

Aggregate Job Search of the Employed, Unemployed, and Non-Participants*

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Abstract

Using data from the American Time Use Survey, I document a substantial rise in the proportion of job seekers who are classified as out of the labor force; nearly 7 million people since 1980. Accounting for these searchers increases the unemployment rate by 5.2 percentage points, rids the unemployment rate of its downward trend, and decrease volatility by 50%. The paper also delivers a total searcher rate, including employed job seekers, and adjusted labor market flows. I show that this miss-classification has a significant impact on the volatility of key labor market statistics and the persistence of unemployment. Finally, estimates of the Phillips curve using the adjusted unemployment rate or total searcher rate shows no sign of a flattening output-inflation relationship in the post-2008 recession period.

Keywords: Unemployment, Labor Force Participation, Worker Flows, Labor Market Volatility, Phillips Curve

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1 Introduction

In 2019 in the United States, 50% of new hires were workers who were previously classified as out of the labor force, 31% came from a different job and only 19% of new hires were from the pool of individuals considered to be actively searching: the unemployed. This startling statistic is not unique to 2019, [Figure 1](#) shows that the proportion of hires coming from non-participation has increased since 1980. In the face of this fact, the unemployment rate, considered by many as the “best single indicator of current labor market conditions”¹, is abysmal at capturing potential hires.

In this paper I show that the fraction of non-participants who are actively searching for a job has increased since 1980. I construct an adjusted unemployment rate and total searcher rate (including employed job seekers) by estimating a monthly probability that an individual is searching for a job, and classifying individuals as job seekers based on these probabilities rather than reported labor market status. I also adjust labor market flows using the estimated probabilities. Finally, I show that misclassification of workers has significantly impacted two well studied aspects of the macro economy: the volatility of the labor market and the Phillips curve.

The main source of data for job search comes from the American Time User Survey (ATUS). Time use diaries from the ATUS allow us to see search effort by all individuals, regardless of labor market status (employed, unemployed, or out of the labor force). Using the 2003-2019 ATUS diaries, I show that nearly 12% of people classified as out of the labor force are actively searching for a job during the month. I show that women, and women with children are more likely to be classified as non-participants, but they are more likely to be searching for a job from outside the labor force than their male counterparts. Similarly education is a strong predictor of labor market status classification, with more educated people less likely to be classified as non-participants but more likely to be searching for a job if classified so.

Next I estimate a model of the probability an individual is searching for a job based on observable characteristics. To estimate these probabilities I use machine learning on the American Time Use data to inform about which demographic characteristics are the strongest predictors of job search. Using the estimated model, I then predict search effort of all individuals in the Current Population Survey. I classify the non-employed as job seekers probabilistically based on the estimated search probability, rather than deterministically based on reported labor market status. I classify then employed as job seekers in the same fashion. Using these classifications I create an adjusted unemployment rate since 1980. The adjusted unemployment rate is on average 5.2 percentage points higher than the standard

¹Yellen, Janet L. Speech at the 2013 National Association for Business Economics Policy Conference. www.federalreserve.gov/newsevents/speech/yellen20130302a.htm

unemployment rate and does not display the same low frequency downward trend observed in the standard unemployment rate. The differences in level is attributed to the large fraction of job seekers classified as non-participants. The difference in trends is attributed to the fact that the fraction of job seekers classified as non-participants has increase from 9.5% in 1980 to 13% in 2020.

Using the predicted search probabilities I also adjust labor market flows. The adjusted labor market flows are substantially different than the standard flows calculated from the matched Current Population Survey. Matching individuals across consecutive months to calculate transition probabilities had become the standard in calculating labor market flows and these flows are published monthly by the Bureau of Labor Statistics. The most notable difference is along the unemployment/non-participation margin. The standard flows suggest that the probability a person leaves unemployment for non-participation (0.21) is nearly as large as the probability he leaves for employment (0.25). The adjusted flows show that the unemployment to non-participation exit probability is only 0.04.

The aggregate statistics presented here, contribute in several aspects to the understanding of how well standard search and matching models can mimic the labor market. First, I show that the miss-classification of workers has a significant impact of the volatility of the unemployment rate, labor market tightness, job finding and separation rates; all of which are between 25%-40% less volatile when accounting for miss-classification. It is a well established fact that the search and matching model cannot generate the perceived labor market volatility in the standard statistics ([Andolfatto, 1996](#); [Shimer, 2005](#)), and many have since adjusted the model in efforts to achieve more volatility.² To the best of my knowledge, this is the first paper to reassess the data in an effort to close the gap. Second, the adjusted flows reduce the unemployment to non-participation exit probability by 80%, implying that unemployment is a much more persistent state than previously thought. The large and volatile movements between unemployment and non-participation in the standard flows are difficult to match using standard calibrations of search and matching models ([Garibaldi and Wasmer, 2005](#)). More recently, [Krusell et al. \(2017\)](#) show that the only way to match such large movements into and out of the labor force is through relatively large transitory shocks to the disutility of search effort.³ The adjusted flows presented here show these large oscillations between unemployment and non-participation are, in fact, due to measurement error, implying that transitory shocks to the disutility of search are not necessary in understanding participation decisions.

I show that miss-classification has a significant impact on broader macroeconomic implications as well, by re-estimating the Phillips curve. Many papers have investigated the flattening of the output-

²See for example [Hall and Milgrom \(2008\)](#); [Hagedorn and Manovskii \(2008\)](#) and many, many thereafter.

³See [Krusell et al. \(2008, 2010, 2011\)](#) for a complete explanation of how aggregate and idiosyncratic shocks affect labor market flows.

inflation relationship following the Great Recession, often using the unemployment gap, measured as the difference between the Congressional Budget Office’s Natural Rate of Unemployment (NRU) and the standard unemployment rate, as a measure for the output gap ([Ball and Mazumder, 2011](#); [Coibion and Gorodnichenko, 2015](#); [Blanchard, 2016](#)). I show that when using the adjusted unemployment rate and total searcher rate instead, there is no flattening in the post-2008 recession period and that pre-recession estimates are similar across all statistics.

The paper contributes to the literature exploring the effects of miss-classification of individuals on both the level of the unemployment rate ([Abowd and Zellner, 1985](#); [Feng and Hu, 2013](#); [Ahn and Hamilton, 2019](#)) and labor market flows ([Abowd and Zellner, 1985](#); [Poterba and Summers, 1986](#); [Elsby et al., 2015](#)). The method used here differs from the existing studies by using data directly on job search to estimate miss-classification probabilities for the non-employed, rather than re-interview surveys conducted in the 1980’s. Similarly to [Ahn and Hamilton \(2019\)](#), I show that miss-classification has increased over time due to demographic changes; which is consistent with [Perry \(1970\)](#); [Flaim \(1979\)](#); [Shimer \(2001\)](#); [Barnichon and Mesters \(2018\)](#) who suggest that demographic changes decreased the unemployment rate by 1-2 percentage points since 1980.

This paper also adds to a small literature focused on constructing a better measure of labor underutilization for the United States.⁴ [Hornstein et al. \(2014\)](#) construct a non-employment index (NEI) in which they weight all non-employed people by the average transition probabilities to employment on a coarse grid of observable characteristics. [Faberman et al. \(2019\)](#) construct a measure of labor market underutilization by differentiating people by the difference between their hours worked, zero if non-employed, and their desired hours worked. I contribute to this literature by estimating labor slack with data on search effort, instead of ex-post transition probabilities, and constructing a total searchers rate which includes employed job seekers. Currently all aggregate measures of total search effort in the economy include the part time employed based on the reason for part time employment. To the best of my knowledge this is the first such measure constructed from reported search effort.

2 Data

The two main data sources are the basic monthly files from the Current Population Survey (CPS) and the American Time Use Survey (ATUS). The CPS is the main source of data used for calculating aggregate statistics regarding the labor force status of U.S. residents. The survey is conducted monthly, the interview unit is based on the address of the household and all members of the household residing at the address are interviewed. A household is in the survey for 4 months, then out for 8 months, and

⁴[Schweitzer \(2003\)](#) and [Jones et al. \(2003\)](#) attempt similar exercises for the United Kingdom.

then back in for 4 months. Given this rotating-panel element of the CPS, in theory three quarters of the each month's sample can be longitudinally linked to the prior month. In practice however, only about two-thirds of the sample can be linked due to households moving. The survey asks a variety of questions related to labor market attachment; then, people are classified as unemployed if they have made at least one active search effort during the past 4 weeks and are available to work. All other non-employed individuals are classified as out of the labor force.⁵

This broad classification of labor force status is advantageous in many respects, but needless to say, not perfect. Misclassification of people across labor market states in the CPS is a well documented fact. [Abowd and Zellner \(1985\)](#) and [Poterba and Summers \(1986\)](#) show that misclassification happens along all margins using data from the reinterview surveys conducted by the CPS on a subset of individuals. The largest error occurs among people that are at first classified as out of the labor force and later reclassified as unemployed. Similarly, [Ahn and Hamilton \(2019\)](#) show that two-thirds of people who were classified as out of the labor force last month and unemployed this month report having an unemployment duration of longer than 4 weeks. [Krueger et al. \(2017\)](#) document that people are more likely to get misclassified as out of the labor force the longer they stay in the survey, which [Halpern-Manners and Warren \(2012\)](#) suggest may be due to the shame carried by admitting, month after month, that they were unable to find a job. However, [Flinn and Heckman \(1983\)](#) find that, at least until the early 1980's, unemployment and out of the labor force are behaviorally distinct states.

Despite measurement issues, the CPS data have become, not only the standard source for labor market stocks, but also the main source for estimating the flows across labor market states. The Bureau of Labor Statistics publishes the flows across labor market states beginning in 1990 and many others have calculated the flows using the linked microdata, see for example [Shimer \(2012\)](#) or [Elsby et al. \(2015\)](#). Beginning with the 1994 redesign of the CPS, [Fallick and Fleischman \(2004\)](#) show that it is possible to observe employment to employment transitions as well.

In what follows, I present evidence using hires that suggests miss-classification problem is large and that the unemployment rate itself is not a good measure of labor market slack. Then using the ATUS I show how miss-classification can be revisited in the time use diaries and used to estimate aggregate search statistics for the United States.

2.1 Hires by Labor Force Status

Figure 1 plots the total number of new hires by previous labor force status, a description of how hires are calculated can be found in Appendix [A.1](#). The figure shows that while total hires out of unemployment

⁵A detailed description of how labor market status is determined can be found at <https://www.bls.gov/cps/definitions.htm>.

have remained stable around 2 million per month since 1980, hires from out of the labor force have nearly doubled. In the 1980's workers that were previously classified as out of the labor force made up 60% of total hires from non-employment and by 2019, they made up 72.5%. As a fraction of all hires, workers who were previously classified as out of the labor force have also been increasing, in the 1990's hires from out of the labor force made up 36% of all hires, and 50% in 2019.⁶ The figure shows that the pool of unemployed does not capture potential workers well in terms of hires.

Over the same time, the total size of the out of the labor force (OLF) group also increase by 50% from 63 million in 1980 to 95 million in 2019. However, the composition of the the OLF pool has also seen large changes since 1980. **Table 1** shows the percent of each demographic group as a total of the OLF population and a total of the hires from the OLF population. Looking across sex, there were only marginal changes in the percent of the total population of OLF by gender. Women made up about 53% of the total OLF population in the 1980's, decreasing to 52% by the 2010's. However, the percent of OLF hires that were women has decreased from 61.4% to 55.4% Decomposing the total OLF population and hires by race shows a shift away from white in both the total OLF population and OLF hires.

Decomposing changes by age and education also show large changes. The pool of OLF have become older; people age 56+ made up about 26% of the pool of OLF in the 1980's and increase by 6 percentage point to 32% by the 2010's. However, the percent of hires from OLF by each age group has changes accordingly. Individuals age 56+ made up about 18% of OLF hires in the 1980's and increased 6 percentage points to 24% by the 2010's.

On the other hand, changes in education show that both the percent of each group and the percent of hires has change since the 1980's. There was a general increase in the education level of the OLF population, with more individuals having completed high school and college in the 2010's than the 1980's. However, whereas people who had completed high school make up more the OLF population in the 2010's, they account for less of the OLF hires in the 2010's. And while less of the OLF population had some college education in the 2010's than in the 1980's, they account for more of the OLF hires in the 2010's than in the 1980's.

2.2 Job Search in the American Time Use Survey

The CPS also asks several questions about job search efforts, however, these questions are limited to people who are classified as unemployed. One the other hand, with the American Time Use Survey,

⁶The decreased level in the job to job transitions after the 2008 recession in the CPS is not consistent with job-to-job transitions from the Longitudinal Employer-Household Dynamics. See [Fujita et al. \(2020\)](#) for a discussion and how some of the difference can be reconciled by the fact that the CPS changed to whom the previous employer question is asked.

job search effort can be observed by all participants. The ATUS, which began as a supplement to the CPS in 2003, randomly selects households that have completed their eighth and final month in the CPS. Selected households are interviewed one time about how they spent their time on the previous day, where they were, and whom they were with. The main goal of the survey is to collect information about how the respondent spent his or her time starting at 4 a.m. the previous day and ending at 4 a.m. on the interview day. Each activity recorded is then coded into over 400 categories.

Of particular interest are the categories devoted to job search which include: job search activities, job interviewing, waiting associated with job search or interview, security procedures related to job search/interviewing, and other job search.⁷ These categories are the focus of this paper as they provide an opportunity to see if, and how much people are searching for a job, regardless of their labor force status. Given these categories, the ATUS has recently become a common data set used to study the cyclical behavior of job search among the unemployed (Mukoyama et al., 2018) and the employed (Ahn and Shao, 2017).

The ATUS interview is conducted between 2 and 5 months after exiting the CPS. Because of the delay between the final CPS interview and the ATUS interview, the questions pertaining to labor force status are asked again, in the same fashion as during the CPS interview, and respondents are classified as employed, unemployed, or out of the labor force accordingly. Regardless of a person's labor market status, if he spent any time searching for a job on the interview day, the time will be recored as job search activity. Therefore, the ATUS data can be used to estimate the probability that any person is searching for a job.

The main disadvantage of using the ATUS to study job search behavior is that people are only surveyed about one day in the month. In what follows, search effort, in terms of minutes per day, is reported both unconditionally and conditional on observing positive search effort. Due to the cross-sectional nature of the ATUS, the probability that a person searches for a job is reported as a daily probability and converted to a monthly probability under the assumption that job search is identical and independent across days.⁸ That is, if p is the daily probability of searching for a job, then the corresponding monthly probability is calculated as $1 - (1 - p)^{30}$. The monthly probability is the probability that the individual spent some time searching on at least one day during the month, this corresponds well to the CPS definition of unemployment of having made at least one active search effort during the past 4 weeks.

Table 2 shows the probability of observing a person searching for a job on a single day, calculated as

⁷Categories 50401-50499 in the ATUS lexicon.

⁸In the subsequent analysis, this restriction will be relaxed and the propensity of search effort can vary across days however remains independent across days, again due to data limitations.

the sample mean of a dummy variable that takes on the value 1 if the person reports spending any time that day looking for a job. Also reported is the corresponding monthly probability. Not surprisingly, people classified as unemployed have the highest probability of searching for a job on a single day, and with almost certainty, search for a job throughout the month. For the employed, those at work have a lower monthly probability (15.8) of searching for a job than those absent from work (32.8). Those classified as out of the labor force also report searching for a job throughout the day, in fact, the probability of observing a person age 25-55 searching for a job throughout the month (24.4) from outside the labor force is higher than the probability of observing an employed person, age 25-55, searching for a job (15.0). The fact that the probability of observing a person classified as out of the labor force searching for a job is positive is further evidence of the measurement issues discussed above.

The intensive margin of search (minutes spent searching) is reported in [Table 3](#). Both the unconditional average and conditional on positive search time is reported. First, across both samples, the unemployed have the highest unconditional search intensity, searching for an average of about 30 minutes per day. The unconditional average for the employed and out of the labor force is mostly below one minute across both samples, stemming from the fact that not even 1 percent of each group is searching for a job on the interview day. However, when looking at the conditional search times, the three groups look strikingly similar. The unemployed, again, have the highest search intensity, spending on average 2.5 hours per day searching for a job. However, conditional on searching for a job, those classified as out of the labor force spend almost as much time per day (2.2 hours) searching as the unemployed. Employed people spend the least amount of time searching for a job, about 2 hours per day. The extensive margin of job search in the ATUS is similar to that presented in [Faberman et al. \(2017\)](#) who use data from the Survey of Consumer Finances, however the intensive margin among job seekers differs between the two surveys.

In the CPS classification scheme, the type of job search activity, active vs passive, also determines if non-employed individuals are classified as unemployed or out of the labor force. [Table 4](#) reports the percent of time each group spends in three types of job search activities. The first activity labeled “Active Job Search” consists of ATUS category number 50401 which includes contacting employers, sending out resumes, ect. The second category is “Interviewing” and the third category “Other” includes all other ATUS categories (50403-50499) which are: waiting time associated with interviewing, security procedures related to job search/interviewing, and all other job search activities not elsewhere specified. Across all three labor market states and both samples, the majority of time is spent in active job search. Employed individuals spend about 14% of the time interviewing whereas, unemployed individuals spend only about 7% of the time interviewing. This is consistent with [Faberman et al. \(2017\)](#), who show that the employed get more interviews than the unemployed. Those classified as out of the labor

force also spend more time interviewing than the unemployed, nearly 11% of their time. The evidence presented in [Table 3](#) and [Table 4](#) suggest that people who are classified as out of the labor force but exert positive search effort, may not be behaving differently from those classified as unemployed.

2.2.1 Who is searching from outside the labor force?

To understand which characteristics make individuals more likely to search while being classified as out of the labor force I run a two stage Heckman selection model on the non-employed. Selection is estimated as the probability of getting classified as out of the labor force on the set of demographic characteristics (sex, married, child, age, race, education) and interaction terms for sex, married and child. The instrument for selection is an indicator for whether or not the ATUS respondent is the same as the respondent in the final CPS interview. [Krueger et al. \(2017\)](#) show that respondents answer question differently based on how long they have been in the CPS, implying that ATUS respondents that were also the CPS respondents may be classified differently from others but this should not affect their search efforts. For selection I run a Probit on an indicator if the non-employed individual was classified as out of the labor force (y_i) on the set of covariates (x_i):

$$P(y_i = 1|x_i) = \phi(\beta x_i + \gamma_d + \gamma_m + \gamma_y) \quad (1)$$

where ϕ is the standard normal p.d.f., γ_d are day of the week fixed effects, γ_m are month fixed effects, and γ_y are year fixed effects. Then on the subset of people classified as out of the labor force I estimated a linear probability model of an indicator if the individual searched for a job on the interview day (s_i), on the set of covariates (x_i) and the estimated selection probability (inverse Mill ratio, λ_i) from the first stage:

$$s_i = \beta x_i + \gamma_d + \gamma_m + \gamma_y + \delta \lambda_i + \varepsilon_i \quad (2)$$

where the fixed effects are as before.

Column (1) of [Table 5](#) shows the parameters from the first stage. Non-employed women, married women, and women with children are all more likely to get classified as out of the labor force than their male counterparts. People with higher levels of education are less likely to get classified as out of the labor force than people with less than a high school degree. The estimated coefficient on the "Same Respondent" dummy is negative and significant, implying that people who were also the CPS respondent are less likely to get classified as out of the labor force.

Column (2) of [Table 5](#) shows the parameter estimates from the linear probability model. The significant coefficient on the inverse Mills ratio implies that there is statistically significant selection

on observables. Conditional on selection, women are more likely to search for a job from out of the labor force. Married men are less likely to be classified as OLF but conditional on selection, they are 20 percentage points more likely to search for a job when they are. Similarly, higher educated people are less likely to be classified as out of the labor force, but conditional on selection they are more likely to search. For example, college educated workers are 31 percentage points more likely to search for a job on the interview day.

3 Estimation and Prediction

The previous section documented an increase in the fraction of hires coming from out of the labor force, suggesting that the CPS definition of unemployment does not capture labor underutilization well. This is also supported by the fact that about 12% of those classified as out of the labor force are searching for a job during the month. The probability that an individual searches for a job while being classified as out of the labor force differs across gender, age and education, demographic characteristics along which individuals classified as out of the labor force have changed substantially since 1980.

In this section I estimate search effort among ATUS respondents based on labor force status and demographic characteristics. Using these estimates I predict the probability that individuals in the CPS are searching for a job during the interview month. Finally, by matching people across consecutive months, I show that the estimated search probability is correlated with job finding probabilities and subsequent hours worked.

3.1 Estimating Search Effort

Search effort (the daily search probability) is predicted from the subset of individuals that participate in the ATUS and predicted for all individuals in the CPS. To maximize the predictive power of the covariates, I use machine learning to choose the set of best predictors. The potential covariates are all demographic variables (a quadratic in age, race, education, sex, marital status, and an indicator for having a child), day of the week fixed effects, a full/part time indicator for the employed, and interactions between sex and education, and all other covariates. The total number of potential covariates for the employed group is 57 and for the unemployed and out of the labor force groups is 51. To ensure that the adjusted unemployment rate is comparable over time, i.e. is not biased by changes in the demographic composition of the pool of job seekers, all demographic variables and day of the week fixed effects enter the machine learning algorithm without penalty, while all interaction terms enter with a standard penalty function described in detail below.

I estimate the daily search probability for each labor market state, including those classified as unemployed. Estimating search probabilities that depend on demographic characteristics for the unemployed, again ensures that the adjusted unemployment rate is not biased by changes in the demographic composition of the unemployed. This also allows for miss-classification error among the U-O margin, that is, workers classified as unemployed that should be classified as out of the labor force.

For each CPS defined labor market status, I run a net-elastic logistic regression, where the dependent variable is an indicator for if the individual spent any time searching for a job on the interview day. Demographic information for each person is collected in the CPS and matched to their ATUS interview. For each individual in the ATUS, let y_i be an indicator that takes on the value one if they spent any time searching for job on the interview day and x_i be the vector of covariates. The probability that the individual searched for a job is modeled using the logistic function,

$$P(y_i = 1|x_i) = \frac{\exp(\beta_0 + x_i'\beta)}{1 + \exp(\beta_0 + x_i'\beta)} \quad (3)$$

and the log-likelihood function is

$$\mathcal{L}(\beta_0, \beta|\{y_i, x_i\}) = \left[\frac{1}{N} \sum_{i=1}^N y_i(\beta_0 + x_i'\beta) - \ln[1 + \exp(\beta_0 + x_i'\beta)] \right] + \lambda \left[(1 - \alpha) \sum_{k \in K} \beta_k^2 + \alpha \sum_{k \in K} |\beta_k| \right] \quad (4)$$

where α is set to 0.95, implying more weight on the LASSO penalty. The tuning parameter, λ , is chosen through cross validation of ten folds of the data with the area under the receiver operating characteristic curve as the selection criteria. The penalty is only over the interaction terms (the set K) to ensure that all demographic variables and day of the week fixed effects are always included in the estimation. Final ATUS weights are used in all calculations.

Table 6 shows the selected covariates and resulting parameter estimates for each labor force status. For the employed group, 5 of the interaction terms are included in the estimation. For the unemployed and out of the labor force groups 17 and 8 interaction terms are included. **Figure 2** plots the receiver operating characteristic curve for each group. The out of the labor force group has the best fit with an area under the curve (AUC) of 0.873, the employed AUC is 0.753 and the unemployed AUC is 0.735.

3.2 Predicting Search Effort

The CPS contains all the same demographic information as the ATUS and labor market status is determined equivalently in both samples. Therefore, although the ATUS sample begins in 2003, search effort can be predicted using the CPS starting in 1980. The estimated search probability is a daily probability, therefore, seven probabilities are predicted for each person in the CPS, one for each day of

the week fixed effect. The predicted daily probability is:

$$\hat{p}_i^d = \frac{\exp(\hat{\beta}_0 + x_i' \hat{\beta})}{1 - \exp(\hat{\beta}_0 + x_i' \hat{\beta})} \quad (5)$$

for $d \in \{1, 2, \dots, 7\}$ where $\hat{\beta}_0$ and $\hat{\beta}$ are the estimated coefficients.

Using the predicted daily probability, the weekly probability that a person is searching for a job is 1 minus the probability he does not search any day during the week, i.e.

$$\hat{p}_i^w = 1 - \sum_{d=1}^7 (1 - \hat{p}_i^d). \quad (6)$$

The monthly probability that he searches for a job is constructed analogously, i.e.

$$\hat{P}_i = 1 - (1 - \hat{p}_i^w)^{4.17} \quad (7)$$

with 4.17 weeks per month. Equation 7 is one minus the cumulative product of not searching each week in the month, that is, it is the probability that person i searched at least once during the month. The probability is in line with the CPS definition of unemployment of having at least one active search effort during the month.

Table 7 reports percentiles of the predicted search probabilities for each labor market state over the entire sample. The unemployed have the highest predicted search probabilities and the distribution of search probabilities is skewed towards one, with the 50th percentile at 0.997. Not surprisingly, many predicted search probabilities are very close to zero for the out of the labor force group, in fact, the 25th percentile is at 0.04. However, the out of the labor force group still has many predicted search probabilities that are larger than the employed group. The 95th percentile of search probabilities for out of the labor force group is 0.44 and 0.35 for the employed group.

3.3 Search Effort and Labor Force Attachment

For the adjusted unemployment rate to be a useful measure of labor underutilization, it should weight non-employed people with a higher labor force attachment more than those with a lower labor force attachment. In term of labor force attachment, three statistics can be calculated from the basic CPS files. First, for the full sample of individuals matched across two consecutive months in the CPS from 1994-2019 an indicator variable that takes on the value one if they found a job (or switched jobs for the employed), that is, a job finding probability is constructed as a measure of labor force attachment. Second, for the subset of non-employed individuals who found a job, their subsequent usual hours

worked is used as a measure of labor force attachment. Third, for the subset of employed individuals who switched jobs, the change their usual hours worked is used as a measure of labor force attachment. For all three statistics, the estimated job search probability is an indicator of labor force attachment if it correlated positively with the outcome.

The correlation between the predicted job search probability and the three measures of labor force attachment is estimated as follows:

$$y_{it} = \beta \hat{P}_{it-1} + \delta_t + \varepsilon_{it} \quad (8)$$

where y_{it} an indicator for new employment, usual hours worked, or change in usual hours worked, \hat{P}_{it-1} is the predicted search effort in the previous month, and δ_t are month by year fixed effects.

The estimated correlation between an indicator for new employment (job finding probability) and predicted search effort are reported in the first two columns of [Table 8](#). The correlation is significantly positive with and without year by month fixed effects and large relative to the mean job finding probability over the sample (0.04). The correlation between search effort and subsequent hours worked for the non-employed job finders subsample is reported in columns (3) and (4) of [Table 8](#); the correlation is positive and significant. The correlation between changes in hours worked and search effort for the subset of employed job switchers is also positive and significant as reported in the last two columns of [Table 8](#). The positive correlation between predicted search effort and job finding probabilities and hours worked suggests that the predicted search effort is indeed a good proxy of labor force attachment.

4 Aggregate Unemployment and Labor Market Flows

4.1 Aggregate Unemployment

Using the monthly predicted probability of search effort for each person, the estimated number of searchers within each CPS defined labor market state, unemployed U^s , employed E^s , and out of the labor force O^s , are constructed as weighted totals:

$$U_t^s = \sum_{i \in U_t} wgt_{it} \times \hat{P}_{it} \quad (9)$$

$$E_t^s = \sum_{i \in E_t} wgt_{it} \times \hat{P}_{it} \quad (10)$$

$$O_t^s = \sum_{i \in O_t} wgt_{it} \times \hat{P}_{it} \quad (11)$$

where U_t , E_t , and O_t are the sets of all individuals in the respective CPS defined labor market state and wgt_{it} is the CPS sampling weight. The total number of people in each state is calculated as the sum of the weights within each group. The resulting series are monthly, seasonally adjusted using the Census X13-ARIMA and then aggregated to a quarterly frequency. A complete description of the process can be found in Appendix A.1.

Figure 3 plots the predicted fraction of people searching in each labor market state (O_t^s/O_t , U_t^s/U_t , E_t^s/E_t), and the population as a whole. The shaded regions depict recessions using the National Bureau of Economic Research's classifications. All series clearly display a counter cyclical patten, with the exception of employed search effort during the 2020 recession. The fraction of people searching while employed rose dramatically during the 2008 recession, increasing by about 1 percentage point from trough to peak. Nearly all those who are unemployed are searching for a job, with the percent varying between 95% and 98%. The fraction of people searching for a job who are classified as out of the labor force has risen by nearly 4 percentage points since 1980, which corresponds to nearly 7 million extra job seekers. The final panel of Figure 3 plots the total fraction of the population that is searching for a job, on average about 16.5% of the population is searching for a job over the sample period.

The fraction of searchers among all three labor force groups in Figure 3 display a clearly counter-cyclical patten, rising during each recession. These results are consistent with Ahn and Shao (2017) who show that the search effort of the employed is countercyclical in the extensive and intensive margin. The unemployed also display countercyclical search effort. This result adds to a growing literature that documents that search effort of the unemployed is countercyclical, such as Shimer (2004), Kudlyak and Faberman (2014) and Mukoyama et al. (2018). However, this stands in contrast to DeLoach and Kurt (2013) who show a-cyclical search effort and Gomme and Lkhagvasuren (2015) who show evidence of pro-cyclical search effort. While the fraction of people searching among people classified as out of the labor force has risen by one third since 1980, the rise is also countercyclical, rising quickly during recessions and flattening or decreasing during expansions.

The adjusted unemployment rate is defined as the ratio of the weighted sum of all non-employed people, weighted by their search probabilities, to the total number of non-employed searchers and employed, that is:

$$\tilde{U}_t = \frac{U_t^s + O_t^s}{U_t^s + O_t^s + E_t^s}. \quad (12)$$

Analogously, the total searcher rate is the ratio of the weighted sum of all people, to the total number of participants,

$$S_t = \frac{U_t^s + O_t^s + E_t^s}{U_t^s + O_t^s + E_t^s}. \quad (13)$$

The total searcher rate is in spirit most similar to the Bureau of Labor Statistic's most inclusive measure

of labor slack, U-6, which includes the unemployed, the marginally attached and all part time employed for economic reasons. However, the total searcher rate may better capture total labor under utilization since it weights individuals by their propensity to begin new employment. Finally, an adjusted measure of labor force participation is constructed by taking the ratio of the weighted average of all non-employed plus all employed to the total population,

$$\tilde{P}_t = \frac{U_t^s + O_t^s + E_t}{U_t + O_t + E_t}. \quad (14)$$

Panel (a) of [Figure 4](#) plots the standard and adjusted unemployment and the total searcher rate. The average standard unemployment rate over the sample is 6.4. Both the average adjusted unemployment rate (11.6) and the average total searcher rate is (24) are higher than the standard unemployment rate. The most notable difference between the standard and adjusted unemployment rate is that the adjusted unemployment rate does not display the same low frequency downward trend that the standard unemployment rate does. Panel (b) plots the standard and adjusted labor force participation rates. The average adjusted participation rate is 3.9 percentage points higher than the standard participation rate. The standard labor force participation rate drops by 4 percentage points from 2000 to 2015. The adjusted participation rate decreases by only 3 percentage points from 2000 to 2015, implying that the increased misclassification among the non-employed can account for 25% of the drop in the standard participation rate.

4.1.1 Accounting for the increase in OLF search effort

Since the OLF search series presented in panel (c) of [Figure 3](#) is constructed using time invariant demographic effects on search probabilities, changes in the series are driven by changes in the demographic composition of the pool of non-participants. To understand which demographic changes had the largest effect on aggregate OLF search effort I construct a counterfactual search series allowing the share of one demographic characteristic to vary at a time. This is done by constructing search effort on 1,000 re-sampled CPS basic monthly files, in which all but one demographic characteristic are fixed at their 1980 shares. The final counterfactual series plotted is the average across the 1,000 series.

[Figure 5](#) plots the resulting fraction of OLF searchers as the percentage point difference from 1980. The series labeled “Aggregate” is the series presented in panel (c) of [Figure 3](#). The figure shows that, changes in sex, race and education contributed to the increase in the search effort of those classified as out of the labor force. For example, the series in which only educational attainment is allowed to vary increases by over 5 percentage points since 1980, implying that the changes in the educational attainment of those classified as out of the labor force had a large effect on the aggregate OLF search

effort holding all else equal. Similarly race and gender had a large effect on OLF search effort. The age of the OLF pool, on the other hand, had no effect on search effort among the group. The counterfactual series where age is the only demographic characteristic allowed to vary does not increase since 1980. In fact, the aging of the pool of the out of the labor force contributed to the decline in search effort after the 2008 recession.

4.2 Labor Market Flows

The standard method used to calculate flows between labor market states uses information on individuals who are matched across consecutive months of the CPS basic monthly files. Using the longitudinally linked data, estimates for transition probabilities are calculated as the fraction of people transitioning across labor market states from month to month.

The approach used here is similar, however, non-employed worker transitions are weighted by the predicted monthly search probability. The probability a worker transitions from employment to unemployment is calculated as:

$$f_{EU} = \frac{\sum_{i \in E_1 N_2} wgt_i \times \hat{P}_{i2}}{\sum_{i \in E_1} wgt_i} \quad (15)$$

where the summation in the numerator is over all workers that are observed in employment in the first month (E_1) and non-employment (CPS defined unemployment and out of the labor force) in the second month (N_2). The summation in the denominator is over all workers in employment in the first month. The weight used in the numerator is the CPS sampling weight times the estimated search probability in the second month. Similarly the transition probability from employment to out of the labor force is calculated as:

$$f_{EO} = \frac{\sum_{i \in E_1 N_2} wgt_i \times (1 - \hat{P}_{i2})}{\sum_{i \in E_1} wgt_i} \quad (16)$$

where the weight used in the numerator is now the CPS sampling weight times the probability the worker is not searching for a job in the second month. The transition probability from unemployment to employment is calculated using all individuals that are not employed in the first month (N_1) and employed in the second month (E_2), weighted by the probability they were searching for a job in the first month. That is,

$$f_{UE} = \frac{\sum_{i \in N_1 E_2} wgt_i \times \hat{P}_{i1}}{\sum_{i \in N_1} wgt_i}. \quad (17)$$

The transition probabilities between unemployment and out of the labor force are calculated slightly differently. Instead of weighting the individual by the search probability each period, workers are weighted by the change in their search probability. If a person remains non-employed for two consecutive months, and his predicted probability of search does not change over those two months then, although

he contributes to both the stock of unemployed and out of the labor force, he does not contribute to the flow between these two states. Alternatively, suppose that a person is not employed in two consecutive months, and his estimated probability of searching is $\hat{P}_1 = 0.3$ in the first month and $\hat{P}_2 = 0.5$ in the second month, then he contributes to the flow from out of the labor force to unemployment by only change in his estimated search probability, that is, with weight 0.2. Therefore, the flow from out of the labor force to unemployment is calculate as

$$f_{OU} = \frac{\sum_{i \in N_1 N_2} wgt_i \times \max\{\hat{P}_{i2} - \hat{P}_{i1}, 0\}}{\sum_{i \in N_1} wgt_i}. \quad (18)$$

Similarly, a person that is not employed in two consecutive months only contributes to the flow from unemployment to out of the labor force if his predicted search probability decreases from the first to the second month. The flow from unemployment to out of the labor force is calculated as

$$f_{UO} = \frac{\sum_{i \in N_1 N_2} wgt_i \times |\min\{\hat{P}_{i2} - \hat{P}_{i1}, 0\}|}{\sum_{i \in N_1} wgt_i}. \quad (19)$$

By construction the flow from out of the labor force to employment is zero.

The resulting transition probabilities are seasonally adjusted and corrected for margin error. The correction for margin error is similar to [Elsby et al. \(2015\)](#) and restricts the flows across labor market states to be consistent with the evolution of the labor market stocks. A detailed description of how this was done can be found in [Appendix A.2](#). In the standard labor market flows data, margin error can arise from movements into the working age population or attrition of households in the matched CPS data; however, [Elsby et al. \(2015\)](#) show that correcting for margin error has little effect on the standard CPS flows. Here the flows and stocks are calculated using estimated search probabilities, so correcting for margin error plays a larger role.

Figure 6 plots the standard and adjusted labor market flows across labor market states. The most notable changes occur along the participation margin. The average flow from out of the labor force to unemployment increases slightly from 0.024 to 0.027 and the average flow from unemployment to out of the labor force decrease from 0.21 to 0.04. The average flow from unemployment to employment decrease slightly from 0.23 to 0.18. The average flow from employment to out of the labor force decreases by more half from 0.026 to 0.008. Using the standard flow calculation workers are almost twice as likely to leave employment for non-participation, whoever the adjusted flows show that this is not true: workers are about 50% more likely to leave employment to unemployment rather than non-participation.

Much of the previous work on labor market flows has focused on adjusting the flows to account

for the misclassification between unemployment and non-participation. [Abowd and Zellner \(1985\)](#) and [Poterba and Summers \(1986\)](#) attempt to understand the amount of measurement error in the CPS classification system by using data from the reinterview surveys conducted by the CPS on a subset of individuals. Unfortunately, the CPS has since stopped conducting reinterview surveys. So more recently, [Elsby et al. \(2015\)](#) match individuals up to three months and recode an individual who is observed as unemployed in the first month, out of the labor force in the second month and unemployed in the third month, as unemployed throughout. Similarly, individuals that are out followed by unemployed and again out, are recoded as out of the labor force for all periods. Indeed, this correction ("deNUNification") decreases the flow between unemployment and out of the labor force but it does not address the issue of movements from out of the labor force directly to employment and vice versa. For example, an individual who is observed as unemployed in the first month, out of the labor force in the second month and employed in the third month is not recoded, therefore, such an individual adds to the flow from unemployment to out of the labor force as well as the flow from out of the labor force to employment.

5 Implications

In this section I show the miss-classification of workers has significantly impacted two well studied aspects of the macro economy. First, it is well known that the work-horse search and matching model cannot match the volatility in labor market tightness through reasonable productivity shocks ([Shimer, 2005](#)). I show that labor market tightness, constructed using the adjusted unemployment rate or total searchers rate, is 24%-38% less volatile. In fact, the labor market is less volatile in several dimensions than previously thought. Second, I show that the much debated flattening of the Phillips Curve is completely absent when using the adjusted unemployment rate, or total searcher rate, as a measure of the output gap.

5.1 Volatility of Labor Market

In assessing the volatility of the labor market I look at 5 statistics: job seeker rates, vacancy rates, labor market tightness, job finding rates, and separation rates. The three job seeker rates I compare are the standard unemployment rate, adjusted unemployment rate and total searcher rate. The vacancy rates I compare are total vacancies per labor market participants. Total vacancies are constructed using the Help Wanted Index taken from [Barnichon \(2010\)](#) from 1980 to November 2000, and total job openings from the Job Openings and Labor Turnover Survey from December 2000 to July 2020. The standard

vacancy rate is calculated as total vacancies per standard number of unemployed and employed. The adjusted vacancy rate is calculated as total vacancies per adjusted number of unemployed and employed.

Standard labor market tightness is calculated as the standard vacancy rate divided by the standard unemployment rate. The adjusted labor market tightness is calculated as the adjusted vacancy rate divided by the adjusted unemployment rate and the total searcher labor market tightness is calculated as the adjusted vacancy rate divided by the total searcher rate. Finally, the standard job finding and job separation rates are the standard U-E and E-U flows and the adjusted job finding and job separation rates are the adjusted U-E and E-U flows presented in [Figure 6](#). I calculate volatility for each series as the standard deviation of the logged quarterly average.

[Table 9](#) reports the volatility of each series. First, the volatility of the standard and adjusted vacancy series are nearly identical; this is not surprising as only the denominator changes. The volatility of the job seeker rates differs substantially. The standard deviation (in log points) of the standard unemployment rate is 0.28 while the standard deviation of the adjusted unemployment rate is 0.15, implying the adjusted unemployment rate is nearly half as volatile. When considering all searchers in the economy, the volatility drops to 0.07, or 133% less volatile. This decrease in volatility is also reflected in labor market tightness. The volatility of standard labor market tightness is 0.46; the volatility of adjusted labor market tightness is 0.35 (24% less) and the volatility of the total searcher labor market tightness is 0.28 (38% less). Finally the adjusted job finding rate is 26% less volatile and the separation rate is 27% less volatile.

The changes in the volatility when correcting for the miss-classification of non-employed individual and when taking employed job seekers into account are large and significant when considering that the standard search and matching model cannot generate large movements in these key labor market statistics. Similarly these changes are stark when considering that the fraction of job seekers in each labor market state is estimated to move counter-cyclically as shown in [Figure 3](#).

5.2 Phillips Curve

The unemployment rate, or unemployment gap, is often used as a measure of labor market utilization in estimating the trade-off between output and inflation, i.e. the Phillips Curve. Since the Great Recession this relationship has gained new interest with many finding that the traditionally strong negative correlation between output and inflation has weakened or even disappeared, the phenomenon referred to as the flattening of the Phillips Curve. Papers investigating the apparent flattening (among many others) include [Ball and Mazumder \(2011\)](#); [Coibion and Gorodnichenko \(2015\)](#); [Blanchard \(2016\)](#); see [McLeay and Tenreyro \(2019\)](#) for a recent review.

I revisit the change in the output-inflation relationship following the Great Recession using the adjusted unemployment rate and the total searcher rate as measures of underutilization. I estimate a Phillips curve, where inflation is determined by the output gap and expected inflation. That is,

$$\pi_t = \phi(x_t - x_t^*) + \gamma E_t[\pi_{t+1}] \quad (20)$$

where π_t is the inflation rate at time t , x_t is a measure of labor market underutilization (the standard unemployment rate, the adjusted unemployment rate or the total searcher rate), x_t^* is the natural rate of each measure, and $E_t[\pi_{t+1}]$ is expected inflation. The parameter of interest is ϕ and how its value changes post the 2008 recession, under different measures of the output gap. I estimate a backwards looking Phillips curve and proxy for inflation expectations with the four quarter average of lagged inflation; that is,

$$E_t[\pi_{t+1}] \equiv \bar{\pi}_t = \frac{1}{4}(\pi_{t-1} + \pi_{t-2} + \pi_{t-3} + \pi_{t-4}). \quad (21)$$

Similar specifications have been used recently by [Stock and Watson \(2019\)](#), [Galí and Gambetti \(2019\)](#) and [Ball and Mazumder \(2011\)](#).

Along with an inflation Phillips curve I estimate a wage Phillips curve, similar to [Galí and Gambetti \(2019\)](#), again comparing the change in the correlation between wage growth and the three measures of the output gap pre and post 2008 recession. The wage Phillips curve is

$$\Delta w_t = \phi(x_t - x_t^*) + \gamma E_t[\pi_{t+1}] \quad (22)$$

where Δw_t is nominal wage growth, and $x_t - x_t^*$ and $E_t[\pi_{t+1}]$ are the same as above.

The natural rate of unemployment, x_t^* , is defined as the rate of unemployment such that inflation remains stable. This level is thought to change over time due to changes in the demographics of the workforce or changes in the structure of the labor market. There exists a vast literature aimed at estimating the natural rate of unemployment, see [Crump et al. \(2019\)](#) for a thorough review. Although there are many ways to estimate the natural rate of unemployment, [Stock and Watson \(2019\)](#) argue that the disappearance of the Phillips curve is robust to whichever measure one chooses. Therefore, the output gap measured using the standard unemployment rate, is estimated as the difference between the standard unemployment rate and the Congressional Budget Offices natural rate of unemployment (NAIRU). Since the adjusted unemployment rate and the total searcher rate are adjusted for demographic changes in the labor force and no longer display the same low frequency downward trend that the standard unemployment rate does, the natural rate for these measures will be estimated using a constant.

The flattening or disappearance of the Phillips curve, in its simplest form, can be estimated with a

change in the parameter ϕ after the start of the great recession. For the standard unemployment rate the change in ϕ is estimated using a similar specification as in [Stock and Watson \(2019\)](#), that is:

$$\pi_t = \phi_1(u_t - u_t^{NAIRU}) + \gamma_1 E_t[\pi_{t+1}] + \phi_2(u_t - u_t^{NAIRU}) \times Post_t + \gamma_2 E_t[\pi_{t+1}] \times Post_t + \varepsilon_t \quad (23)$$

where $Post_t = \mathbb{I}\{t > 2007.25\}$ is an indicator that takes on the value one after the first quarter of 2007 and the parameter ϕ_2 estimates the change in the Phillips curve. For the adjusted unemployment rate and the total searcher rate the specification is:

$$\pi_t = \alpha_1 + \phi_1 x_t + \gamma_1 E_t[\pi_{t+1}] + \alpha_2 \times Post_t + \phi_2 x_t \times Post_t + \gamma_2 E_t[\pi_{t+1}] \times Post_t + \varepsilon_t \quad (24)$$

where $\alpha_1 = -\phi_1 x^*$ and $\alpha_2 = -\phi_2 x^*$, and ϕ_2 estimates the change in the Phillips curve. Identical specifications are run for the wage Phillips curve.

Table 10 reports the results from the regressions. The sample period is 1980Q1 to 2019Q4. The first three columns show the results from the wage Phillips curve for which Δw_t is the annualized growth rate of average hourly earning of Production and Nonsupervisory Employees⁹ and $E_t[\pi_{t+1}]$ is the average of the previous 4 quarters of inflation growth constructed using the annualized growth rate of the Personal Consumption Expenditure Index (PCE). Column (1) shows what has been documented as the flattening of the wage Phillips curve with the estimated effect of the unemployment rate gap on wage growth significantly decreasing post 2007, similar to what [Galí and Gambetti \(2019\)](#) find. Columns (2) and (3) report the estimated parameters when using the adjusted unemployment rate and the total searcher rate as measures of the output gap. Contrary to the standard unemployment rate gap, there is no significant change in the slope of the wage Phillips curve when using these alternative measures, seen by the small and statistically insignificant estimates on the $\tilde{U} \times Post$ and $S \times Post$ variables. The correlation between the adjusted unemployment rate and the total searcher rate, and wage inflation (-0.065 and -0.08) are statistically indistinguishable from the pre-recession estimated correlation with the standard unemployment rate gap and wage inflation (-0.072).

The estimated parameters of the inflation Phillips curve are reported in columns (4)-(6). Column (4) shows the inflation Phillips curve estimated with the standard unemployment gap. The slope coefficient before the Great Recession is estimated at -0.31 and there is a large change in the post recessionary period, these estimates are consistent with [Stock and Watson \(2019\)](#) and many others. Column (5) shows the estimated coefficients of the Phillips curve estimated using the adjusted unemployment rate and a time invariant natural rate for the adjusted unemployment rate. In the pre-recession period the

⁹<https://fred.stlouisfed.org/series/AHETPI>

estimated slope coefficient is -0.272, similar to the standard unemployment gap coefficient. However, the slope does not change significantly in the post recession period, again, seen by the small and statistically insignificant estimate on the $\tilde{U} \times Post$ coefficient in column (5). When using the total search rate as a measure of labor slack, the pre-recession slope is estimated at -0.476, about 50% larger than the size of the correlation with the standard unemployment gap in the pre-recession period. The change in the slope when using the total searcher rate is also statistically insignificant. This suggests that the flattening of the Phillips curve may be, in part, due to measurement issues in the output gap. The result stands in contrast to [Stock and Watson \(2019\)](#), who argue that the flattening of the Phillips curve is present regardless of which measure of inflation and the output gap is used.

6 Conclusion

I document a rise in hires from out of the labor force and, using the American Time Use Survey, a large proportion of individual searching actively while being classified as out of the labor force, and argue that the standard unemployment rate therefore does a poor job capture the churning of the labor market. Using estimates from the American Time Use Survey to predict search effort in the CPS, I construct an adjusted unemployment rate, a total searcher rate, and labor market flows since 1980. There has been a substantial increase in the proportion of people classified as out of the labor force that are predicted to be actively seeking employment, which is attributed to increase educational attainment. Adding these job seekers into the unemployment rate nearly doubles its level, rids the unemployment rate of the low frequency downward trend, and significantly decreases overall labor market volatility. The adjusted flows show that unemployment is a much more persistent state than previously thought and that workers leaving employment are more likely to leave for unemployment rather than non-participation. An application to the Phillips Curve shows that when using the adjusted unemployment rate rather than the standard unemployment rate as a measure of the output gap, there is no post-2008 flattening.

7 Tables

Table 1: Out of the Labor Force Demographics by Decade

	1980-1989	1990-1999	2000-2009	2010-2018
Men				
% of population	47.4	47.9	48.2	48.3
% of hires	38.6	41.1	42.7	44.6
Women				
% of population	52.6	52.1	51.8	51.7
% of hires	61.4	58.9	57.3	55.4
Age 16-24				
% of population	19.8	16.6	16.3	15.4
% of hires	41.1	38.5	36.8	34.2
Age 25-55				
% of population	54.1	58.2	56.7	52.1
% of hires	41.2	43.3	43.8	41.9
Age 56+				
% of population	26.1	25.2	27.1	32.5
% of hires	17.7	18.1	19.3	23.9
White				
% of population	86.3	84.1	81.9	78.8
% of hires	85.6	82.1	79.2	76.2
Black				
% of population	11.0	11.7	11.9	12.5
% of hires	11.3	12.8	13.6	14.1
Other				
% of population	2.7	4.2	6.2	8.7
% of hires	3.1	5.1	7.1	9.7
Less than HS				
% of population	23.4	20.8	16.8	13.1
% of hires	25.7	21.6	17.7	12.7
High School				
% of population	8.9	35.0	32.4	30.4
% of hires	9.6	7.9	7.3	6.9
Some College				
% of population	48.6	23.9	26.3	27.4
% of hires	48.2	55.4	56.9	58.1
College				
% of population	3.9	13.5	16.4	18.8
% of hires	4.5	4.5	5.4	6.1
Advanced Degree				
% of population	15.4	6.8	8.1	10.3
% of hires	12.1	10.6	12.6	16.2

Table 2: Search Effort by Labor Force Status: Extensive Margin

	Age 16+		Age 25-55	
	Daily Probability	Monthly Probability	Daily Probability	Monthly Probability
Employed	0.600	16.518	0.542	15.044
At Work	0.571	15.784	0.514	14.324
Absent	1.315	32.774	1.253	31.496
Unemployed	17.394	99.676	23.192	99.964
On Layoff	5.810	83.399	6.185	85.271
Looking	18.639	99.795	25.671	99.986
Out of the Labor Force	0.399	11.303	0.930	24.445

Notes: Summary statistics calculated from the pooled 2003-2019 ATUS. The daily probability of observing search within a group is calculated as the mean across the group of a binary variable that takes on the value 1 if the individuals engaged in any positive amount of job search the day of the interview. Job search activities are coded as categories 50401-50499 in the ATUS lexicon. Monthly probabilities are calculated as $1 - (1 - p)^{30}$, assuming 30 days per month and that the probability of searching for a job is the same every day.

Table 3: Search Effort by Labor Force Status: Minutes Per Day

	Age 16+		Age 25-55	
	Unconditional	Conditional	Unconditional	Conditional
Employed	0.702	116.862	0.661	121.922
At Work	0.613	107.453	0.587	114.105
Absent	2.820	214.411	2.546	203.194
Unemployed	25.868	148.719	36.479	157.292
On Layoff	8.41	144.747	9.593	155.103
Looking	27.744	148.852	40.398	157.369
Out of the Labor Force	0.530	132.788	1.305	140.287

Notes: Summary statistics calculated from the pooled 2003-2019 ATUS. Unconditional job search minutes and conditional and minutes conditional on positive search time.

Table 4: Search Effort by Labor Force Status: Percent of Time by Activity

	Age 16+			Age 25-55		
	E	U	O	E	U	O
Active Job Search	81.3	90.7	85.0	81.1	92.5	90.4
Interviewing	14.3	7.0	10.9	13.5	5.1	5.1
Other	4.4	2.2	4.1	5.3	2.4	4.6
N	622	1,431	208	453	1,015	131

Notes: Summary statistics calculated from the pooled 2003-2019 ATUS. Conditional on searching, the table show the percent of time spent in Active Job Search (050481), Interviewing (050403), and Other (050404-050499).

Table 5: Search Effort of People who are Out of the Labor Force

	Selection Probit	Searching LPM
Female	0.089 (0.000)	0.042 (0.066)
Married	-0.152 (0.000)	0.212 (0.075)
Child	-0.067 (0.000)	-0.406 (0.102)
Age	0.033 (0.000)	0.012 (0.004)
Race - Black	-0.286 (0.000)	0.206 (0.065)
Race - Other	0.144 (0.000)	0.369 (0.098)
High School	-0.344 (0.000)	-0.045 (0.067)
Some College	-0.343 (0.000)	0.180 (0.069)
College	-0.379 (0.000)	0.311 (0.080)
Advanced Degree	-0.435 (0.000)	0.221 (0.096)
Female \times Married	0.377 (0.000)	-0.079 (0.098)
Female \times Child	0.206 (0.000)	0.196 (0.116)
Same Respondent	-0.094 (0.000)	
Inverse Mills		3.923 (0.398)
Diary Day FE	✓	✓
Month FE	✓	✓
Year FE	✓	✓

Table 6: Logit Parameters

Parameter	Employed	Unemployed	Out of the Labor Force	Parameter	Employed	Unemployed	Out of the Labor Force
Monday	0.358	0.512	0.520	Female \times Full Time	0.398	Not Included	Not Included
Saturday	-0.563	-0.843	-0.410	College \times Age		-0.004	
Sunday	-0.550	-0.674	-0.257	High School \times Age			
Thursday	0.341	0.288	0.604	Less than HS \times Age		0.016	
Tuesday	0.275	0.441	0.635	Some College \times Age		0.019	
Wednesday	0.247	0.672	1.014	College \times Married		-0.056	
Female	-0.670	-0.379	-0.415	High School \times Married			
Age	-0.027	-0.003	-0.059	Less than HS \times Married		0.176	0.240
College	-0.157	-0.048	-0.065	Some College \times Married		0.295	
High School	-0.843	-0.771	-1.041	College \times Child		0.276	
Less than HS	-1.296	-2.169	-1.318	High School \times Child			
Some College	-0.640	-1.670	-0.305	Less than HS \times Child			-0.574
Married	-0.541	-0.101	0.595	Some College \times Child	0.152	0.350	0.058
Child	0.048	-0.207	-0.289	College \times Race - Other	-0.635	-0.839	
Race - Other	-0.656	0.183	-0.175	High School \times Race - Other		-0.740	
Race - White	-0.640	-0.129	-0.921	Less than HS \times Race - Other			-1.515
Full Time	-1.397	Not Included	Not Included	Some College \times Race - Other		-0.598	-2.003
Female \times Age				College \times Race - White			
Female \times College				High School \times Race - White			
Female \times High School		-0.053		Less than HS \times Race - White			
Female \times Less than HS				Some College \times Race - White			
Female \times Some College		0.221		College \times Full Time		Not Included	Not Included
Female \times Married		-0.304	-0.803	High School \times Full Time		Not Included	Not Included
Female \times Child	-0.101	-0.074		Less than HS \times Full Time		Not Included	Not Included
Female \times Race - Other		0.295	0.029	Some College \times Full Time	0.105	Not Included	Not Included
Female \times Race - White		0.110	-0.255	Constant	-1.729	-0.329	-1.446

Note: A total of 57 parameters we included in the Logit estimation for the Employed group and 51 for the Unemployed and Out of the labor force. The first 17 (18 for employed) are always included and the remaining interaction terms are chosen with a net-elastic logit with a weight of 0.95 on the LASSO penalty,. The regularization parameter is chosen using cross-validation of 10 folds.

Table 7: Percentiles of Predicted Search Effort

	Employed	Unemployed	Out of the Labor Force
5th Percentile	0.0285	0.8637	0.0002
10th Percentile	0.0398	0.9181	0.0006
25th Percentile	0.0654	0.9770	0.0044
50th Percentile	0.1086	0.9967	0.0444
75th Percentile	0.1655	0.9998	0.1557
90th Percentile	0.2601	1.0000	0.3073
95th Percentile	0.3504	1.0000	0.4405

Table 8: Correlation between Search Effort and Labor Force Attachment: 1994-2019

	Job Finding Prob.		Hours Worked		Change in Hours	
Search Probability	0.190 (0.000)	0.191 (0.000)	7.728 (0.067)	7.838 (0.067)	22.335 (0.250)	22.275 (0.249)
Mean	0.037	0.037	30.33	30.33	0.33	0.33
Month \times Year FE		✓		✓		✓
Observations	17608693	17608693	345967	345967	188130	188130
Sample	Full	Full	Nonemployed Job Finders	Nonemployed Job Finder	Employed Job Switchers	Employed Job Switchers

Table 9: Standard Deviations of Logged Statistics 1980-2020

	Job Seekers	Vacancy Rate	Tightness	Job Finding Rate	Separation Rate
Standard	0.279	0.236	0.459	0.163	0.218
Adjusted	0.146	0.238	0.347	0.120	0.158
Total Searcher	0.065		0.284		

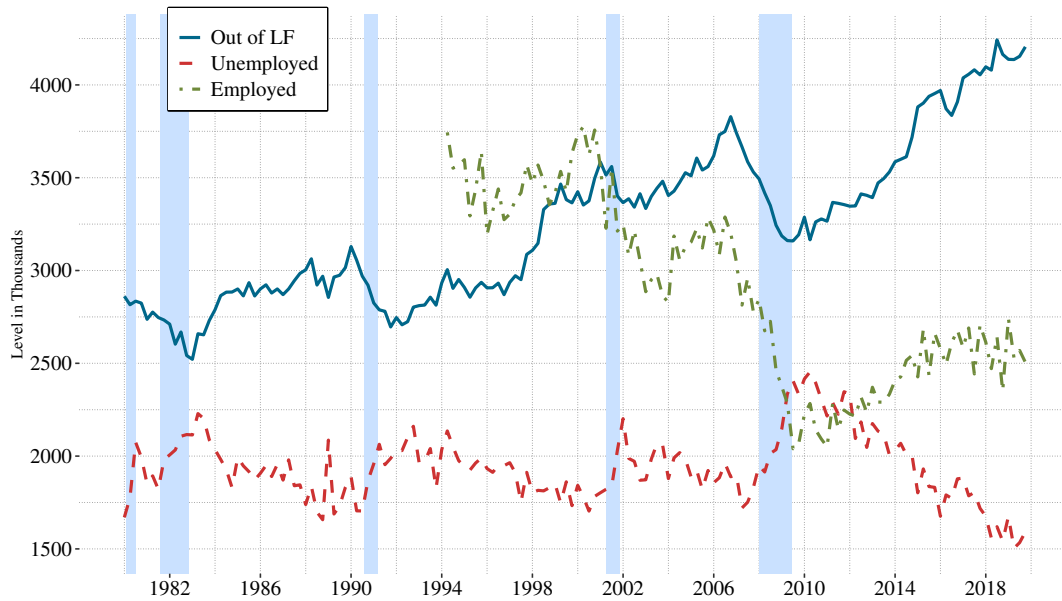
Table 10: Correlations with Inflation and Nominal Wage Growth

	1980Q1 - 2019Q4					
	Average Hourly Earnings			Consumer Price Index		
	(1)	(2)	(3)	(4)	(5)	(6)
$U\text{-Gap}$	-0.073** (0.029)			-0.242 (0.172)		
$U\text{-Gap} \times \text{Post}$	0.086* (0.050)			0.668 (0.399)		
\tilde{U}		-0.065*** (0.018)			-0.272** (0.136)	
$\tilde{U} \times \text{Post}$		0.004 (0.024)			-0.097 (0.456)	
S			-0.080*** (0.019)			-0.476*** (0.167)
$S \times \text{Post}$			0.009 (0.027)			0.029 (0.553)
$\bar{\pi}_{t-1}^{PCE}$	0.211*** (0.025)	0.088*** (0.026)	0.064*** (0.022)			
$\bar{\pi}_{t-1}^{PCE} \times \text{Post}$	0.029 (0.071)	-0.087** (0.035)	-0.069** (0.031)			
$\bar{\pi}_{t-1}^{CPI}$				0.972*** (0.050)	0.626*** (0.134)	0.593*** (0.128)
$\bar{\pi}_{t-1}^{CPI} \times \text{Post}$				-0.431 (0.289)	-0.898** (0.410)	-0.880** (0.413)
Intercept		0.009*** (0.002)	0.021*** (0.004)		0.048*** (0.016)	0.130*** (0.040)
Post		0.003 (0.003)	0.002 (0.006)		0.052 (0.065)	0.036 (0.145)

Notes: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are heteroskedastic and autocorrelation consistent. $U\text{-Gap}$ refers to the difference of the standard unemployment rate and the Congressional Budget Office's natural rate of unemployment (NRU). $\bar{\pi}_{t-1}^x$ is the four quarter average of the year-over-year inflation rate measured by the Consumer Price Index ($x=CPI$) or the Personal Consumption Expenditure Index ($x=PCE$). Post is an indicator variable that takes on the value 1 after 2007Q2.

8 Figures

Figure 1: Total Hires by Labor Market Status



Note: The figure plots the number of new hires from each labor market state in thousands. Details of how these values are calculated can be found in Appendix A.1.

Figure 2: Receiver Operating Characteristic Curve

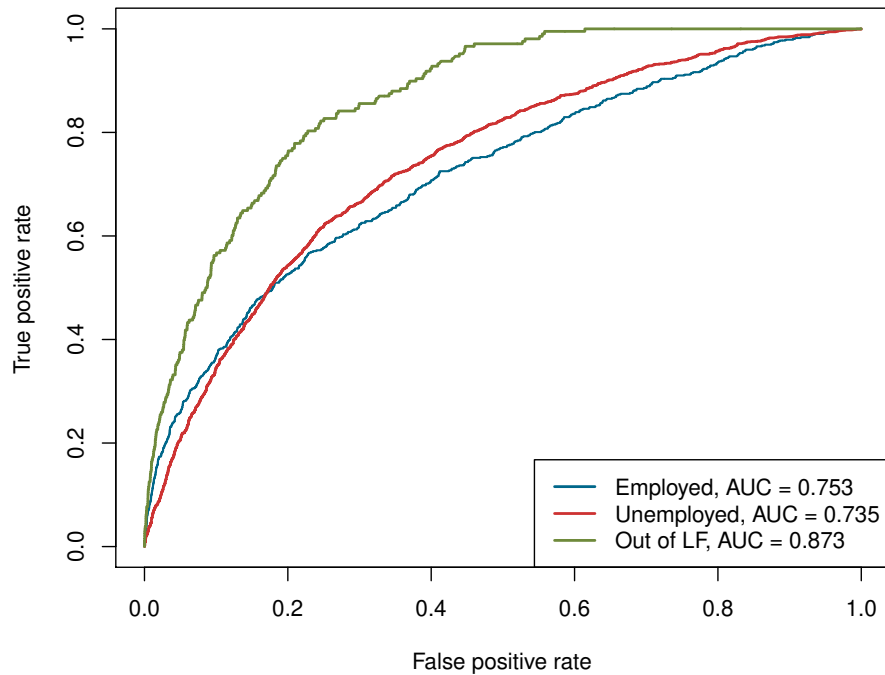
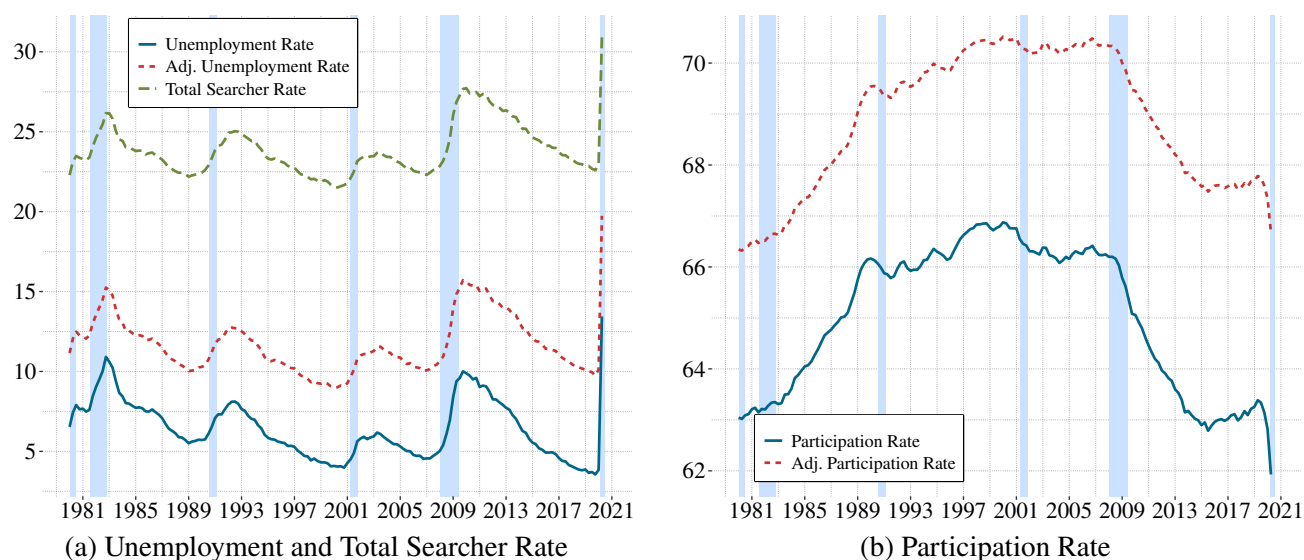


Figure 3: Fraction of Job Searchers



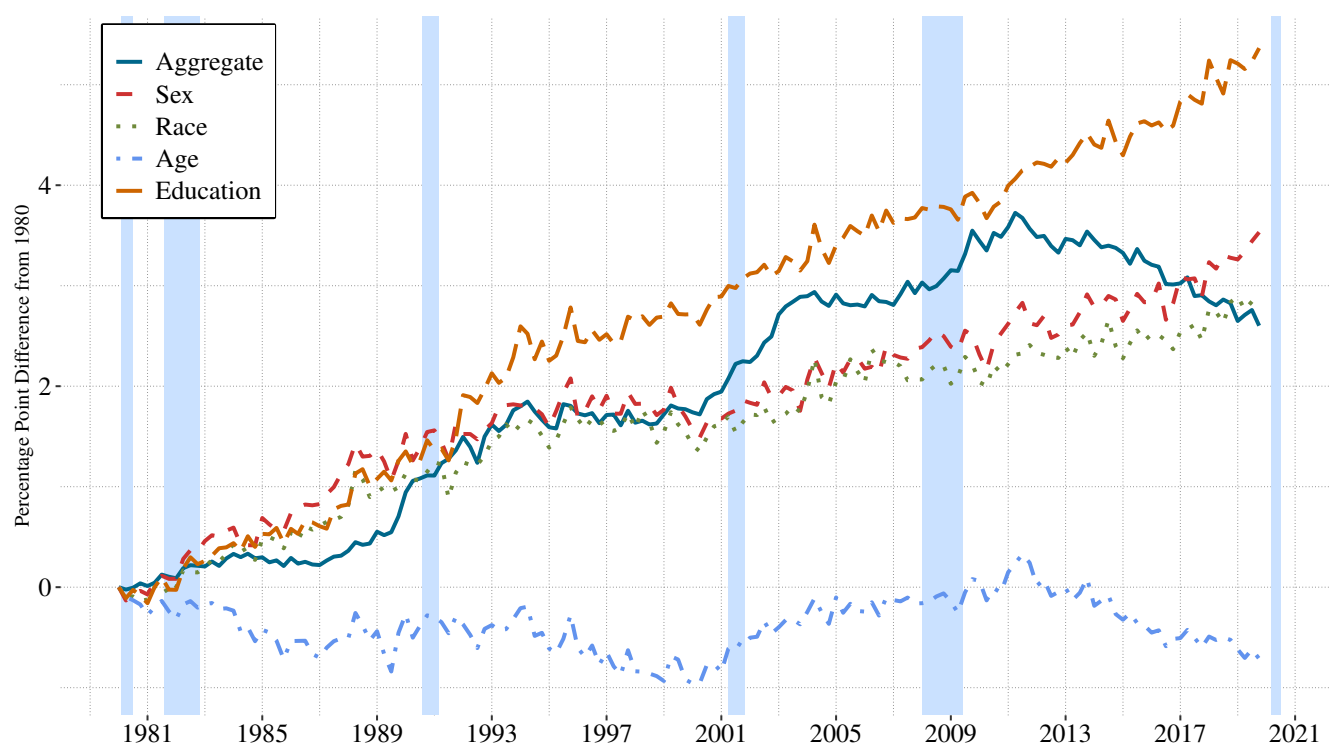
Note: The figure plots the quarterly fraction of job searchers among the employed, unemployed, out of the labor force and total population.

Figure 4: Unemployment, Total Searcher and Participation Rate



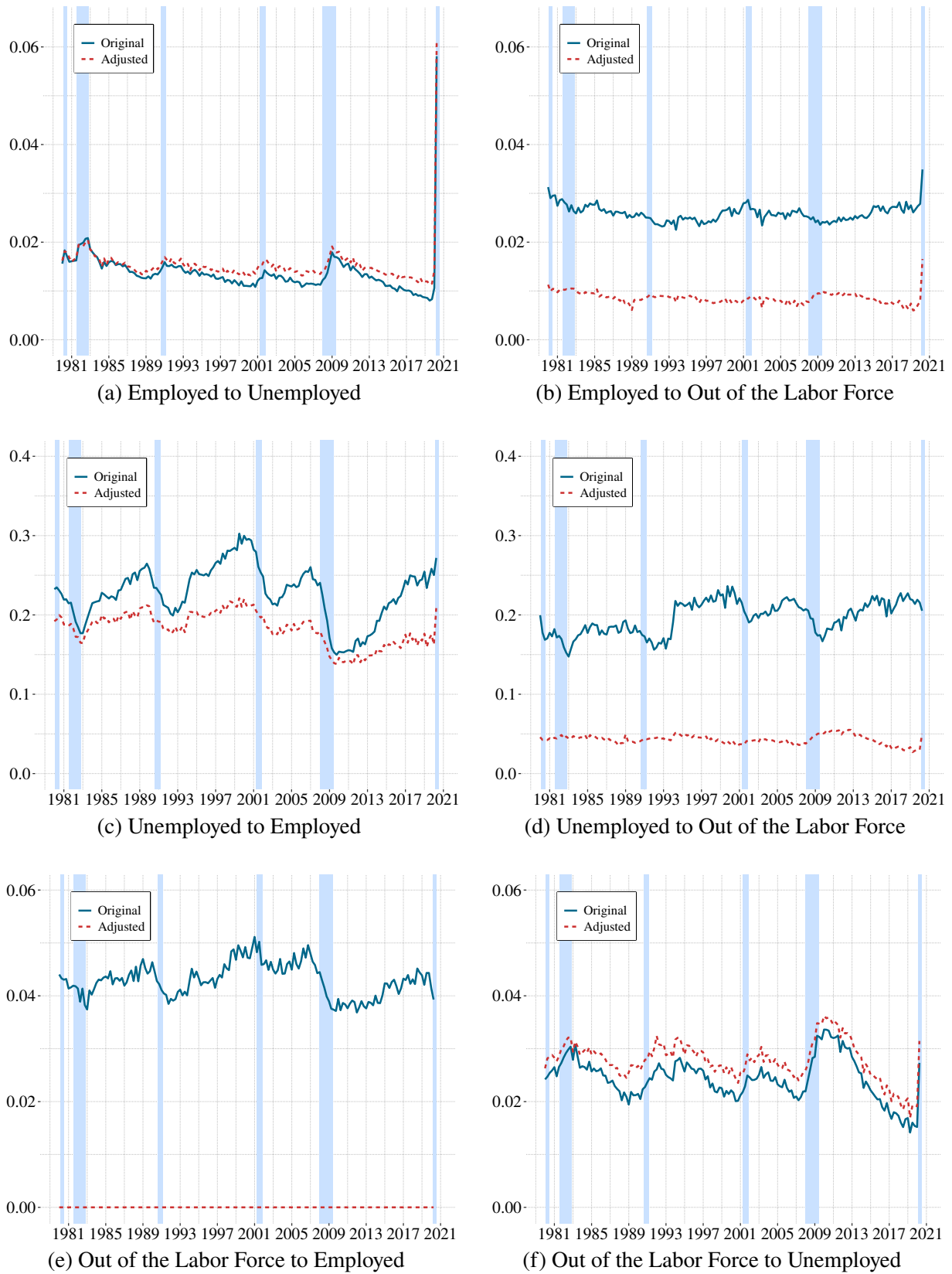
Note: The figure plots the quarterly original and adjusted unemployment rate and participation rate.

Figure 5: Decomposition of Out of the Labor Force Searching for a Job



Note: The figure plots the fraction of out of the labor force that is searching for a job ("Aggregate") along with the four counterfactual series if the fraction of each demographic group remained at its 1980's values. Each series is graphed as a percentage point difference from the 1980 value.

Figure 6: Labor Market Flows



Note: The figure plots the seasonally adjusted quarterly original and adjusted flows across labor market states.

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A Appendix

A.1 Data Cleaning and Aggregation

A.1.1 CPS Demographic Data

The main data used to calculate the number of individuals and flows across the labor market come from the basic monthly files of the Current Population Survey from January 1979 to December 2018. Over this period several changes to the demographic variables used to predict the search probability occurred. First, whether or not a child was present in the home was not asked prior to 1984 and between January 1994 to October 1999. For the years prior to 1984, the indicator for if a child was present is replaced with the sample mean in 1984. During the months between January 1994 and October 1999, the indicator is replaced with the average of the sample average in 1993 and 2000. Second, the education variable changed from a continuous measure to a discrete degree based measure in 1992. The education variable is made consistent using the method described in [Jaeger \(1997\)](#).

A.1.2 Aggregation

After predicting and aggregating several of the series have a small discrete jump at different dates. The employed searchers series as a discrete jump in January 1989 and January 1994. The jump is removed from the series by multiplying a constant term to the series pre January 1994 and pre January 1989 such that the ratio of January 1989 to December 1988 and January 1994 to December 1993 are equal to the average January/December ratio over the whole sample. The unemployment and search unemployment series are adjusted December 1983 to January 1984 and March to May 1995 in the same fashion. The out of the labor force series is adjusted December 1988 to January 1989 and December 2002 to January 2003. The out of the labor force search series is adjusted January to February 1984, December 1988 to January 1989, January to February 1989, December 1991 to January 1992, December 1993 to January 1994, and November to December 1999. All series are seasonally adjusted using the X-13ARIMA-SEATS seasonal adjustment program provided by the Census Bureau, available at: <https://www.census.gov/srd/www/x13as/>.

A.1.3 Flows and Hires

Flows of the labor market are calculated using the matched basic monthly files of the Current Population Survey. The matching and flow calculation files are taken from Robert Shimer and modified; the original programs are available at: <https://sites.google.com/site/robertshimer/research/flows>. The flows are first seasonally adjusted using X-13, and then corrected for margin error similar to [Elsby et al. \(2015\)](#) (described in detail below).

Hires plotted in [Figure 1](#) are calculated using the match basic monthly files and calculating the total number of hires by previous labor force status (including employed at a different employer starting in 1994) per total population of the matched monthly files. That fraction is then multiplied by the total population of the basic monthly files. The series are seasonally adjusted using X-13ARIMA and averaged to quarterly values. The percent of hires in each demographic group in [Table 1](#) are created analogously.

A.2 Margin Error Adjustment

The margin error adjustment is similar to [Elsby et al. \(2015\)](#). First define new stocks of unemployed and out of the labor force as:

$$\tilde{U}_t = U_t^S + O_t^S$$

$$\tilde{O}_t = O_t - O_t^S$$

. Then let $S_t = [E_t \ \tilde{U}_t]$ be the vector containing the number of employed in unemployed and let $\Delta S_t = [E_t \ \tilde{U}_t]' - [E_{t-1} \ \tilde{U}_{t-1}]'$ be the change in the current state vector. The change in the current state vector can be written as:

$$\Delta S_t = \begin{bmatrix} -E_{t-1} & -E_{t-1} & \tilde{U}_{t-1} & 0 & 0 \\ E_{t-1} & 0 & -\tilde{U}_{t-1} & -\tilde{U}_{t-1} & \tilde{O}_{t-1} \end{bmatrix} \times \begin{bmatrix} p_{E\tilde{U}} \\ p_{E\tilde{O}} \\ p_{\tilde{U}E} \\ p_{\tilde{U}\tilde{O}} \\ p_{\tilde{O}\tilde{U}} \end{bmatrix}$$

$$\Delta S_t = \mathbf{X}_{t-1} \mathbf{p}$$

Where p_{jk} is probability that an individual transitions from labor market state j to k . Notice that here, individuals can not directly transition from out of the labor force to employment. The estimated vector of transition probabilities, denoted $\hat{\mathbf{p}}$, has a covariance matrix proportional to the matrix that is consistently estimated using:

$$\mathbf{W} = \begin{bmatrix} \frac{\hat{p}_{E\tilde{U}}(1-\hat{p}_{E\tilde{U}})}{E_{t-1}} & -\frac{\hat{p}_{E\tilde{U}}\hat{p}_{E\tilde{O}}}{E_{t-1}} & 0 & 0 & 0 \\ -\frac{\hat{p}_{E\tilde{U}}\hat{p}_{E\tilde{O}}}{E_{t-1}} & \frac{\hat{p}_{E\tilde{O}}(1-\hat{p}_{E\tilde{O}})}{E_{t-1}} & 0 & 0 & 0 \\ 0 & 0 & \frac{\hat{p}_{\tilde{U}E}(1-\hat{p}_{\tilde{U}E})}{\tilde{U}_{t-1}} & -\frac{\hat{p}_{\tilde{U}E}\hat{p}_{\tilde{U}\tilde{O}}}{\tilde{U}_{t-1}} & 0 \\ 0 & 0 & -\frac{\hat{p}_{\tilde{U}E}\hat{p}_{\tilde{U}\tilde{O}}}{\tilde{U}_{t-1}} & \frac{\hat{p}_{\tilde{U}\tilde{O}}(1-\hat{p}_{\tilde{U}\tilde{O}})}{\tilde{U}_{t-1}} & 0 \\ 0 & 0 & 0 & 0 & \frac{\hat{p}_{\tilde{O}\tilde{U}}(1-\hat{p}_{\tilde{O}\tilde{U}})}{\tilde{O}_{t-1}} \end{bmatrix}$$

The vector of transition probabilities, \mathbf{p} , is chosen to minimize the weighted least square to the estimated transition probabilities and restricted to match the observed changes in labor market states. That is:

$$\mathbf{p} = \text{argmin } (\mathbf{p} - \hat{\mathbf{p}})' \mathbf{W}^{-1} (\mathbf{p} - \hat{\mathbf{p}}) \quad \text{s.t. } \Delta S_t = \mathbf{X}_{t-1} \mathbf{p}$$

The Lagrangian is

$$\mathcal{L} = (\mathbf{p} - \hat{\mathbf{p}})' \mathbf{W}^{-1} (\mathbf{p} - \hat{\mathbf{p}}) - 2\mu [\Delta S_t - \mathbf{X}_{t-1} \mathbf{p}]$$

where μ is the vector of Lagrange multipliers. The solution is,

$$\begin{bmatrix} \mathbf{p} \\ \mu \end{bmatrix} = \begin{bmatrix} \mathbf{W} & \mathbf{X}'_{t-1} \\ \mathbf{X}_{t-1} & 0 \end{bmatrix}^{-1} \times \begin{bmatrix} \mathbf{W}\hat{\mathbf{p}} \\ \Delta\mathbf{S}_t \end{bmatrix}.$$

Since all objects on the right-hand side are known, the above equation gives the solution to the margin error adjusted probabilities \mathbf{p} .