Flow Study in Relativistic Nuclear Collisions by Fourier Expansion of Azimuthal Particle Distributions

S. Voloshin^{1,*}, Y. Zhang²

¹ University of Pittsburgh, Pittsburgh, PA 15260

² State University of New York, Stony Brook, NY 11794

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We propose a new method to study transverse flow effects in relativistic nuclear collisions by Fourier analysis of the azimuthal distribution on an eventby-event basis in relatively narrow rapidity windows. The distributions of Fourier coefficients provide direct information on the magnitude and type of flow. Directivity and two dimensional sphericity tensor, widely used to analyze flow, emerge naturally in our approach, since they correspond to the distributions of the first and second harmonic coefficients, respectively. The role of finite particle fluctuations and particle correlations is discussed.

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I. INTRODUCTION

In the study of specific processes such as flow phenomena, it is very important to find variables which are both easy to work with and have clear physical interpretations. Many different methods were proposed for the study of flow effects in relativistic nuclear collisions, of which the most commonly used are directivity [1] and sphericity (three dimensional [2]) or two dimensional [3]) tensor. In the current paper we propose to use a more general method, which includes as a part both directivity and two dimensional sphericity methods in a natural way and gives a clear physical meaning to the analysis. Moreover, it provides ways to investigate complicated event shapes, which may not be described by an ellipsoid through three dimensional sphericity analysis.

There exist two different approaches in flow analysis. The first one is to fit p_t and/or

^{*}On leave from Moscow Engineering Physics Institute, Moscow, 115409, Russia

dN/dy distributions of different particles by assuming local thermal equilibrium and hydrodynamical expansion. In this case some information about initial conditions, equation-of-state, expansion dynamics, freeze-out temperature can be extracted. (For recent analysis of such type of CERN SPS and BNL AGS data see [4,5]). In spite of sometimes very good description of experimental data this approach has essential plague: it is very model dependent. Just the very existing of thermal equilibrium is still not well established. The alternative approach is to study the azimuthal event shapes (which are sensitive to flow) from experimental data without any model in mind. All the methods mentioned in the first paragraph including the method we are going to discuss in this paper are falling into this category. The information provided in this way can be used for comparison with different models: hydro-dynamical, statistical, cascade, multi-string model, etc. The study of a possible azimuthal anisotropy of the multiparticle production in high energy nucleus-nucleus collisions just has begun, and first indications of it have been seen at AGS [6], where the anisotropy of particle production in one region was noted to correlate with the reaction plane angle defined in another region.

In our approach we investigate the shapes of azimuthal distributions, and try to detect and study of anisotropy of different kinds. The origin of the anisotropy could be different: hydro-dynamical flow due to pressure gradients, shadowing, both, etc. What they have in common is some collective behavior in the evolution of multiparticle production process. Consequently, we often use the term transverse collective flow for the phenomena we study, although we do not necessarily mean the hydro-dynamical flow. Our method is most useful for detecting an anisotropic collective flow. Though it can be used for a study of radial flow as well (see the short discussion in the Section II) along with other methods such as an analysis of p_t spectra. In order to detect complex three dimensional event shapes, we slice longitudinal variables, e.g., (pseudo)-rapidity, into different windows, and perform Fourier analysis in each window. Fourier coefficients of different harmonics reflect the different type of anisotropy (transverse collective flow). A three dimensional event shape can be obtained by correlating and combining the Fourier coefficients in different longitudinal windows. Some ideas of this approach have been employed to analyze Au on Au data of E877 collaboration at BNL AGS, when for the first time in this energy range transverse flow signals have been

observed [7,8].

We present the Fourier expansion method in Section II. We study the fluctuations due to finite particle multiplicity in section III. In section IV, we address correlation between different windows and different harmonics. Finally, relations to other existing methods are discussed in section V.

II. FOURIER EXPANSION OF AZIMUTHAL DISTRIBUTIONS

The particle azimuthal distributions can be constructed from different quantities such as transverse momentum, multiplicity, or transverse energy in relatively narrow (pseudo)rapidity windows. To get more detail of the event shapes one would like to divide the whole (pseudo)-rapidity acceptance into more windows. But the limitation is that there should be sufficient number of particles in each window. We will discuss this problem in next section. If needed, it is possible to introduce different weights for different particles, as is often done in the analysis of flow phenomena at lower energies. It is also possible to study specific functions of p_t , multiplicity, etc; for example, if one wants to enhance the contribution from particles with high p_t , one can study $p_t^2(\phi)$ instead of $p_t(\phi)$, or select particles with p_t larger than certain value. In this way the method can also be used for a study of radial flow. To study flow effects in future RHIC experiments, where jet production can be important and will be a background to flow study, one can chose the range $p_t < c\langle p_t^{jet} \rangle$, where constant c can be chosen from experiments. Or one can do just the opposite to study anisotropy due to jets. With particle identification, one can get the azimuthal function for certain type of particles, e.g., analyze protons and pions separately. In the following to be specific we generally discuss an azimuthal distribution of transverse momentum in certain rapidity window, but the procedure for other variables remains the same.

We denote the azimuthal distribution function of the quantity under study with $r(\phi)$, i.e., $dp_T(\phi)/d\phi$, where $p_T(\phi)$ is the total transverse momentum of particles emitted at azimuthal angle ϕ . The function $r(\phi)$ can be constructed from experimental data event by event and be written in the form of Fourier expansion since $r(\phi)$ is a periodical function:

$$r(\phi) = \frac{x_0}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} [x_n \cos(n\phi) + y_n \sin(n\phi)].$$
 (1)

The coefficients in the Fourier expansion of $r(\phi)$ are integrals of $r(\phi)$ with weights proportional to $\cos(n\phi)$ or $\sin(n\phi)$. For the case of a finite number of particles, the integrals become simple sums over particles found in the appropriate rapidity window:

$$x_n = \int_0^{2\pi} r(\phi) \cos(n\phi) d\phi = \sum_{\nu} r_{\nu} \cos(n\phi_{\nu}), \tag{2}$$

$$y_n = \int_0^{2\pi} r(\phi) \sin(n\phi) d\phi = \sum_{\nu} r_{\nu} \sin(n\phi_{\nu}),$$
 (3)

where ν runs over all particles, and ϕ_{ν} is the azimuthal angle of the ν -th particle. Without any flow and neglecting fluctuations (we study the fluctuations in Section III) the function $r(\phi)$ is constant, $r(\phi) = x_0/(2\pi) = 1/(2\pi) \sum_{\nu} r_{\nu}$. All Fourier coefficients except a_0 are zero.

The appearance of transverse anisotropic flow in nuclear collision is expected from a non-zero value of an impact parameter. We define the longitudinal or beam direction as z-axis, and the transverse plane as x-y plane. The impact parameter vector in x-y plane (from the center of the target nucleus to the center of the beam nucleus) together with z-axis defines the reaction plane, and the reaction plane angle ψ_r ($0 \le \psi_r \le 2\pi$) is the angle between x-axis and reaction plane. Each non-zero pair of the Fourier coefficients x_n and y_n give the value of non-zero component of the corresponding harmonic, defined as $v_n = \sqrt{x_n^2 + y_n^2}$, and the angle ψ_n ($0 \le \psi_n < 2\pi/n$) of the "n-th type" flow.

$$x_n = v_n \cos(n\psi_n),\tag{4}$$

$$y_n = v_n \sin(n\psi_n). \tag{5}$$

If flow exists, $r(\phi)$ is no longer a constant, and the shape of the distribution is no longer a circle centered at zero (see Fig. 1). Then the first harmonic coefficients correspond to an overall shift of the distribution in the transverse plane; such flow we call directed flow. The magnitude of flow is v_1 . With $r(\phi)$ being the transverse momentum distribution, coefficients x_1 and y_1 are x and y components of the total transverse momentum of the particles produced in the window, while v_1 is the magnitude of the total vector sum of transverse momenta. In the case of negligible fluctuations the direction of flow due to symmetry is to coincide with the reaction plane angle $\psi_1 = \psi_r$ (repulsive flow), or point in the opposite direction $\psi_1 = \psi_r + \pi$ (attractive flow).

The non-zero second harmonic describes the eccentricity of an ellipse-like distribution. If one approximates the distribution by an ellipse as shown in Fig. 1, then the second coefficient v_2 carries information on the magnitude of the eccentricity. Quantitatively it is the difference between the major and the minor axis. The orientation of major axis ψ_2 (or, $\psi_2 + \pi$, which give the same orientation for an ellipse) can be only ψ_r or $\psi_r + \pi/2$. In the case $\psi_2 = \psi_r$, the major axis lies within the reaction plane; while $\psi_2 = \psi_r + \pi/2$ indicates that the orientation of the ellipse is perpendicular to the reaction plane, which is the case for squeeze-out flow and may be expected in mid-rapidity window.

Coefficient v_3 could be non-zero for asymmetric nuclear collisions and represent the asymmetry of the flow due to different sizes of colliding nuclei, if the distribution can be approximately described as triangle-type distribution. In this case the symmetry dictates that the flow angle ψ_3 (or one of $\psi_3 + 2n\pi/3$, n = 1, 2) gets value ψ_r or $\psi_r + \pi/3$.

Forth – a rectangle-type deformation amplitude v_4 could be non-zero for rapidity windows close to the center of mass, where both squeeze-out, emitting preferentially more particles in the direction perpendicular to the reaction plane, and "side-splash" effects can be important. The flow angle ψ_4 would be ψ_r if the origin of the rectangle-type flow is the case as discussed above. But $\psi_4 = \psi_r + \pi/4$ can not be excluded from the naive symmetric reaction plane argument.

We see that Fourier coefficients corresponding to different harmonics have a clear physical meaning. Perhaps the most interesting and unique feature of this approach would be to correlate flow effects between different types. But before trying to do that, we have to answer the question of how to deal with the finite multiplicity fluctuations, which have always been a big obstacle in flow analysis. Because it is difficult to resolve the reaction plane angle event by event due to fluctuations, all methods based on the determination of the reaction plane on an event-by-event basis remains rather ambiguous. It becomes very important to detect flow without using the information on the exact position of the reaction plane in each event. When consider the fluctuations, the value of v_n becomes a distribution instead of a single number for events of the same characterization. Below we show that the distribution in v_n is sensitive to anisotropic flow and can be used for flow detection. Information about the reaction plane position could be used if one considers the relative

angular distance between the reaction plane and some other plane, also defined in the same event. For analysis of this kind we propose to study the correlations between reactions plane angles defined in different windows, or between reaction plane angles derived, possibly, from the same window but using different harmonics.

III. FINITE MULTIPLICITY FLUCTUATIONS

In this section we discuss problems arising in the analysis due to finite multiplicity fluctuations. Since two particle correlations are weak [9], below we treat all particles in the same event as being totally independent following certain distribution with variance σ_0^2 . If two particle correlations are non-negligible we expect larger fluctuations, but the general procedure remains valid. Although we study finite multiplicity fluctuations, we consider the case when the total number of particles in selected window is relatively large $(N \gg 1)$. Under these assumptions, in the absence of flow the probability distribution of vectors (v_1, ψ_1) is Gaussian from the central limit theorem:

$$\frac{d^2w}{dv_1d\psi_1} = \frac{1}{2\pi\sigma^2} \exp(-\frac{v_1^2}{2\sigma^2}),\tag{6}$$

with the peak of the distribution centered at the center of the coordinates. The variance of the distribution is:

$$\sigma^2 = \sigma_x^2 = \sigma_y^2 = N \langle r_i^2 \rangle \langle \cos^2(\phi_i) \rangle = \frac{N \langle r_i^2 \rangle}{2} \equiv N \sigma_0^2, \tag{7}$$

where N is the number of particles in the window under study.

For simplicity, we begin our study with flow of the first type, namely directed ("side-splash") flow. With the appearance of flow, each event can be characterized by the magnitude of flow and the flow angle. For events with the same parameters of flow the center of the distribution (6) will be shifted to the point characterized by (\tilde{v}_1, ψ_r) in the polar coordinate system. It is reasonable to assume that in shifted distributions the statistical fluctuations (7) are the same. We have chosen here \tilde{v}_1 as the parameter relevant to the magnitude of flow, which is directly related to the impact parameter of the collision. On the other hand, it is known from Monte Carlo studies that the impact parameter is strongly correlated with global variables used to measure the centrality of events (transverse energy,

multiplicity, zero degree energy, etc.). Using these correlations one can select events with the same magnitude of flow experimentally. Below we assume that the distribution in \tilde{v}_1 is narrow and can be treated as a constant for events within the same centrality bins.

The distribution in the flow angle ψ_1 should be flat. That can be used experimentally to justify the equalization and correction applied to detectors. We can integrate over $\psi'_1 = \psi_1 - \psi_r$, which gives:

$$\frac{dw}{v_1 dv_1} = \frac{1}{2\pi\sigma^2} \int_0^{2\pi} d\psi_1' \exp(-\frac{\tilde{v}_1^2 + v_1^2 - 2\tilde{v}_1 v_1 \cos(\psi_1')}{2\sigma^2}) = \frac{1}{\sigma^2} \exp(-\frac{\tilde{v}_1^2 + v_1^2}{2\sigma^2}) I_0(\frac{\tilde{v}_1 v_1}{\sigma^2}), \quad (8)$$

where I_0 is the modified Bessel function. The shape of the distribution (8) for a few values of parameters are shown in Fig. 2, where we plot dw/xdx using the notation $x \equiv v_1/\sigma$ and $\tilde{x} \equiv \tilde{v}_1/\sigma$. The extrema are defined by the solution of the equation

$$\tilde{v}_1 I_1(v_1 \tilde{v}_1 / \sigma^2) - v_1 I_0(v_1 \tilde{v}_1 / \sigma^2) = 0.$$
(9)

It follows from Eq. 9 that the distribution (Eq. 8) has a local minimum at $v_1 = 0$ if $\tilde{v}_1/\sigma > \sqrt{2}$. Thus the observation of such a minimum in the data is a sufficient condition for the appearance of flow. But such a minimum is not required for the extraction of \tilde{v}_1 and σ by fitting Eq. (8) to data.

It will be shown in Section V C that in the case of the analysis of azimuthal p_t distributions \tilde{v}_1 equals $N\langle p_x\rangle$, where $\langle p_x\rangle$ is the mean transverse momentum gained by a particle due to transverse flow (x-z) plane is the reaction plane here). In this case

$$\frac{\tilde{v}_1}{\sigma} = \frac{N\langle p_x \rangle}{\sqrt{N\langle p_t^2 \rangle/2}} = \frac{\langle p_x \rangle \sqrt{2N}}{\sqrt{\langle p_t^2 \rangle}}.$$
 (10)

We see from Eq.(10) that, one has to slice the rapidity windows experimentally to accept reasonably large multiplicity (N) in each window to make sure that \tilde{v}_1/σ is not too small, and an available statistics allows to perform a reliable fit. The "relatively narrow" rapidity windows discussed in the text are precisely chosen this way.

Applying the same consideration to the distribution of the coefficients corresponding to higher harmonics, one can see that they should have the same behavior (Eqs. 7–8) but with their own parameters \tilde{v}_i . Note, that because of $\langle \cos^2(\phi) \rangle = \langle \cos^2(n\phi) \rangle$ one has the same values of σ for all harmonics.

The functional form (8), which allows to have a local minimum of the distribution at zero, was proposed almost a decade ago for the distribution of flow polar angles extracted from the three dimensional sphericity matrix [10]. For the two dimensional transverse sphericity matrix a comprehensive study was done in Ref. [3,11]. Here we want to emphasize that the same functional behavior with the same parameter responsible for finite multiplicity fluctuations is valid for the distribution of Fourier coefficients of any harmonic order. Therefore one would expect more stable and sensible results fitting simultaneously all $dw/v_n dv_n$ distributions with free parameters \tilde{v}_n (n=1,2,3,4,...) and the same parameter σ .

In order to be able to compare the magnitudes of flow in different rapidity windows or centrality bins, or to compare different experiments, it is useful to consider the values of \tilde{v}_n normalized to $\tilde{v}_0 = \langle x_0 \rangle = N \langle p_t \rangle$, that is \tilde{v}_n/\tilde{v}_0 . The ratio \tilde{v}_1/\tilde{v}_0 has a clear physical meaning as the ratio of transverse momentum gained by a particle due to transverse flow to the mean transverse momentum (x-z plane being the reaction plane):

$$\frac{\tilde{v}_1}{\tilde{v}_0} = \frac{\langle p_x \rangle}{\langle p_t \rangle}.\tag{11}$$

IV. CORRELATIONS

The parameter \tilde{v}_n and σ can be extracted from experimental data by fitting the distributions $dw/v_n dv_n$ by the functional form of Eq. 8. The non-zero \tilde{v}_n reflects the existence and magnitude of flow of "n-th type". It is interesting to study whether the detected flow in one rapidity window is correlated with flow in a different rapidity window, or whether the flow of one kind of particles is correlated with the flow of another particle type. The study of some of such kind of correlations was proposed as proof of an existence of flow in Ref. [1] (where the authors discuss in our notation the correlations of ψ_1 from different rapidity windows) and in Ref. [11] (correlations of ψ_2). In our more general approach along with these correlations we can study very interesting correlations between flow of different types defining the overall flow picture. In general, we can study the correlations between any rapidity windows and any harmonics, $(v'_m, m\psi'_m)$ and $(v_n, n\psi_n)$. Denoting the relative angle by $\phi = \psi'_m - \psi_n$ and averaging over the reaction plane angle we have the distribution:

$$\frac{d^3w}{dv_m'^2 dv_n^2 d\phi} = \int \frac{d^4w}{dv_m'^2 dv_n'^2 d\psi_m' d\psi_n} d\psi_m' d\psi_n \, \delta(\phi - \psi_m' + \psi_n) =$$

$$= \frac{1}{4(2\pi\sigma^2)^2} \exp\left(-\frac{v_m'^2 + \tilde{v}_m'^2}{2\sigma^2} - \frac{v_n^2 + \tilde{v}_n^2}{2\sigma^2}\right) \int_0^{2\pi} d\psi \, \exp\left(+\frac{v_m'\tilde{v}_m'}{\sigma^2}\cos(\psi) + \frac{v_n\tilde{v}_n}{\sigma^2}\cos(\phi + \psi)\right), \tag{12}$$

The observation of correlation expressed in Eq. 12 could be very helpful for understanding the nature of flow and get a global picture of three dimensional event shapes. The correlations between flow of different types in the same rapidity window can be biased by auto-correlations due to finite multiplicity. To avoid the problem one can study the correlations using as a reference a flow angle defined in another rapidity window. The other way to suppress these pseudo-correlations could be to calculate higher harmonic term after removal of lower harmonic terms event-by-event, e.g., to perform a shift of the center of particle distribution to the point (v_1, ψ_1) and then calculate $(v_2, 2\psi_2)$.

The correlation between different rapidity windows can be numerically calculated using the fit parameters to the distributions (8). In this case a comparison with experimental data will be a check of the consistency of the method. Note that in spite of rather complicated form (12), the real calculation can be easily done numerically. Some analytical calculations for specific parameters can be done using the technique presented in the Appendix to Ref. [11].

V. RELATION TO OTHER METHODS

A. Directivity and sphericity

The vector \mathbf{Q} , which is often referred as directivity, was originally introduced in the flow analysis by Danielewicz and Odyniec [1]:

$$\mathbf{Q} = \sum_{\nu=1}^{N} \omega_{\nu} \mathbf{p}_{t;\nu},\tag{13}$$

where ν is particle index and ω_{ν} is a weight. It coincides with the vector $\mathbf{v}_1 \equiv (v_1, \psi_1)$ in the Fourier analysis by taking $\omega_{\nu} = 1$. One of the results of Ref. [1] concerning the width of the distribution in \mathbf{Q} is directly related to the modification of the original Gaussian distribution due to flow, which we discussed above.

The relation of the second harmonic coefficients to the usual parameterization of two dimensional sphericity tensor is also straightforward. Two dimensional transverse sphericity tensor, which is relevant to our study, has three independent parameters, and usually is written in the form:

$$S_{ij} \equiv \sum_{\nu=1}^{N} \omega_{\nu} p_{i;\nu} p_{j;\nu} = \frac{S}{2} \begin{pmatrix} 1 + \epsilon \cos 2\theta & \epsilon \sin 2\theta \\ \epsilon \sin 2\theta & 1 - \epsilon \cos 2\theta \end{pmatrix}, \tag{14}$$

where i, j = x, y. If we chose $\omega_{\nu} = r_{\nu}/(p_{t;\nu})^2$ the sphericity tensor becomes

$$S_{ij} = \sum_{\nu=1}^{N} r_{\nu} \cos \phi_{\nu,i} \cos \phi_{\nu,j}, \tag{15}$$

where $\phi_{\nu,i}$ is an angle between the momentum of ν -th particle and i-th axis. In this case $S = \text{tr}S_{ij} = \sum_{\nu} r_{\nu} = v_0$. By computing the eigenvalues one can derive that the eccentricity $\epsilon = v_2/v_0$ and thus is directly connected to the magnitude of the second harmonic coefficient in Fourier analysis.

B. Reaction plane determination

Experimentally, to study the flow in the reaction plane (so called side-splash flow), traditionally a directivity (an angle ψ_1) is calculated from the whole available acceptance to define the reaction plane. To define the accuracy of the procedure one should consider the distribution in an angle ψ which is defined as $\psi = \psi_r - \psi_1$:

$$\frac{dw}{d\psi} = \int \frac{vdv}{2\pi\sigma^2} \exp\left(-\frac{\tilde{v}_1^2 + v_1^2 - 2\tilde{v}_1 v_1 \cos\psi}{2\sigma^2}\right) =
= \frac{1}{2\pi} \left[\exp\left(-\frac{\tilde{v}_1^2}{2\sigma^2}\right) + \frac{\tilde{v}_1 \cos\psi\sqrt{\pi}}{\sigma\sqrt{2}} \exp\left(-\frac{\tilde{v}_1^2 \sin^2\psi}{2\sigma^2}\right) (1 + \exp\left(\frac{\tilde{v}_1 \cos\psi}{\sigma\sqrt{2}}\right))\right].$$
(16)

As long as the parameters of flow are determined by fitting the experimental distribution to the form (8), Eq.(16) can be used to answer the question of how close the angle ψ_1 is to the true reaction plane angle ψ_r . It can be generalized to n-th type flow angle dispersion by replacing v_1 with v_n , and ψ_1 with $n\psi_n$.

C. Mean p_x

Any transverse flow would influence the p_t distribution of observed particles. Theoretical calculations [12] usually present a mean projection of the particle transverse momentum into the reaction plane $\langle p_x \rangle$ as a function of rapidity (it is assumed here that x-z plane is the reaction plane). The projection can be non-zero due to transverse flow. Generally, as discussed above, the directivity is used to define the reaction plane, and then in different rapidity bins transverse momenta are projected into this plane. (One should be careful here to avoid autocorrelation. For details see Ref. [1].) Below we use the notation $p_{x'}$ for such a projection instead of p_x to note that it is not the projection into true reaction plane. Due to the fluctuations in ψ_1 around the true reaction plane angle $\langle p_{x'} \rangle$ does not equal $\langle p_x \rangle$. In our approach, we get our fitting parameter \tilde{v}_1 without determining of the reaction plane on an event-by-event basis. Taking into account that $\mathbf{v}_1 = \sum_{\nu} \mathbf{p}_{t;\nu}$, one can see that \tilde{v}_1 is exactly the true $\langle p_x \rangle$, multiplied by the total number of particles in the rapidity region.

$$\langle p_x \rangle N = \frac{1}{2\pi\sigma^2} \int d\mathbf{v}_1 v_1 \cos(\widehat{\mathbf{v}_1, \tilde{\mathbf{v}}_1}) \exp(-\frac{(\mathbf{v}_1 - \tilde{\mathbf{v}}_1)^2}{2\sigma^2}) = \tilde{v}_1.$$
 (17)

The relation between $\langle p_x \rangle$ and $\langle p_{x'} \rangle$ is very simple [1] $\langle p_x \rangle = \langle p_{x'} \rangle \langle \cos(\psi_{xx'}) \rangle$, where $\psi_{xx'}$ is the difference between true and defined reaction plane angles $\psi_{xx'} = \psi_r - \psi_1$. In our approach

$$\langle \cos(\psi_{xx'}) \rangle = \frac{1}{2\pi\sigma^2} \int d\mathbf{v}_1 \cos(\widehat{\mathbf{v}_1}, \widehat{\mathbf{v}}_1) \exp(-\frac{(\mathbf{v}_1 - \widehat{\mathbf{v}}_1)^2}{2\sigma^2}) =$$

$$= \frac{1}{\sqrt{2\pi}} \frac{\widetilde{v}_1}{\sigma} \exp(-\frac{\widetilde{v}_1^2}{2\sigma^2}) {}_1F_1(3/2; 2; \frac{\widetilde{v}_1^2}{2\sigma^2}), \tag{18}$$

where ${}_{1}F_{1}(\alpha;\beta;\gamma)$ is a confluent hypergeometric function. The behavior of $\langle \cos(\psi_{xx'}) \rangle$ as a function of \tilde{v}_{1}/σ is shown in Fig.3.

D. Azimuthal two-particle correlations

An azimuthal two-particle correlation function $C(\psi) = P_{cor}(\psi)/P_{uncor}(\psi)$, where ψ is the angle between two particles, was proposed in Refs. [13,14] to study transverse flow as a method which is free from uncertainty in the reaction plane determination. Here $P_{cor}(\psi)$ is observed pair distribution and $P_{uncor}(\psi)$ is the distribution generated by "event"

mixing". The formalism used above for Fourier analysis permits us to estimate the azimuthal two-particle correlations in the same way as it was done for the distributions of Fourier coefficients. The origin of azimuthal correlations lies in the overall shift of the particle distribution in \mathbf{p}_t -space due to flow. The parameter which controls the magnitude of the correlations is $\langle p_x \rangle / \sqrt{\langle p_t^2 \rangle}$. The correlations increase with increasing values of this parameter. Note that the corresponding parameter (which defines the magnitude of the effect) in our approach is $\tilde{v}_1/\sigma = N\langle p_x \rangle (y)/\sqrt{N\langle p_t^2 \rangle/2}$, which is \sqrt{N} times larger. The difference, as always, originates from the fact that in the study of collective phenomena the fluctuations in the collective variables are less than the fluctuations in variables describing individual particles.

If one assumes a Gaussian form for the one-particle p_t distribution, the expression for $C(\psi)$ coincides with the expression for the correlation between two flow angles defined in different regions, after a substitution of corresponding parameters. The statement made above about finite multiplicity fluctuations becomes explicit in this case.

VI. CONCLUSION

To summarize we propose to study transverse anisotropic collective flow by analysis of azimuthal distributions in different rapidity regions. We show that Fourier expansion of the azimuthal distributions is an appropriate tool for such an analysis. It is shown that the distributions in $v_n = \sqrt{x_n^2 + y_n^2}$, where x_n and y_n are n—th harmonic Fourier coefficients, are sensitive to transverse flow. The classification of flow of different types leading to a deformation of azimuthal distribution corresponding to different Fourier harmonics appears naturally in this method. The important feature of the approach is that it is free from uncertainties in a determination of the reaction plane on an event-by-event basis. The dispersion of the flow angles is derived through the discussion of the finite multiplicity fluctuations. Correlations in the flow angles determined for flow of different types are proposed as a technique for flow analysis. It is shown that many existing methods are within our framework, either the first or the second harmonics. But our approach provide additional information, from which a three dimensional event shape can be established.

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FIGURE CAPTIONS

- 1. Function $r(\phi)$ (solid line). Approximation by ellipse (dashed line).
- 2. The distribution dw/xdx for different values of \tilde{x} . The curves marked by 1,2,3, and 4 correspond to $\tilde{x}=0.5,\,1.0,\,1.5,\,2.0$, respectively.
- 3. $\langle \cos(\phi_{xx'}) \rangle$ as a function of \tilde{v}_1/σ_s

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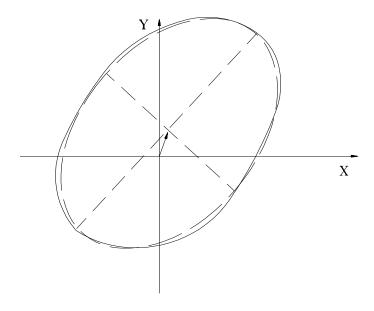


Figure 1:

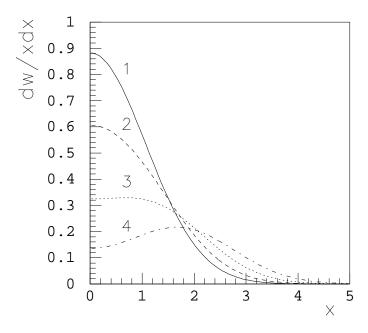


Figure 2:

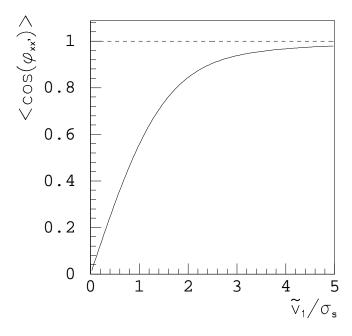


Figure 3: