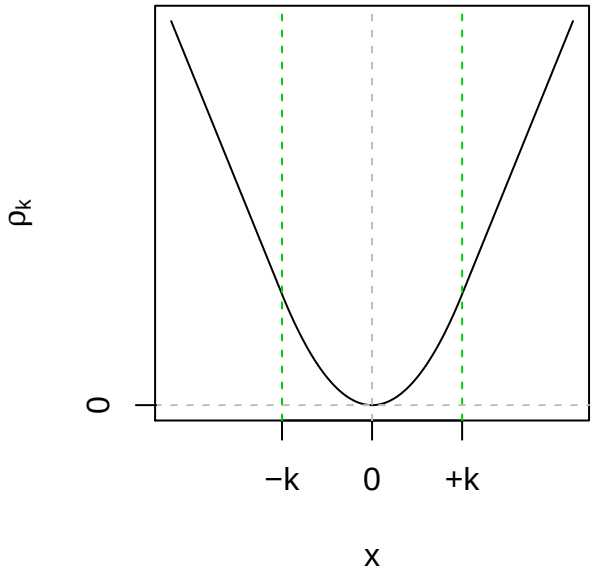
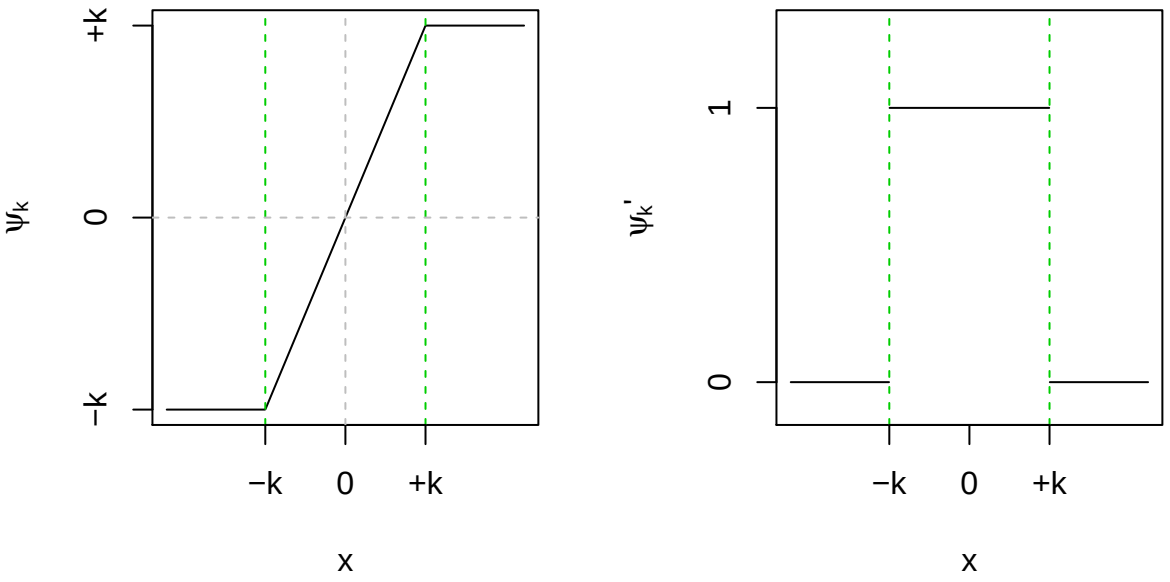
Smoothed Huber Loss

Huber loss, a robust loss function proposed by Huber in *Robust Estimation of a Location Parameter* (Annals of Statistics, 1964) is defined piecewise by

$$\rho_k(x) = \begin{cases} \frac{1}{2}x^2 & |x| \le k \\ k\,|x| - \frac{1}{2}k^2 & |x| > k \end{cases}$$



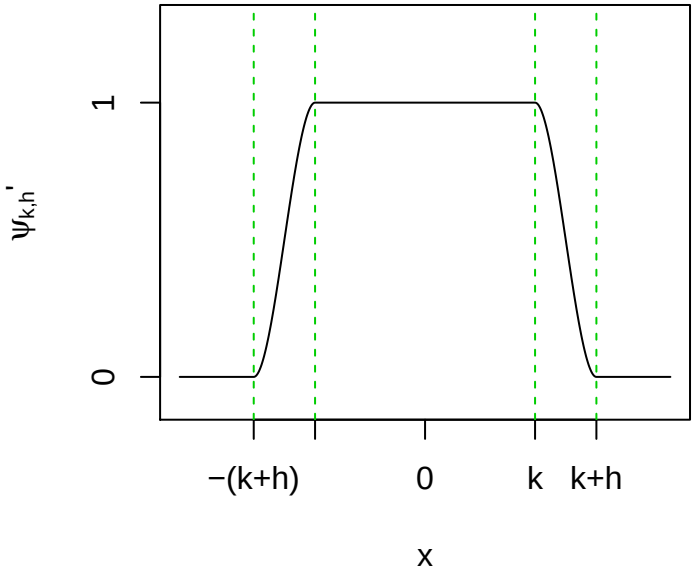
Denoting the first and second derivatives of the Huber loss as ψ_k and $\psi_k^{'}$ respectively, the shape of these functions is demonstrated below:



Notice that the second derivative has discontinuities at $x = -k$ and $x = k$.

Suppose instead that we want a similar function that decays smoothly from 1 to 0 over an interval $(k, k + h)$

(and likewise $(-k, -(k+h))$). The functions in this repository achieve this by interpolating with a piecewise section of a third-degree polynomial, which results in a continuously differentiable second derivative.



From this smoothed second derivative, the corresponding first derivative of the smoothed Huber loss and the smoothed Huber loss itself are also uniquely determined by the requirement that both be continuously differentiable.