Answers to Test 301: BCBE DEDB DEDB

Serial Number: 301 Name:

ECE 3300 Fall 2016 (Signals, Systems, and Transforms): Exam III

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in COURSE; you may leave SECTION blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. Circle your answers on the test and bubble in the corresponding answers on your scantron. The examination lasts 60 minutes and you may use three sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

Question 1: Determine x[n] if  $X(e^{j\omega}) = \frac{4e^{-2j\omega}}{2+e^{-j\omega}}$ . The answer has the form  $A(\frac{1}{B})^{n-C}u[n-C]$ . What is A+B+C? Choose the closest answer. *Hint:* Work from the fact that, if -1 < a < 1,  $a^nu[n] \leftrightarrow \frac{1}{1-ae^{-j\omega}}$ .

A: 10.

B: 2.

C: 8.

D: 4.

E: 6.

Question 2: Suppose  $\tilde{x}(t)$  has fundamental cycle x(t) and fundamental period  $T_0 = 4\pi$ . If the Fourier transform of the fundamental cycle is  $X(j\omega) = \frac{4\pi}{\omega^2} \sin^2(\frac{\pi\omega}{2})$ , determine the Fourier series coefficient  $x_1$ . Choose the closest answer.

A: 6.

B: 8.

C: 2.

D: 10.

E: 4.

Question 3: Suppose  $|X(j\omega)| = \sqrt{9 - \omega^2} \operatorname{rect}(\frac{\omega}{6})$ . Determine the percentage of energy in the frequency band (0,1). Choose the closest answer. Hint: Recall that  $rect(\frac{\omega}{2a}) = 1$  for  $-a < \omega < a$  and 0 otherwise.

A: 30%.

B: 50%.

C: 40%.

D: 60%.

E: 70%.

Question 4: Suppose  $x[n] = (\frac{1}{2})^n u[n] - (\frac{1}{3})^n u[n]$ . Determine the magnitude of the Fourier transform. The answer has the form  $|X(e^{j\omega})| = \frac{A}{\sqrt{B-\cos\omega}\sqrt{C-\frac{2}{3}\cos(\omega)}}$ . What is A+B+C? Choose the closest answer.

B: 4.

C: 2.

D: 1.

E: 3.

**Question 5**: Suppose x[n] is a real, even signal, and suppose  $X(e^{j\omega}) = (\frac{1}{2} - \cos \omega)(\frac{\sqrt{3}}{2} + \cos \omega)$ . For  $0 < \omega < \pi$ , in which of the following regions is  $\angle X(e^{j\omega})$  equal to zero?

A:  $\frac{\pi}{6} < \omega < \frac{2\pi}{3}$ .

 $B: \angle X(e^{j\omega})$  is never equal to zero over  $0 < \omega < \pi$ .

C:  $0 < \omega < \frac{\pi}{3}$  and  $\frac{5\pi}{6} < \omega < \pi$ .

$$\begin{split} &D\colon \, \tfrac{\pi}{3} < \omega < \tfrac{5\pi}{6}. \\ &E\colon \, 0 < \omega < \tfrac{\pi}{6} \text{ and } \tfrac{2\pi}{3} < \omega < \pi. \end{split}$$

**Question 6**: Determine the Fourier transform of  $(e^{-t} + e^{-2t})^2 u(t)$ . The answer has the form  $\frac{A}{B+j\omega} + \frac{C}{D+j\omega} + \frac{E}{F+j\omega}$ . What is A+B+C+D+E+F? Choose the closest answer. *Hint:* Expand the square.

A: 14.

B: 15.

C: 12.

D: 11.

E: 13.

Question 7: Suppose  $\tilde{x}(t)$  has period  $T_0 = 4$  and Fourier series coefficients  $x_k = \frac{k^2}{|k|+4}$ . One of the terms in the Fourier transform of  $\tilde{x}(t)$  is  $A\delta(\omega - 2\pi)$ . What is A? Choose the closest answer.

 $A: \pi$ .

 $B: 3\pi.$ 

 $C: 5\pi.$ 

 $D: 4\pi.$ 

 $E: 2\pi.$ 

Question 8: Given that, if a > 0,  $\sqrt{t}e^{-at}u(t) \leftrightarrow \frac{\sqrt{\pi}}{2} \frac{1}{(a+j\omega)^{3/2}}$ , determine the Fourier transform of  $\int_{-\infty}^{t} \sqrt{\tau}e^{-2\tau}u(\tau)d\tau$ . The answer has the form  $\frac{\sqrt{\pi}}{2} \frac{1}{j\omega(2+j\omega)^{3/2}} + \frac{\pi^A}{B}\delta(\omega)$ . What is A + B? Choose the closest answer.

A: 8.

B: 7.

C: 6.

D: 9.

E: 10.

Question 9: Suppose p[n] is a sequence of numbers such that  $\sum_{n=1}^{\infty} p[n]a^n = \ln(G(1-a))$ , where  $G(\cdot)$  is a function. Consider the signal  $x[n] = (\frac{1}{2})^n p[n] u[n-1]$ . Determine  $X(e^{j\omega})$ .

the signal  $x[n] = (\frac{1}{2})^n p[A: \frac{\ln(G(1-e^{j\omega}))}{1-\frac{1}{2}e^{-j\omega}}]$ .  $B: \ln(G(1-\frac{1}{2}e^{j\omega}))$ .  $C: \frac{\ln(G(1-e^{-j\omega}))}{1-\frac{1}{2}e^{-j\omega}}$ .  $D: \ln(G(1-\frac{1}{2}e^{-j\omega}))$ .  $E: (\frac{1}{2})^n \ln(G(1-e^{j\omega}))$ .

Question 10: Suppose x[n] has Fourier transform  $X(e^{j\omega})$ . Determine the Fourier transform of y[n] = x[n+2] - x[n] + x[n-2]. The answer has the form  $(A\cos(B\omega) + C)X(e^{j\omega})$ . What is A + B + C? Choose the closest answer.

A: 2.

B: 4.

C: 5.

D: 1.

E: 3.

Question 11: Given that x(t) has Fourier transform  $\frac{\frac{1}{4}}{(1+2j\omega)^4}$ , determine the Fourier transform of tx(t). The answer has the form  $\frac{A+jB}{(1+2j\omega)^C}$ , where A, B, and C are real. What is A+B+C? Choose the closest answer. A: 9.

B: 6.

C: 8.

D: 7.

E: 10.

Question 12: Suppose  $\tilde{x}(t)$  has period  $T_0 = \pi$  and Fourier series coefficients  $x_k = jk$  if  $-4 \le k \le 4$  and  $x_k = 0$  otherwise. Determine the percentage of power in the frequency band [3,7]. Choose the closest answer.

A: 30%.

B: 40%.

C: 10%.

D: 50%.

E: 20%.