Mixed-event corrections for two-particle angular correlations

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ALICE

Two-particle angular correlations

- Published papers:
  - Pb-Pb:
    - I_{AA} (PRL108, 092301 (2012))
    - Fourier decomposition (PLB708, 249 (2012))
  - pp:
    - minijets (JHEP09, 049 (2013))
    - underlying event (JHEP07, 116 (2012))
  - p-Pb:
    - ridge (PLB719, 29 (2013))
    - PID ridge (PLB726, 164 (2013))
- Ongoing analyses (PAG TWiki)
  - minijets in p-Pb
  - untriggered correlations (analysis note)
  - PID correlations
  - large-$\Delta\eta$ correlations (μ-h, VZERO, TZERO)

Not a complete list: (Q-) cumulants, FB correlations and potentially more ...

Two-particle angular correlations

- Published papers:
  - Pb-Pb:
    - I_{AA} — $1/N_{trig}\ dN/d\Delta\phi$
    - Fourier decomposition — $C(\Delta\eta,\Delta\phi)$
  - pp:
    - minijets — $1/N_{trig}\ dN/d\Delta\phi$
    - underlying event — $1/N_{trig}\ dN/d\Delta\phi$
  - p-Pb:
    - ridge — $1/N_{trig}\ d^2N/d\Delta\phi d\Delta\eta$
    - PID ridge — $1/N_{trig}\ d^2N/d\Delta\phi d\Delta\eta$
- Ongoing analyses:
  - minijets in p-Pb — $1/N_{trig}\ dN/d\Delta\phi$
  - untriggered correlations — $C(\Delta\eta,\Delta\phi)$
  - PID correlations — $1/N_{trig}\ dN/d\Delta\phi$
  - large-$\Delta\eta$ correlations — $1/N_{trig}\ d^2N/d\Delta\phi d\Delta\eta$

"1D method"
- $1/N_{trig}\ dN/d\Delta\phi$

mixed events for systematic checks

ambiguities within ALICE:
e.g. jet yields cannot directly be compared

"2D methods"
- $C(\Delta\eta,\Delta\phi)$
- $1/N_{trig}\ d^2N/d\Delta\phi d\Delta\eta$

mixed events required

→ compare the two 2D methods

ALICE Correlation vs. associated yield

- Fourier decomposition (extract v_n from two-particle correlations)
  - only relative modulation required
  - absolute yields not important
  - → a true correlation function is used: $C(\Delta\eta,\Delta\phi)$

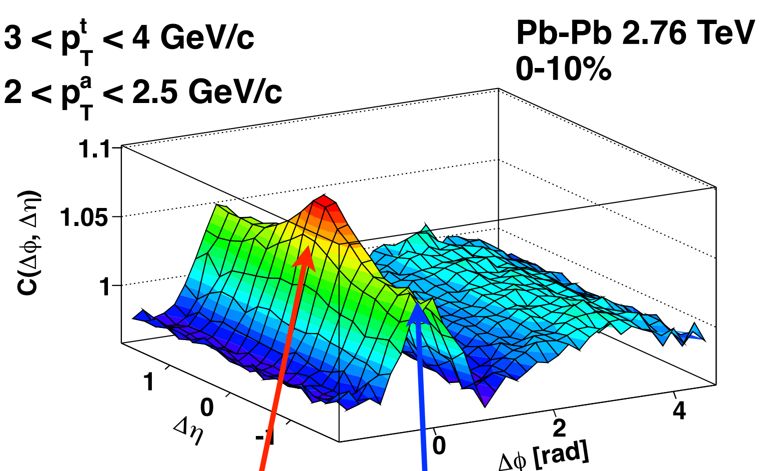
- Jet-associated yields
  - normalization matters when integrating yields
  - → define per-trigger associated yield: $1/N_{trig}\ d^2N/d\Delta\phi\ d\Delta\eta$

ALICE Correlation function method

Same and mixed event pair distributions are normalized to their integral and divided:

$$C(\Delta\phi, \Delta\eta) \equiv \frac{N_{\text{mixed}}}{N_{\text{same}}} \times \frac{N_{\text{same}}(\Delta\phi, \Delta\eta)}{N_{\text{mixed}}(\Delta\phi, \Delta\eta)}$$

no correlation would be flat at C=1

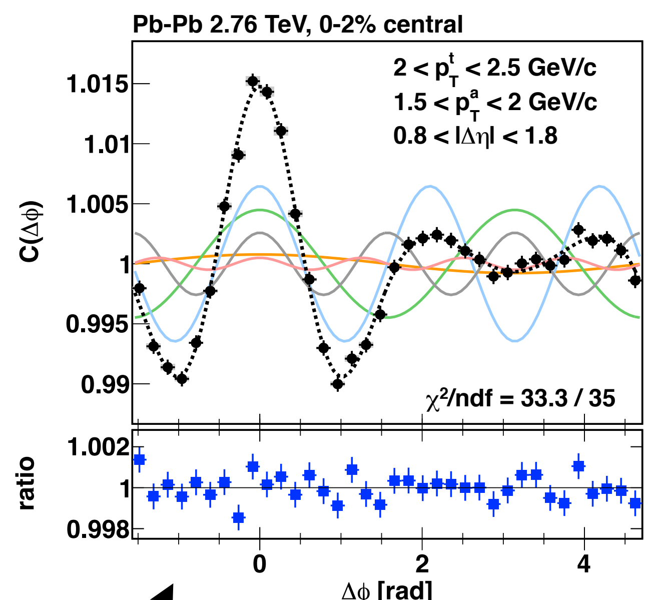


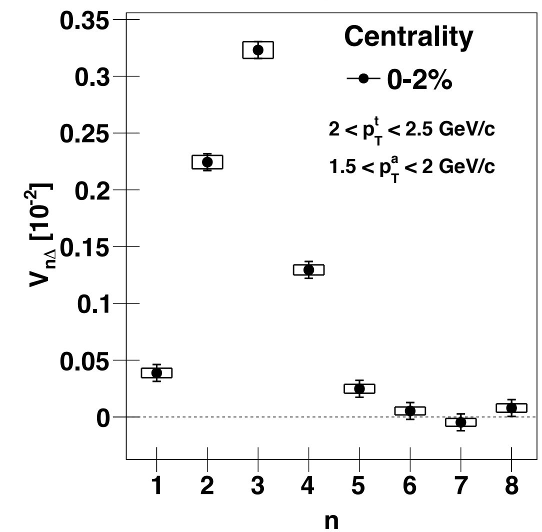
Separate jet and flow by selecting Δη ranges

Project to Δφ for 0.8<|Δη|<1.8

Extract Fourier coefficients from the projection:

$$V_{n\Delta} \equiv \langle \cos(n\Delta\phi) \rangle = \sum_i C_i \cos(n\Delta\phi_i) / \sum_i C_i.$$





Fourier decomposition

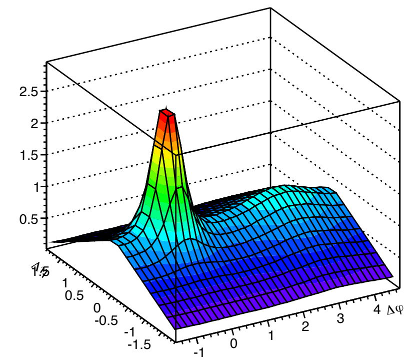
$$\frac{dN^{\text{pairs}}}{d\Delta\phi} \propto 1 + \sum_{n=1}^{\infty} 2V_{n\Delta}(p_T^t, p_T^a) \cos(n\Delta\phi).$$

For a correlation function mixed events are required by construction

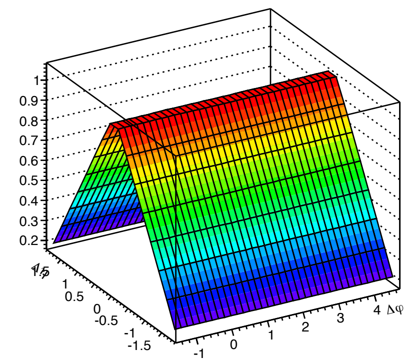
Still based on assumptions, e.g. that η gap separates jet and flow

Associated yield method

- Same-event pair distribution $$S(\Delta\eta, \Delta\varphi) = \frac{1}{N_{\rm trig}} \frac{{\rm d}^2 N_{\rm same}}{{\rm d}\Delta\eta\, {\rm d}\Delta\varphi}$$
  - normalized per trigger (per ensemble, not per event!)
  - Δη dependence dominated by pair acceptance



- Mixed-event pair distribution $$B(\Delta\eta, \Delta\varphi) = \frac{1}{B(0,0)} \frac{{\rm d}^2 N_{\rm mixed}}{{\rm d}\Delta\eta\, {\rm d}\Delta\varphi}$$
  - to correct for pair acceptance and efficiency
  - normalized to 1 around (Δη,Δϕ)=(0,0), where pair acceptance is 1 by construction
  - *S* and *B* are corrected for single-particle efficiency

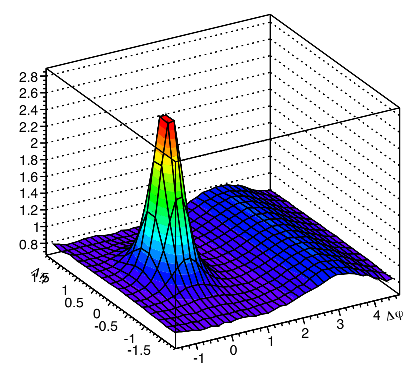


- Corrected associated yield per trigger

$$\frac{1}{N_{\rm trig}} \frac{{\rm d}^2 N_{\rm assoc}}{{\rm d}\Delta\eta\, {\rm d}\Delta\varphi} = \frac{S(\Delta\eta, \Delta\varphi)}{B(\Delta\eta, \Delta\varphi)}$$

  - short-hand notation can be misleading: CMS uses the same notation, but a different normalization, that leads to a multiplicity bias

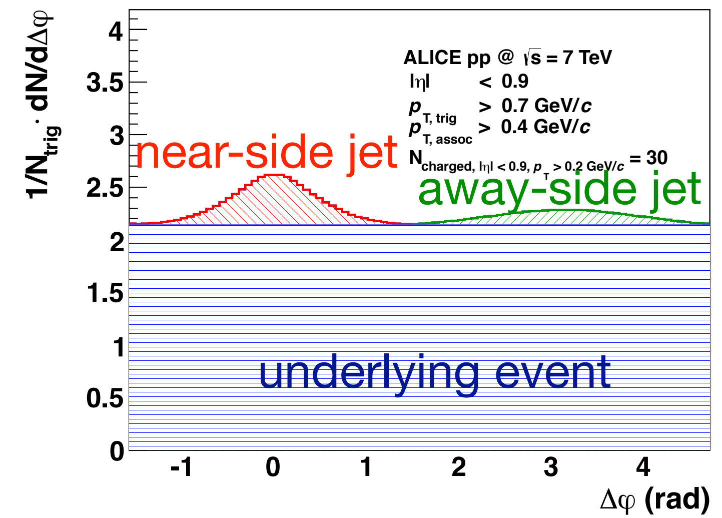
$$\frac{1}{\sum N_{\rm trig}} \sum \frac{{\rm d}^2 N_{\rm assoc}}{{\rm d}\Delta\eta\, {\rm d}\Delta\varphi} \text{ vs. } \sum \frac{1}{N_{\rm trig}} \frac{{\rm d}^2 N_{\rm assoc}}{{\rm d}\Delta\eta\, {\rm d}\Delta\varphi}$$



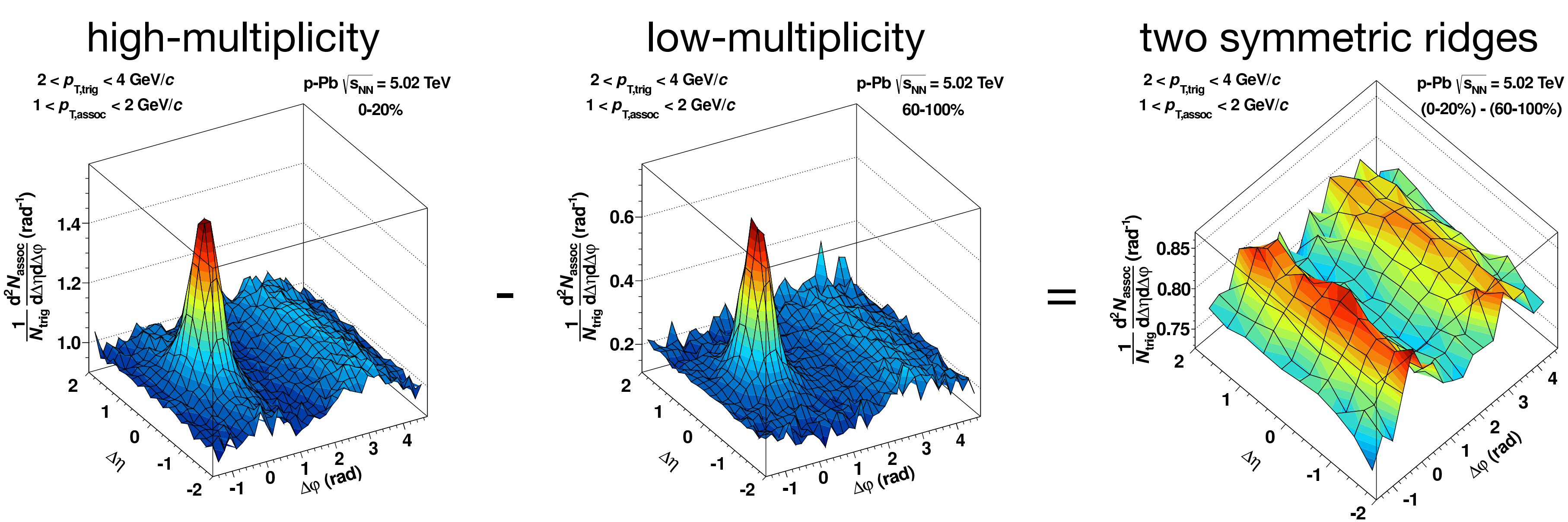
  - single-particle efficiency correction and $N_{\rm trig}$ normalization → A. Morsch, APW Puebla

Associated yield method

- Extract yields
  - integrate jet and underlying event yields
  - proper normalization required (see previous slide)



- First used in ALICE in p-Pb analysis: two-dimensional subtraction method
  - assume same jet component in high- and low-multiplicity p-Pb collisions
  - high-multiplicity events have additional two-ridge structure



Mixed events ↔ pair acceptance

- Focus on η
  - ϕ acceptance in ALICE and spectrum d*N*/dϕ are flat
- Mixed events are used to correct for two-particle acceptance
  - data-driven approach (no simulation needed)
  - all cuts (e.g. two-track cuts) can be applied as for signal
  - autocorrelation of single-particle distribution

$$B(\Delta\eta) \propto \int \mathrm{d}\eta f(\eta) f(\eta - \Delta\eta)$$
$$f(\eta) = \mathrm{acc}(\eta) \cdot \mathrm{eff}(\eta) \cdot \mathrm{d}N/\mathrm{d}\eta$$

- Two-particle acceptance
  - autocorrelation of single-particle acceptance
  - MC required for single-particle acceptance

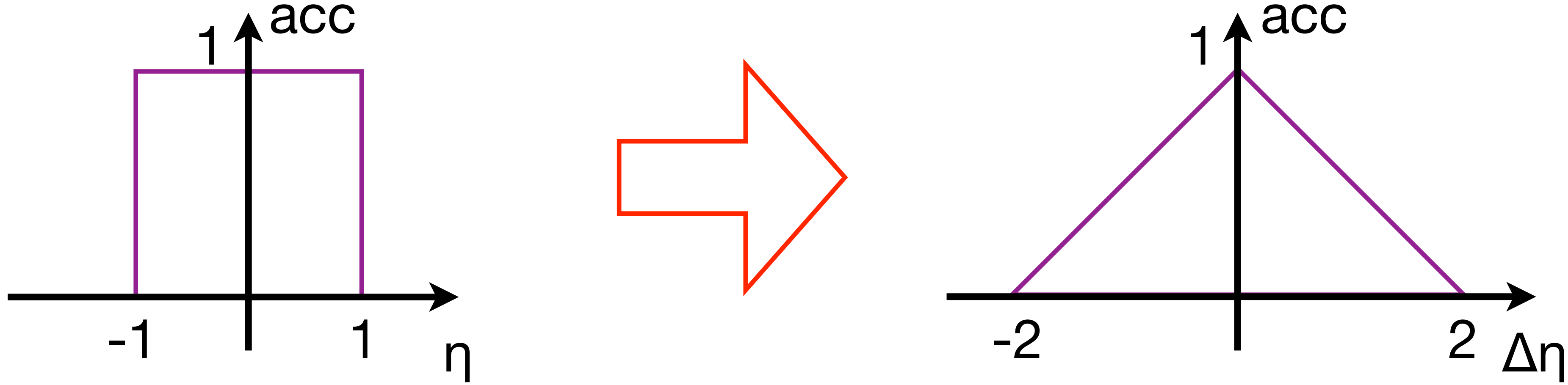
$$A(\Delta\eta) \propto \int \mathrm{d}\eta a(\eta) a(\eta - \Delta\eta)$$
$$a(\eta) = \mathrm{acc}(\eta) \cdot \mathrm{eff}(\eta)$$

- The two approaches are equivalent
  - when the underlying spectrum d*N*/dη is uniform
  - cf.: A. Morsch, APW in Puebla

Method Mixed events ↔ pair acceptance

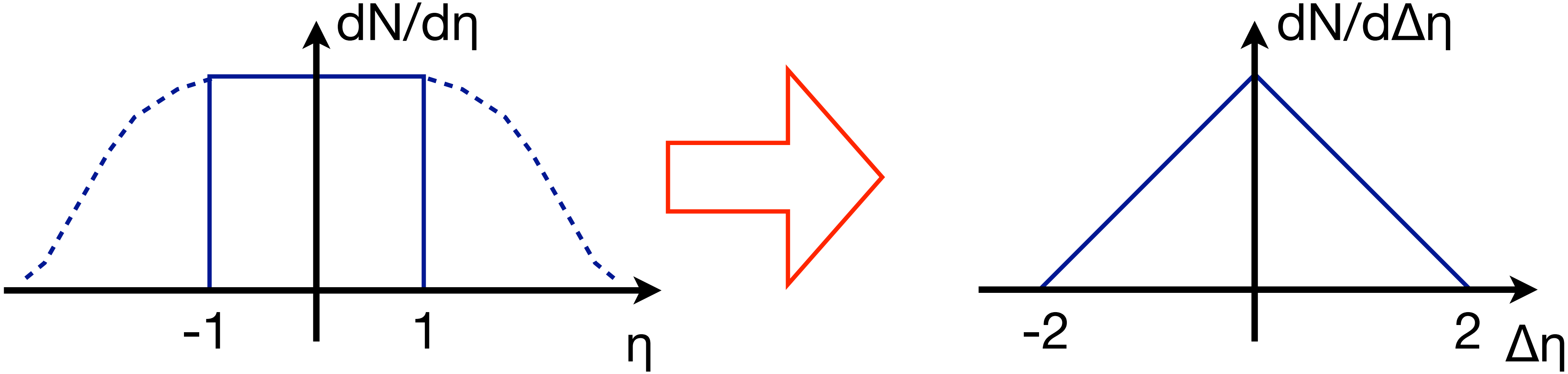
single-particle acceptance (efficiency=1)

pair acceptance



single-particle distribution

uncorrelated pair distribution

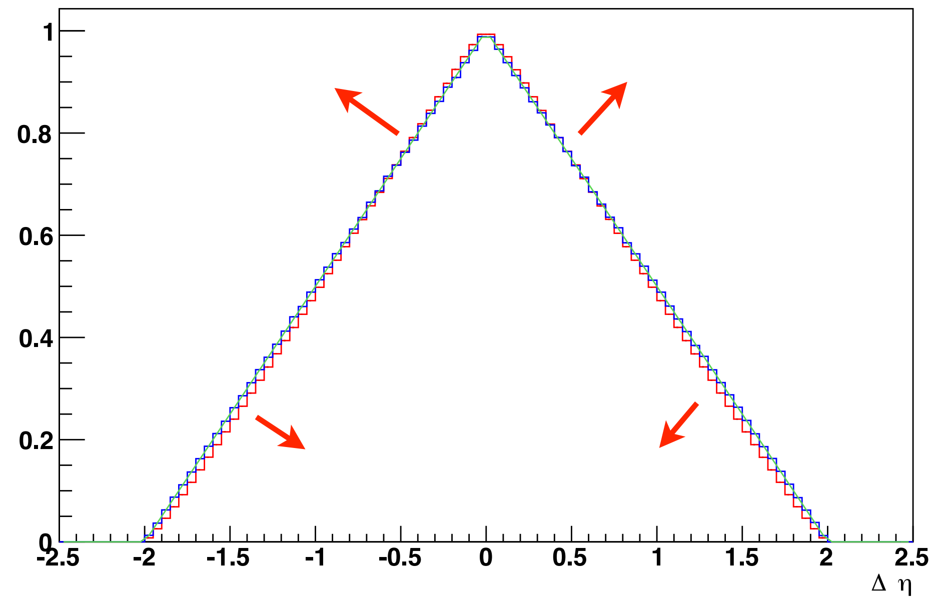


In case of flat acceptance and flat spectrum, the two methods give the same background Δη distribution

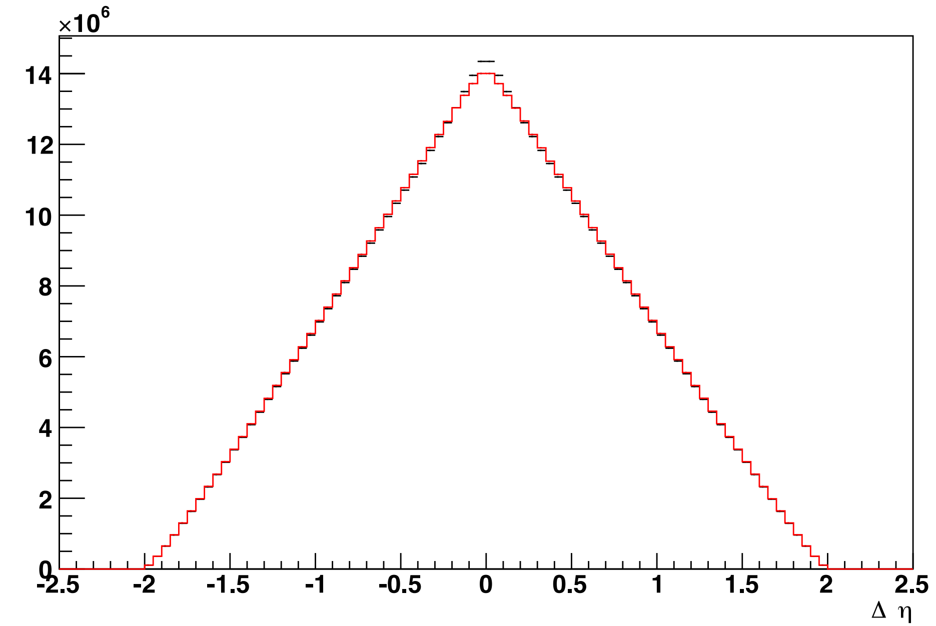
Mixed events ↔ pair acceptance

Toy MC with $\mathrm{d}N/\mathrm{d}\eta \propto 1 + 0.25\eta$

subtle effect: mixed events deviate from triangular shape

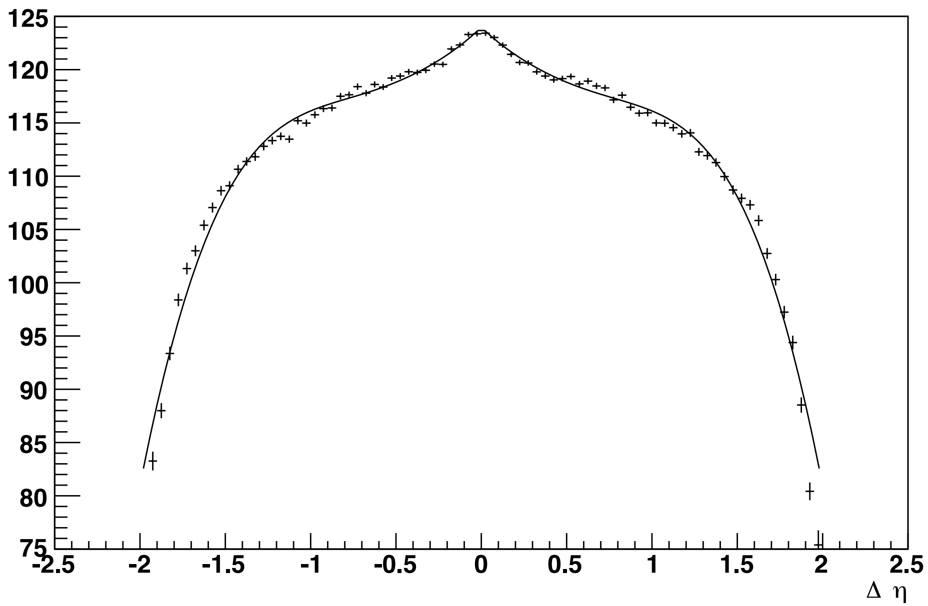


mixed events and signal have the same shape outside of (added, Gaussian) jet peak

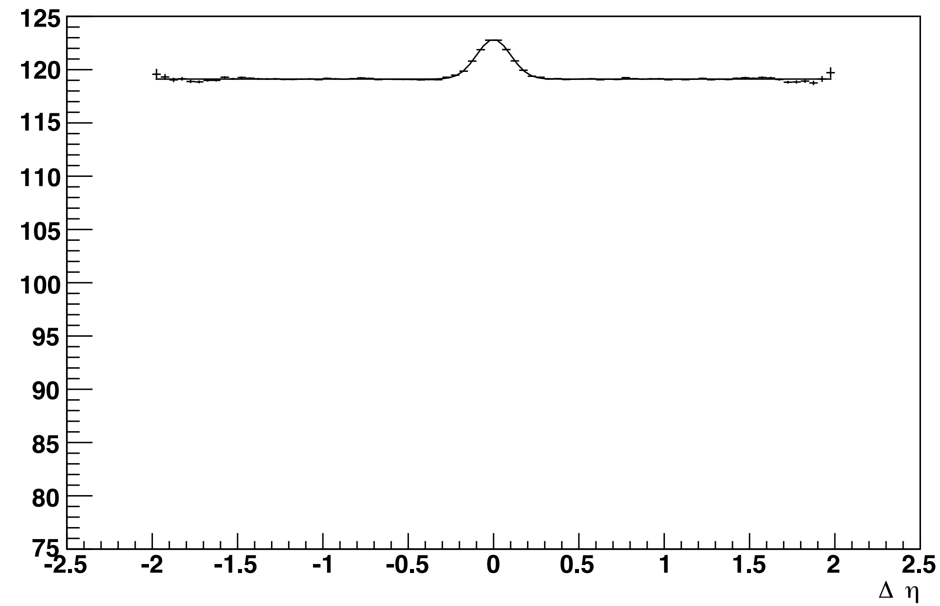


Toy MC　Mixed events ↔ pair acceptance

signal/acceptance



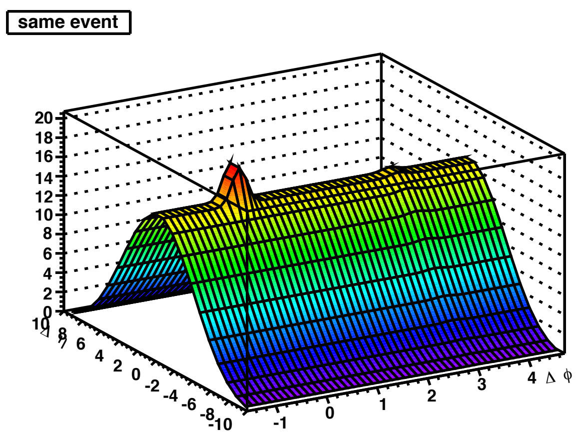
signal/mixed events



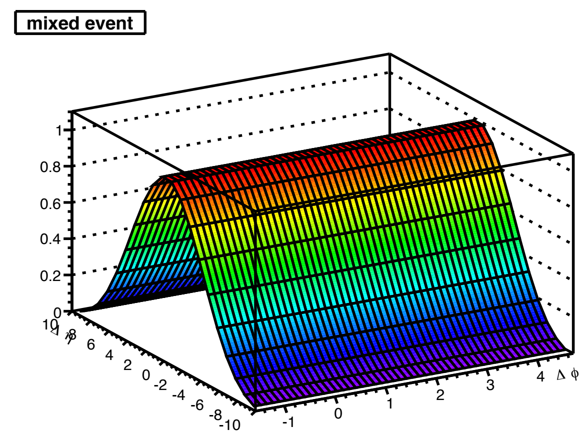
- In both cases, the signal peak is recovered
  - only a Gaussian near-side jet peak is introduced in the toy MC
- The acceptance-corrected distribution also shows a “correlation” caused by the background spectrum
- Mixed events make uncorrelated part flat but distort signal (minimal in the case of small Δη extent of signal)

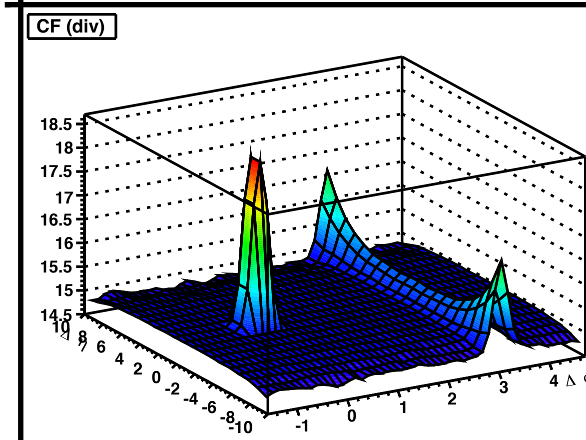
Toy MC Mixed events ↔ pair acceptance

$$S\left(\Delta\eta,\Delta\varphi\right)=\frac{1}{N_{\mathrm{trig}}}\frac{\mathrm{d}^2N_{\mathrm{same}}}{\mathrm{d}\Delta\eta\,\mathrm{d}\Delta\varphi}$$



$$B\left(\Delta\eta,\Delta\varphi\right)=\frac{1}{B\left(0,0\right)}\frac{\mathrm{d}^2N_{\mathrm{mixed}}}{\mathrm{d}\Delta\eta\,\mathrm{d}\Delta\varphi}$$



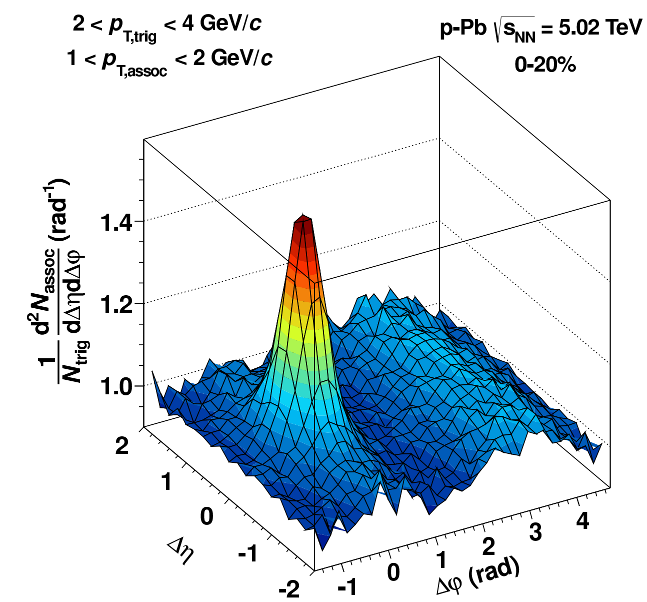


$$\frac{1}{N_{\mathrm{trig}}}\frac{\mathrm{d}^2N_{\mathrm{assoc}}}{\mathrm{d}\Delta\eta\,\mathrm{d}\Delta\varphi}=\frac{S\left(\Delta\eta,\Delta\varphi\right)}{B\left(\Delta\eta,\Delta\varphi\right)}$$

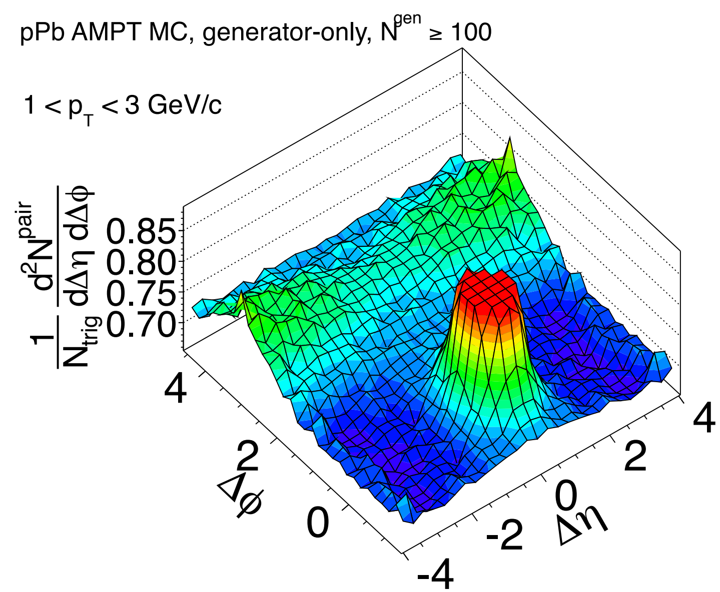
- Two-dimensional toy MC
  - near- and away-side jets
  - on top of non-uniform uncorrelated background
  - expected (by construction): a flat away side jet structure
- Signal / mixed event distribution grows towards large Δη
  - uncorrelated background is flat
  - away-side jet structure is distorted due to the underlying dN/dη spectrum

Conclusion Mixed events ↔ pair acceptance

- Mixed events can be used to correct for two-particle acceptance
  - in case of uniform underlying spectra, the two are equivalent
  - in ALICE midrapidity acceptance, no large effect, even in p-Pb



- Limits of the method
  - distortions arise when background dN/dη distribution changes considerably over studied η range (e.g. $\Delta\eta > \sigma_\eta$)
  - CMS large-$\Delta\eta$ analysis potentially shows a wing effect

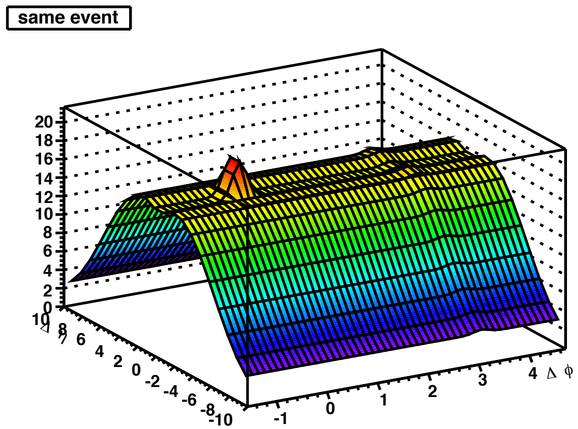


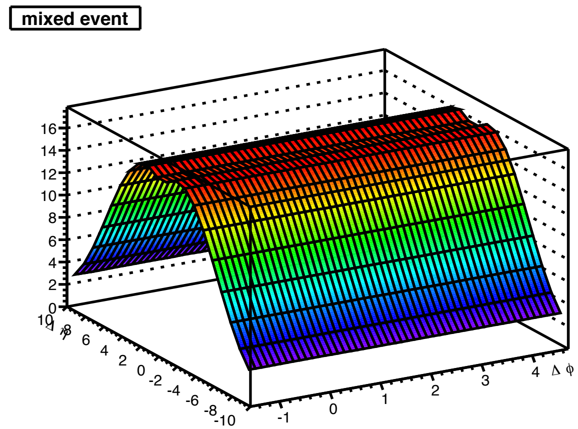
CMSPublic/PhysicsResultsHIN12015

- Other methods?

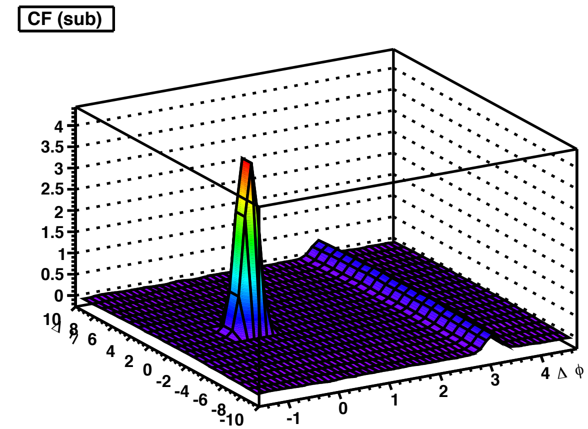
MC Model Correlation function (sub)

$$S'(\Delta\eta, \Delta\varphi) = \frac{\mathrm{d}^2 N_{\mathrm{same}}/\mathrm{d}\Delta\eta\,\mathrm{d}\Delta\varphi}{A \times A} \qquad B'(\Delta\eta, \Delta\varphi) = \alpha \cdot \frac{\mathrm{d}^2 N_{\mathrm{mixed}}/\mathrm{d}\Delta\eta\,\mathrm{d}\Delta\varphi}{A \times A}$$





normalization so that
S'(Δϕ=π) = *B*'(Δϕ=π)



$S' - B'$

- Subtraction method
  - pair acceptance is constructed out of the single-track acceptances: *A*A*
  - same and mixed event pair distributions are first corrected by *A*A*, then subtracted
  - flat uncorrelated part and no large-Δη distortions!
- Has been tested in p-Pb μ-h analysis
  - no large effect expected, no large difference to standard method observed

Method — Mixed events ↔ pair acceptance

PHYSICAL REVIEW C **88**, 064907 (2013)

Event mixing does not reproduce single-particle acceptance convolutions for nonuniform pseudorapidity distributions

Lingshan Xu,[1] Chin-Hao Chen,[2] and Fuqiang Wang[1]
[1]*Department of Physics, Purdue University, West Lafayette, Indiana 47907, USA*
[2]*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*

- Recent paper
  - confirms expectations about mixed events
  - gives analytic solution for uncorrelated pair distribution

To see this, we take the simple example of a measured single-particle density to be linear in η,

$$\frac{dN}{d\eta} \propto 1 + k\frac{\eta}{\eta_m}, \tag{1}$$

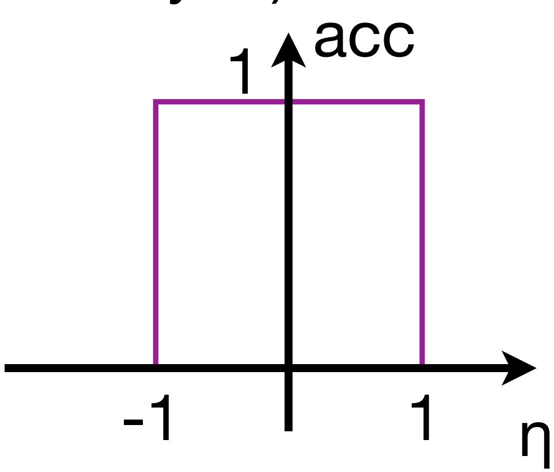
where $\pm\eta_m$ are the acceptance limits. The combinatorial two-particle density distribution will be

$$\begin{aligned}\frac{dN}{d\Delta\eta} &\propto \int_{\eta_1}\int_{\eta_2}\left(1+k\frac{\eta_1}{\eta_m}\right)\left(1+k\frac{\eta_2}{\eta_m}\right) \\ &\quad\times \delta(\eta_2-\eta_1-\Delta\eta)d\eta_1 d\eta_2 \\ &= \int_{\max(-\eta_m,-\eta_m-\Delta\eta)}^{\min(\eta_m,\eta_m-\Delta\eta)}\left(1+k\frac{\eta_1}{\eta_m}\right)\left(1+k\frac{\eta_1+\Delta\eta}{\eta_m}\right)d\eta_1 \\ &= (2\eta_m-|\Delta\eta|)\left\{1+\frac{1}{6}k^2\left[2-2\frac{|\Delta\eta|}{\eta_m}-\left(\frac{\Delta\eta}{\eta_m}\right)^2\right]\right\}.\end{aligned} \tag{2}$$

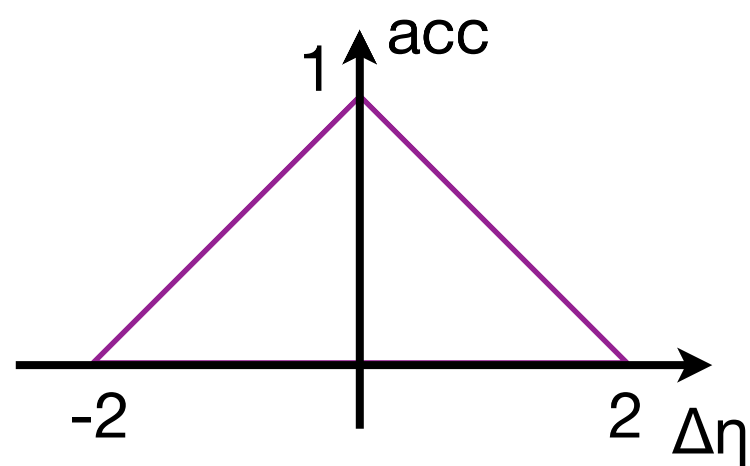
acceptance (triangle) distortion due to spectrum

Method Mixed events ↔ pair acceptance

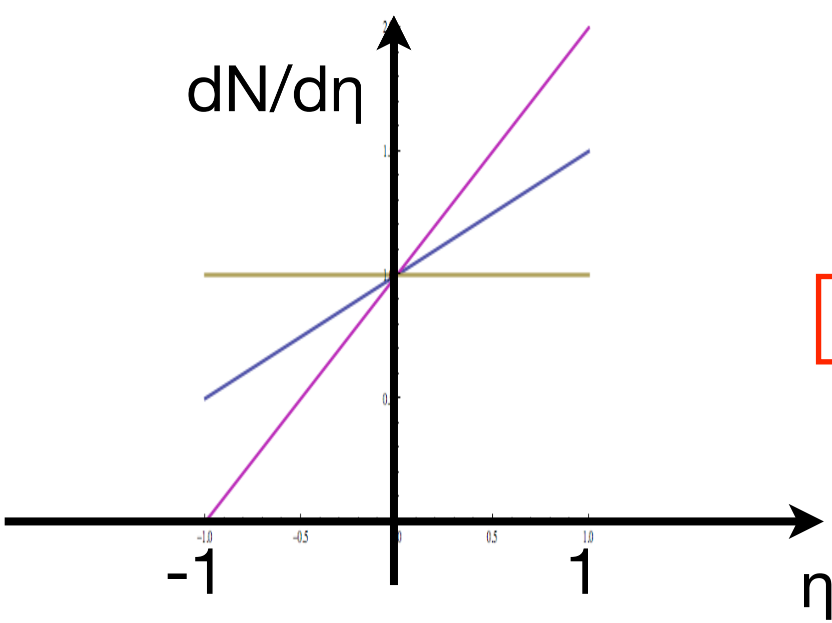
single-particle acceptance
(efficiency=1)



pair acceptance

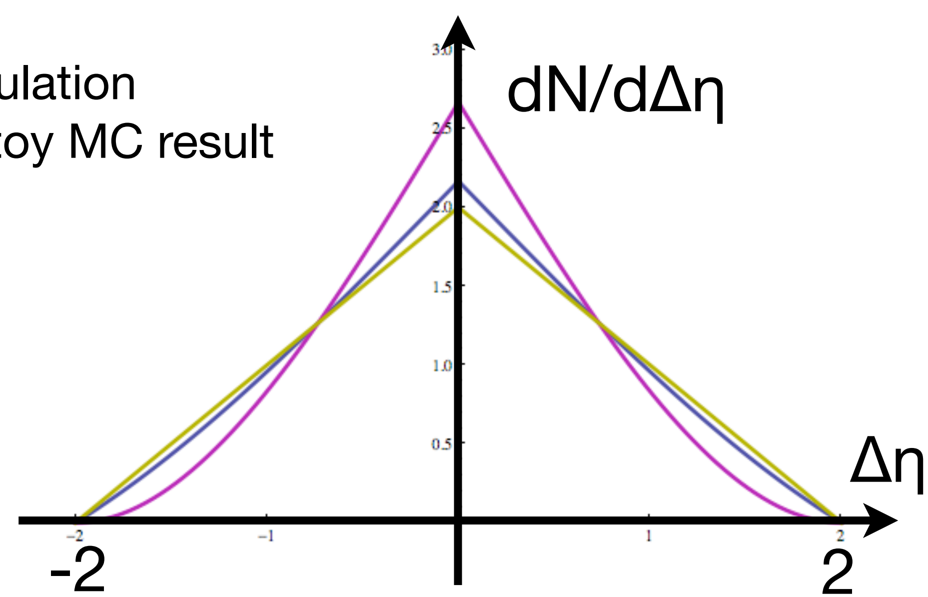


single-particle distribution



analytic calculation
agrees with toy MC result

pair distribution



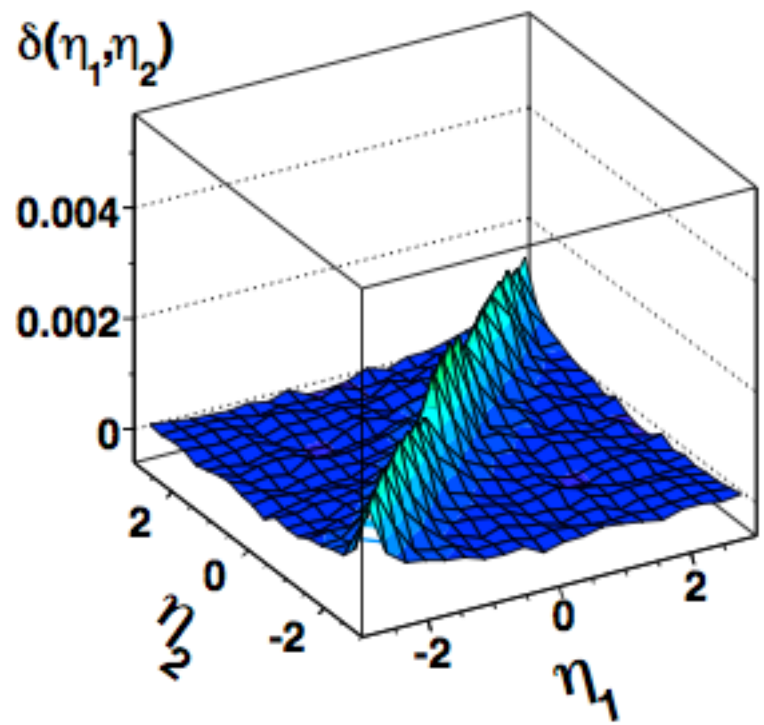
Method Event mixing

Correcting Correlation Function Measurements

Shantam Ravan, Prabhat Pujahari, Sidharth Prasad, and Claude A. Pruneau
Department of Physics and Astronomy, Wayne State University

- Another recent paper
  - "correlation function cannot be decomposed into correlated and uncorrelated parts"
    in disagreement with ALICE practice and observations
  - methods discussed mainly based on $C(\Delta\eta,\Delta\phi)$ and suggests correction procedures
  - not suitable for p-Pb analyses, where we deal with additive jet+collective signals
    - assumption of pure correlation that cannot be decomposed
    - normalization
  - reminds us to have a look at the correlation as a function of η_1,η_2 as e.g. PHOBOS did: are we averaging over $<\eta>$ correctly?

PHOBOS PRC81, 034915 (2010)



Conclusion Two-particle angular correlations

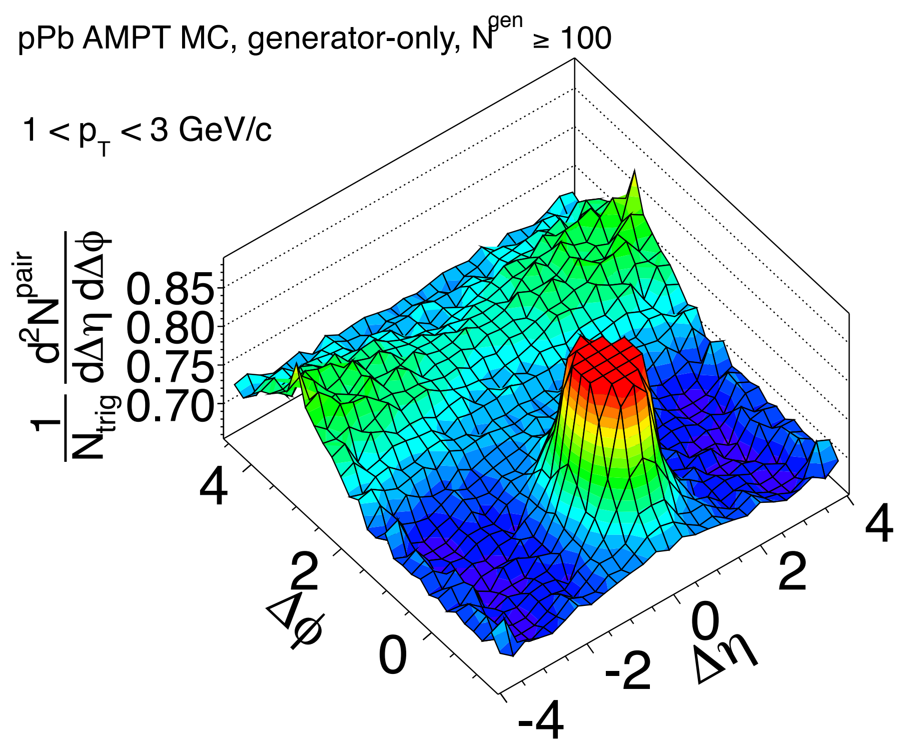
- Recent publications disagree on the use of mixed events
  - Wang et al.: two-particle acceptance should be corrected using convoluted single-particle acceptance instead of mixed events
  - Pruneau & Co.: that's a misconception - mixed events are needed for correlation function
- Two-particle angular correlation analyses in ALICE
  - adequate methods in use in ALICE to extract flow coefficients and jet yields
  - suitable for p-Pb and for large $\Delta\eta$
  - different contributions (flow-like, jet-like, uncorrelated) can be quantified using $\Delta\eta$ cuts or subtraction method
in contrast to strict interpretation from Pruneau et al. "decomposition is non-sensical"
- Every analysis method is based on pre-assumptions that need to be kept in mind

End.

Introduction Large-$\Delta\eta$ distortions

- Default definition of correlation function (division by mixed events) has been used in multiple ALICE publications
- Works fine for the ALICE central barrel midrapidity acceptance
- Distortions arise when background dN/dη distribution changes considerably over studied η range e.g. $\Delta\eta > \sigma_\eta$

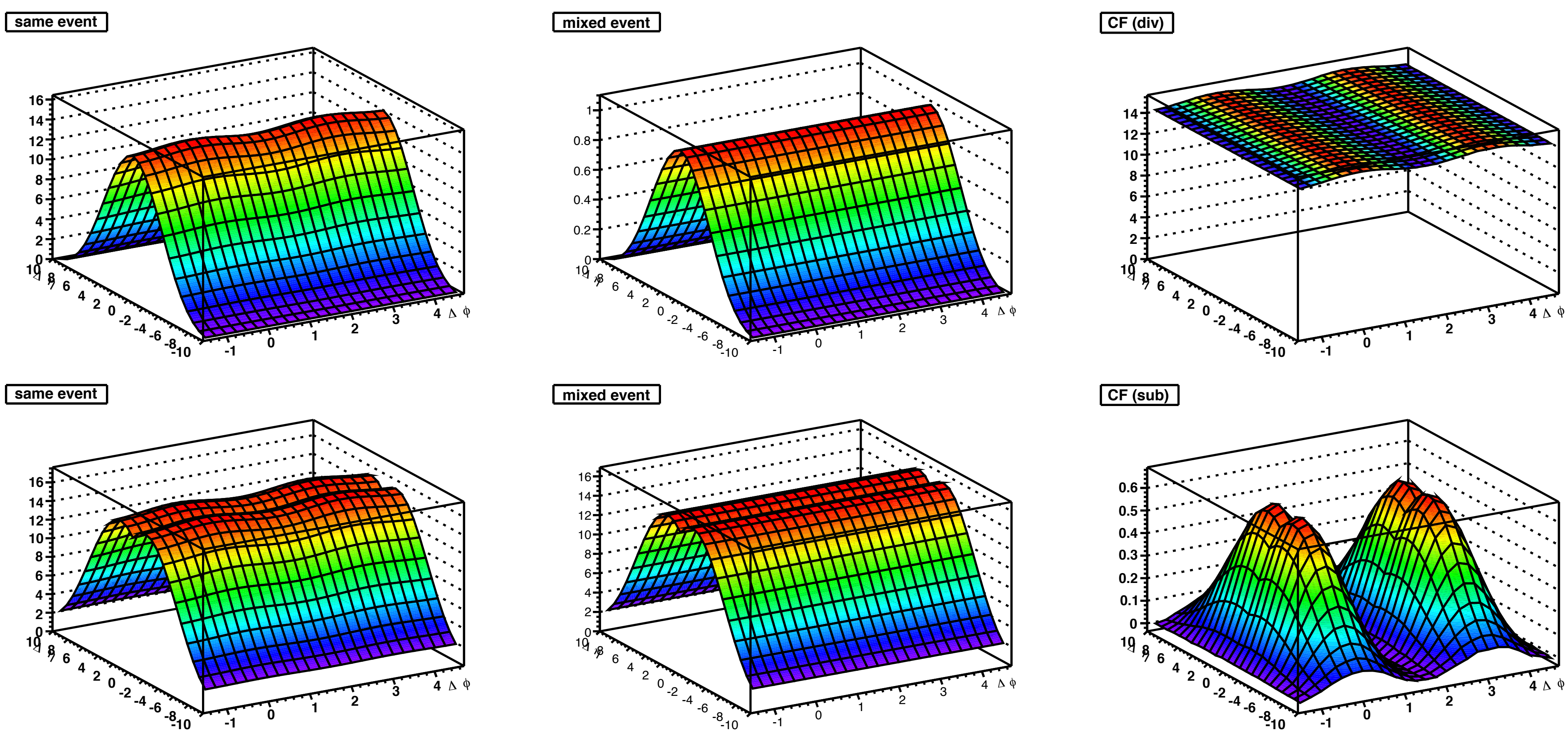
(cf. Dos and don'ts: A. Morsch APW Puebla)



CMSPublic/PhysicsResultsHIN12015

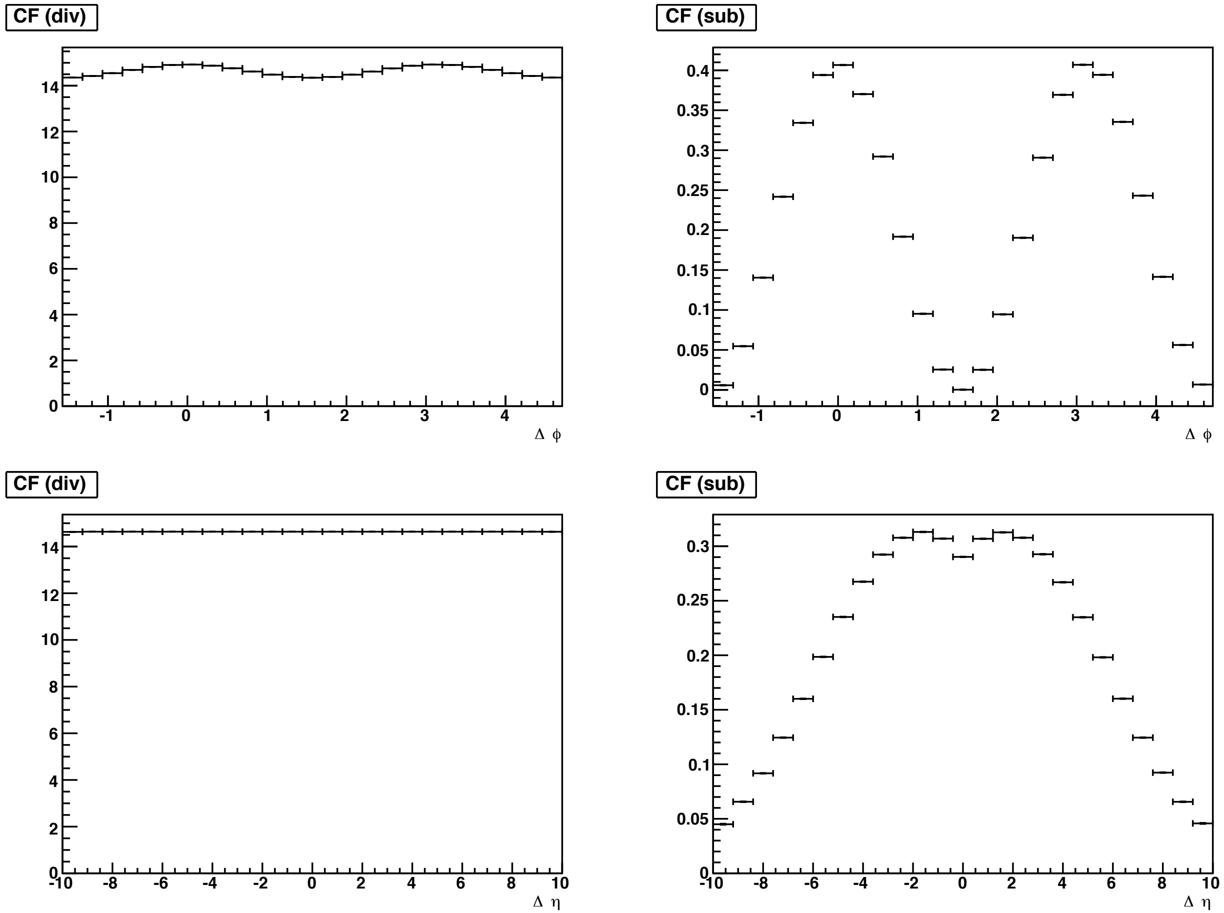
Correlation function for flow

- Different results for the two CF methods
  - in div method, the flow signal is invariant in Δη
  - in sub method, a “camel back” structure remains in Δη



Wide η Correlation function for flow

- In 1D
  - div method appears fine
  - sub method: how to extract v_N at all, as we have no baseline?



Intractable Factors of Two and Averaging

- What is measured for jet-like correlations in symmetric bins ?
- Assume event (*i*) composed of sum of independent N^i sources emitting n_{ij} correlated particles.
- With our way of averaging:

$$
\begin{aligned}
\frac{N_{pair}}{N_{trig}} &= \frac{\sum_{i=1}^{N_{evt}} \sum_{j=1}^{N^i_{source}} \frac{1}{2} n_{ij}(n_{ij}-1)}{\sum_{i=1}^{N_{evt}} \sum_{j=1}^{N^i_{source}} n_{ij}} \\
&= \frac{N_{evt}\langle N_{source}\rangle \frac{1}{2}\langle n(n-1)\rangle}{N_{evt}\langle N_{source}\rangle\langle n\rangle} \\
&= \frac{1}{2}\frac{\langle n(n-1)\rangle}{\langle n\rangle}
\end{aligned}
$$

no source/multiplicity dependence

DO !

<n(n-1)>/<n>

- The ratio <*n*(*n*-1)>/<*n*> can be related to the source size <*n*> and the width σ_n of the particle distribution:

$$\sigma_n^2 = \langle n^2 \rangle - \langle n \rangle^2$$

Hence,

$$\frac{\langle n(n-1) \rangle}{\langle n \rangle} + 1 = \langle n \rangle + \frac{\sigma_n^2}{\langle n \rangle}$$

no factor of 2

("effective cluster size" K_{eff})

For large <*n*> and Poissonian type emission: $K_{eff} \rightarrow$ <*n*> + 1
which is <n> with the trigger condition.

- A more general limit exists for <*n*> small and monotonically falling probability distribution p_n:

Define factorial moments as:

$$f_i = \frac{1}{i!}\langle n(n-1)(n-2)....(n-i+1)\rangle = \langle \binom{n}{i} \rangle$$

In particular,

mean of binomial coefficient

$$\begin{aligned} f_0 &= \langle 1 \rangle = 1 \\ f_1 &= \langle n \rangle \\ f_2 &= \frac{1}{2}\langle n(n-1)\rangle \end{aligned}$$

That's what you get from asymmetric bins.

Number of associated particles under the trigger condition:

$$N_{ass} = \frac{\langle n \rangle}{1-p_0} - 1 = \frac{\sum_{i=2}^{\infty}(-1)^i f_i}{\sum_{i=1}^{\infty}(-1)^{i+1} f_i}$$

2, 3, 4, …, n – particle correlations

In first order:

$$N_{ass} \approx \hat{f}_2 = \frac{f_2}{f_1} = \frac{1}{2}\frac{\langle n(n-1)\rangle}{\langle n \rangle}$$

Factor 2 needed here
to match asymmetric bins

DO !

The expression is exact for self-similar (jet-like) emission = geometric distribution

Corrections with single particle efficiencies

For for the simplest case, efficiency ϵ for all tracks. Assume N particles have been produced, then the probability to reconstruct n particles is:

$$P_n = \binom{N}{n} \epsilon^n (1-\epsilon)^{N-n}$$

Mean number of combinations

binomial coefficients mate !

$$\begin{aligned}
\langle f_k \rangle &= \sum_{n=k}^{N} \binom{n}{k}\binom{N}{n} \epsilon^n (1-\epsilon)^{N-n} \\
&= \frac{1}{k!} \sum_{n=k}^{N} \frac{N!}{(N-n)!(n-k)!} \epsilon^n (1-\epsilon)^{N-n} \\
&= \binom{N}{k} \epsilon^k \sum_{n=0}^{N-k} \frac{(N-k)!}{(N-k-n)!(n)!} \epsilon^n (1-\epsilon)^{N-n} \\
&= \binom{N}{k} \epsilon^k \sum_{n=0}^{N-k} \binom{N-k}{n} \epsilon^n (1-\epsilon)^{N-k-n} \\
&= \binom{N}{k} \epsilon^k
\end{aligned}$$

Weighting each particle with $1/\epsilon$ gives:

$$\langle f_k \rangle_{\text{corrected}} = \binom{N}{k}$$

DO !

(but watch assumption)

Alternative Averaging: Average of Ratios Used by CMS

$$\frac{N_{pair}}{N_{trig}} = \frac{1}{N_{evt}} \sum_{i}^{N_{evt}} \frac{\sum_{j=1}^{N^i_{source}} n_{ij}(n_{ij}-1)}{\sum_{j=1}^{N^i_{source}} n_{i,j}} \tag{2}$$

It is impossible to simplify this expression for the general case. The result depends on the distribution of number of sources. This can be seen by considering two limiting cases.

(1) $N_{source} = 1$

$$\frac{N_{pair}}{N_{trig}} = \langle n-1 \rangle|_{n>0} = \frac{\langle n \rangle}{1-p_0} - 1 \tag{3}$$

The average of ratios measures the number of additional particles under the trigger condition. This is usually called the *number of associated particles* N_{ass}

(2) N_{source} **large** In this case the source average and the event average are equal and the average of ratios is equal to the ratio of averages.

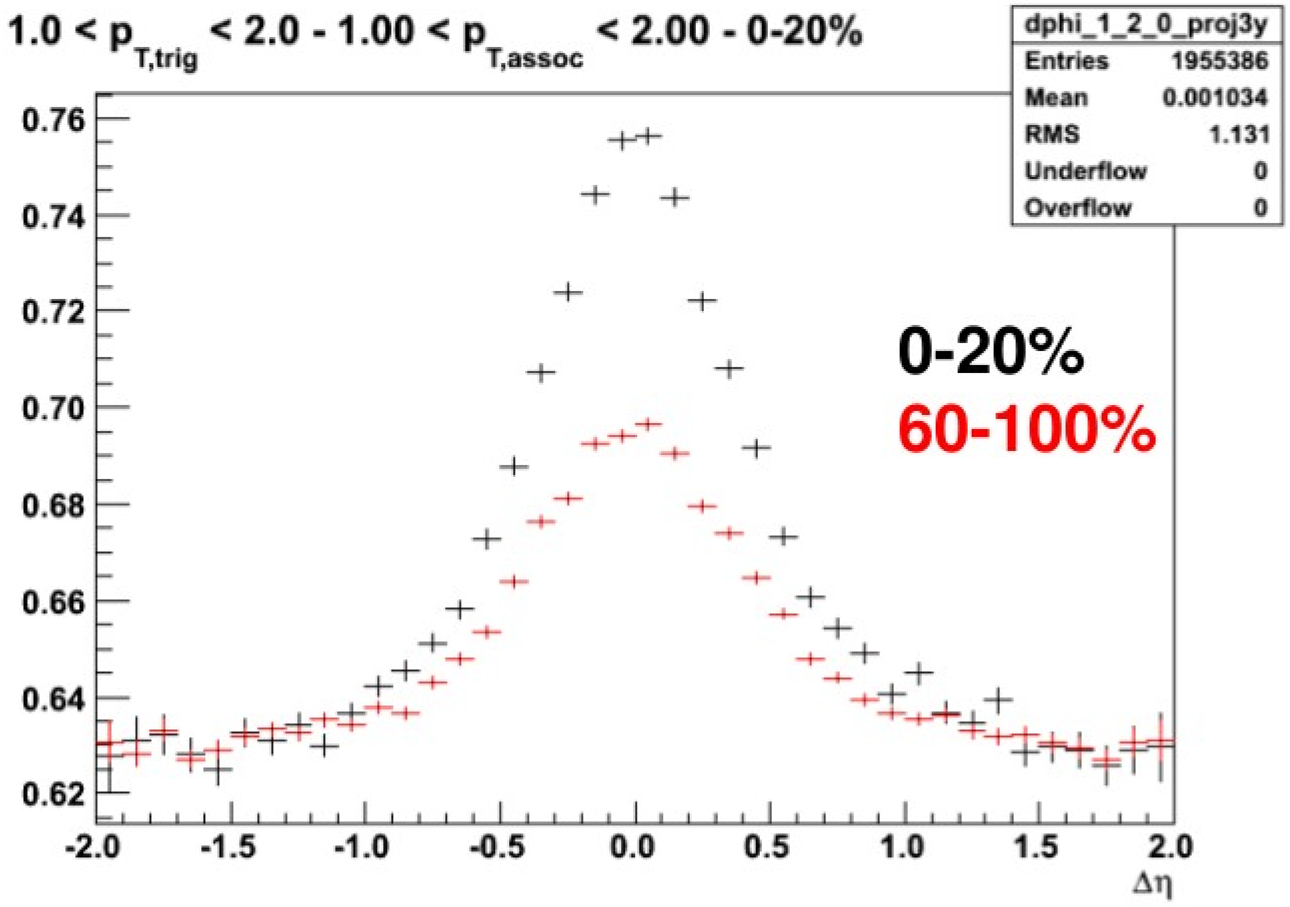
$$\begin{aligned} \frac{N_{pair}}{N_{trig}} &= \frac{1}{N_{evt}} \sum_{i}^{N_{evt}} \frac{\sum_{j=1}^{N^i_{source}} n_{i,j}(n_{ij}-1)}{\sum_{j=1}^{N^i_{source}} n_{i,j}} \\ &= \frac{\langle n(n-1) \rangle}{\langle n \rangle} \end{aligned} \tag{4}$$

Typically factor 2
multiplicity dependence.

DON'T

Average of ratios

ALICE pA √s=5.02 TeV



Strong multiplicity dependence not seen with the ratio of averages.

Event counting

Numerator of (3) in terms of number of events N_n with n triggers:

$$\sum_{n=2}^{\infty} N_n n - \sum_{n=2}^{\infty} N_n = N_{ev}\langle n\rangle - N_1 - (N_{ev} - N_1 - N_0) = N_{ev}\langle n\rangle - (N_{ev} - N_0)$$

Naturally divided by (N_{ev} – N_0) to obtain

$$\frac{N_{ev}\langle n\rangle}{N_{ev} - N_0} - 1 = \frac{\langle n\rangle}{1 - \frac{N_0}{N_{ev}}} = \frac{\langle n\rangle}{1 - p_0} - 1$$

Instead CMS divides by (N_{ev} - N_1 - N_0) to obtain … ?

DON'T

however, it cancels part of the bias induced by the averaging procedure

Mixed event correction

$$B(\Delta\eta,\Delta\varphi)=\alpha\,\mathrm{d}^2 N_{\mathrm{mixed}}/\mathrm{d}\Delta\eta\,\mathrm{d}\Delta\varphi$$

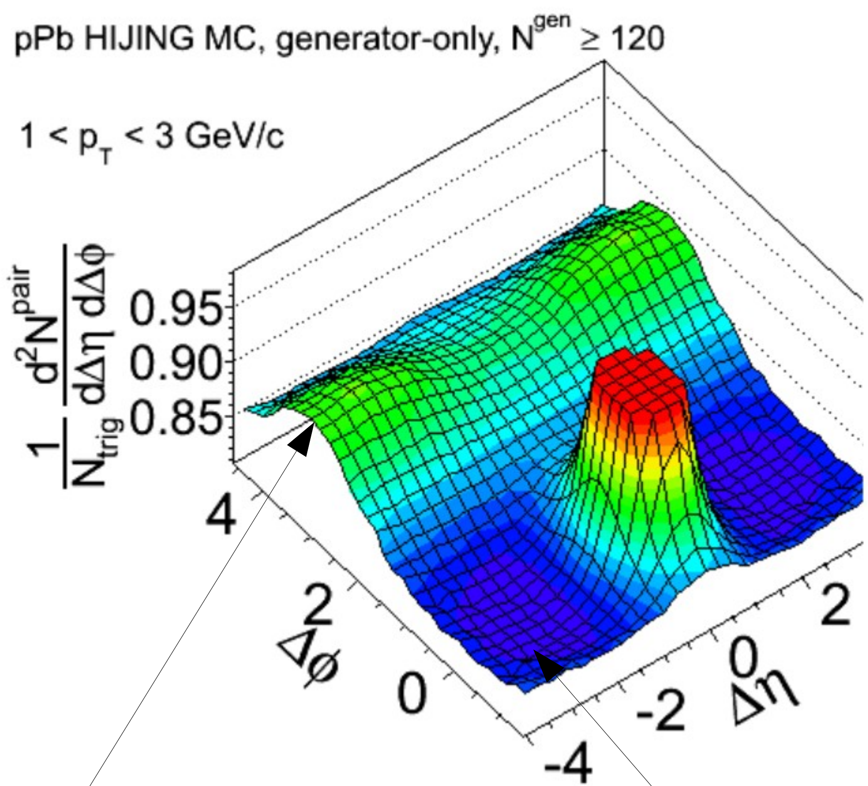
B is an autocorrelation function:

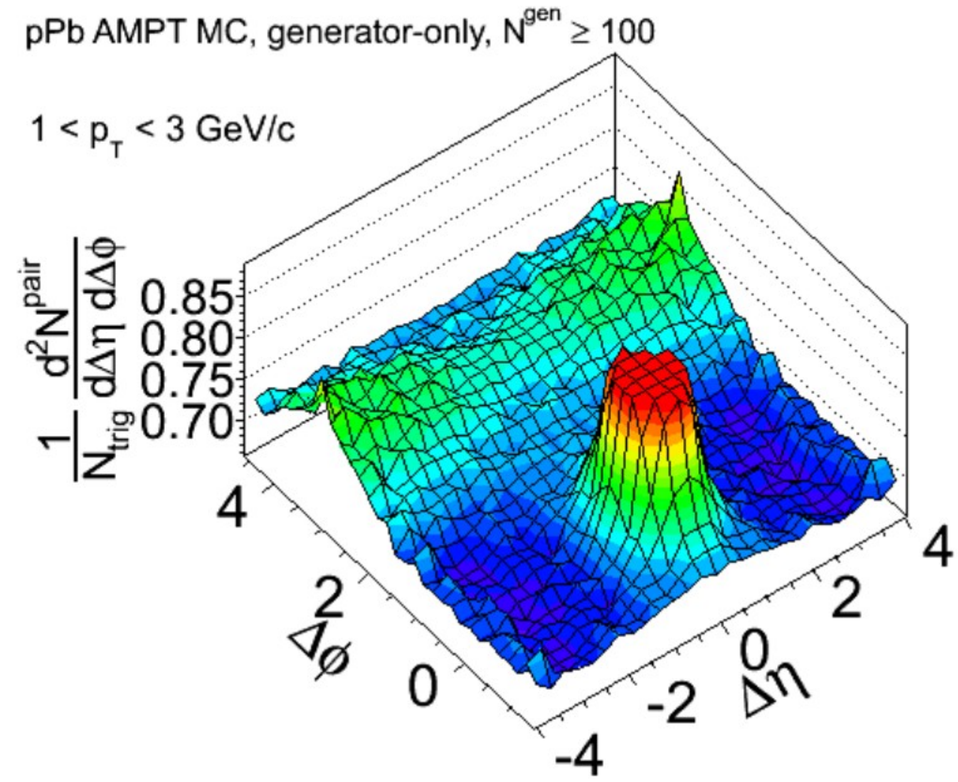
$$B(\Delta\eta,\Delta\varphi)\propto\int d\eta\, f(\eta) f(\eta-\Delta\eta)$$

$$f(\eta)=acceptance(\eta)\,efficiency(\eta)\frac{dN}{d\eta}(\eta)$$

The last term assures a flat uncorrelated background even if dN/dη varies within the acceptance. However, it also distorts the signal in case it is additive.

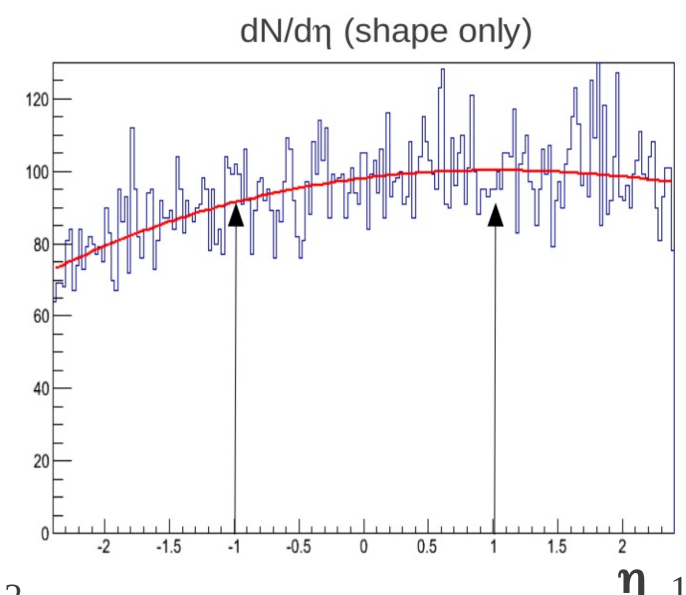
https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsHIN12015





wings due to mixed event correction

flat uncorrelated background



DON'T

but ok for ALICE acceptance