Progress report 07/28/2009: SoftSUSY modification

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ABSTRACT: In this progress report, we explain how and what we have modified in SoftSUSY v3.0.7 for implementing the deflected mirage mediation scenario (two-scale model) in detail.

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1. Background

To perform a serious analysis of the low energy SUSY phenomenology of a given high-scale model, it is necessary to have a very good RG running and spectrum calculator. There are several good programs in the market, but most do not allow for possible threshold corrections at intermediate scale, which are necessary in deflected mirage mediation. We will call the class of models with intermediate-scale threshold corrections as two-scale models. Our deflected mirage mediation is one typical example.

In this work, we modify a commercial RG-running code so that it can be used to analyze such two-scale models. We also incorporate the particular threshold corrections and beta function coefficients that arise in deflected mirage mediation in the code.

The commercially available codes for soft-term RGE are listed in the following table.

code name	specialty
Suspect	Fortran77, popular, used in SUSYHIT
SoftSUSY	C++, loop correction is fully implemented
SPheno	Fortran90, development seems stopped
CPSuperH	Fortran, can treat CP violation

Among these codes, we have chosen to modify SoftSUSY [1, 2]. This decision was made for the following reasons:

- In deflected mirage mediation, a wino LSP can naturally arise, which can result in a highly degenerate chargino and neutralino. In such cases, loop corrections to physical mass spectrum are important. To our knowledge, only SPheno and SoftSUSY treat this issue correctly.
- SoftSUSY is designed with the most modular structure among the codes.
- Ian-Woo is familiar with C++ and SoftSUSY, and has previously modified the SoftSUSY code.

2. Structure of RG running code: SoftSUSY

The RG running codes for SUSY soft terms typically have the following iteration structure:

- **Step 1.** initialize memory, fixed experimental values (like low energy gauge couplings, quark masses), etc.
- **Step 2.** guess the GUT scale by running g_1 and g_2 up from their known values at the EW scale. (Here, only the MSSM spectrum is assumed, and no EW threshold corrections are considered.)
- **Step 3.** apply the boundary conditions at the GUT scale.
- **Step 4.** run all the parameters down to the EW scale¹.
- **Step 5.** calculate the physical spectrum using the EW scale parameters.
- **Step 6.** calculate the EW threshold corrections due to the sparticles.
- **Step 7.** obtain the GUT scale by running g_1 and g_2 up including the EW threshold corrections.
- **Step 8.** iterate steps 3 through 7 until the GUT scale and the soft spectrum approach a certain fixed point.

To implement the procedure for *two-scale models*, we have to modify step 2, step 4 and step 7, as follows:

- **Newstep 2.** guess the GUT scale by running g_1 and g_2 up from the EW scale with modified beta functions above the intermediate threshold scale, $M_{\text{threshold}}$.
- Newstep 4-1. run all the parameters down to the intermediate scale.
- Newstep 4-2. apply the threshold correction at $M_{\text{threshold}}$.
- Newstep 4-3. run all the parameters down to the EW scale.
- **Newstep 7.** obtain the GUT scale by running g_1 and g_2 up including the EW threshold corrections, using the modified beta functions above $M_{\text{threshold}}$.

One complication is that SoftSUSY "remembers" the current RG scale. Hence, when we run the parameters to some scale, we have to consider whether the current scale is below or above $M_{\rm threshold}$ and whether the running process passes through $M_{\rm threshold}$. Our approach is to split the RG running routine run into run_high and run_low, which indicate running above $M_{\rm threshold}$ and running below $M_{\rm threshold}$, respectively. We then have two kinds of threshold boundary conditions, deflectedmirageBcsThresholdUp and deflectedmirageBcsThresholdDown, depending on whether the current RG running is

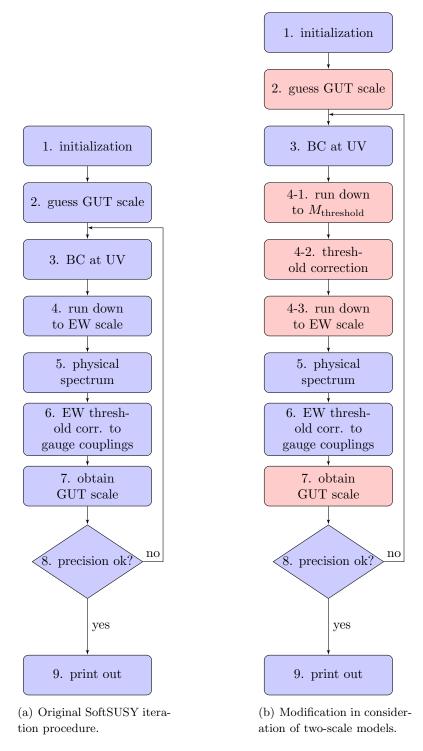


Figure 1: Comparison of the flowcharts of iteration scheme for original SoftSUSY and the modification for two-scale models.

towards the UV or the IR. The threshold corrections are added when running down and

subtracted when running up.

SoftSUSY is designed in an object-oriented way. Its main functions are modularized in terms of classes and objects, and common behavior between objects are hierarchically structured using inheritance. Here, we summarize the relevant parts for MSSM RG running in the SoftSUSY code.

class	header file	implementation	comment
RGE	rge.h	rge.cpp	generic interface class
			for RG running
MssmSusy	susy.h	susy.cpp	subclass of RGE implementing
			supersymmetric MSSM
SoftParsSusy	softpars.h	softpars.cpp	subclass of MssmSusy implementing
			MSSM with soft terms
MssmSoftsusy	softsusy.h	softsusy.cpp	subclass of SoftParsSusy with
			low energy spectrum calculation

RGE provides common virtual² methods: (i) run which make the parameters run from a certain input scale to another scale, and (ii) runto, which run from the current scale to another scale, respectively. run and runto call beta for obtaining the RG equations, and call the Runge-Kutta integration routine, callRK for the actual calculation. beta is undefined at the level of the class RGE, which makes RGE an abstract class that defines a common interface for all the RG running procedure.

MssmSusy inherits RGE, and declares renormalizable superpotential and gauge couplings for the supersymmetric part of the MSSM (i.e. no soft terms). In this class, beta is defined as the β -functions for the supersymmetric parameters. For the gauge couplings, the β -functions are given by $B_a^{(1)}$ at the one-loop level and by $B_{ab}^{(2)}$ and $C_a^{u,d,e}$ at the two-loop level, where the indices a,b run over $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gauge groups, as defined in [3]. The method setBetas does this job. For the Yukawa couplings and supersymmetric bilinear terms, the RG evolution results from the anomalous dimensions. The anomalous dimensions are calculated in anomalousDimension, which in turn calls getOneLpAnom and getTwoLpAnom for one-loop and two-loop anomalous dimensions, respectively, as the names suggest.

SoftParsMssm inherits MssmSusy, but also includes the soft supersymmetry breaking parameters of the MSSM. The β -function for the soft parameters are given in beta2.

MssmSoftsusy is the class in which the main iteration routine for the spectrum calcuation exists. The main routine that calculates the low energy spectrum using RGE for the softly broken MSSM is lowOrg. The iteration in this procedure is separated as a different method, itLowsoft. During iteration, SoftSUSY calculates the EW scale threshold corrections for the gauge couplings by calling the method sparticleThresholdCorrections of MssmSoftsusy. In the design of SoftSUSY, the UV boundary conditions are given in the form of a functional argument of lowOrg method as a user input.

¹The EW scale is usually chosen as $M_S = \sqrt{M_{\tilde{t}_1} M_{\tilde{t}_2}}$.

² "Virtual" means late binding in C++

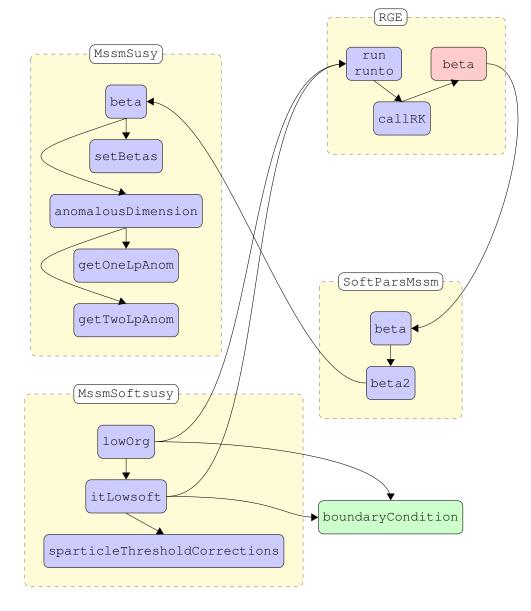


Figure 2: Function call scheme in SoftSUSY.

In Fig. 2, we show how SoftSUSY routines call another routine in the RG iteration process. We omit the detailed explanation here.

3. Beta function coefficient

SoftSUSY uses exactly the same formulas presented in [3].

We consider the new contributions to the beta functions from additional SU(5) complete multiplets. At one-loop, their contributions only involve gauge interactions. We will not include the effect of any Yukawa couplings among the MSSM matter fields and additional matter. Though they can be present at two loops, we will assume they are small.

We will now summarize the new contributions to the running that result from the additional matter fields. The beta functions that are modified are

 $\begin{array}{lll} \beta_g^{(L)}: & \text{the β-function coefficient for the gauge couplings,} \\ \beta_M^{(L)}: & \text{the β-function coefficient for the gaugino masses,} \\ \gamma_i^{(L)j}: & \text{the anomalous dimension matrix,} \\ \left[\beta_h^{(L)}\right]^{ijk}: & \text{the β-function coefficient for the trilinear couplings,} \\ \left[\beta_b^{(L)}\right]^{ij}: & \text{the β-function coefficient for the bilinear couplings,} \\ \left[\beta_{m^2}^{(L)}\right]^{j}: & \text{the β-function coefficient for the soft scalar masses.} \end{array}$

In the above, L denotes the loop order and i, j, k denote flavor and generation indices. The schematic expressions for the beta function coefficients are given by Eq. (2.1)-(2.20) in [3]. The relevant parts for the additional matter field contributions are

$$\beta_g^{(1)} \to g^3 \{ S(R) - 3C(G) \}$$
 (3.1)

$$\beta_q^{(2)} \to g^5 \left\{ -6[C(G)]^2 + 2C(G)S(R) + 4S(R)C(R) \right\}$$
 (3.2)

$$\beta_M^{(1)} \to g^2 \left\{ 2S(R) - 6C(G) \right\} M$$
 (3.3)

$$\beta_M^{(2)} \to g^2 \left\{ -24[C(G)]^2 + 8C(G)S(R) + 16S(R)C(R) \right\} M$$
 (3.4)

$$\gamma_i^{(2)j} \to 2\delta_i^j g^4 \left\{ C(i)S(R) + 2C(i)^2 - 3C(G)C(i) \right\}$$
 (3.5)

$$\left[\beta_h^{(2)}\right]^{ijk} \to (2h^{ijk} - 8MY^{ijk})g^4[C(k)S(R) + 2C(k)^2 - 3C(G)C(k)] + (k \leftrightarrow i) + (k \leftrightarrow j)$$
(3.6)

$$\left[\beta_b^{(2)}\right]^{ij} \to (2b^{ij} - 8\mu^{ij}M)g^4[C(i)S(R) + 2C(i)^2 - 3C(G)C(i)] + (i \leftrightarrow j)$$
 (3.7)

$$\left[\beta_{m^{2}}^{(2)}\right]_{i}^{j} \to 8g^{4}t_{i}^{Aj}\operatorname{Tr}\left[t^{A}C(r)m^{2}\right]
+\delta_{i}^{j}g^{4}MM^{\dagger}\left[24C(i)S(R) + 48C(i)^{2} - 72C(G)C(i)\right]
+8\delta_{i}^{j}g^{4}C(i)\left\{\operatorname{Tr}\left[S(r)m^{2}\right] - C(G)MM^{\dagger}\right\},$$
(3.8)

where C(i) is the quadratic Casimir of the representation i, S(R) is the Dynkin index summed over all chiral multiplets, and S(R)C(R) is the sum of the Dynkin indices weighted by the quadratic Casimir. Note that there are no additional corrections for the one-loop anomalous dimension and beta functions except for the gauge couplings and the gaugino masses. The above shorthand notation must be treated correctly when we consider product groups. More precisely,

$$g^{5}S(R)C(R) \to \sum_{b} g_{a}^{3}g_{b}^{2}S_{a}(R)C_{b}(R),$$
 (3.9)

$$16g^4S(R)C(R)M \to 8\sum_b g_a^2 g_b^2 S_a(R)C_b(R)(M_a + M_b), \tag{3.10}$$

$$g^4 C(r)^2 \to \sum_a \sum_b g_a^2 g_b^2 C_a(r) C_b(r),$$
 (3.11)

$$48g^{4}MM^{\dagger}C(i)^{2} \rightarrow \sum_{a}\sum_{b}g_{a}^{2}g_{b}^{2}C_{a}(i)C_{b}(i)\left[32M_{a}M_{a}^{\dagger}+8M_{a}M_{b}^{\dagger}+8M_{b}M_{a}^{\dagger}\right],(3.12)$$

$$g^4 t_i^{Aj} \text{Tr} \left[t^A C(r) m^2 \right] \to \sum_a \sum_b g_a^2 g_b^2 (t_a^A)_i^j \text{Tr} \left[t_a^A C_b(r) m^2 \right],$$
 (3.13)

$$g^4 C(i) \text{Tr} [S(r)m^2] \to \sum_a \sum_b g_a^2 g_b^2 (q_a)_i \text{Tr} [q_a q_b m^2],$$
 (3.14)

where the last rule applies only to U(1) gauge groups.

We have developed a program for obtaining the above contributions from additional SU(5) multiplets in haskell. Currently, we have obtained the contributions from $\mathbf{5}$, $\bar{\mathbf{5}}$ pairs, $\mathbf{10}$, $\overline{\mathbf{10}}$ pairs and $\mathbf{24}$ s. In a later progress note, we will explain further details of the program[4].

For now, we present the resultant expressions. In what follows, we denote the number of $\mathbf{5}$, $\bar{\mathbf{5}}$ pairs by N_5 , the number of $\mathbf{10}$, $\overline{\mathbf{10}}$ pairs by N_{10} and the number of $\mathbf{24}$ s by N_{24} .

For one loop gauge couplings, the new contribution is simply N_{index} :

$$\frac{\Delta \beta_{g1}^{(1)}}{g_1^3} = \frac{\Delta \beta_{g2}^{(1)}}{g_2^3} = \frac{\Delta \beta_{g3}^{(1)}}{g_3^3} = N_5 + 3N_{10} + 5N_{24} \equiv N_{\text{index}}, \tag{3.15}$$

and hence the gaugino masses are

$$\frac{\Delta \beta_{M1}^{(1)}}{g_1^2 M_1} = \frac{\Delta \beta_{M2}^{(1)}}{g_2^2 M_2} = \frac{\Delta \beta_{M3}^{(1)}}{g_3^2 M_3} = 2N_{\text{index}}.$$
 (3.16)

At two loops, it is convenient to define

$$\Delta B_{ab}^{(2)} = \begin{pmatrix} \frac{7}{15} & \frac{9}{5} & \frac{32}{15} \\ \frac{3}{5} & 7 & 0 \\ \frac{4}{15} & 0 & \frac{34}{3} \end{pmatrix} N_5 + \begin{pmatrix} \frac{23}{5} & \frac{3}{5} & \frac{48}{5} \\ \frac{1}{5} & 21 & 16 \\ \frac{6}{5} & 6 & 34 \end{pmatrix} N_{10} + \begin{pmatrix} \frac{50}{3} & 30 & \frac{160}{3} \\ 10 & 90 & 32 \\ \frac{20}{3} & 12 & \frac{460}{3} \end{pmatrix} N_{24}.$$
 (3.17)

The new contributions for two loop beta functions of the gauge couplings and gaugino masses are given by

$$\Delta \beta_{g_a}^{(2)} = \sum_{b=1}^{3} \left[\Delta B_{ab}^{(2)} \right] g_a^3 g_b^2, \tag{3.18}$$

$$\Delta \beta_{M_a}^{(2)} = 2 \sum_{b=1}^{3} \left[\Delta B_{ab}^{(2)} \right] g_a^2 g_b^2 (M_a + M_b). \tag{3.19}$$

The additional contributions to the two loop anomalous dimensions are

$$\Delta \gamma_{h1}^{(2)} = \Delta \gamma_{h2}^{(2)} = \Delta \gamma_l^{(2)} = \left\{ \frac{3}{10} g_1^4 + \frac{3}{2} g_2^4 \right\} N_{\text{index}}, \tag{3.20}$$

$$\Delta \gamma_e^{(2)} = \frac{6}{5} g_1^4 N_{\text{index}}, \tag{3.21}$$

$$\Delta \gamma_q^{(2)} = \left\{ \frac{1}{30} g_1^4 + \frac{3}{2} g_2^4 + \frac{8}{3} g_3^4 \right\} N_{\text{index}}, \tag{3.22}$$

$$\Delta \gamma_d^{(2)} = \left\{ \frac{2}{15} g_1^4 + \frac{8}{3} g_3^4 \right\} N_{\text{index}}, \tag{3.23}$$

$$\Delta \gamma_u^{(2)} = \left\{ \frac{8}{15} g_1^4 + \frac{8}{3} g_3^4 \right\} N_{\text{index}}.$$
 (3.24)

The contributions to the two loop beta functions for the soft scalar mass-squared parameters are

$$\Delta \beta_{m^2(H2)}^{(2)} = \left\{ \frac{18}{5} g_1^4 M_1^2 + 18 g_2^4 M_2^2 \right\} N_{\text{index}}, \tag{3.25}$$

$$\Delta \beta_{m^2(H1)}^{(2)} = \left\{ \frac{18}{5} g_1^4 M_1^2 + 18 g_2^4 M_2^2 \right\} N_{\text{index}}, \tag{3.26}$$

$$\Delta\beta_{m^2(q)}^{(2)} = \left\{ \frac{2}{5} g_1^4 M_1^2 + 18 g_2^4 M_2^2 + 32 g_3^4 M_3^2 \right\} N_{\text{index}}, \tag{3.27}$$

$$\Delta \beta_{m^2(l)}^{(2)} = \left\{ \frac{18}{5} g_1^4 M_1^2 + 18 g_2^4 M_2^2 \right\} N_{\text{index}}, \tag{3.28}$$

$$\Delta \beta_{m^2(u)}^{(2)} = \left\{ \frac{32}{5} g_1^4 M_1^2 + 32 g_3^4 M_3^2 \right\} N_{\text{index}}, \tag{3.29}$$

$$\Delta \beta_{m^2(d)}^{(2)} = \left\{ \frac{8}{5} g_1^4 M_1^2 + 32 g_3^4 M_3^2 \right\} N_{\text{index}}, \tag{3.30}$$

$$\Delta \beta_{m^2(e)}^{(2)} = \frac{72}{5} g_1^4 M_1^2 N_{\text{index}}, \tag{3.31}$$

The beta functions of the trilinear terms and the bilinear terms are modified as follows.

$$\Delta \beta_{h_u}^{(2)} = \left\{ \left[\frac{13}{15} g_1^4 + 3g_2^4 + \frac{16}{3} g_3^4 \right] h_u + \left[-\frac{52}{15} g_1^4 M_1 - 12g_2^4 M_2 - \frac{64}{3} g_3^4 M_3 \right] Y_u \right\} N_{\text{index}},$$

$$\Delta \beta_{h_d}^{(2)} = \left\{ \left[\frac{7}{15} g_1^4 + 3g_2^4 + \frac{16}{3} g_3^4 \right] h_d \right\}$$
(3.32)

$$+ \left[-\frac{28}{15} g_1^4 M_1 - 12 g_2^4 M_2 - \frac{64}{3} g_3^4 M_3 \right] Y_d \} N_{\text{index}}, \tag{3.33}$$

$$\Delta \beta_{h_e}^{(2)} = \left\{ \left[\frac{9}{5} g_1^4 + 3g_2^4 \right] h_e + \left[-\frac{36}{5} g_1^4 M_1 - 12g_2^4 M_2 \right] Y_e \right\} N_{\text{index}}.$$
 (3.34)

$$\Delta \beta_B^{(2)} = \left\{ \left[\frac{3}{5} g_1^4 + 3g_2^4 \right] B + \left[-\frac{12}{5} g_1^4 M_1 - 12g_2^4 M_2 \right] \mu \right\} N_{\text{index}}$$
 (3.35)

4. Sample results

In this section, we present particle spectra and RG running for the test input parameter sets. The results are obtained by using the program dmmsoftsusy for which the usage is provided in the appendix A. The test parameter sets, Sample 1 and Sample 2, are shown in Table 1. We assume that both higgs fields have the same modular weight n_H and all matter fields have the same modular weight n_M for simplicity. In addition, we assume that the messenger pairs are $\mathbf{5}$, $\bar{\mathbf{5}}$ representations, and do not consider other possibilities.

Set	n_H	n_M	α_m	α_g	N_5	M_0	$M_{ m mess}$	$\tan \beta$	$\operatorname{sgn} \mu$
Sample 1	1	$\frac{1}{2}$	1	1	3	1000	10^{10}	10	+1
Sample 2	1	$\frac{\overline{1}}{2}$	1	0.5	1	2000	10^{4}	20	+1

Table 1: Sample parameter sets chosen for testing the new code. All the mass parameters are in GeV. We do not consider the cases with nonvanishing N_{10} and N_{24} in this test.

	San	nple 1		Sample 2			
h	113	H, A	793	h	122	H, A	1721
$ ilde{g}$	182	H^\pm	797	$ ilde{g}$	2120	H^\pm	1723
χ_1^0	382	χ_2^0	447	χ_1^0	1232	χ_2^0	1388
χ_3^0	730	χ_4^0	742	χ_3^0	1643	χ_4^0	1657
$\begin{array}{c} \chi_3^0 \\ \chi_1^{\pm} \end{array}$	382	χ_2^{\pm}	744	χ_1^{\pm}	1390	χ_2^{\pm}	1666
$ ilde{u}_L$	1040	\tilde{u}_R	999	\tilde{u}_L	2694	\tilde{u}_R	2612
$ ilde{d}_L$	1046	$ ilde{d}_R$	1003	$ ilde{d}_L$	2705	$ ilde{d}_R$	2610
$ ilde{e}_L$	784	$ ilde{e}_R$	714	$ ilde{e}_L$	1645	\tilde{e}_R	1475
$ ilde{ u}_e$	780			$ ilde{ u}_e$	1643		
$ ilde{t}_1$	757	$ ilde{t}_2$	964	\tilde{t}_1	2142	$ ilde{t}_2$	2495
$ ilde{b}_1$	935	$ ilde{b}_2$	999	$ ilde{b}_1$	2468	$ ilde{b}_2$	2564
$ ilde{ au}_1$	708	$ ilde{ au}_2$	783	$ ilde{ au}_1$	1431	$ ilde{ au}_2$	1629
$\tilde{ u}_{ au}$	778			$ ilde{ u}_{ au}$	1624		

Table 2: The MSSM particle mass spectrum for Sample 1 and Sample 2. All masses are in GeV.

The numerical results for the physical spectrum in each case are given in Table 2. In Fig. 3 and 4, we present the RG running of the soft parameters for Sample 1 and Sample 2, respectively. Note that in the figures, we show $m_i^2/\sqrt{\left|m_i^2\right|}$ for the soft scalar mass squared parameters.

A. Manual for dmmsoftsusy

In this section, we explain how to use the user interface program dmmsoftsusy. To run dmmsoftsusy, enter

% dmmsoftsusy inputfile outputfile

in the terminal. The input file look like the following.

run on 08/13/09

method scan scanmethod lattice option omega on

comment can be written with # at the beginning of a line.

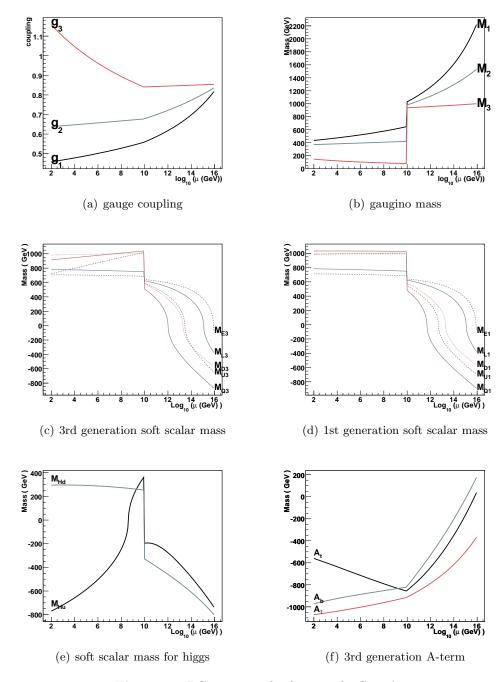


Figure 3: RG running of soft terms for Sample 1

```
\rm MO = [1000.0, 2000.0, 100.0] # GeV Unit ( this is also a comment. ) alpham= 1.0 alphag = 0.0 \rm nM = 0.5 \rm nH = 1.0
```

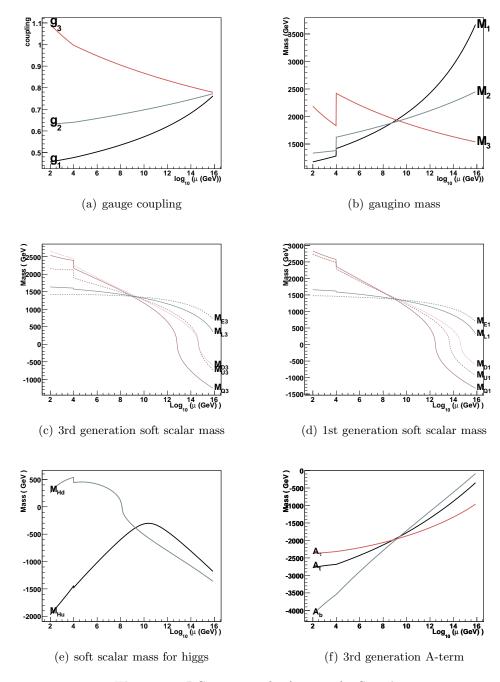


Figure 4: RG running of soft terms for Sample 2

```
N5 = 3

N10= 0

N24 = 0

sgnMu = +1

tanb = 10.0

Mmess = [1.0e5, 1.0e7, 10.0]
```

parameter	name	parameter	name
M_0	MO	α_m	alpham
$M_{3/2}$	m32	$lpha_g$	alphag
$M_{ m mess}$	Mmess	an eta	tanb
$sgn(\mu)$	sgnMu	N_5	N5
N_{10}	N10	N_{24}	N24
$n_{Q,U,D,L,E}$	${\tt nQ,nU,nD,nL,nE}$	n_{H_u,H_d}	nHu , nHd
n_{matter}	nM	$n_{ m higgs}$	nH

Table 3: Parameters for dmmsoftsusy.

We can add comments in the input file using #. The input file starts with method. Currently, dmmsoftsusy provides three methods single, rgrun and scan. By single method, one can generate a single spectrum and write the spectrum to the output file. By rgrun method, one can generate RG running of soft parameters for a single parameter set and record the RG running to the output file. If we choose scan, dmmsoftsusy will scan parameters. For the scan method which is given by scanmethod, two options are available, random and lattice. The meaning of each scan method is clear by their names. For random, the range of the parameters that we want to scan must be given as a pair

```
parameter = [ from , to ]
```

where parameter denotes the name of the parameter, and from and to mean the lower limit and the upper limit of the range, respectively. For lattice, the range of the scanning parameters must be given as a triple

```
parameter = [ from , to , inc ]
```

where inc specifies an incremental amount for the lattice scan. If more than one parameters are scanned in lattice method, dmmsoftsusy will loop over all the specified scanning variables. In the current version, scannable continuous parameters are M_0 , α_m , α_g , M_{mess} and $\tan \beta$.

In Table. 3, we show the name of parameters used in dmmsoftsusy. The modular weights for matter and higgs can be collectively given by nM and nH, respectively.

References

- [1] B. C. Allanach, Comput. Phys. Commun. 143, 305 (2002) [arXiv:hep-ph/0104145].
- [2] B. C. Allanach and M. A. Bernhardt, arXiv:0903.1805 [hep-ph].
- [3] S. P. Martin and M. T. Vaughn, Phys. Rev. D 50, 2282 (1994) [Erratum-ibid. D 78, 039903 (2008)] [arXiv:hep-ph/9311340].
- [4] Ian-Woo Kim, Yongyan Rao, work in progress.