CHAPTER

Digital Signals and Systems

3

CHAPTER OUTLINE

OBJECTIVES:

This chapter introduces notations for digital signals and special digital sequences that are widely used in this book. The chapter continues to study some properties of linear systems such as time invariance, BIBO (bounded-in and bounded-out) stability, causality, impulse response, difference equations, and digital convolution.

3.1 DIGITAL SIGNALS

In our daily lives, analog signals appear in forms such as speech, audio, seismic, biomedical, and communications signals. To process an analog signal using a digital signal processor, the analog signal must be converted in to a digital signal, that is, analog-to-digital conversion (ADC) must take place, as discussed in Chapter 2. Then the digital signal is processed via digital signal processing (DSP) algorithm(s).

A typical digital signal $x(n)$ is shown in Figure 3.1, where both the time and the amplitude of the digital signal are discrete. Notice that the amplitudes of the digital signal samples are given and sketched only at their corresponding time indices, where $x(n)$ represents the amplitude of the nth sample and n is the time index or sample number. From Figure 3.1, we learn that

$x(0)$: zeroth sample amplitude at sample number $n = 0$,
$x(1)$: first sample amplitude at sample number $n = 1$,

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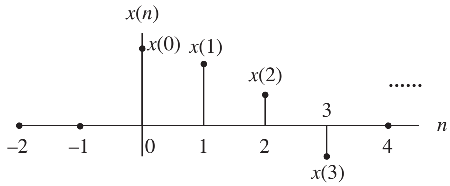

FIGURE 3.1

Digital signal notation.

$x(2)$: second sample amplitude at sample number $n = 2$,
$x(3)$: third sample amplitude at sample number $n = 3$, and so on.

Furthermore, Figure 3.2 illustrates the digital samples whose amplitudes are the discrete encoded values represented in the digital signal (DS) processor. Precision of the data is based on the number of bits used in the DSP system. The encoded data format can be either an integer if a fixed-point DS processor is used or a floating-point number if a floating-point DP processor is used. As shown in Figure 3.2 for the floating-point DS processor, we can identify the first five sample amplitudes at their time indices as follows:

$x(0) = 2.25$
$x(1) = 2.0$
$x(2) = 1.0$
$x(3) = -1.0$
$x(4) = 0.0$
……

Again, note that each sample amplitude is plotted using a vertical bar with a solid dot. This notation is well accepted in DSP literature.

3.1.1 Common Digital Sequences

Let us study some special digital sequences that are widely used. We define and plot each of them as follows:

Unit-impulse sequence (digital unit-impulse function):

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \tag{3.1}$$

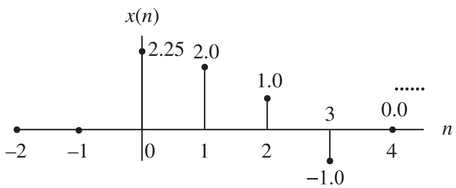

FIGURE 3.2

Plot of the digital signal samples.

The plot of the unit-impulse function is given in Figure 3.3. The unit-impulse function has unit amplitude at only $n = 0$ and zero amplitude at other time indices.

Unit-step sequence (digital unit-step function):

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{3.2}$$

The plot is given in Figure 3.4. The unit-step function has unit amplitude at $n = 0$ and at all the positive time indices, and zero amplitude at all negative time indices.

The shifted unit-impulse and unit-step sequences are displayed in Figure 3.5. As shown in the figure, the shifted unit-impulse function $\delta(n-2)$ is obtained by shifting the unit-impulse function $\delta(n)$ to the right by two samples, and the shifted unit-step function $u(n-2)$ is achieved by shifting the unit-step function $u(n)$ to the right by two samples; similarly, $\delta(n+2)$ and $u(n+2)$ are acquired by shifting $\delta(n)$ and $u(n)$ two samples to the left, respectively.

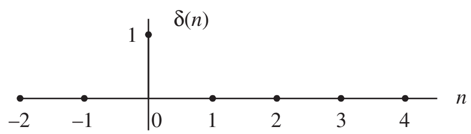

FIGURE 3.3

Unit-impulse sequence.

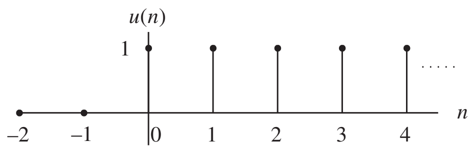

FIGURE 3.4

Unit-step sequence.

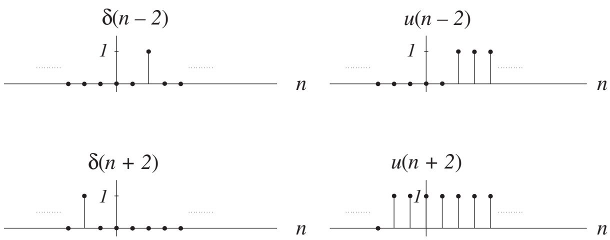

FIGURE 3.5

Shifted unit-impulse and unit-step sequences.

Sinusoidal and exponential sequences are depicted in Figures 3.6 and 3.7, respectively. For the sinusoidal sequence $x(n) = A\cos(0.125\pi n)u(n)$, and $A = 10$, we can calculate the digital values for the first eight samples and list their values in Table 3.1.

For the exponential sequence $x(n) = A(0.75)^n u(n)$, the calculated digital values for the first eight samples with $A = 10$ are listed in Table 3.2.

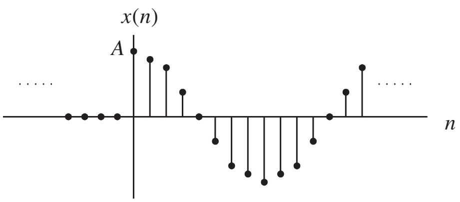

FIGURE 3.6

Plot of samples of the sinusoidal function.

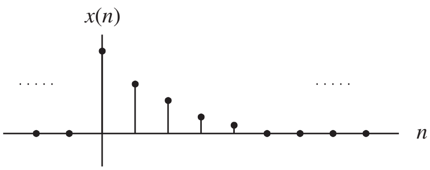

FIGURE 3.7

Plot of samples of the exponential function.

Table 3.1 Sample Values Calculated from the Sinusoidal Function

n	$x(n) = 10\cos(0.125\pi n)u(n)$
0	10.0000
1	9.2388
2	7.0711
3	3.8628
4	0.0000
5	–3.8628
6	–7.0711
7	–9.2388

Table 3.2 Sample Values Calculated from the Exponential Function

n	$10(0.75)^n u(n)$
0	10.0000
1	7.5000
2	5.6250
3	4.2188
4	3.1641
5	2.3730
6	1.7798
7	1.3348

EXAMPLE 3.1

Sketch the following sequence

$$x(n) = \delta(n+1) + 0.5\delta(n-1) + 2\delta(n-2)$$

Solution:

According to the shift operation, $\delta(n+1)$ is obtained by shifting $\delta(n)$ to the left by one sample, while $\delta(n-1)$ and $\delta(n-2)$ are yielded by shifting $\delta(n)$ to right by one sample and two samples, respectively. Using the amplitude of each impulse function, we obtain the sketch in Figure 3.8.

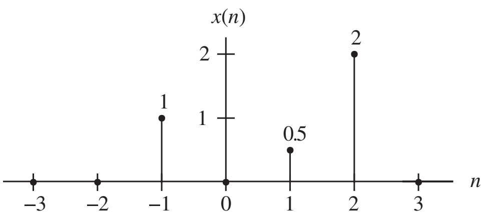

FIGURE 3.8

Plot of digital sequence in Example 3.1.

3.1.2 Generation of Digital Signals

Given the sampling rate of a DSP system to sample the analytical function of an analog signal, the corresponding digital function or digital sequence (assuming its sampled amplitudes are encoded to have finite precision) can be found. The digital sequence is often used to

1. Calculate the encoded sample amplitude for a given sample number n.
2. Generate the sampled sequence for simulation.

The procedure to develop the digital sequence from its analog signal function is as follows. Assuming that an analog signal $x(t)$ is uniformly sampled at the time interval of $\Delta t = T$, where T is the sampling period, the corresponding digital function (sequence) $x(n)$ gives the *instant encoded values* of the analog signal $x(t)$ at all the time instants $t = n\Delta t = nT$ and can be achieved by substituting time $t = nT$ into the analog signal $x(t)$, that is,

$$x(n) = x(t)|_{t=nT} = x(nT) \tag{3.3}$$

Also notice that for sampling the unit-step function $u(t)$, we have

$$u(t)|_{t=nT} = u(nT) = u(n) \tag{3.4}$$

The following example will demonstrate the use of Equations (3.3) and (3.4).

EXAMPLE 3.2

Assume we have a DSP system with a sampling time interval of 125 microseconds.

a. Convert each of following analog signals $x(t)$ to a digital signal $x(n)$:
  1. $x(t) = 10e^{-5{,}000t}u(t)$
  2. $x(t) = 10\sin(2{,}000\pi t)u(t)$

b. Determine and plot the sample values from each obtained digital function.

Solution:

a. Since $T = 0.000125$ seconds in Equation (3.3), substituting $t = nT = n \times 0.000125 = 0.000125\,n$ into the analog signal $x(t)$ expressed in (1) leads to the digital sequence

$$1.\quad x(n) = x(nT) = 10e^{-5{,}000\times 0.000125n}u(nT) = 10e^{-0.625n}u(n)$$

Similarly, the digital sequence for (2) is achieved as follows:

$$2.\quad x(n) = x(nT) = 10\sin(2{,}000\pi \times 0.000125n)u(nT) = 10\sin(0.25\pi n)u(n)$$

b. The first five sample values for (1) are calculated and plotted in Figure 3.9.

$$x(0) = 10e^{-0.625\times 0}u(0) = 10.0$$
$$x(1) = 10e^{-0.625\times 1}u(1) = 5.3526$$

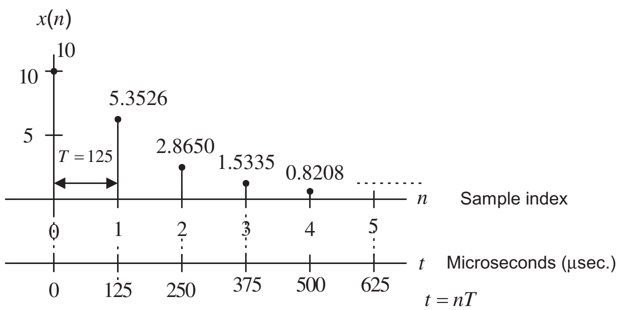

FIGURE 3.9

Plot of the digital sequence for (1) in Example 3.2.

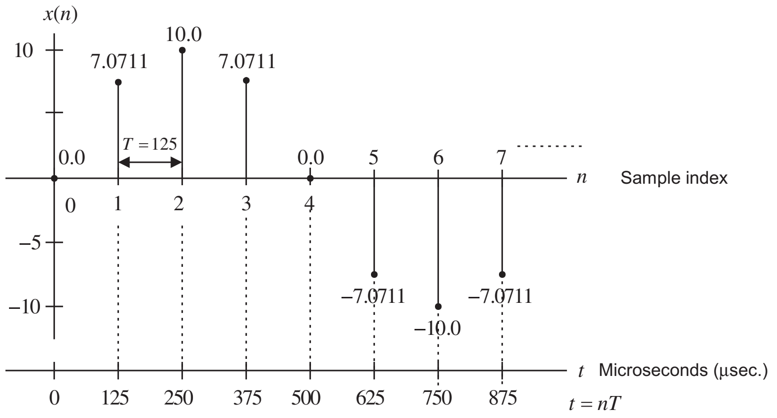

FIGURE 3.10

Plot of the digital sequence for (2) in Example 3.2.

$$
\begin{aligned}
x(2) &= 10e^{-0.625\times 2}u(2) = 2.8650\\
x(3) &= 10e^{-0.625\times 3}u(3) = 1.5335\\
x(4) &= 10e^{-0.625\times 4}u(4) = 0.8208
\end{aligned}
$$

The first eight amplitudes for (2) are computed and sketched in Figure 3.10.

$$
\begin{aligned}
x(0) &= 10\sin(0.25\pi\times 0)u(0) = 0\\
x(1) &= 10\sin(0.25\pi\times 1)u(1) = 7.0711\\
x(2) &= 10\sin(0.25\pi\times 2)u(2) = 10.0\\
x(3) &= 10\sin(0.25\pi\times 3)u(3) = 7.0711\\
x(4) &= 10\sin(0.25\pi\times 4)u(4) = 0.0\\
x(5) &= 10\sin(0.25\pi\times 5)u(5) = -7.0711\\
x(6) &= 10\sin(0.25\pi\times 6)u(6) = -10.0\\
x(7) &= 10\sin(0.25\pi\times 7)u(7) = -7.0711
\end{aligned}
$$

3.2 LINEAR TIME-INVARIANT, CAUSAL SYSTEMS

In this section, we study linear time-invariant causal systems and focus on properties such as linearity, time-invariance, and causality.

3.2.1 Linearity

A linear system is illustrated in Figure 3.11, where $y_1(n)$ is the system output using an input $x_1(n)$, and $y_2(n)$ the system output with an input $x_2(n)$.

Figure 3.11 illustrates that the system output due to the weighted sum inputs $\alpha x_1(n) + \beta x_2(n)$ is equal to the same weighted sum of the individual outputs obtained from their corresponding inputs, that is,

$$y(n) = \alpha y_1(n) + \beta y_2(n) \tag{3.5}$$

where α and β are constants.

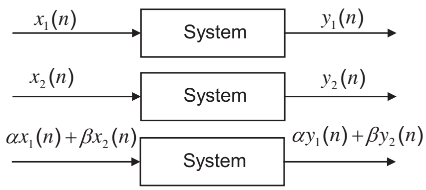

FIGURE 3.11

Digital linear system.

For example, assuming a digital amplifier is represented by $y(n) = 10x(n)$, the input is multiplied by 10 to generate the output. The inputs $x_1(n) = u(n)$ and $x_2(n) = \delta(n)$ generate the outputs

$$y_1(n) = 10u(n), \text{ and } y_2(n) = 10\delta(n), \text{ respectively}$$

If, as described in Figure 3.11, we apply the combined input $x(n)$ to the system, where the first input multiplied by a constant 2 while the second input multiplied by a constant 4,

$$x(n) = 2x_1(n) + 4x_2(n) = 2u(n) + 4\delta(n)$$

then the system output due to the combined input is obtained as

$$y(n) = 10x(n) = 10(2u(n) + 4\delta(n)) = 20u(n) + 40\delta(n) \tag{3.6}$$

If we verify the weighted sum of the individual outputs, we see that

$$2y_1(n) + 4y_2(n) = 20u(n) + 40\delta(n) \tag{3.7}$$

Comparing Equations (3.6) and (3.7) verifies that

$$y(n) = 2y_1(n) + 4y_2(n) \tag{3.8}$$

Hence, the system $y(n) = 10x(n)$ is a linear system. The linearity means that the system obeys the superposition principle, as shown in Equation (3.8). Let us verify a system whose output is a square of its input,

$$y(n) = x^2(n)$$

Applying the inputs $x_1(n) = u(n)$ and $x_2(n) = \delta(n)$ to the system leads to

$$y_1(n) = u^2(n) = u(n), \textit{ and } y_2(n) = \delta^2(n) = \delta(n)$$

It is very easy to verify that $u^2(n) = u(n)$ and $\delta^2(n) = \delta(n)$.

We can determine the system output using a combined input, which is the weighed sum of the individual inputs with constants 2 and 4, respectively. Using algebra, we see that

$$\begin{aligned} y(n) &= x^2(n) = (4x_1(n) + 2x_2(n))^2 \\ &= (4u(n) + 2\delta(n))^2 = 16u^2(n) + 16u(n)\delta(n) + 4\delta^2(n) \\ &= 16u(n) + 20\delta(n) \end{aligned} \tag{3.9}$$

Note that we use the fact that $u(n)\delta(n) = \delta(n)$, which can be easily verified.

Again, we express the weighted sum of the two individual outputs with the same constants 2 and 4 as

$$4y_1(n) + 2y_2(n) = 4u(n) + 2\delta(n) \tag{3.10}$$

It is obvious that

$$y(n) \neq 4y_1(n) + 2y_2(n) \tag{3.11}$$

Hence, the system is a nonlinear system, since the linear property, superposition, does not hold, as shown in Equation (3.11).

3.2.2 Time Invariance

A time-invariant system is illustrated in Figure 3.12, where $y_1(n)$ is the system output for the input $x_1(n)$. Let $x_2(n) = x_1(n - n_0)$ be the shifted version of $x_1(n)$ by n_0 samples. The output $y_2(n)$ obtained with the shifted input $x_2(n) = x_1(n - n_0)$ is equivalent to the output $y_2(n)$ acquired by shifting $y_1(n)$ by n_0 samples, $y_2(n) = y_1(n - n_0)$.

This can simply be viewed as the following:

> If the system is time invariant and $y_1(n)$ is the system output due to the input $x_1(n)$, then the shifted system input $x_1(n - n_0)$ will produce a shifted system output $y_1(n - n_0)$ by the same amount of time n_0.

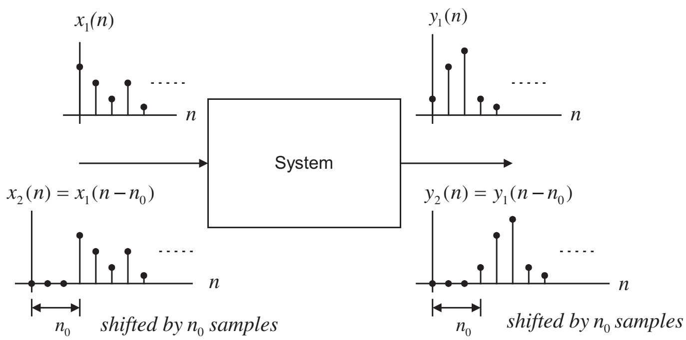

FIGURE 3.12

Illustration of the linear time-invariant digital system.

EXAMPLE 3.3

Determine whether the linear systems

a. $y(n) = 2x(n-5)$
b. $y(n) = 2x(3n)$

are time invariant.

Solution:
a. Let the input and output be $x_1(n)$ and $y_1(n)$, respectively; then the system output is $y_1(n) = 2x_1(n-5)$. Again, let $x_2(n) = x_1(n-n_0)$ be the shifted input and $y_2(n)$ be the output due to the shifted input. We determine the system output using the shifted input as

$$y_2(n) = 2x_2(n-5) = 2x_1(n-n_0-5)$$

Meanwhile, shifting $y_1(n) = 2x_1(n-5)$ by n_0 samples leads to

$$y_1(n-n_0) = 2x_1(n-5-n_0)$$

We can verify that $y_2(n) = y_1(n-n_0)$. Thus the shifted input of n_0 samples causes the system output to be shifted by the same n_0 samples. The system is thus time invariant.
b. Let the input and output be $x_1(n)$ and $y_1(n)$, respectively; then the system output is $y_1(n) = 2x_1(3n)$. Again, let the input and output be $x_2(n)$ and $y_2(n)$, where $x_2(n) = x_1(n-n_0)$, a shifted version, and the corresponding output is $y_2(n)$. We get the output due to the shifted input $x_2(n) = x_1(n-n_0)$ and note that $x_2(3n) = x_1(3n-n_0)$:

$$y_2(n) = 2x_2(3n) = 2x_1(3n-n_0)$$

On the other hand, if we shift $y_1(n)$ by n_0 samples, and replace n in $y_1(n) = 2x_1(3n)$ by $n-n_0$, we obtain

$$y_1(n-n_0) = 2x_1(3(n-n_0)) = 2x_1(3n-3n_0)$$

Clearly, we know that $y_2(n) \neq y_1(n-n_0)$. Since the system output $y_2(n)$ using the input shifted by n_0 samples is not equal to the system output $y_1(n)$ shifted by the same n_0 samples, the system is not time invariant.

3.2.3 Causality

A causal system is the one in which the output $y(n)$ at time n depends only on the current input $x(n)$ at time n, and its past input sample values such as $x(n-1)$, $x(n-2)$,…. Otherwise, if a system output depends on future input values such as $x(n+1)$, $x(n+2)$, …, the system is noncausal. The noncausal system cannot be realized in real time.

EXAMPLE 3.4

Determine whether the systems

a. $y(n) = 0.5x(n) + 2.5x(n-2)$, for $n \geq 0$
b. $y(n) = 0.25x(n-1) + 0.5x(n+1) - 0.4y(n-1)$, for $n \geq 0$

are causal.

Solution:
a. Since for $n \geq 0$, the output $y(n)$ depends on the current input $x(n)$ and its past value $x(n-2)$, the system is causal.
b. Since for $n \geq 0$, the output $y(n)$ depends on the current input $x(n)$ and its future value $x(n+1)$, the system is noncausal.

3.3 DIFFERENCE EQUATIONS AND IMPULSE RESPONSES

Now we study the difference equation and its impulse response.

3.3.1 Format of the Difference Equation

A causal, linear, time-invariant system can be described by a difference equation having the following general form:

$$y(n) + a_1 y(n-1) + \cdots + a_N y(n-N) = b_0 x(n) + b_1 x(n-1) + \cdots + b_M x(n-M) \tag{3.12}$$

where $a_1, \ldots, a_N$, and $b_0, b_1, \ldots, b_M$ are the coefficients of the difference equation. Equation (3.12) can also be written as

$$y(n) = -a_1 y(n-1) - \cdots - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + \cdots + b_M x(n-M) \tag{3.13}$$

or

$$y(n) = -\sum_{i=1}^{N} a_i y(n-i) + \sum_{j=0}^{M} b_j x(n-j) \tag{3.14}$$

Notice that $y(n)$ is the current output, which depends on the past output samples $y(n-1), \ldots, y(n-N)$, the current input sample $x(n)$, and the past input samples, $x(n-1), \ldots, x(n-N)$.

We will examine the specific difference equations in the following examples.

EXAMPLE 3.5

Given the difference equation

$$y(n) = 0.25 y(n-1) + x(n)$$

identify the nonzero system coefficients.

Solution:

Comparison with Equation (3.13) leads to

$$b_0 = 1$$

$$-a_1 = 0.25, \quad \text{that is,} \quad a_1 = -0.25$$

EXAMPLE 3.6

Given a linear system described by the difference equation

$$y(n) = x(n) + 0.5 x(n-1)$$

determine the nonzero system coefficients.

Solution:

By comparing Equation (3.13), we have

$$b_0 = 1 \quad \text{and} \quad b_1 = 0.5$$

3.3.2 System Representation Using Its Impulse Response

A linear time-invariant system can be completely described by its unit-impulse response, which is defined as the system response due to the impulse input $\delta(n)$ with zero initial conditions, depicted in Figure 3.13.

With the obtained unit-impulse response $h(n)$, we can represent the linear time-invariant system as shown in Figure 3.14.

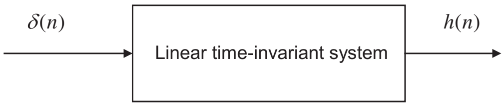

FIGURE 3.13

Unit-impulse response of a linear time-invariant system.

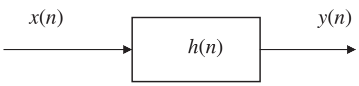

FIGURE 3.14

Representation of a linear time-invariant system using the impulse response.

EXAMPLE 3.7

Assume we have a linear time-invariant system

$$y(n) = 0.5x(n) + 0.25x(n-1)$$

with an initial condition $x(-1) = 0$.

a. Determine the unit-impulse response $h(n)$.
b. Draw the system block diagram.
c. Write the output using the obtained impulse response.

Solution:

a. According to Figure 3.13, let $x(n) = \delta(n)$, then

$$h(n) = y(n) = 0.5x(n) + 0.25x(n-1) = 0.5\delta(n) + 0.25\delta(n-1)$$

Thus, for this particular linear system, we have

$$h(n) = \begin{cases} 0.5 & n = 0 \\ 0.25 & n = 1 \\ 0 & elsewhere \end{cases}$$

b. The block diagram of the linear time-invariant system is shown in Figure 3.15.

c. The system output can be rewritten as

$$y(n) = h(0)x(n) + h(1)x(n-1)$$

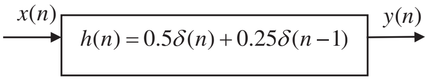

FIGURE 3.15

The system block diagram for Example 3.7.

From the result Example 3.7, it is noted that the difference equation does not have the past output terms, $y(n-1), \ldots, y(n-N)$, that is, the corresponding coefficients $a_1, \ldots, a_N$, are zeros, and the impulse response $h(n)$ has a finite number of terms. We call this system a *finite impulse response* (FIR) system. In general, Equation (3.12) contains the past output terms and the resulting impulse response $h(n)$ has an infinite number of terms. We can express the output sequence of a linear time-invariant system using its impulse response and inputs as

$$y(n) = \cdots + h(-1)x(n+1) + h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \cdots \tag{3.15}$$

Equation (3.15) is called the *digital convolution sum*, which will be explored in a later section. We can verify Equation (3.15) by substituting the impulse sequence $x(n) = \delta(n)$ to get the impulse response

$$h(n) = \cdots + h(-1)\delta(n+1) + h(0)\delta(n) + h(1)\delta(n-1) + h(2)\delta(n-2) + \cdots$$

where $\ldots h(-1), h(0), h(1), h(2) \ldots$ are the amplitudes of the impulse response at the corresponding time indices. Now let us look at another example.

EXAMPLE 3.8

Consider the difference equation

$$y(n) = 0.25y(n-1) + x(n) \quad \text{for} \quad n \geq 0 \quad \text{and} \quad y(-1) = 0$$

a. Determine the unit-impulse response $h(n)$.

b. Draw the system block diagram.

c. Write the output using the obtained impulse response.

d. For a step input $x(n) = u(n)$, verify and compare the output responses for the first three output samples using the difference equation and digital convolution sum (Equation (3.15)).

Solution:

a. Let $x(n) = \delta(n)$, then

$$h(n) = 0.25h(n-1) + \delta(n)$$

To solve for $h(n)$, we evaluate

$h(0) = 0.25h(-1) + \delta(0) = 0.25 \times 0 + 1 = 1$

$h(1) = 0.25h(0) + \delta(1) = 0.25 \times 1 + 0 = 0.25$

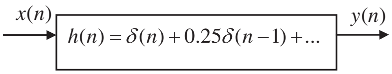

FIGURE 3.16

The system block diagram for Example 3.8.

$h(2) = 0.25h(1) + \delta(2) = 0.25 \times 0.5 + 0 = 0.0625$

....

With the calculated results, we can predict the impulse response as

$$h(n) = (0.25)^n u(n) = \delta(n) + 0.25\delta(n-1) + 0.0625\delta(n-2) + \cdots$$

b. The system block diagram is given in Figure 3.16.

c. The output sequence is a sum of infinite terms expressed as

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \cdots$$
$$= x(n) + 0.25x(n-1) + 0.0625x(n-2) + \cdots$$

d. From the difference equation and using the zero initial condition, we have

$y(n) = 0.25y(n-1) + x(n)$ for $n \geq 0$ and $y(-1) = 0$

$n = 0$, $y(0) = 0.25y(-1) + x(0) = u(0) = 1$

$n = 1$, $y(1) = 0.25y(0) + x(1) = 0.25 \times u(0) + u(1) = 1.25$

$n = 2$, $y(2) = 0.25y(1) + x(2) = 0.25 \times 1.25 + u(2) = 1.3125$

....

Applying the convolution sum in Equation (3.15) yields

$$y(n) = x(n) + 0.25x(n-1) + 0.0625x(n-2) + \cdots$$

$$n = 0, \quad \begin{aligned} y(0) &= x(0) + 0.25x(-1) + 0.0625x(-2) + \cdots \\ &= u(0) + 0.25 \times u(-1) + 0.125 \times u(-2) + \cdots = 1 \end{aligned}$$

$$n = 1, \quad \begin{aligned} y(1) &= x(1) + 0.25x(0) + 0.0625x(-1) + \cdots \\ &= u(1) + 0.25 \times u(0) + 0.125 \times u(-1) + \cdots = 1.25 \end{aligned}$$

$$n = 2, \quad \begin{aligned} y(2) &= x(2) + 0.25x(1) + 0.0625x(0) + \cdots \\ &= u(2) + 0.25 \times u(1) + 0.0625 \times u(0) + \cdots = 1.3125 \end{aligned}$$

...

Comparing the results, we verify that a linear time-invariant system can be represented by the convolution sum using its impulse response and input sequence. Note that we verify only the causal system for simplicity, and the principle works for both causal and noncausal systems.

Notice that this impulse response $h(n)$ contains an infinite number of terms in its duration due to the past output term $y(n-1)$. Such a system as described in the preceding example is called an *infinite impulse response* (IIR) system, which will be studied in later chapters.

3.4 BOUNDED-IN AND BOUNDED-OUT STABILITY

We are interested in designing and implementing stable linear systems. A stable system is one for which every bounded input produces a bounded output (BIBO). There are many other stability definitions. To find the stability criterion, consider the linear time-invariant representation with all the inputs reaching the maximum value M for the worst case. Equation (3.15) becomes

$$y(n) = M(\cdots + h(-1) + h(0) + h(1) + h(2) + \cdots) \tag{3.16}$$

Using the absolute values of the impulse response leads to

$$y(n) < M(\cdots + |h(-1)| + |h(0)| + |h(1)| + |h(2)| + \cdots) \tag{3.17}$$

If the absolute sum in Equation (3.17) is a finite number, the product of the absolute sum and the maximum input value is therefore a finite number. Hence, we have a bounded input and bounded output. In terms of the impulse response, a linear system is stable if the sum of its absolute impulse response coefficients is a finite number. We can apply Equation (3.18) to determine whether a linear time-invariant system is stable or not stable, that is,

$$S = \sum_{k=-\infty}^{\infty} |h(k)| = \cdots + |h(-1)| + |h(0)| + |h(1)| + \cdots < \infty \tag{3.18}$$

Figure 3.17 describes a linear stable system, where the impulse response decreases to zero in a finite amount of time so that the summation of its absolute impulse response coefficients is guaranteed to be finite.

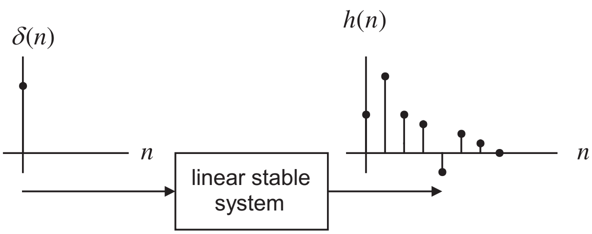

FIGURE 3.17

Illustration of stability of the digital linear system.

EXAMPLE 3.9

Given the linear system in Example 3.8,

$$y(n) = 0.25y(n-1) + x(n) \quad \text{for} \quad n \geq 0 \quad \text{and} \quad y(-1) = 0$$

which is described by the unit-impulse response

$$h(n) = (0.25)^n u(n)$$

determine whether this system is stable.

Solution:
Using Equation (3.18), we have

$$S = \sum_{k=-\infty}^{\infty} |h(k)| = \sum_{k=-\infty}^{\infty} \left|(0.25)^k u(k)\right|$$

Applying the definition of the unit-step function $u(k) = 1$ for $k \geq 0$, we have

$$S = \sum_{k=0}^{\infty} (0.25)^k = 1 + 0.25 + 0.25^2 + \cdots$$

Using the formula for a sum of the geometric series (see Appendix G),

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a},$$

where $a = 0.25 < 1$, we conclude

$$S = 1 + 0.25 + 0.25^2 + \cdots = \frac{1}{1-0.25} = \frac{4}{3} < \infty$$

Since the summation is a finite number, the linear system is stable.

3.5 DIGITAL CONVOLUTION

Digital convolution plays an important role in digital filtering. As we verified in the last section, a linear time-invariant system can be represented using a digital convolution sum. Given a linear time-invariant system, we can determine its unit-impulse response $h(n)$, which relates the system input and output. To find the output sequence $y(n)$ for any input sequence $x(n)$, we write the digital convolution shown in Equation (3.15) as

$$\begin{aligned} y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \\ &= \cdots + h(-1)x(n+1) + h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \cdots \end{aligned} \tag{3.19}$$

The sequences $h(k)$ and $x(k)$ in Equation (3.19) are interchangeable. Hence, we have an alternative form:

$$\begin{aligned} y(n) &= \sum_{k=-\infty}^{\infty} x(k)h(n-k) \\ &= \cdots + x(-1)h(n+1) + x(0)h(n) + x(1)h(n-1) + x(2)h(n-2) + \cdots \end{aligned} \tag{3.20}$$

Using conventional notation, we express the digital convolution as

$$y(n) = h(n) * x(n) \tag{3.21}$$

Note that for a causal system, which implies an impulse response of

$$h(n) = 0 \quad \text{for} \quad n < 0 \tag{3.16}$$

the lower limit of the convolution sum begins at 0 instead of ∞, that is

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k) = \sum_{k=0}^{\infty} x(k)h(n-k) \tag{3.22}$$

We will focus on evaluating the convolution sum based on Equation (3.20). Let us examine first a few outputs from Equation (3.20):

$$\begin{aligned}
y(0) &= \textstyle\sum_{k=-\infty}^{\infty} x(k)h(-k) = \cdots + x(-1)h(1) + x(0)h(0) + x(1)h(-1) + x(2)h(-2) + \cdots \\
y(1) &= \textstyle\sum_{k=-\infty}^{\infty} x(k)h(1-k) = \cdots + x(-1)h(2) + x(0)h(1) + x(1)h(0) + x(2)h(-1) + \cdots \\
y(2) &= \textstyle\sum_{k=-\infty}^{\infty} x(k)h(2-k) = \cdots + x(-1)h(3) + x(0)h(2) + x(1)h(1) + x(2)h(0) + \cdots \\
&\ldots..
\end{aligned}$$

We see that the convolution sum requires the sequence $h(n)$ to be reversed and shifted. The graphical, formula, and table methods for evaluating the digital convolution will be discussed via several examples. To evaluate the convolution sum graphically, we need to apply the reversed sequence and shifted sequence. The reversed sequence is defined as follows: if $h(n)$ is the given sequence, $h(-n)$ is the reversed sequence. The reversed sequence is a mirror image of the original sequence, assuming the vertical axis as the mirror. Let us study the reversed sequence and shifted sequence via the following example.

EXAMPLE 3.10

Consider a sequence

$$h(k) = \begin{cases} 3, & k = 0, 1 \\ 1, & k = 2, 3 \\ 0 & elsewhere \end{cases}$$

where k is the time index or sample number.

a. Sketch the sequence $h(k)$ and reversed sequence $h(-k)$.
b. Sketch the shifted sequences $h(-k+3)$ and $h(-k-2)$.

Solution:
a. Since $h(k)$ is defined, we plot it in Figure 3.18. Next, we need to find the reversed sequence $h(-k)$. We examine the following:

$k > 0$, $h(-k) = 0$
$k = 0$, $h(-0) = h(0) = 3$
$k = -1$, $h(-k) = h(-(-1)) = h(1) = 3$
$k = -2$, $h(-k) = h(-(-2)) = h(2) = 1$
$k = -3$, $h(-k) = h(-(-3)) = h(3) = 1$

One can verify that $k \le -4$, $h(-k) = 0$. Then the reversed sequence $h(-k)$ is shown as the second plot in Figure 3.18.

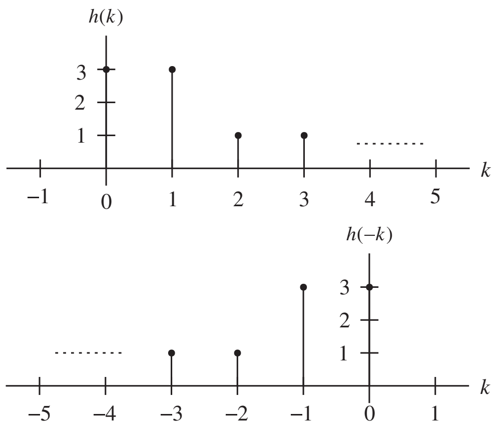

FIGURE 3.18

Plots of the digital sequence and its reversed sequence in Example 3.10.

As shown in the sketches, $h(-k)$ is just a mirror image of the original sequence $h(k)$.

b. Based on the definition of the original sequence, we know that $h(0) = h(1) = 3$, $h(2) = h(3) = 1$, and the others are zeros. The time indices correspond to the following:

$$-k+3 = 0,\ k = 3$$
$$-k+3 = 1,\ k = 2$$
$$-k+3 = 2,\ k = 1$$
$$-k+3 = 3,\ k = 0$$

Thus we can sketch $h(-k+3)$ as shown in Figure 3.19.

Similarly, $h(-k-2)$ is shown in Figure 3.20.

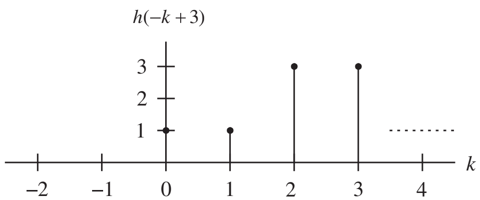

FIGURE 3.19

Plot of the sequence $h(-k+3)$ in Example 3.10.

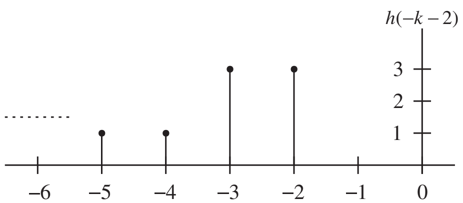

FIGURE 3.20

Plot of the sequence $h(-k-2)$ in Example 3.10.

We can get $h(-k+3)$ by shifting $h(-k)$ to the right by three samples, and we can obtain $h(-k-2)$ by shifting $h(-k)$ to the left by two samples.

In summary, given $h(-k)$, we can obtain $h(n-k)$ by shifting $h(-k)$ n samples to the right or the left, depending on whether n is positive or negative.

Once we understand the shifted sequence and reversed sequence, we can perform digital convolution of the two sequences $h(k)$ and $x(k)$, defined in Equation (3.20) graphically. From that equation, we see that each convolution value $y(n)$ is the sum of the products of two sequences $x(k)$ and $h(n-k)$, the latter of which is the shifted version of the reversed sequence $h(-k)$ by $|n|$ samples. Hence, we can summarize the graphical convolution procedure in Table 3.3.

We illustrate the digital convolution sum in the following example.

Table 3.3 Digital Convolution Using the Graphical Method

Step 1. Obtain the reversed sequence $h(-k)$.
Step 2. Shift $h(-k)$ by $|n|$ samples to get $h(n-k)$. If $n \geq 0$, $h(-k)$ will be shifted to right by n samples; but if $n < 0$, $h(-k)$ will be shifted to the left by $|n|$ samples.
Step 3. Perform the convolution sum, which is the sum of the products of two sequences $x(k)$ and $h(n-k)$, to get $y(n)$.
Step 4. Repeat Steps 1 to 3 for the next convolution value $y(n)$.

EXAMPLE 3.11

Using the sequences defined in Figure 3.21, evaluate the digital convolution

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

a. By the graphical method.
b. By applying the formula directly.

Solution:

a. To obtain $y(0)$, we need the reversed sequence $h(-k)$; and to obtain $y(1)$, we need the reversed sequence $h(1-k)$, and so on. Using the technique we have discussed, sequences $h(-k)$, $h(-k+1)$, $h(-k+2)$, $h(-k+3)$, and $h(-k+4)$ are obtained and plotted in Figure 3.22.
Again, using the information in Figures 3.21 and 3.22, we can compute the convolution sum as

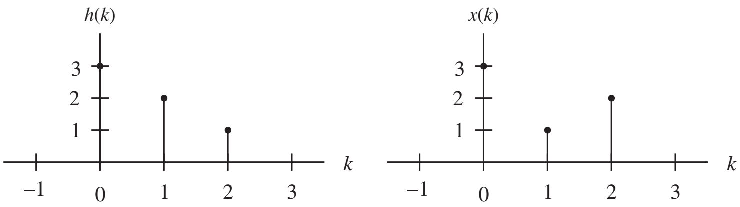

FIGURE 3.21

Plots of digital input sequence and impulse sequence in Example 3.11.

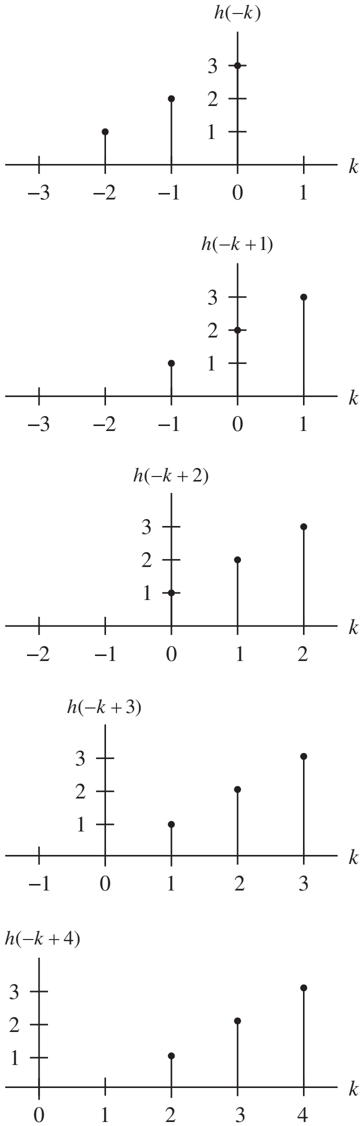

FIGURE 3.22

Illustration of convolution of two sequences $x(k)$ and $h(k)$ in Example 3.11.

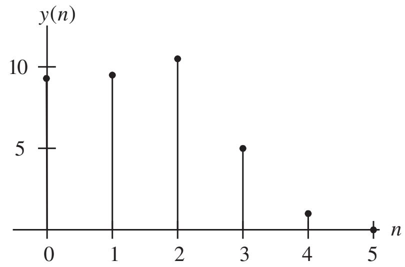

FIGURE 3.23

Plot of the convolution sum in Example 3.11.

sum of product of $x(k)$ and $h(-k)$: $y(0) = 3 \times 3 = 9$
sum of product of $x(k)$ and $h(1-k)$: $y(1) = 1 \times 3 + 3 \times 2 = 9$
sum of product of $x(k)$ and $h(2-k)$: $y(2) = 2 \times 3 + 1 \times 2 + 3 \times 1 = 11$
sum of product of $x(k)$ and $h(3-k)$: $y(3) = 2 \times 2 + 1 \times 1 = 5$
sum of product of $x(k)$ and $h(4-k)$: $y(4) = 2 \times 1 = 2$
sum of product of $x(k)$ and $h(5-k)$: $y(n) = 0$ for $n > 4$, since sequences $x(k)$ and $h(n-k)$ do not overlap.

Finally, we sketch the output sequence $y(n)$ in Figure 3.23.

b. Applying Equation (3.20) with zero initial conditions leads to

$$y(n) = x(0)h(n) + x(1)h(n-1) + x(2)h(n-2)$$
$$n = 0,\ y(0) = x(0)h(0) + x(1)h(-1) + x(2)h(-2) = 3 \times 3 + 1 \times 0 + 2 \times 0 = 9$$
$$n = 1,\ y(1) = x(0)h(1) + x(1)h(0) + x(2)h(-1) = 3 \times 2 + 1 \times 3 + 2 \times 0 = 9$$
$$n = 2,\ y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0) = 3 \times 1 + 1 \times 2 + 2 \times 3 = 11$$
$$n = 3,\ y(3) = x(0)h(3) + x(1)h(2) + x(2)h(1) = 3 \times 0 + 1 \times 1 + 2 \times 2 = 5$$
$$n = 4,\ y(4) = x(0)h(4) + x(1)h(3) + x(2)h(2) = 3 \times 0 + 1 \times 0 + 2 \times 1 = 2$$
$$n \geq 5,\ y(n) = x(0)h(n) + x(1)h(n-1) + x(2)h(n-2) = 3 \times 0 + 1 \times 0 + 2 \times 0 = 0$$

In simple cases such as this example, it is not necessary to use the graphical or formula methods. We can compute the convolution by treating the input sequence and impulse response as number sequences and sliding the reversed impulse response past the input sequence, cross-multiplying, and summing the nonzero overlap terms at each step. The procedure and calculated results are listed in Table 3.4.

Table 3.4 Convolution Sum Using the Table Method

k :	**−2**	**−1**	**0**	**1**	**2**	**3**	**4**	**5**	
$x(k)$:			**3**	**1**	**2**				
$h(-k)$:	1	2	3						$y(0) = 3 \times 3 = 9$
$h(1-k)$		1	2	3					$y(1) = 3 \times 2 + 1 \times 3 = 9$
$h(2-k)$			1	2	3				$y(2) = 3 \times 1 + 1 \times 2 + 2 \times 3 = 11$
$h(3-k)$				1	2	3			$y(3) = 1 \times 1 + 2 \times 2 = 5$
$h(4-k)$					1	2	3		$y(4) = 2 \times 1 = 2$
$h(5-k)$						1	2	3	$y(5) = 0$ (no overlap)

We can see that the calculated results using all the methods are consistent. The steps using the table method are summarized in Table 3.5.

Table 3.5 Digital Convolution Steps via the Table Method

Step 1. List the index k covering a sufficient range.
Step 2. List the input $x(k)$.
Step 3. Obtain the reversed sequence $h(-k)$, and align the rightmost element of $h(n-k)$ to the leftmost element of $x(k)$.
Step 4. Cross-multiply and sum the nonzero overlap terms to produce $y(n)$.
Step 5. Slide $h(n-k)$ to the right by one position.
Step 6. Repeat Step 4; stop if all the output values are zero or if required.

EXAMPLE 3.12

Convolve the following two rectangular sequences using the table method:

$$x(n) = \begin{cases} 1 & n = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad h(n) = \begin{cases} 0 & n = 0 \\ 1 & n = 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

Solution:
Using Table 3.5 as a guide, we list the operations and calculations in Table 3.6. Note that the output should show the trapezoidal shape.

Table 3.6 Convolution Sum in Example 3.12

k **:**	**−2**	**−1**	**0**	**1**	**2**	**3**	**4**	**5**	
$x(k)$:			**1**	**1**	1				
$h(-k)$:	1	1	0						$y(0) = 0$ (no overlap)
$h(1-k)$		1	1	0					$y(1) = 1 \times 1 = 1$
$h(2-k)$			1	1	0				$y(2) = 1 \times 1 + 1 \times 1 = 2$
$h(3-k)$				1	1	0			$y(3) = 1 \times 1 + 1 \times 1 = 2$
$h(4-k)$					1	1	0		$y(4) = 1 \times 1 = 1$
$h(n-k)$						1	1	0	$y(n) = 0,\ n \geq 5$ (no overlap) Stop

Let us examine convolving a finite long sequence with an infinite long sequence.

EXAMPLE 3.13

A system representation using the unit-impulse response for the linear system

$$y(n) = 0.25y(n-1) + x(n) \quad \text{for} \quad n \geq 0 \quad \text{and} \quad y(-1) = 0$$

was determined in Example 3.8 as

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

where $h(n) = (0.25)^n u(n)$. For a step input $x(n) = u(n)$, determine the output response for the first three output samples using the table method.

Solution:

Using Table 3.5 as a guide, we list the operations and calculations in Table 3.7. As expected, the output values are the same as those obtained in Example 3.8.

Table 3.7 Convolution Sum in Example 3.13.

k:	−2	−1	0	1	2	3	…	
$x(k)$:			**1**	**1**	**1**	1	…	
$h(-k)$:	0.0625	0.25	1					$y(0) = 1 \times 1 = 1$
$h(1-k)$		0.0625	0.25	1				$y(1) = 1 \times 0.25 + 1 \times 1 = 1.25$
$h(2-k)$			0.0625	0.25	1			$y(2) = 1 \times 0.0625 + 1 \times 0.25 + 1 \times 1 = 1.3125$
								Stop as required

3.6 SUMMARY

1. Digital signal samples are sketched using their encoded amplitude versus sample numbers with vertical bars topped by solid circles located at their sampling instants, respectively. The impulse sequence, unit-step sequence, and their shifted versions are sketched in this notation.
2. The analog signal function can be sampled to its digital (discrete-time) version by substituting time $t = nT$ into the analog function, that is,
   $x(n) = x(t)|_{t=nT} = x(nT)$
   The digital function values can be calculated for the given time index (sample number).
3. The DSP system we wish to design must be a linear, time-invariant, causal system. Linearity means that the superposition principle exists. Time invariance requires that the shifted input generate the corresponding shifted output in the same amount of time. Causality indicates that the system output depends on only its current input sample and past input sample(s).
4. The difference equation describing a linear, time-invariant system has a format such that the current output depends on the current input, past input(s), and past output(s) in general.

5. The unit-impulse response can be used to fully describe a linear, time-invariant system. Given the impulse response, the system output is the sum of the products of the impulse response coefficients and corresponding input samples, called the digital convolution sum.
6. BIBO is a type of stability in which a bounded input will produce a bounded output. A BIBO system requires that the sum of the absolute impulse response coefficients be a finite number.
7. The digital convolution sum, which represents a DSP system, is evaluated in three ways: the graphical method, evaluation of the formula, and the table method. The table method is found to be most effective.

3.7 PROBLEMS

3.1. Sketch each of the following special digital sequences:

a. $5\delta(n)$

b. $-2\delta(n-5)$

c. $-5u(n)$

d. $5u(n-2)$

3.2. Calculate the first eight sample values and sketch each of the following sequences:

a. $x(n) = 0.5^n u(n)$

b. $x(n) = 5\sin(0.2\pi n)u(n)$

c. $x(n) = 5\cos(0.1\pi n + 30^0)u(n)$

d. $x(n) = 5(0.75)^n \sin(0.1\pi n)u(n)$

3.3. Sketch each of the following special digital sequences:

a. $8\delta(n)$

b. $-3.5\delta(n-4)$

c. $4.5u(n)$

d. $-6u(n-3)$

3.4. Calculate the first eight sample values and sketch each of the following sequences:

a. $x(n) = 0.25^n u(n)$

b. $x(n) = 3\sin(0.4\pi n)u(n)$

c. $x(n) = 6\cos(0.2\pi n + 30^0)u(n)$

d. $x(n) = 4(0.5)^n \sin(0.1\pi n)u(n)$

3.5. Sketch the following sequences:

a. $x(n) = 3\delta(n+2) - 0.5\delta(n) + 5\delta(n-1) - 4\delta(n-5)$

b. $x(n) = \delta(n+1) - 2\delta(n-1) + 5u(n-4)$

3.6. Given the digital signals $x(n)$ in Figures 3.24 and 3.25, write an expression for each digital signal using the unit-impulse sequence and its shifted sequences.

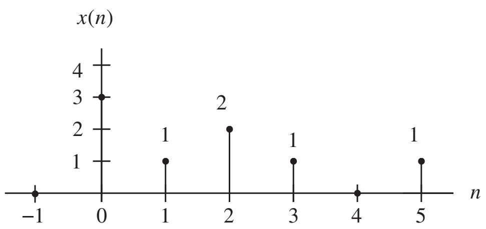

FIGURE 3.24

The first digital signal in Problem 3.6.

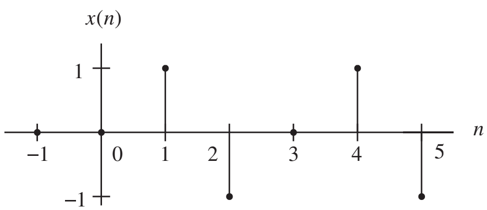

FIGURE 3.25

The second digital signal in Problem 3.6.

3.7. Sketch the following sequences:

a. $x(n) = 2\delta(n+3) - 0.5\delta(n+1) - 5\delta(n-2) - 4\delta(n-5)$

b. $x(n) = 2\delta(n+2) - 2\delta(n+1) + 5u(n-3)$

3.8. Given the digital signals $x(n)$ in Figures 3.26 and 3.27, write an expression for each digital signal using the unit-impulse sequence and its shifted sequences.

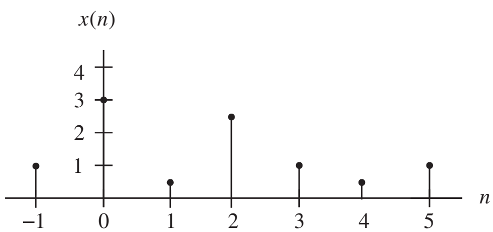

FIGURE 3.26

The first digital signal in Problem 3.8.

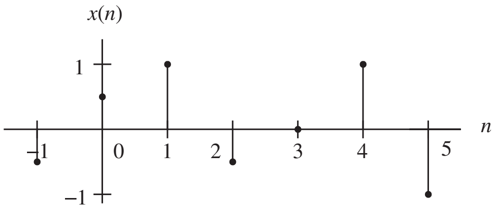

FIGURE 3.27

The second digital signal in Problem 3.8.

3.9. Assume that a DS processor with a sampling time interval of 0.01 second converts the following analog signals $x(t)$ to a digital signal $x(n)$; determine the digital sequence for each of the analog signals.

a. $x(t) = e^{-50t}u(t)$

b. $x(t) = 5\sin(20\pi t)u(t)$

c. $x(t) = 10\cos(40\pi t + 30^0)u(t)$

d. $x(t) = 10e^{-100t}\sin(15\pi t)u(t)$

3.10. Determine which of the following systems is a linear system.

a. $y(n) = 5x(n) + 2x^2(n)$

b. $y(n) = x(n-1) + 4x(n)$

c. $y(n) = 4x^3(n-1) - 2x(n)$

3.11. Assume that a DS processor with a sampling time interval of 0.005 second converts each of the following analog signals $x(t)$ to a digital signal $x(n)$; determine the digital sequence for each of the analog signals.

a. $x(t) = e^{-100t}u(t)$

b. $x(t) = 4\sin(60\pi t)u(t)$

c. $x(t) = 7.5\cos(20\pi t + 60^0)u(t)$

d. $x(t) = 20e^{-200t}\sin(60\pi t)u(t)$

3.12. Determine which of the following systems is a linear system.

a. $y(n) = 4x(n) + 8x^3(n)$

b. $y(n) = x(n-3) + 3x(n)$

c. $y(n) = 5x^2(n-1) - 3x(n)$

3.13. Determine which of the following linear systems is time invariant.

a. $y(n) = -5x(n-10)$

b. $y(n) = 4x(n^2)$

3.14. Determine which of the following linear systems is causal.

a. $y(n) = 0.5x(n) + 100x(n-2) - 20x(n-10)$

b. $y(n) = x(n+4) + 0.5x(n) - 2x(n-2)$

3.15. Determine the causality for each of the following linear systems.

a. $y(n) = 0.5x(n) + 20x(n-2) - 0.1y(n-1)$

b. $y(n) = x(n+2) - 0.4y(n-1)$

c. $y(n) = x(n-1) + 0.5y(n+2)$

3.16. Find the unit-impulse response for each of the following linear systems.

a. $y(n) = 0.5x(n) - 0.5x(n-2)$; for $n \geq 0$, $x(-2) = 0$, $x(-1) = 0$

b. $y(n) = 0.75y(n-1) + x(n)$; for $n \geq 0$, $y(-1) = 0$

c. $y(n) = -0.8y(n-1) + x(n-1)$; for $n \geq 0$, $x(-1) = 0$, $y(-1) = 0$

3.17. Determine the causality for each of the following linear systems.

a. $y(n) = 5x(n) + 10x(n-4) - 0.1y(n-5)$

b. $y(n) = 2x(n+2) - 0.2y(n-2)$

c. $y(n) = 0.1x(n+1) + 0.5y(n+2)$

3.18. Find the unit-impulse response for each of the following linear systems.

a. $y(n) = 0.2x(n) - 0.3x(n-2)$; for $n \geq 0$, $x(-2) = 0$, $x(-1) = 0$

b. $y(n) = 0.5y(n-1) + 0.5x(n)$; for $n \geq 0$, $y(-1) = 0$

c. $y(n) = -0.6y(n-1) - x(n-1)$; for $n \geq 0$, $x(-1) = 0$, $y(-1) = 0$

3.19. For each of the following linear systems, find the unit-impulse response, and draw the block diagram.

a. $y(n) = 5x(n-10)$

b. $y(n) = x(n) + 0.5x(n-1)$

3.20. Determine the stability of the following linear system.

$$y(n) = 0.5x(n) + 100x(n-2) - 20x(n-10)$$

3.21. For each of the following linear systems, find the unit-impulse response, and draw the block diagram.

a. $y(n) = 2.5x(n-5)$

b. $y(n) = 2x(n) + 1.2x(n-1)$

3.22. Determine the stability for the following linear system.

$$y(n) = 5x(n) + 30x(n-3) - 10x(n-20)$$

3.23. Determine the stability for each of the following linear systems.

a. $y(n) = \sum_{k=0}^{\infty} 0.75^k x(n-k)$

b. $y(n) = \sum_{k=0}^{\infty} 2^k x(n-k)$

3.24. Determine the stability for each of the following linear systems.

a. $y(n) = \sum_{k=0}^{\infty}(-1.5)^k x(n-k)$

b. $y(n) = \sum_{k=0}^{\infty}(-0.5)^k x(n-k)$

3.25. Given the sequence

$$h(k) = \begin{cases} 2, & k = 0, 1, 2 \\ 1, & k = 3, 4 \\ 0 & elsewhere, \end{cases}$$

where k is the time index or sample number,

a. sketch the sequence $h(k)$ and the reverse sequence $h(-k)$;

b. sketch the shifted sequences $h(-k+2)$ and $h(-k-3)$.

3.26. Given the sequence

$$h(k) = \begin{cases} -1 & k = 0, 1 \\ 2 & k = 2, 3 \\ -2 & k = 4 \\ 0 & elsewhere \end{cases}$$

where k is the time index or sample number,

a. sketch the sequence $h(k)$ and the reverse sequence $h(-k)$;

b. sketch the shifted sequences $h(-k+1)$ and $h(-k-2)$.

3.27. Using the sequence definitions

$$h(k) = \begin{cases} 2, & k = 0, 1, 2 \\ 1, & k = 3, 4 \\ 0 & elsewhere \end{cases} \quad and\ x(k) = \begin{cases} 2, & k = 0 \\ 1, & k = 1, 2 \\ 0 & elsewhere \end{cases}$$

evaluate the digital convolution

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

a. using the graphical method;

b. using the table method;

c. applying the convolution formula directly.

3.28. Using the sequence definitions

$$x(k) = \begin{cases} -2, & k = 0, 1, 2 \\ 1, & k = 3, 4 \\ 0 & elsewhere \end{cases} \quad and\ h(k) = \begin{cases} 2, & k = 0 \\ -1, & k = 1, 2 \\ 0 & elsewhere \end{cases}$$

evaluate the digital convolution

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

a. using the graphical method;

b. using the table method;

c. applying the convolution formula directly.

3.29. Convolve the two rectangular sequences

$$x(n) = \begin{cases} 1 & n = 0, 1 \\ 0 & \textit{otherwise} \end{cases} \quad \textit{and } h(n) = \begin{cases} 0 & n = 0 \\ 1 & n = 1, 2 \\ 0 & \textit{otherwise} \end{cases}$$

using the table method.