a very special one. It is recognized to have its limits, and there are better approaches to give greater insight into the nature of neutrino oscillations. In particular, a wave-packet approach is more often implemented and this approach is far superior to the standard derivation we have used in terms of generality. This method takes into consideration the properties of localization of the particles involved, and also considers the possible decoherence of neutrinos. However, it is a far more rigorous approach involving some ideas from advanced quantum mechanics, and is beyond the scope of our present discussion. The standard derivation formula however is useful in the sense that it gives us an idea of certain general aspects of neutrinos without requiring as much mathematical rigor and complexities. It turns out that this formula can be obtained as the limit of negligible decoherence effects in the wave-packet approach to neutrino oscillations. The current method of approach to neutrino oscillations on a more advanced scale are implementations of quantum field theory (QFT) as this proves to be even superior to the wave-packet approach. This method takes into consideration the processes of production and detection of the neutrinos, as well as consistent normalization of the wave function. In addition to this, we have also neglected the study of neutrino oscillations under matter effects. This can be done quite easily however, only bringing in additional complexities. Neutrino oscillations take place as a result of their intrinsic property- the mass eigenstates do not coincide with flavor eigenstates. This is what makes neutrinos so special. Some other fundamental particles such as charged leptons for example, cannot show this oscillatory behavior as their mass eigenstate necessarily coincides with the flavor eigenstate of the lepton. Neutrino oscillations are a phenomenon that occur on a macroscopic scale. It is interesting that such a phenomenon can be examined to such a great extent by simply using the framework of quantum mechanics, which usually can be replaced with classical mechanics as systems move to a macroscopic scale.

References

1) 'On the quantum mechanics of Neutrino oscillation' by Boris Kayser

2) 'Theory of Neutrino Oscillations in Framework of Quantum Mechanics' by David Walther Schönleber

3) https://neutrino-history.in2p3.fr/neutrino-oscillation/ for information on Neutrino Oscillations

4) 'On theories of neutrino oscillations' by Daniel Krupkke

5) https://en.wikipedia.org/wiki/Pontecorvo
for basic information regarding PMNS matrix.