Dividing by mixed-events for acceptance correction is wrong

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We point out that the mixed-event method for two-particle acceptance correction, widely used in particle correlation measurements at RHIC and LHC, is wrong. The correct acceptance should be the convoluted distribution from two single-particle efficiency×acceptance distributions. The error of the mixed-event method, which guarantees a uniform $\Delta\eta$ two-particle combinatorial density, is however small in correlation analyses where the two particles are integrated over an extended η range. With one particle fixed in η and the right acceptance correction, the background-subtracted correlated pair density may reveal a $\Delta\eta$ dependence. This has important physics implication, and may provide crucial information to disentangle physics mechanisms for the recently observed long-range ridge correlation in asymmetric p+Pb collisions at the LHC.

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I. TWO-PARTICLE ACCEPTANCE

Two-particle correlations are a valuable tool to study heavy-ion collisions [1–3]. Correlation functions are often formed by particle pair density in real events divided by that from mixed events, where the two particles are taken from two different events. With proper normalization the deviation of the correlation function from unity reveals correlations between two particles. This technique applies to situations where all particle pairs are correlated, for example, in HBT interferometry [4] and anisotropic flow correlations [5].

One unique case of correlations is jet angular correlations. The study of jet correlations has provided wealth of information about relativistic heavy-ion collisions [1–3]. The object of interest is a cluster of particles–not all particles in the event–that are correlated due to their common origin of parton fragmentation. The interest is often to find number of correlated particles and their angular distributions. The correlations are often formed in terms of the particle pair azimuthal angle difference, $\Delta\phi$ and pseudo-rapidity difference, $\Delta\eta$. The mixed-event technique is widely used, not as the uncorrelated baseline as in HBT or anisotropic flow analysis, but to correct for two-particle acceptance. For example, the mixed-event two-particle density is approximately triangular in $\Delta\eta$, and is used for $\Delta\eta$ acceptance correction after normalized to 100% at $\Delta\eta = 0$. This mixed-event method for two-particle acceptance correction is, however, wrong. The correct two-particle acceptance should be the convolution of two single-particle acceptances (or efficiency functions).

In the widely used mixed-event acceptance correction, the single particle distributions are mistaken as detection efficiencies, implying that the true distributions are always uniform. This does not make a big error for mid-rapidity particles in symmetric collisions, where particle density is nearly uniform in η. By using mixed-event technique, the "corrected" two-particle correlation signal is guaranteed to be uniform except regions of correlation signals. The truth, of course, may not be uniform as the single particle η distribution at mid-rapidity is not strictly uniform.

The mixed-event acceptance correction could be worse for asymmetric collisions, such as proton-nucleus (p+Pb) collisions, where the single-particle η density is nonuniform even at mid-rapidity. The corrected two-particle correlations will, again, be uniform by definition except the $\Delta\eta$ regions of correlation signals. But this clearly is wrong. It is easy to see in the following simple example. Suppose in a jet-correlation study, the high-p_T trigger particles are fixed at $\eta = 0$ and all other particles are paired with the trigger to form angular correlation functions. There will be a peak at zero due to jet correlations and the underlying background will be as same as the measured single particle η distribution. The two-particle acceptance correction to be applied should be the single particle efficiency as function of η (because the trigger particle is always at $\eta = 0$). After correction, the signal should be the real jet signal on top of the real background which is the true single particle density which may not be uniform in η. If the mixed-event $\Delta\eta$ distribution (which is the single particle η distribution in this simple case) is used as the acceptance correction, then the corrected pair density will be completely flat in $\Delta\eta$ except for regions of correlation signal. Clearly, this will be wrong.

In real data analysis, often all pairs of trigger and associated particles are used. In asymmetric collisions, neither the trigger nor the associated particle η density is uniform. However, averaging over all trigger and associated particles, the non-uniformities in the single $dN/d\eta$ distributions become second order effect in the two-particle density distribution in $dN/d\Delta\eta$. To see this, we take the simple example of a linear single particle density,

$$\frac{dN}{d\eta} \propto 1 + k\frac{\eta}{\eta_m}\,, \tag{1}$$

where $\pm\eta_m$ are the acceptance limits. The two-particle density distribution will be

$$\begin{aligned}\frac{dN}{d\Delta\eta} &= \int_{\eta_1}\int_{\eta_2}\left(1+k\frac{\eta_1}{\eta_m}\right)\left(1+k\frac{\eta_2}{\eta_m}\right)\delta(\eta_2-\eta_1-\Delta\eta)d\eta_1 d\eta_2 \\ &= \int_{\max(-\eta_m,-\eta_m-\Delta\eta)}^{\min(\eta_m,\eta_m-\Delta\eta)}\left(1+k\frac{\eta_1}{\eta_m}\right)\left(1+k\frac{\eta_1+\Delta\eta}{\eta_m}\right)d\eta_1 \\ &= (2\eta_m \pm \Delta\eta)\left[1+\frac{1}{6}k^2\left(2\pm 2\frac{\Delta\eta}{\eta_m}-\left(\frac{\Delta\eta}{\eta_m}\right)^2\right)\right], \qquad (2)\end{aligned}$$

where the $\pm$ corresponds to the $\Delta\eta < 0$ and $\Delta\eta > 0$ case, respectively. The effect of non-uniformity is quadratic in k. When $k << 1$ which is typically the case for mid-rapidity region, the two-particle density is approximately triangular in $\Delta\eta$. In most data analyses, the mixed-event technique is used for two-particle acceptance correction and the final correlated yield is uniform by definition as we discussed. If the correct acceptance is used, the resultant two-particle density does not deviate much from a uniform distribution. So the error due to mixed-event two-particle acceptance correction is small.

II. PHYSICS IMPLICATION

We have so far focused on a pure technical point, that two-particle acceptance correction should not be obtained from mixed-events, but rather from convolution of two single particle efficiency functions. Now we want to turn to an important physics implication of this point.

Surprisingly, a strong large-$\Delta\eta$ small-$\Delta\phi$ correlation is observed in p+Pb collisions at the LHC [6–8]. This is called the "ridge" following the observation in heavy-ion collisions [9–11]. The strength of the ridge in p+Pb is as strong as that in heavy-ion collisions, where the ridge is considered to be a consequence of anisotropic flow. As we discussed, the observed ridge in p+Pb collisions is uniform by construction of the mixed-event technique, and will likely remain uniform even using the correct two-particle acceptance corrections because the trigger and associated particles in a wide η range are averaged. However, the extra feature of the nonuniform single particle η distribution in p+Pb collisions may be essential to unravel the underlying physics mechanisms for the ridge formation. In order for any $\Delta\eta$-dependent ridge to be observable, one needs to fix the trigger particle η within a narrow bin and study correlations of associated particles, $dN/d\Delta\eta \approx dN/d\eta_{assoc}$.

There are two leading models for the physics mechanism of the ridge in small systems. One is hydrodynamics where the initial geometry anisotropy is converted into final-state anisotropic particle distributions by hydrodynamic evolutions [12]. The anisotropic particle distributions result in an enhanced two-particle density at $\Delta\phi = 0$, the ridge. In this picture, the ridge strength is proportional to the underlying background pair density:

$$\frac{d^2N}{d\Delta\eta d\Delta\phi} = \frac{dN(\Delta\eta)}{d\Delta\eta}\left(1+\sum_{n=1} 2V_n \cos n\Delta\phi\right). \qquad (3)$$

Because the underlying event baseline $dN(\Delta\eta)/d\Delta\eta$ depends on $\Delta\eta$, a measurement of the signal $d^2N/d\Delta\eta d\Delta\phi$ will be informative about the nature of the ridge. Of course, the anisotropic harmonic V_n (product of the two single particle v_n's) may also depend on $\Delta\eta$. Thus, a measurement of $d^2N/d\Delta\eta d\Delta\eta$ may not uniquely confirms or refutes the hydrodynamic explanation, but should provide an important extra information. In this respect, it would be crucial to have predictions of the η dependences of v_2 and v_3 from hydrodynamical calculations.

The second model is the color glass condensate (CGC) [13]. The CGC framework gives particular predictions of the two-gluon production process as a function of $\Delta\eta$. A calculation for Au+Au collisions is given in Ref. [14] which attributes the ridge to a net effect of the CGC enhanced two-gluon production and the strong collective radial flow. Calculations of the $\Delta\eta$ dependence of the ridge in p+Pb collisions are not yet available but should be extremely valuable.

Presumably, different physics mechanisms would yield different $\Delta\eta$ dependences of the ridge correlations, as well as different energy dependences. Measurements of the ridge as a function of $\Delta\eta$ at both the LHC and RHIC should, therefore, put stringent constraints on models.

Experimentally, the two-particle acceptance should be taken from the convolution of two single-particle efficiencies. For a fixed narrow trigger particle η, the two-particle acceptance is approximately the single particle efficiency for associated particles. One should obtain the near-side raw yield vs. $\Delta\eta$ and subtract the combinatorial background by, for example, the zero-yield-at-minimum (ZYAM) procedure [15]. The ZYAM magnitude as a function of $\Delta\eta$ may be treated as the underlying background particle pair density. One then examine whether the background-subtracted signal is uniform in $\Delta\eta$, proportional to the background pair $\Delta\eta$ density in the ZYAM region, or of any other shape.

To examine the ridge yield relative to the combinatorial background, one may not even need to do two-particle acceptance correction. One can simply form the ratio of the pair density from real events to that from mixed-events (i.e. the original correlation analysis), but with fixed narrow trigger particle η bin, and examine the shape of the correlation function in the ridge region of large $\Delta\eta$. One may not even need mixed-events, but treating the pair density in the ZYAM $\Delta\phi$ region as the combinatorial background, and examine the ratio of the pair density in the ridge region at small $\Delta\phi$ to that in the ZYAM region, both of a function of $\Delta\eta$.

The idea of $\Delta\eta$ dependence of the ridge can be applied to asymmetric heavy-ion collisions, for example, the available Cu+Au collision data from RHIC.

In summary, we point out the misconception in two-particle acceptance correction by the mixed-event technique. The mixed-event acceptance correction is, in principle, wrong and guarantees a uniform $\Delta\eta$ pair density (correlation) signal. The correct two-particle acceptance should be the convoluted distribution from two single-particle efficiency$\times$acceptance distributions. The effect of the improper mixed-event correction is, however, small in present experimental data where both the trigger and associated particles are integrated over extended η ranges. With a fixed narrow η range for trigger particles and the proper acceptance correction, the correlation signal may reveal a $\Delta\eta$ dependence, especially for asymmetric collision systems such as p+Pb recently available at the LHC. The $\Delta\eta$ dependence may be crucial to discriminate physics models for the observed ridge.

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