Artificial Vision

Course Summary - Master's Degree

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Abstract

This document contains a summary of the Artificial Vision course syllabus for the Master's Degree. It includes a summary of the main topics covered during the sessions, as well as additional explanations and extensions of the concepts and techniques referenced in class. The purpose of this document is to serve as study material and reference for the course contents.

Contents

1 The Nyquist-Shannon Sampling Theorem

The Nyquist-Shannon sampling theorem, also known as the sampling theorem, is a fundamental principle in signal processing and digital image processing. It establishes the conditions under which a continuous signal can be perfectly reconstructed from its discrete samples.

1.1 Statement of the Theorem

Theorem 1.1 (Nyquist-Shannon Sampling Theorem)**.** If a function $x(t)$ contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced $\frac{1}{2B}$ seconds apart. In other words, a band-limited signal can be perfectly reconstructed from its samples if the sampling frequency f_s satisfies:

$$f_s \geq 2f_{\max} \tag{1}$$

where $f_{\max}$ is the highest frequency component in the signal. The frequency $f_N = \frac{f_s}{2}$ is called the **Nyquist frequency**, and $2f_{\max}$ is called the **Nyquist rate**.

1.2 Understanding Sampling in the Time Domain

To understand the theorem, consider a continuous signal $x(t)$ that we wish to sample at regular intervals. The sampling process can be visualized as multiplying the continuous signal by a train of Dirac delta functions:

$$x_s(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \tag{2}$$

where $T_s = \frac{1}{f_s}$ is the sampling period.
Figure 1 illustrates three different sampling scenarios:

- **Adequate sampling** ($f_s > 2f_{\max}$): The signal can be perfectly reconstructed.
- **Nyquist rate sampling** ($f_s = 2f_{\max}$): The minimum sampling rate that theoretically allows perfect reconstruction.
- **Insufficient sampling** ($f_s < 2f_{\max}$): Aliasing occurs, and the original signal cannot be recovered.

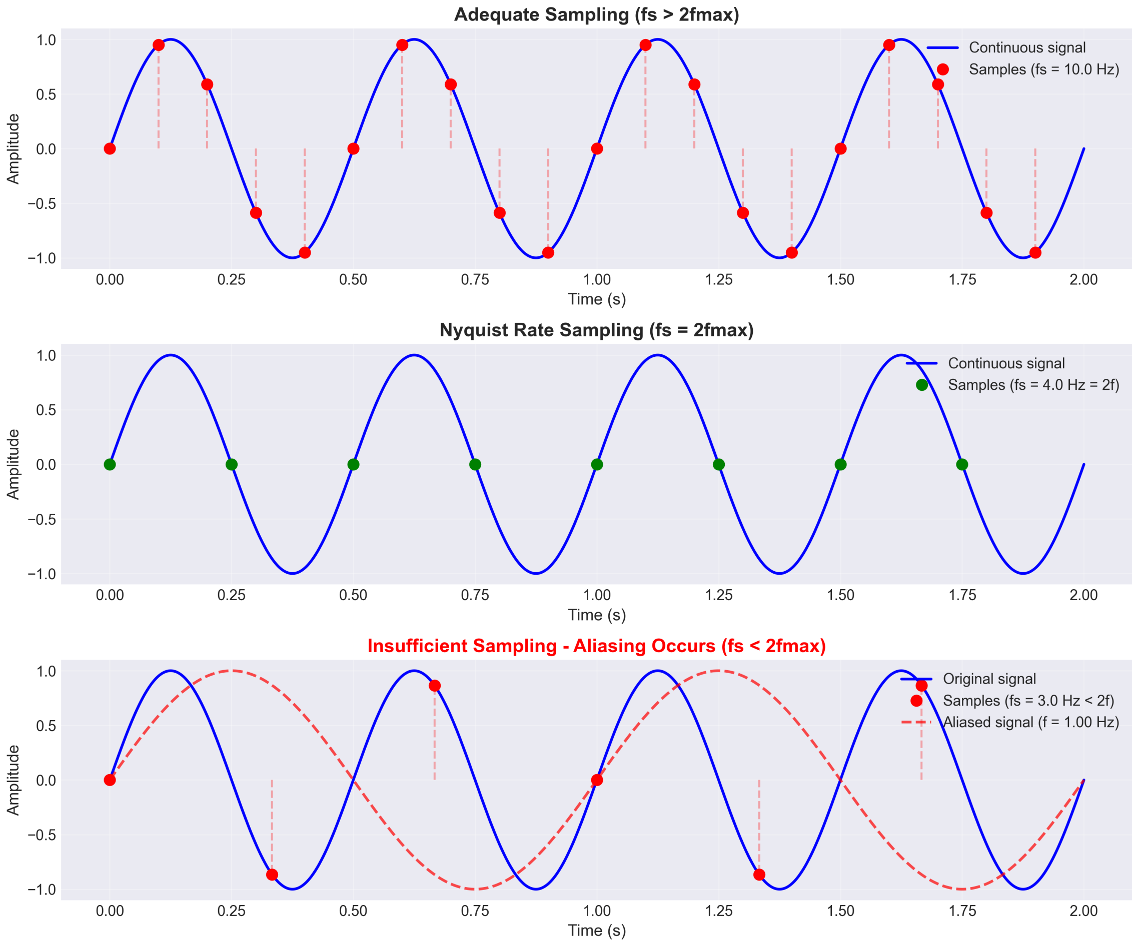

Figure 1: Comparison of different sampling rates: adequate sampling (top), Nyquist rate (middle), and insufficient sampling leading to aliasing (bottom).

1.3 Frequency Domain Interpretation

In the frequency domain, sampling creates periodic replicas of the original spectrum. The Fourier transform of the sampled signal $X_s(f)$ consists of copies of the original spectrum $X(f)$ shifted by integer multiples of the sampling frequency:

$$X_s(f) = f_s \sum_{k=-\infty}^{\infty} X(f - kf_s) \tag{3}$$

For perfect reconstruction, these replicas must not overlap. This requires that the sampling frequency satisfies the Nyquist criterion. Figure 2 shows how sampling affects the frequency spectrum:

- When $f_s > 2f_{\max}$, the replicas are separated and can be filtered out.
- When $f_s < 2f_{\max}$, the replicas overlap, causing **aliasing**—high frequencies appear as lower frequencies in the sampled signal.

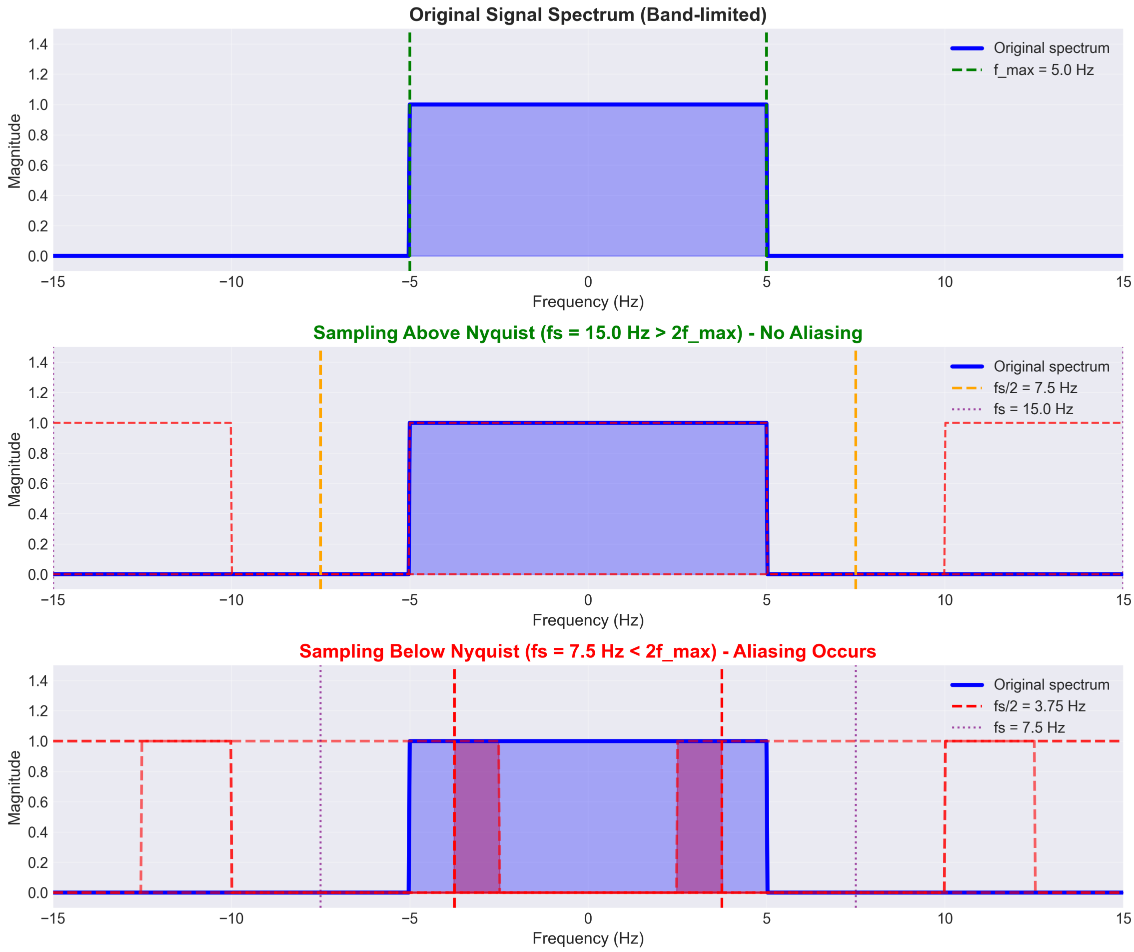

Figure 2: Frequency domain representation: original spectrum (top), sampling above Nyquist rate with separated replicas (middle), and sampling below Nyquist rate with overlapping replicas causing aliasing (bottom).

1.4 The Nyquist Criterion

The relationship between the signal's maximum frequency and the required sampling frequency can be visualized as shown in Figure 3. The region above the line $f_s = 2f_{\max}$ represents safe sampling conditions where no aliasing occurs.

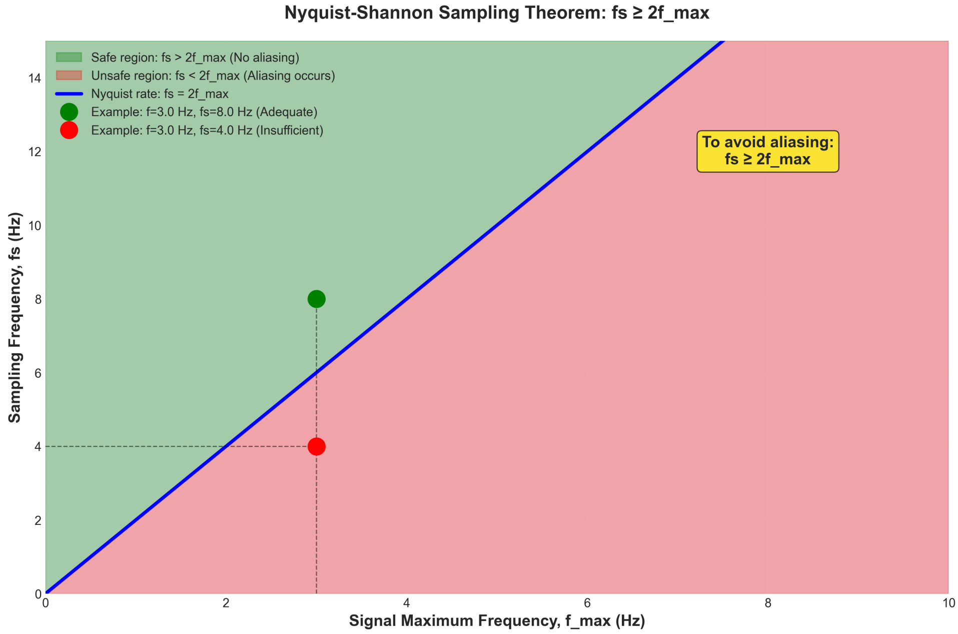

Figure 3: Visual representation of the Nyquist criterion. The green region (above the blue line) represents safe sampling conditions, while the red region (below the line) leads to aliasing.

1.5 Applications in Computer Vision

The Nyquist-Shannon theorem has critical implications in computer vision:

- **Image acquisition**: Digital cameras must sample at a rate sufficient to capture the highest spatial frequencies in the scene. Insufficient sampling leads to aliasing artifacts such as moiré patterns.
- **Image resampling**: When resizing or rotating images, care must be taken to avoid aliasing by proper filtering before downsampling.
- **Feature detection**: Understanding sampling theory helps in designing filters and feature detectors that work correctly with discrete image data.

1.6 Anti-aliasing

To prevent aliasing when sampling, an **anti-aliasing filter** (low-pass filter) is typically applied before sampling to remove frequency components above the Nyquist frequency. This ensures that the signal is band-limited and satisfies the conditions of the sampling theorem.

In practice, perfect reconstruction requires:

1. Band-limiting the signal to frequencies below $f_N = \frac{f_s}{2}$ using an anti-aliasing filter.
2. Sampling at a rate $f_s \geq 2f_{\text{max}}$.
3. Reconstructing using an ideal low-pass filter (sinc interpolation) or appropriate interpolation methods.