Competitive/Directed Search

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Random vs Directed vs Competitive

- **Random Search:** Unemployed workers and firms with vacancies bump into each other at random.
- Burdett-Mortenson (on the job search) is an example with wage posting
  - firms differ in terms of the wage they offer
  - workers know the overall wage distribution
  - workers can not pick out which firms offer which wage
  - workers randomly get an offer from the wage distribution
- DMP is an example with wage bargaining
  - workers randomly meet firms
  - after they match they bargain over the wage

Random vs Directed vs Competitive

- **Directed Search:** Workers observe wages and then decide which jobs to apply to, matching process is model
- Burdett, Shi, Wright (2001) is an example we have seen
  - firms take workers application strategies into consideration when choosing which wage to post
  - workers choose probabilities of applying based on posted wages
  - this gives us a micro foundation for the matching function
- **Competitive Search:** Workers observe wages and then decide which jobs to apply to, matching process is not specifically modeled, sometimes refers to the type of equilibrium
- Directed and competitive are often used interchangeably

Random vs Directed vs Competitive

- The difference is information and commitment about wages
- **Random search:**
  - worker has no information prior to applying
  - randomly bumps into a firm with a vacancy
  - wage is either
    1. bargained over (DMP)
    2. firm makes a take it or leave it offer (BM)
- **Directed/Competitive search**
  - Firms post a wage with full commitment
  - Workers choose which wage to apply to
  - Wage is exactly what was posted

A simple directed/competitive search model

- **Time** is continuous
- **Workers**
  - homogeneous
  - search for jobs when unemployed
  - get b when unemployed
  - discount at rate r
- **Firms**
  - homogeneous
  - post vacancies at cost κ
  - post wages under full commitment to max profits
  - produce y when job is filled
  - discount at rate r

A simple directed/competitive search model

- **Matches**
  - CRS matching function for each wage posted
  - matching function depend on tightness $u(w)/v(w) = \theta(w)$, where $u(w)$ is the number of unemployed that apply to wage w and $v(w)$ is the number of jobs open at wage w
  - firms meet workers at rate $q(\theta(w))$
  - worker meet firms at rate $p(\theta(w)) = \theta(w)q(\theta(w))$
  - matches end at rate δ

A simple directed/competitive search model

- **Equilibrium**
  - workers and firms are all identical so only one wage will be posted (drop the w indexing from everything)
  - equilibrium consists of w^* and θ^* such that
    - free entry drives value of a vacancy to 0
    - firms profits are maximized

    and a value of unemployment $r\bar{U}$ such that the worker is no better off applying to any other job.
- **Comparison to DMP with bargaining**
  - job creation curve
  - bargaining solution

A simple directed/competitive search model

- Workers value functions

$$rU = b + p(\theta)[E(w) - U]$$

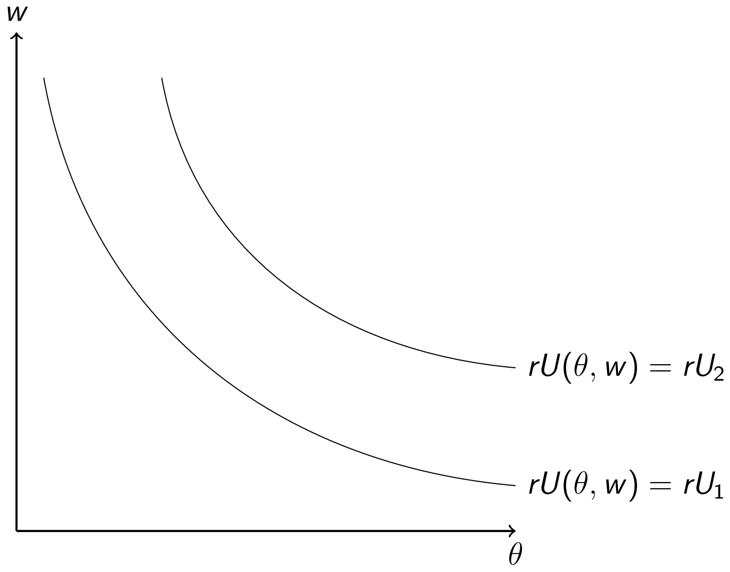
$$rE(w) = w + \delta[U - E(w)]$$

- Combine these two

$$rU(\theta, w) = \frac{(r+\delta)b + p(\theta)w}{r + \delta + p(\theta)}$$

- $rU(\theta, w)$ is increasing in w and θ and quasi-concave

$rU(\theta, w)$ indifference curves for $U_1 < U_2$



A simple directed/competitive search model

- Firms value functions

$$rV = -\kappa + q(\theta)[J(w) - V]$$

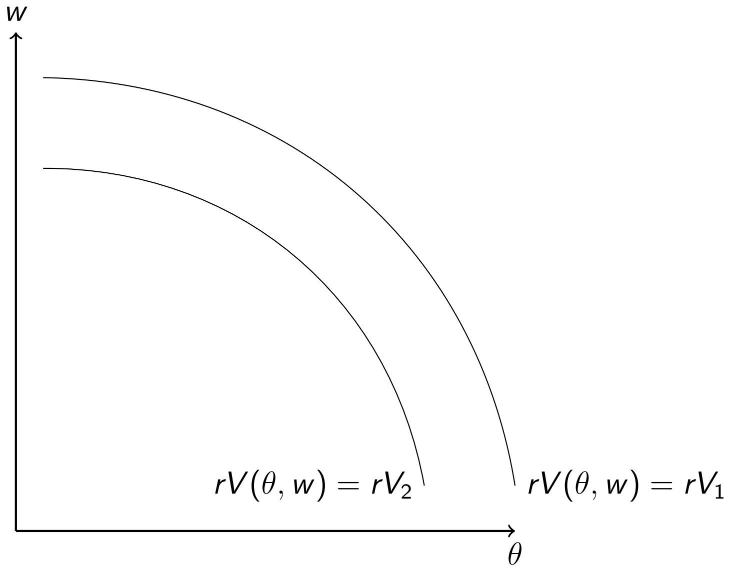
$$rJ(w) = y - w + \delta[V - J(w)]$$

- Combine these two

$$rV(\theta, w) = \frac{-(r+\delta)\kappa + q(\theta)[y - w]}{r + \delta + q(\theta)}$$

- $rV(\theta, w)$ is decreasing in w and θ and quasi-convex

$rV(\theta, w)$ indifference curves for $V_1 < V_2$



A simple directed/competitive search model

- By free entry we have $V = 0$ and

$$J = \frac{y - w}{r + \delta}$$

- Firm maximizes expected profits s.t. worker receiving some "market value" of unemployment $r\bar{U}$

$$\max_{w,\theta} \frac{q(\theta)[y - w]}{r + \delta} \text{ s.t. } rU(\theta, w) \geq r\bar{U}$$

- Plug in workers unemployment value to eliminate w and $q(\theta) = \theta p(\theta)$

$$\max_{\theta} \ \frac{q(\theta)[y - r\bar{U}]}{r + \delta} - \theta[r\bar{U} - b]$$

A simple directed/competitive search model

- FOC for θ give us

$$q'(\theta^*)\frac{y - r\bar{U}}{r+\delta} = r\bar{U} - b$$

- From the free entry condition we also know that

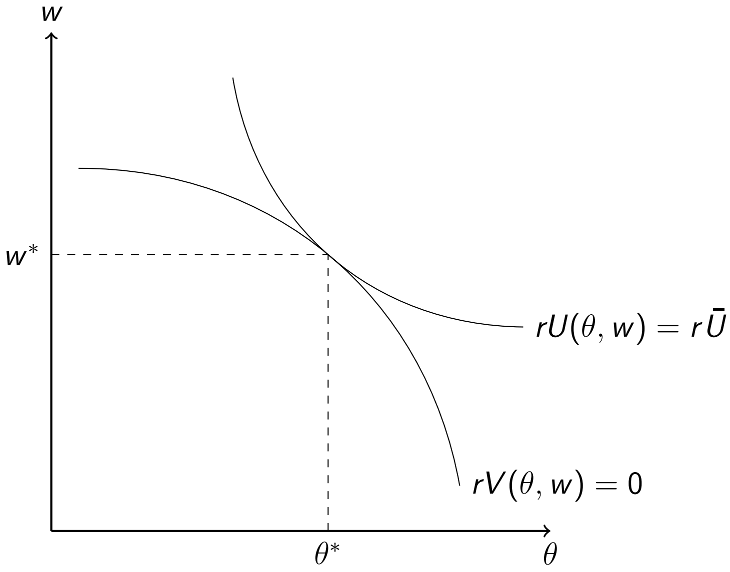
$$\frac{\kappa}{q(\theta^*)} = \frac{y - w^*}{r+\delta}$$

- And from the workers value of unemployment we know

$$r\bar{U} = \frac{(r+\delta)b + p(\theta^*)w^*}{r+\delta+p(\theta^*)}$$

- We have three unknowns $(w^*, \theta^*, r\bar{U})$ and three equations

Equilibrium



Equilibrium Wage

- The equilibrium wage comes out to be

$$w^* = \frac{q'(\theta^*)\theta^*}{q(\theta^*)} y + \left(1 - \frac{q'(\theta^*)\theta^*}{q(\theta^*)}\right) r\bar{U}$$

- The wage from DMP with bargaining

$$w = \gamma y + (1-\gamma) rU$$

- Hosios condition says that bargaining is efficient when γ is equal to the elasticity of the job filling rate, $q(\theta)$

$$w = \frac{q'(\theta)\theta}{q(\theta)} y + \left(1 - \frac{q'(\theta)\theta}{q(\theta)}\right) rU$$

Efficiency

- Directed/competitive search is efficient
  - with wage posting and directed search the surplus of the job is split efficiently
  - the Hosios condition hold endogenously
- Why?
  - competition between firms maximizes the workers utility
  - free entry implies firms always break even
- Is directed/competitive always efficient?
  - generally true, with CRS matching function
  - but not always the case, for example not with private information, Guerrieri & Shimer (2018)

Heterogeneity

- Now let's see what happens when we have firm heterogeneity
- Assume firms differ in their productivity y_j
- **Random Search** with wage posting
  - no wage distribtuion
  - all firms post b
  - workers indifferent between working and not working
  - no reason to search
- **Competitive Search:** Moen (1997)
  - firms post different wages
  - this creates "submarkets"

An alternative to profit maxing

- Until now we assumed firms posted wages by maxing profits
- Alternative approach is to assume there exists a market maker
  - decides how many markets there are
  - what wage must be posted in each market
  - chooses these optimally by
    1. (1) workers go to the market that gives them the highest value of unemployment
    2. (2) firms break even in each market
- This alternative gives the same equilibrium conditions

Moen 1997: Environment

- **Time** is continuous
- **Markets**
  - there exits $m \geq 1$ submarket, indexed by i
  - each market has a distinct wage w_i
  - there exits a matching function for each market $M(u_i, v_i)$ where u_i and v_i are the number of searchers and vacancies in each market, $\theta_i = v_i / u_i$
  - $p(\theta_i)$ job finding rate in market i
  - $q(\theta_i)$ job filling rate in market i

Moen 1997: Environment

- **Workers**
  - homogeneous
  - direct their search to one market
  - get b when unemployed
  - discount at rate r
- **Firms**
  - pays a cost χ to open a vacancy
  - draw productivity from a discrete probability distribution F with mass points at $\{y_1, ...y_n\}$
  - choose which market to post vacancy in
  - pay κ to search for a worker
  - discount at rate r
- Job destruction at rate δ

Moen 1997: Workers

- Workers value of unemployment if searching in market i

$$rU_i = b + p(\theta_i)[E(w_i) - U_i]$$

- Worker chooses submarket that maximizes unemployment value

$$U = \max\{U_1, ..., U_m\}$$

- Workers value of employment in market i

$$rE(w_i) = w_i + \delta[U - E(w_i)]$$

Moen 1997: Workers

- Since workers are identical, any submarket that attracts workers must offer the same unemployment value, and it must be the maximum U

- Substituting U and $E(w_i)$ in for the value of unemployment in market i
$$rU = \frac{(r+\delta)b + w_i p(\theta_i)}{r+\delta+p(\theta_i)}$$
$$p(\theta_i) = \frac{rU - b}{w_i - rU}(r+\delta)$$

Moen 1997: Workers

$$p(\theta_i) = \frac{rU - b}{w_i - rU}(r + \delta)$$

- $w_i \to rU$ the gain from employment goes to zero so $\theta_i \to \infty$
- $w_i \to \infty$ the gain from employment goes to ∞ so $\theta_i \to 0$
- $w_i < rU$ no workers will show up in that market, market maker shuts it down
- workers are indifferent between waiting a long time to find a high paying job or finding a low paying job quickly
- so submarkets are classified by w_i and corresponding θ_i

Moen 1997: Firms

- After paying χ a firm draws a productivity y_j
- Then decides which submarket (w_i, θ_i) to post the vacancy in
- Value of posting a vacancy in market i

$$rV(y_j, w_i, \theta_i) = -\kappa + q(\theta_i)[J(y_j, w_i) - V(y_j, w_i, \theta_i)]$$

- Value of a filled job in market i

$$rJ(y_j, w_i) = y_j - w_i - \delta J(y_j, w_i)$$

- Plugging J into V

$$(r+\delta)V(y_j, w_i, \theta_i) = q(\theta_i)\frac{y_j - w_i}{r+\delta} - \kappa$$

- Firm chooses submarket that maximizes $V(y_j, w_i, \theta_i)$

Moen 1997: Equilibrium

- Equilibrium objects
  - the set of open submarkets $\mathcal{I}$
  - the wage in each submarket $w_i^* \ \forall i \in \mathcal{I}$
  - the labor market tightness in each market $\theta_i^* \ \forall i \in \mathcal{I}$
  - the number of unemployed in each market $u_i^* \ \forall i \in \mathcal{I}$
  - a value of unemployment U

Moen 1997: Equilibrium Wages

- First, from the worker's indifference condition

$$p(\theta_i) = \frac{rU - b}{w_i - rU}(r + \delta)$$

  we have can solve for tightness as a function fo the wage $\theta(w_i)$

- A set of equilibrium wages W^* is optimal if there does not exists a wage w' such that

$$V(y_j, w', \theta(w')) > V(y_j, w^*, \theta(w^*))$$

  for all $w^* \in W^*$.

Moen 1997: Equilibrium Wages

- This implies the market maker chooses wages such that

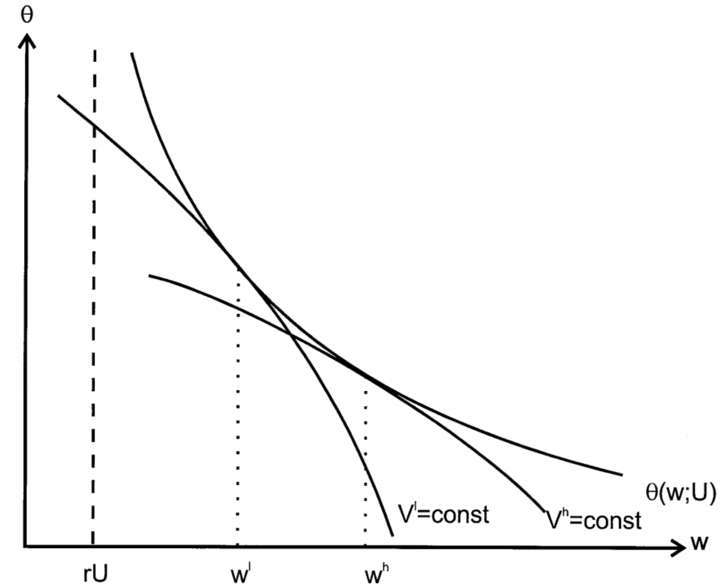
$$w_i^* = \underset{w_i}{\operatorname{argmax}}\ V(y_j, w_i, \theta(w_i))$$

- Each submarket has one type of productivity and each submarket maximizes the value of posting a vacancy in that market for a given productivity

- Will index everything by i, productivity and markets

- Again from the workers indifference we now have θ_i^*

Moen 1997: Equilibrium Wages



Moen 1997: Equilibrium Number of Markets

- We know that each productivity will form a separate market
- There are n productivities in the distribution
- All submarkets such that $w_i \geq rU$ will remain open
- Let ι denote the lowest submarket open

Moen 1997: Equilibrium

- So far we have
  - w_i^* from maximizing the vacancies
  - θ_i^* from the worker indifference condition
  - $\mathcal{I} = \{\iota, ..., n\}$ from the reservation wage
- All of these still depend on U

Moen 1997: Free Entry

- Free entry implies that the expected value of opening a vacancy will be equal to the cost of opening it χ

- The expected value of opening a vacancy

$$\bar{V}(U) = \sum_{i=\iota(U)}^{n} f_i V(y_i, w_i^*(U), \theta_i^*(U))$$

- In equilibrium it will be that

$$\bar{V}(U) = \chi$$

Moen 1997: Beveridge Curve

- Just as in DMP, to pin down for u_i^* given θ_i^* we need the Beveridge curve
- The outflow of unemployment in market i

$$u_i p(\theta_i)$$

- The inflow to unemployment in market i

$$(1-u)\delta\frac{f_i}{1-F_\iota}$$

where $F_\iota = Pr[y \leq y_i]$ and $u = \sum_{i=\iota}^{n} u_i$

- In equilibrium, inflow = outflow, given θ_i^* we can solve for u_i^*

$$u_i p(\theta_i) = (1-u)\delta\frac{f_i}{1-F_\iota}$$

Moen 1997: Summary of Equilibrium

- Free entry

$$\bar{V}(U) = \chi$$

- Vacancy maximizing

$$w^* = \underset{w}{\operatorname{argmax}} \; V(y_i, w, \theta(w)) \;\; \forall i \geq \iota$$

- Worker indifference

$$p(\theta_i) = \frac{rU - b}{w_i - rU}(r + \delta) \;\; \forall i \geq \iota$$

- Beveridge Curve

$$u_i p(\theta_i) = (1 - u)\delta \frac{f_i}{1 - F_\iota} \;\; \forall i \geq \iota$$

- Aggregate unemployment

$$u = \sum_{i=\iota}^{n} u_i$$

Moen 1997: Properties of the Equilibrium

- The equilibrium is not necessarily unique
  - the vacancy maximizing equation may have more than one solution
- U is unique, every equilibrium will have the same value of unemployment for the workers
- All equilibria are optimal
  - the distribution of unemployed and vacancies across submarkets is efficient
  - the total number of vacancies opened is efficient

Directed vs Random in the data

- Godoy & Moen (2015): competitive search with on the job search (Garibaldi, Moen, and Sommervoll (2016))
    - if search is random, the ratio of probabilities of observing a worker at $w_1 > w_{previous}$ to $w_2 > w_{previous}$ is independent of $w_{previous}$
    - show in Danish data that this is not true
- Engelhardt & Rupert (2016): test the specification of directed search, if workers search in different submarkets of different productivities, if the wage satisfies the Hosios condition in each market.

Directed vs Random in the data

- Braun, Engelhardt, Griffy, Rupert (2020): show that the job finding rate and wage are not independent.
  - show that UI increases, decrease job finding rates differentially across the wage distribution.
- Lenz & Moen (WP): have a model where a parameter μ determines how directed a workers search is. Estimate μ on Danish data.
  - $\mu \to 0$ fully directed
  - $\mu \to \infty$ fully random