

Revisiting Unemployment with an Intensive Margin*

Christine Braun[†]

University of Warwick

January 12, 2021

Abstract

The unemployment rate is the most used single indicator of labor market conditions, but its measure is black and white, lacking any notion of intensity. This paper introduces a continuous unemployment rate; a measure in which people are weighted by their relative search effort. The measure of relative search effort is a monthly probability of exerting search effort, estimated from the American Time Use Survey. The paper delivers a continuous unemployment rate, as well as adjusted labor market flows, for the United States from 1980 onward. On average, the continuous unemployment rate is 1.4 percentage points higher than the standard unemployment rate. While the standard unemployment rate displays a low frequency downward trend, the continuous unemployment rate does not. This difference is attributed to the upward trend in the search effort of individuals classified as out of the labor force; this upward can be accounted for by their increasing educational attainment. The continuous unemployment rate is used to re-estimate the Phillips curve. Contrary to estimates with the standard unemployment rate, the estimated Phillips curve with the continuous unemployment rate shows no sign of a flattening output-inflation relationship in the post-2008 recession period.

Keywords: Unemployment, Labor Force Participation, Worker Flows, Phillips Curve

*I want to thank David Dorn, Nir Jaimovich, Tim Kehoe, Roland Ratholet, and Thijs van Rens, for invaluable input at early stages of the project. I also want to thank seminar participants at the University of California, Santa Barbara, University of Zurich, University of Warwick, University of Munich, University College London, Paris School of Economics, and the University of British Columbia. All remaining errors are my own.

[†]Department of Economics, University of Warwick. christine.braun@warwick.ac.uk.

1 Introduction

Summarizing a complex system, such as the labor market, in one comprehensive statistic, is a daunting task. For decades, the most undisputed candidate for this statistic has been the unemployment rate. Over 120 years after the first attempts to measure unemployment at a national level, Federal Reserve Chair, Janet Yellen, stated “the unemployment rate is probably the best single indicator of current labor market conditions.”¹ Despite being the most influential statistic in assessing the health of the labor market, the modern concept of unemployment in the United States has remained unchanged since 1940, when unemployment was first aligned with the notion of active job search.²

Today, a person is considered unemployed if they are available to work and have had at least one active search attempt in the last 4 weeks. This discrete view of labor market attachment implies that all unemployed people contribute with equal weight to the unemployment rate. In reality, the degree to which people are attached to the labor force varies. Some people may be searching for only part time work or accept only jobs with only certain characteristics, while others may be searching broadly and accept any job offered to them. People differ in terms of their labor force attachment or unemployment intensity. The current measure of unemployment lacks any notion of intensity – a concept widely accepted when measuring employment as total hours worked or full time equivalents.

In this paper, I create a new continuous unemployment rate, in which people are weighted by their relative search intensity. The measure of search intensity is a monthly probability of exerting search effort, estimated from the American Time Use Survey. All non-employed individuals contribute to the continuous unemployment rate and those with higher search probabilities enter with more weight. I argue that the estimated search intensity is a predictor of labor force attachment by showing that search intensity is positively correlated with the job finding probability and subsequent hours worked. Along with the continuous unemployment rate, I also construct a measure of total search effort in the economy, i.e., including those searching while employed, and adjusted labor market flows.

The continuous unemployment rate is on average 1.4 percentage points higher than the standard unemployment rate and is nearly half as volatile as the standard unemployment rate. The increase in the level of the continuous unemployment rate is no surprise in light of the fact that nearly 70% of new hires every month are people who were previously classified as out of the labor force. In 2017, an average of 4.1 million hires per month came from out of the labor force, that is nearly 2.5 times as many hires as from unemployment and 1.4 times as from employment.³ The level of any statistic used to assess the

¹Yellen, Janet L. Speech at the 2013 National Association for Business Economics Policy Conference. www.federalreserve.gov/newsevents/speech/yellen20130302a.htm

²See Card (2011) for a brief history of the theory and measurement of unemployment.

³Fujita et al. (2019) show that the number for the hires from employment are downward biased by a change in the current population Survey that occurred between 2007 and 2009.

health of the labor market is, in general, not as important as it is for that statistic to be comparable over time, and across space. Unfortunately, this is not the case with the standard unemployment rate. The percent of people classified as out of the labor force who are actively seeking a job was about 9% in 1980 and increased rapidly in every subsequent recession, reaching over 15% in 2009. The increase in the percent of active job seekers who are classified as out of the labor force is entirely attributed to the increase in the education levels of the out of the labor force pool.

The adjusted labor market flows are substantially different than the standard flows calculated from the matched Current Population Survey (CPS). Matching individuals across consecutive months to calculate transition probabilities had become the standard in calculating labor market flows and these flows are published monthly by the Bureau of Labor Statistics. The most notable difference is along the unemployment/non-participation margin. The standard flows suggest that the probability a person leaves unemployment for non-participation (0.21) is nearly as large as the probability he leaves for employment (0.25). The adjusted flows show that the unemployment exit probability to non-participation is only 0.05, implying that unemployment is a more persistent state. Several papers have tried to understand the large oscillations between unemployment and non-participation. Using the reinterview surveys conducted by the CPS, [Abowd and Zellner \(1985\)](#) and [Poterba and Summers \(1986\)](#) show that the largest margin of misclassification is along the unemployment/out-of-the-labor-force margin; however, their adjustments decrease the flow from unemployment to out of the labor force by half and it remains 50% higher than the flow constructed here. Similarly, [Elsby et al. \(2015\)](#) correct labor market transitions to get ride of oscillations between non-participation and unemployment, but conclude that unemployment to non-participation exit rate remains above 0.1. These large and volatile movements between unemployment and non-participation are difficult to match using standard calibrations of search and matching models ([Garibaldi and Wasmer, 2005](#)). More recently, [Krusell et al. \(2017\)](#) show that the only way to match such large movements into and out of the labor force is through relatively large transitory shocks to the disutility of search effort.⁴ I argue that, unless matching a distribution of labor market flows, a more accurate aggregate flow target is the continuous measure presented here.

Finally, as an application for the continuous unemployment rate, I re-estimate the Phillips Curve and investigate the apparent flattening that has occurred after the Great Recession using both the continuous unemployment rate and total search effort. Many papers have investigated the changing output-inflation relationship following the Great Recession, often using the unemployment gap, measured as the difference between the Congressional Budget Office's Natural Rate of Unemployment (NRU) and the standard unemployment rate, as a measure for the output gap ([Ball and Mazumder, 2011](#); [Coibion](#)

⁴See [Krusell et al. \(2008, 2010, 2011\)](#) for a complete explanation of how aggregate and idiosyncratic shocks affect labor market flows.

and [Gorodnichenko, 2015](#); [Blanchard, 2016](#)). I estimate a wage Phillips Curve and an inflation Phillips Curve using instead, the continuous unemployment rate and total search effort and show that there is no flattening in the post 2008 recession period. This suggests that the change in the output-inflation relationship may, in part, be driven by measurement issues. This result stands in contrast to [Stock and Watson \(2019\)](#) who argue that the flattening is robust to different measures of the output gap.

There exist two empirical challenges when attempting to create a consistent aggregate labor market statistic. First, long run trends in the demographic composition of the pool of job seekers make the standard unemployment rate hard to compare across time. Many papers have worked to ride the standard unemployment rate of this bias ([Perry, 1970](#); [Flaim, 1979](#); [Shimer, 2001](#); [Barnichon and Mesters, 2018](#)), finding that demographic changes decreased the unemployment rate by 1.2-2.2 percentage points since 1980. Second, the misclassification errors pointed out by [Abowd and Zellner \(1985\)](#) and later explored by [Feng and Hu \(2013\)](#) and [Ahn and Hamilton \(2019\)](#), suggest that the true unemployment rate may be somewhere between 2 and 4 percentage points higher than the reported one. What remains constant across all the corrections is the extensive definition of unemployment. Here, the continuous unemployment rate deals with both challenges simultaneously. First, the weights are demographic specific but the effect of demographics does not change over time. Second, the weight is calculated for all non-employed individuals, so misclassification between unemployment and non-participation is not an issue.

The continuous unemployment rate adds to a small literature focused on constructing a better measure of labor underutilization for the United States.⁵ [Hornstein et al. \(2014\)](#) construct a non-employment index (NEI) in which they weight all non-employed people by the average transition probabilities to employment on a coarse grid of observable characteristics. [Faberman et al. \(2019\)](#) construct a measure of labor market underutilization by differentiating people by the difference between their hours worked, zero if non-employed, and their desired hours worked. The measure is constructed for the entire population and tries to measure labor market slack as the difference between hours available and actual hours worked. The method proposed in this paper is advantageous because the continuous unemployment rate is created using the same data used to create the official labor market statistics for the United States, the Current Population Survey, and a publicly available supplement of the survey, the American Time Use Survey. The measure can be reconstructed for any demographic group or region and does not rely on the outcome of job search.

⁵[Schweitzer \(2003\)](#) and [Jones et al. \(2003\)](#) attempt similar exercises for the United Kingdom.

2 Estimating Search Effort

Unlike the standard unemployment rate, which counts all unemployed equally, the continuous unemployment rate weights all non-employed people by the probability they exert positive search effort in a given month. The two main sources of data used to estimate the probability that a person exerts positive search effort are the basic monthly files of the Current Population Survey (CPS) January 1980 - August 2020 and the time diaries from the American Time Use Survey (ATUS) January 2003 - December 2018.

2.1 Data Sources

The CPS is the main source of data used for calculating aggregate statistics regarding the labor force status of U.S. residents. The survey is conducted monthly, the interview unit is based on the address of the household and all members of the household residing at the address are interviewed. A household is in the survey for 4 months, then out for 8 months, and then back in for 4 months. Given this rotating-panel element of the CPS, in theory three quarters of the each month's sample can be longitudinally linked to the prior month. In practice however, only about two-thirds of the sample can be linked due to households moving. The survey asks a variety of questions related to labor market attachment; then, people are classified as unemployed if they have made at least one active search effort during the past 4 weeks and are available to work. All other non-employed individuals are classified as out of the labor force.⁶

This broad classification of labor force status is advantageous in many respects, but needless to say, not perfect. Misclassification of people across labor market states in the CPS is a well documented fact. [Abowd and Zellner \(1985\)](#) and [Poterba and Summers \(1986\)](#) show that misclassification happens along all margins using data from the reinterview surveys conducted by the CPS on a subset of individuals. The largest error occurs among people that are at first classified as out of the labor force and later reclassified as unemployed. Similarly, [Ahn and Hamilton \(2019\)](#) show that two-thirds of people who were classified as out of the labor force last month and unemployed this month report having an unemployment duration of longer than 4 weeks. [Krueger et al. \(2017\)](#) document that people are more likely to get misclassified as out of the labor force the longer they stay in the survey, which [Halpern-Manners and Warren \(2012\)](#) suggest may be due to the shame carried by admitting, month after month, that they were unable to find a job. However, [Flinn and Heckman \(1983\)](#) find that, at least until the early 1980's, unemployment and out of the labor force are behaviorally distinct states.

⁶A detailed description of how labor market status is determined can be found at <https://www.bls.gov/cps/definitions.htm>.

Despite measurement issues, the CPS data have become, not only the standard source for labor market stocks, but also the main source for estimating the flows across labor market states. The Bureau of Labor Statistics publishes the flows across labor market states beginning in 1990 and many others have calculated the flows using the linked microdata, see for example [Shimer \(2012\)](#) or [Elsby et al. \(2015\)](#). Beginning with the 1994 redesign of the CPS, [Fallick and Fleischman \(2004\)](#) show that it is possible to observe employment to employment transitions as well.

The CPS also asks several questions about job search efforts, however, these questions are limited to people who are classified as unemployed. On the other hand, with the American Time Use Survey (ATUS), job search effort can be observed by all participants. The ATUS, which began as a supplement to the CPS in 2003, randomly selects households that have completed their eighth and final month in the CPS. Selected households are interviewed one time about how they spent their time on the previous day, where they were, and whom they were with. The main goal of the survey is to collect information about how the respondent spent his or her time starting at 4 a.m. the previous day and ending at 4 a.m. on the interview day. For each activity reported, the interviewer asks how long the activity lasted. For most activities, the interviewer also asks who was in the room or accompanied the respondent during the activity and where the activity took place. The activities are then coded into over 400 categories.

Of particular interest are the categories devoted to job search which include: job search activities, job interviewing, waiting associated with job search or interview, security procedures related to job search/interviewing, and other job search.⁷ These categories are the focus of this paper as they provide an opportunity to see if, and how much people are searching for a job, regardless of their labor force status. Given these categories, the ATUS has recently become a common data set used to study the cyclical behavior of job search among the unemployed ([Mukoyama et al., 2018](#)) and the employed ([Ahn and Shao, 2017](#)).

The ATUS interview is conducted between 2 and 5 months after exiting the CPS. Because of the delay between the final CPS interview and the ATUS interview, the questions pertaining to labor force status are asked again, in the same fashion as during the CPS interview, and respondents are classified as employed, unemployed, or out of the labor force accordingly. Regardless of a person's labor market status, if he spent any time searching for a job on the interview day, the time will be recorded as job search activity. Therefore, the ATUS data can be used to estimate the probability that any person is searching for a job.

The main disadvantage of using the ATUS to study job search behavior is that people are only surveyed about one day in the month. In what follows, search effort, in terms of minutes per day, is reported both unconditionally and conditional on observing positive search effort. Due to the cross-

⁷Categories 50401-50499 in the ATUS lexicon.

sectional nature of the ATUS, the probability that a person searches for a job is reported as a daily probability and converted to a monthly probability under the assumption that job search is identical and independent across days.⁸ That is, if p is the daily probability of searching for a job, then the corresponding monthly probability is calculated as $1 - (1 - p)^{30}$.

Table 1 shows the probability of observing a person searching for a job on a single day, calculated as the sample mean of a dummy variable that takes on the value 1 if the person reports spending any time that day looking for a job. Also reported is the corresponding monthly probability. Not surprisingly, people classified as unemployed have the highest probability of searching for a job on a single day, and with almost certainty, search for a job throughout the month. For the employed, those at work have a lower monthly probability (15.9) of searching for a job than those absent from work (35.7). Those classified as out of the labor force also report searching for a job throughout the day, in fact, the probability of observing a person age 25-55 searching for a job throughout the month (24.4) from outside the labor force is higher than the probability of observing an employed person, age 25-55, searching for a job (15.1). The fact that the probability of observing a person classified as out of the labor force searching for a job is positive is further evidence of the measurement issues discussed above.

The intensive margin of search (minutes spent searching) is reported in **Table 2**. Both the unconditional average and conditional on positive search time is reported. First, across both samples, the unemployed have the highest unconditional search intensity, searching for an average of about 30 minutes per day. The unconditional average for the employed and out of the labor force is mostly below one minute across both samples, stemming from the fact that no more than 25 percent of each group is searching for a job. However, when looking at the conditional search times, the three groups look strikingly similar. The unemployed, again, have the highest search intensity, spending on average 2.5 hours per day searching for a job. However, conditional on searching for a job, those classified as out of the labor force spend almost as much time per day (2.2 hours) searching as the unemployed. Employed people spend the least amount of time searching for a job, about 2 hours per day.

In the CPS classification scheme, the type of job search activity, active vs passive, also determines if non-employed individuals are classified as unemployed or out of the labor force. **Table 3** reports the percent of time each group spends in three types of job search activities. The first activity labeled "Active Job Search" consists of ATUS category number 50401 which includes contacting employers, sending out resumes, ect. The second category is "Interviewing" and the third category "Other" includes all other ATUS categories (50403-50499) which are: waiting time associated with interviewing, security procedures related to job search/interviewing, and all other job search activities not elsewhere specified.

⁸In the subsequent analysis, this restriction will be relaxed and the propensity of search effort can vary across days however remains independent across days, again due to data limitations.

Across all three labor market states and both samples, the majority of time is spent in active job search. Employed individuals spend about 14% of the time interviewing whereas, unemployed individuals spend only about 6% of the time interviewing. Consistent with the fact that people classified as out of the labor force should not to be actively searching for a job, they spend up to 5% of the reported job search time in the “Other” category; however, they spend the majority of the time actively searching for a job, between 85% and 90%. The evidence presented in [Table 2](#) and [Table 3](#) suggest that people who are classified as out of the labor force but exert positive search effort, may not be behaving differently from those classified as unemployed.

[Table 4](#) gives the summary statistics of demographic characteristics, education, and the day of the week each interview occurred by labor force status. The demographic and education information is matched from the respondents CPS interview. Women, as well as married women, make up a larger proportion of the unemployed and out of the labor force. College educated and people with advanced degrees represent a larger proportion among the employed than the unemployed and out of the labor force. Average age also differs across the three labor market states as expected; unemployed being on average the youngest and out of the labor forces the oldest.

2.2 Estimating Search Effort

The ultimate goal is the predict search effort of all the respondents of the CPS which will be used to weight people when creating aggregate unemployment statistics. The evidence presented in [Table 2](#) suggests that, conditional on searching, the intensive margin of search does not differ much across CPS defined labor market states. [Table 1](#) shows that the driving differences in search effort is the extensive margin. Therefore, the probability that someone is searching for a job over the month is predicted and ultimately used as the weight.

Search effort is predicted from the subset of individuals that participate in the ATUS and predicted for all individuals in the CPS. To maximize the predictive power of the covariates, I use machine learning to choose the set of best predictors. The potential covariates are all demographic variables (a quadratic in age, race, education, sex, marital status, and an indicator for having a child), day of the week fixed effects, a full/part time indicator for the employed, a measure describing the state of the economy, state fixed effects, and interactions between sex and education, and all other covariates. The total number of potential covariates for the employed group is 64 and for the unemployed and out of the labor force groups is 58. To ensure that the continuous unemployment rate is comparable over time, i.e. is not biased by changes in the demographic composition of the pool of job seekers, all demographic variables, the state of the economy, state fixed effects and day of the week fixed effects

enter the machine learning algorithm without penalty, while all interaction terms enter with a standard penalty function described in detail below.

For each CPS defined labor market status, I run a net-elastic logistic regression, where the dependent variable is an indicator for if the individual spent any time searching for a job on the interview day. Demographic information for each person is collected in the CPS and matched to their ATUS interview. For the measure of the state of the economy, I use the monthly state Coincidence Index (CI) provided by the Federal Reserve Bank of Philadelphia as a proxy for the state level state of the economy in which the worker is searching.⁹ The state CI is created using a dynamic single factor model that

combine[s] four state-level indicators to summarize current economic conditions in a single statistic. The four state-level variables in each coincident index are nonfarm payroll employment, average hours worked in manufacturing by production workers, the unemployment rate, and wage and salary disbursements deflated by the consumer price index (U.S. city average).

The trend of the state level index is set to the trend of the state's Gross Domestic Product. The final economy variable is constructed by removing a linear trend from the log of the coincidence index per capita of each state.

For each individual in the ATUS, let y_i be an indicator that takes on the value one if they spent any time searching for job on the interview day and x_i be the vector of covariates. The probability that the individual searched for a job is modeled using the logistic function,

$$P(y_i = 1|x_i) = \frac{\exp(\hat{\beta}_0 + x_i' \hat{\beta})}{1 - \exp(\beta_0 + x_i' \beta)} \quad (1)$$

and the likelihood function is

$$\min_{\beta_0, \beta} - \left[\frac{1}{N} \sum_{i=1}^N y_i(\beta_0 + x_i' \beta) - \ln[1 - \exp(\beta_0 + x_i' \beta)] \right] + \lambda \left[(1 - \alpha) \sum_{k \in K} \beta_k^2 + \alpha \sum_{k \in K} |\beta_k| \right] \quad (2)$$

where α is set to 0.95, implying more weight on the LASSO penalty and the tuning parameter, λ , is chosen through cross validation of ten folds of the data with the area under the receiver operating characteristic curve as the selection criteria. The penalty is only over the interaction terms (the set K) to ensure that all demographic variables, day of the week fixed effects and the state of the economy are always included in the estimation. Final ATUS weights are used in all calculations.

Table 5 shows the selected covariates and resulting parameter estimates for each labor force status. For the employed group, 13 of the interaction terms are included in the estimation. For the unemployed

⁹<https://www.philadelphiafed.org/research-and-data/regional-economy/indexes/coincident/>

and out of the labor force groups 9 and 16 interaction terms are included. **Figure 1** plots the receiver operating characteristic curve for each labor force group. The out of the labor force group has the best fit with an area under the curve (AUC) of 0.891, the employed AUC is 0.775 and the unemployed AUC is 0.745.

2.3 Predicting Search Effort

The CPS contains all the same demographic information as the ATUS and labor market status is determined equivalently in both samples. Therefore, although the ATUS sample begins in 2003, search effort can be predicted using the CPS starting in 1980. The estimated search probability is a daily probability, therefore, seven probabilities are predicted for each person in the CPS, one for each day of the week fixed effect. The predicted daily probability is:

$$\hat{p}_i^d = \frac{\exp(\beta_0 + x_i' \beta)}{1 - \exp(\hat{\beta}_0 + x_i' \hat{\beta})} \quad (3)$$

for $d \in \{1, 2, \dots, 7\}$ where $\hat{\beta}_0$ and $\hat{\beta}$ are the estimated coefficients.

Using the predicted daily probability, the weekly probability that a person is searching for a job is 1 minus the probability he does not search any day during the week, i.e.

$$\hat{p}_i^w = 1 - \sum_{d=1}^7 (1 - \hat{p}_d). \quad (4)$$

The monthly probability that he searches for a job is constructed analogously, i.e.

$$\hat{P}_i = 1 - (1 - \hat{p}_i^w)^{4.17} \quad (5)$$

with 4.17 weeks per month.

Table 6 reports percentiles of the predicted search probabilities for each labor market state over the entire sample. The unemployed have the highest predicted search probabilities and the distribution of search probabilities is skewed towards one, with the 50th percentile at 0.994. Not surprisingly, many predicted search probabilities are very close to zero for the out of the labor force group, in fact, the 25th percentile is at 0.001 which is an order of magnitude smaller than even the 10th percentile of the employed group (0.02). However, the out of the labor force group still has many predicted search probabilities that are larger than the employed group. The 95th percentile of search probabilities for out of the labor force group is 0.47 and 0.37 for the employed group.

2.4 Search Effort and Labor Force Attachment

For the continuous unemployment rate to be a useful measure of labor underutilization, it should weight non-employed people with a higher labor force attachment more than those with a lower labor force attachment. In term of labor force attachment, three statistics can be calculated from the basic CPS files. First, for the full sample of individuals matched across two consecutive months in the CPS from 1994-2019 an indicator variable that takes on the value one if they found a job (or switched jobs for the employed), that is, a job finding probability is constructed as a measure of labor force attachment. Second, for the subset of non-employed individuals who found a job, their subsequent usual hours worked is used as a measure of labor force attachment. Third, for the subset of employed individuals who switched jobs, the change their usual hours worked is used as a measure of labor force attachment. For all three statistics, the estimated job search probability is an indicator of labor force attachment if it correlated positively with the outcome.

The correlation between the predicted job search probability and the three measures of labor force attachment is estimated as follows:

$$y_{it} = \beta \hat{P}_{it-1} + \delta_t + \varepsilon_{it} \quad (6)$$

where y_{it} an indicator for new employment, usual hours worked, or change in usual hours worked, \hat{P}_{it-1} is the predicted search effort in the previous month, and δ_t are month by year fixed effects.

The estimated correlation between an indicator for new employment (job finding probability) and predicted search effort are reported in the first two columns of [Table 7](#). The correlation is significantly positive with and without year by month fixed effects and large relative to the mean job finding probability over the sample (0.04). The correlation between search effort and subsequent hours worked for the non-employed job finders subsample is reported in columns (3) and (4) of [Table 7](#); the correlation is positive and significant. The correlation between changes in hours worked and search effort for the subset of employed job switchers is also positive and significant as reported in the last two columns of [Table 7](#). The positive correlation between predicted search effort and job finding probabilities and hours worked suggests that the predicted search effort is indeed a good proxy of labor force attachment.

3 Unemployment and Labor Market Flows

3.1 The Continuous Unemployment Rate

Using the monthly predicted probability of search effort for each person, the estimated number of searchers within each CPS defined labor market state, unemployed U^s , employed E^s , and out of the labor force O^s , are constructed as weighted totals:

$$U_t^s = \sum_{i \in U_t} wgt_{it} \times \hat{P}_{it} \quad (7)$$

$$E_t^s = \sum_{i \in E_t} wgt_{it} \times \hat{P}_{it} \quad (8)$$

$$O_t^s = \sum_{i \in O_t} wgt_{it} \times \hat{P}_{it} \quad (9)$$

where U_t , E_t , and O_t are the sets of all individuals in the respective CPS defined labor market state and wgt_{it} is the CPS sampling weight. The total number of people in each state is calculated as the sum of the weights within each group. The resulting series are monthly, seasonally adjusted using the Census X13-ARIMA and then aggregated to a quarterly frequency. A complete description of the process can be found in Appendix A.1.

Figure 2 plots the predicted fraction of people searching in each labor market state (O_t^s/O_t , U_t^s/U_t , E_t^s/E_t). The shaded regions depict recessions using the National Bureau of Economic Research's classifications. All three series clearly display a counter cyclical patter. The fraction of people searching while employed rose dramatically during the 2008 recession, increasing by about 1 percentage point from trough to peak. Although declining after the end of the recession, the fraction of employed job seekers remains above its trough. Nearly all those who are unemployed are searching for a job, with the percent varying between 95% and 97.5%. The fraction of people searching for a job who are classified as out of the labor force displays large counter-cyclical movements, rising 4 percentage point during the 2008 recession, and a slow moving upward trend. The final panel of **Figure 2** plots the total fraction of the population that is searching for a job. The general shape is close to the the standard unemployment rate, defined as $U/(U + E)$. However, the level of the total searcher rate is much higher than the unemployment rate, and the total searcher rate does not display the same downward trend that the standard unemployment rate does.

The fraction of searchers among all three labor force groups in **Figure 2** display a clearly counter-cyclical patter, rising sharply during each recession. These results are consistent with [Ahn and Shao \(2017\)](#) who show that the search effort of the employed is countercyclical in the extensive and

intensive margin. The unemployed also display countercyclical search effort. This result adds to a growing literature that documents that search effort of the unemployed is countercyclical, such as [Shimer \(2004\)](#), [Kudlyak and Faberman \(2014\)](#) and [Mukoyama et al. \(2018\)](#). However, this stands in contrast to [DeLoach and Kurt \(2013\)](#) who show a-cyclical search effort and [Gomme and Lkhagvasuren \(2015\)](#) who show evidence of pro-cyclical search effort. While the fraction of people searching among people classified as out of the labor force has risen by nearly two thirds since 1980, the rise is also countercyclical, rising quickly during recessions and flattening or decreasing during expansions.

The continuous unemployment rate is defined as the ratio of the weighted sum of all non-employed people, weighted by their search probabilities, to the total number of working age people, that is:

$$\tilde{U}_t = \frac{U_t^s + O_t^s}{U_t + O_t + E_t}. \quad (10)$$

Contrary to the standard unemployment rate, the denominator of the continuous unemployment rate is the entire working age population since, in theory, each could be employed. Analogously, the continuous total searcher rate is the ratio of the weighted sum of all people, to the total number,

$$S_t = \frac{U_t^s + O_t^s + E_t^s}{U_t + O_t + E_t}. \quad (11)$$

The total searcher rate is in spirit most similar to the Bureau of Labor Statistic's most inclusive measure of labor slack, U-6, which includes the unemployed, the marginally attached and all part time employed for economic reasons. However, the total searcher rate may better capture total labor under utilization since it weights individuals by their propensity to begin new employment. Finally, a continuous measure of labor force participation is constructed by taking the ratio of the weighted average of all non-employed plus all employed to the total population,

$$\tilde{P}_t = \frac{U_t^s + O_t^s + E_t}{U_t + O_t + E_t}. \quad (12)$$

Panel (a) of [Figure 3](#) plots the standard and continuous unemployment and the total searcher rate. The average standard unemployment rate over the sample is 6.3. Both the average continuous unemployment rate (7.6) and the average total searcher rate is (15.9) are higher than the standard unemployment rate. The most notable difference between the standard and continuous unemployment rate is that the continuous unemployment rate does not display the same low frequency downward trend that the standard unemployment rate does. Panel (b) plots the standard and continuous labor force participation rates. The average continuous participation rate is 3.6 percentage points higher than the standard participation rate. The most notable difference between the standard and continuous

participation rates is that, while the standard participation rate falls during both the 2008 recession and the 2020 recession, the continuous participation rate rises.

3.2 Labor Market Flows

The standard method used to calculate flows between labor market states uses information on individuals who are matched across consecutive months of the CPS basic monthly files. The basic monthly files are composed of eight rotations groups. Households in the first through third and fifth through seventh month in the sample will be surveyed in the following month and can thus be linked across month, in theory three quarters of the sample can be longitudinally linked. However, in practice, only about two thirds of the sample can be linked due to attrition. Using the longitudinally linked data, estimates for transition probabilities are calculated as the fraction of workers transitioning across labor market states from month to month.

The approach used here is similar, however, non-employed worker transitions are weighted by the predicted monthly search probability. The probability a worker transitions from employment to unemployment is calculated as:

$$f_{EU} = \frac{\sum_{i \in E_1 N_2} wgt_i \times \hat{P}_{i2}}{\sum_{i \in E_1} wgt_i} \quad (13)$$

where the summation in the numerator is over all workers that are observed in employment in the first month (E_1) and non-employment (CPS defined unemployment and out of the labor force) in the second month (N_2). The summation in the denominator is over all workers in employment in the first month. The weight used in the numerator is the CPS sampling weight times the estimated search probability in the second month. Similarly the transition probability from employment to out of the labor force is calculated as:

$$f_{EO} = \frac{\sum_{i \in E_1 N_2} wgt_i \times (1 - \hat{P}_{i2})}{\sum_{i \in E_1} wgt_i} \quad (14)$$

where the weight used in the numerator is now the CPS sampling weight times the probability the worker is not searching for a job in the second month. The transition probability from unemployment to employment is calculated using all individuals that are not employed in the first month (N_1) and employed in the second month (E_2), weighted by the probability they were searching for a job in the first month. That is,

$$f_{UE} = \frac{\sum_{i \in N_1 E_2} wgt_i \times \hat{P}_{i1}}{\sum_{i \in N_1} wgt_i}. \quad (15)$$

The transition probabilities between unemployment and out of the labor force are calculated slightly differently. Instead of weighting the individual by the search probability each period, workers are weighted by the change in their search probability. If a person remains non-employed for two consecutive

month, and his predicted probability of search does not change over those two months then, although he contributes to both the stock of unemployed and out of the labor force, he does not contribute to the flow between these two states. Alternatively, suppose that a person is not employed in two consecutive months, and his estimated probability of searching is $\hat{P}_1 = 0.3$ in the first month and $\hat{P}_2 = 0.5$ in the second month, then he contributes to the flow from out of the labor force to unemployment by only change in his estimated search probability, that is, with weight 0.2. Therefore, the flow from out of the labor force to unemployment is calculate as

$$f_{OU} = \frac{\sum_{i \in N_1 N_2} wgt_i \times \max\{\hat{P}_{i2} - \hat{P}_{i1}, 0\}}{\sum_{i \in N_1} wgt_i}. \quad (16)$$

Similarly, a person that is not employed in two consecutive months only contributes to the flow from unemployment to out of the labor force if his predicted search probability decrease from the first to the second month. The flow from unemployment to out of the labor force is calculated as

$$f_{UO} = \frac{\sum_{i \in N_1 N_2} wgt_i \times |\min\{\hat{P}_{i2} - \hat{P}_{i1}, 0\}|}{\sum_{i \in N_1} wgt_i}. \quad (17)$$

By construction the flow from out of the labor force to employment is zero.

The resulting transition probabilities are seasonally adjusted and corrected for margin error. The correction for margin error is similar to [Elsby et al. \(2015\)](#) and restricts the flows across labor market states to be consistent with the evolution of the labor market stocks. A detailed description of how this was done can be found in the Appendix. In the standard labor market flows data, margin error can arise from movements into the working age population or attrition of households in the matched CPS data; however, [Elsby et al. \(2015\)](#) show that correcting for margin error has little effect on the standard CPS flows. Here the flows and stocks are calculated using estimated search probabilities, so correcting for margin error plays a larger role and decreases the estimated flow from unemployment to out of the labor force by 43%.

Figure 4 plots the standard and continuous labor market flows across labor market states for the full sample. The most notable changes occur along the participation margin. The average flow from out of the labor force to unemployment increases slightly from of 0.027 to 0.030 and the average flow from unemployment to out of the labor force decrease from and average of 0.21 to 0.05. The average flow from unemployment to employment decrease slightly from an average of 0.25 to 0.20. The average flow from employment to out of the labor force decreases by more half from 0.028 to 0.01. The standard flow from employment to out of the labor force is nearly twice as high as the standard flow from employment to unemployment; the continuous flow from employment to out of the labor force is

less than the continuous flow from employment to unemployment.

Much of the previous work on labor market flows has focused on adjusting the flows to account for the misclassification between unemployment and non-participation. [Abowd and Zellner \(1985\)](#) and [Poterba and Summers \(1986\)](#) attempt to understand the amount of measurement error in the CPS classification system by using data from the reinterview surveys conducted by the CPS on a subset of individuals. They show that misclassification happens along all margins, however the largest error occurs among individuals that are at first classified as out of the labor force and later reclassified as unemployed. However, the methods they propose continue to treat unemployment simply as an extensive margin. The benefit of the method proposed here is again, that all individuals, regardless of their labor market state when not employed contribute to each flow, so misclassification is not an issue.

Unfortunately, the CPS has since stopped conducting reinterview surveys. So more recently, [Elsby et al. \(2015\)](#) match individuals up to three months and recode an individual who is observed as unemployed in the first month, out of the labor force in the second month and unemployed in the third month, as unemployed throughout. Similarly, for individuals that are out followed by unemployed and again out, are recoded as out of the labor force for the entire period. Indeed, this correction ("deNUNification") decreases the flow between unemployment and out of the labor force but it does not address the issue of movements from out of the labor force directly to employment and vice versa. For example, an individual who is observed as unemployed in the first month, out of the labor force in the second month and employed in the third month is not recoded, therefore, such an individual adds to the flow from unemployment to out of the labor force as well as the flow from out of the labor force to employment.

4 Decomposing Changes Over Time

This section decomposes changes in the demographic characteristics of the pool of people who are classified as out of the labor force (OLF) and how these demographic changes have influenced the aggregate out of the labor force search effort over time. The demographic characteristics of focus are, sex, age, race and education level. First, changes in the demographic composition of the out of the labor force group and hires coming from out of the labor force are documented. Second, changes in the share of each demographic group, i.e. between group changes, and changes in search effort within groups are explored.

Unfortunately, since data on search effort is only available from 2003 to 2018, and the aggregate OLF search effort series is projected back to 1980, the effects of demographic characteristics was held fixed over time in the estimation process presented in [section 2](#). Therefore, within group changes

occurring since 1980 can not be explored. However, it is possible to see which demographic shifts in the OLF population had the largest effect on aggregate OLF search effort.

4.1 Demographic changes in OLF population and OLF hires

Figure 5 plots the total number of new hires by labor force status. The figure shows that while total hires out of unemployment have remained stable around 2 million per month since 1980, hires from out of the labor force have increased by about 46% from 2.8 million in 1980 to 4.2 million in 2018. Over the same time, the total size of the out of the labor force group also increase by 50% from 63 million in 1980 to 95 million in 2018. However, the composition of the the OLF pool has also seen large changes since 1980.

Table 8 shows the percent of each demographic group as a total of the OLF population and a total of the hires from the OLF population. Looking across sex, there were only marginal changes in the percent of the total population as well as total hires. Women made up about 53% of the total OLF population and OLF hires in the 1980's, decreasing to 52% by the 2010's. Similarly, decomposing the total OLF population and hires by race shows only small changes occurring over the past 40 year.

Decomposing changes by age and education show much larger changes. The pool of OLF have become older; people age 56+ made up about 26% of the pool of OLF in the 1980's and increase by 6 percentage point to 32% by the 2010's. However, the percent of hires from OLF by each age group has changes accordingly. Individuals age 56+ made up about 18% of OLF hires in the 1980's and increased 6 percentage points to 24% by the 2010's.

On the other hand, changes in education show that both the percent of each group and the percent of hires has change since the 1980's. There was a general increase in the education level of the OLF population, with more individuals having completed high school and college in the 2010's than the 1980's. However, whereas people who had completed high school make up more the OLF population in the 2010's, they account for less of the OLF hires in the 2010's. And while less of the OLF population had some college education in the 2010's than in the 1980's, they account for more of the OLF hires in the 2010's than in the 1980's.

4.2 Decomposing changes in OLF search effort

In order to decompose within group changes in search effort, one needs estimates of how search effort has changed over time by demographic group. To do this, search effort in the ATUS is re-estimated with time varying demographic effects. Then search effort is predicted in the CPS from 2003 to 2018. This allows for both within and between group changes in search effort to be explored from 2003 to

2018.

First, the probability of observing a person searching for a job in the ATUS is re-estimated to include time varying demographic effects. That is, the probability that someone is searching for a job is modeled using the logistic function, where the covariates included are those chosen by the net-elastic logistic regression estimation in [section 2](#) and a linear time trend times each demographic variable (sex, age, age squared, race indicators and education level indicators):

$$P(y_i = 1|x_i) = \frac{\exp(\beta_0 + x_i^{ML'} \beta_1 + t \times x_i^{DEM'} \beta_3)}{1 - \exp(\beta_0 + x_i^{ML'} \beta_1 + t \times x_i^{DEM'} \beta_3)} \quad (18)$$

where x_i^{ML} is the set of covariates selected in the net-elastic logistic regression estimation, x_i^{DEM} are the demographic covariates for sex, race, education and age, and t is a yearly linear time trend, such that $t = 0$ corresponds to the year 2003. Search effort is predicted analogously to that in [section 2](#).

To begin, [Figure 6](#) compares the fraction of the out of the labor force group predicted to be searching for a job using the time varying coefficients for the demographic group to the “Fixed coefficients” series discussed in [section 3](#) and plotted in [Figure 2](#) panel (c). The two series are nearly identical.

To explore within group changes in the search effort of the OLF population, search effort in each month is predicted on 1,000 re-sampled CPS basic month files, such that, the fraction of women, each race category, each education category, and three age categories (16-24,25-55,56+) are held constant at their January 2003 values. The mean of each 1,000 OLF search series is used. The resulting aggregate OLF search effort series therefore varies only due to changes in search effort within demographic groups. Second, between group changes are observed by predicting search effort for each month holding the time varying coefficients equal to their 2003 values. The resulting OLF search series therefore varies only due to changes between groups.

Panel (a) of [Figure 7](#) plots the Aggregate series (when both the demographic composition as well as the effects of demographics are allowed to vary over time), the Between variation, and Within variation series. The Within series follows the Aggregate measure almost perfectly until 2014 when the Within series flattens, implying that within group changes in search effort did not have a significant effect on aggregate OLF search until 2014. Post 2014, the decline in aggregate search effort is largely due to between group changes in search effort. Changes between groups had a larger effect on aggregate search effort over the great recession. The Between series shows that compositional changes in the pool of OLF, increased aggregate search effort of the OLF by nearly 2 percentage points over the recession.

Panel (b) of [Figure 7](#) decomposes the within group changes further by allowing only one of the effects of demographic characteristics on search effort to vary over time and holding the demographic composition fixed at 2003. For example, the line labeled “sex” in panel (b) is constructed by allowing

only the effect of sex on search effort to vary over time and holding the demographic composition fixed to 2003. The figure shows that the within group changes in search effort by sex and race declined since 2003 whereas within group search effort by age and education increased since 2003. The within age group effect on search effort is the largest, implying that if nothing but search effort by age had changed since 2003, aggregate OLF search effort would have increase by 20 percentage points.

Panel (c) of [Figure 7](#) decomposes the between group changes further by holding all but one demographic composition fixed. For example, the line labeled “sex” in panel (c) is constructed allowing only the fraction of women in the OLF pool to vary over time, and holding the effect of demographic characteristics on search effort constant at their 2003 values. The resulting series shows that if only the fraction of women in the pool of OLF had changed, aggregate OLF search effort would have risen 1 percentage point more during the recession and remained higher afterwards. Changes in the educational composition of the pool of unemployed increased aggregate OLF search effort by about 1.75 percentage points during the recession.

As mentioned above, the same within and between group decomposition in OLF search effort can not be done going back to 1980. In fact, since the OLF search series presented in panel (c) of [Figure 2](#) is constructed without time varying demographic effects, changes in the series are driven by between group changes. It is possible however, to understand which demographic changes had the largest effect on aggregate OLF search effort. To do so, the aggregate search series are constructed allowing the share of one demographic characteristic to vary at a time. As before this is done by constructing search effort on 1,000 re-sampled CPS basic monthly files and averaging across the 1,000 resulting series. [Figure 8](#) plots the resulting series. The figure shows that, changes in sex, race and education contributed to the increase in the search effort of the OLF pool through the mid 1990’s after which only changes in education contributed to the increase in the aggregate search effort of the OLF group. In fact, the counterfactual series where education is the only demographic that changes follows the aggregate series closely until the end of the 2008 recession, suggesting that the increased educational attainment of people classified as out of the labor force can account for the increase in OLF search effort since 1980. On the other hand, if only the age composition of the pool of OLF would have changed, their aggregate search effort would have fallen slightly since the 1980s.

5 An Application: The Phillips Curve

The unemployment rate, or unemployment gap, is often used as a measure of labor market utilization in estimating the trade-off between output and inflation, i.e. the Phillips Curve. Since the Great Recession this relationship has gained new interest with many finding that the traditionally strong

negative correlation between output and inflation has weakened or even disappeared, the phenomenon referred to as the flattening of the Phillips Curve. Papers investigating the apparent flattening (among many others) include [Ball and Mazumder \(2011\)](#); [Coibion and Gorodnichenko \(2015\)](#); [Blanchard \(2016\)](#); see [McLeay and Tenreyro \(2019\)](#) for a recent review.

I revisit the change in the output-inflation relationship following the Great Recession using the continuous unemployment rate and the total searcher rate as measures of underutilization. I estimate a Phillips curve, where inflation is determined by the output gap and expected inflation. That is,

$$\pi_t = \phi(x_t - x_t^*) + \gamma E_t[\pi_{t+1}] \quad (19)$$

where π_t is the inflation rate at time t , x_t is a measure of labor market underutilization (the standard unemployment rate, the continuous unemployment rate or the total searcher rate), x_t^* is the natural rate of each measure, and $E_t[\pi_{t+1}]$ is expected inflation. The parameter of interest is ϕ and how its value changes post the 2008 recession, under different measures of the output gap. I estimate a backwards looking Phillips curve and proxy for inflation expectations with the four quarter average of lagged inflation; that is,

$$E_t[\pi_{t+1}] \equiv \bar{\pi}_t = \frac{1}{4}(\pi_{t-1} + \pi_{t-2} + \pi_{t-3} + \pi_{t-4}). \quad (20)$$

Similar specifications have been used recently by [Stock and Watson \(2019\)](#), [Galí and Gambetti \(2019\)](#) and [Ball and Mazumder \(2011\)](#).

Along with an inflation Phillips curve I estimate a wage Phillips curve, similar to [Galí and Gambetti \(2019\)](#), again comparing the change in the correlation between wage growth and the three measures of the output gap pre and post 2008 recession. The wage Phillips curve is

$$\Delta w_t = \phi(x_t - x_t^*) + \gamma E_t[\pi_{t+1}] \quad (21)$$

where Δw_t is nominal wage growth, and $x_t - x_t^*$ and $E_t[\pi_{t+1}]$ are the same as above.

The natural rate of unemployment, x_t^* , is defined as the rate of unemployment such that inflation remains stable. This level is thought to change over time due to changes in the demographics of the workforce or changes in the structure of the labor market. There exists a vast literature aimed at estimating the natural rate of unemployment, see [Crump et al. \(2019\)](#) for a thorough review. Although there are many ways to estimate the natural rate of unemployment, [Stock and Watson \(2019\)](#) argue that the disappearance of the Phillips curve is robust to whichever measure one chooses. Therefore, for the output gap measured using the standard unemployment rate is estimated as the difference between the standard unemployment rate and the Congressional Budget Offices natural rate of unemployment

(NAIRU). Since the continuous unemployment rate and the total searcher rate are adjusted for demographic changes in the labor force and no longer display the same low frequency downward trend that the standard unemployment rate does, the natural rate for these measures will be estimated using a constant.

The flattening or disappearance of the Phillips curve, in its simplest form, can be estimated with a change in the parameter ϕ after the start of the great recession. For the standard unemployment rate the change in ϕ is estimated using a similar specification as in [Stock and Watson \(2019\)](#), that is:

$$\pi_t = \phi_1(u_t - u_t^{NAIRU}) + \gamma_1 E_t[\pi_{t+1}] + \phi_2(u_t - u_t^{NAIRU}) \times Post_t + \gamma_2 E_t[\pi_{t+1}] \times Post_t + \varepsilon_t \quad (22)$$

where $Post_t = \mathbb{I}\{t > 2007.25\}$ is an indicator that takes on the value one after the first quarter of 2007 and the parameter ϕ_2 estimates the change in the Phillips curve. For the continuous unemployment rate and the total searcher rate the specification is:

$$\pi_t = \alpha_1 + \phi_1 x_t + \gamma_1 E_t[\pi_{t+1}] + \alpha_2 \times Post_t + \phi_2 x_t \times Post_t + \gamma_2 E_t[\pi_{t+1}] \times Post_t + \varepsilon_t \quad (23)$$

where $\alpha_1 = -\phi_1 x^*$ and $\alpha_2 = -\phi_2 x^*$, and ϕ_2 estimates the change in the Phillips curve. Identical specifications are run for the wage Phillips curve.

Table 9 reports the results from the regressions. The sample period is 1980Q1 to 2019Q4. The first three columns show the results from the wage Phillips curve for which Δw_t is the annualized growth rate of average hourly earning of Production and Nonsupervisory Employees¹⁰ and $E_t[\pi_{t+1}]$ is the average of the previous 4 quarters of inflation growth constructed using the annualized growth rate of the Personal Consumption Expenditure Index (PCE). Column (1) shows what has been documented as the flattening of the wage Phillips curve with the estimated effect of the unemployment rate gap on wage growth significantly decreasing post 2007, similar to what [Galí and Gambetti \(2019\)](#) find. Columns (2) and (3) report the estimated parameters when using the continuous unemployment rate and the total searcher rate as measures of the output gap. Contrary to the standard unemployment rate gap, there is no significant change in the slope of the wage Phillips curve when using these alternative measures, seen by the small and statistically insignificant estimates on the $\tilde{U} \times Post$ and $S \times Post$ variables. The correlation between the continuous unemployment rate and the total searcher rate, and wage inflation (-0.070 and -0.071) are nearly identical to the pre-recession estimated correlation with the standard unemployment rate gap and wage inflation (-0.072).

The estimated parameters of the inflation Phillips curve are reported in columns (4)-(6). Column (4)

¹⁰<https://fred.stlouisfed.org/series/AHETPI>

shows the inflation Phillips curve estimated with the standard unemployment gap. The slope coefficient before the Great Recession is estimated at -0.31 and there is a large change in the post recessionary period, these estimates are consistent with [Stock and Watson \(2019\)](#) and many others. Column (5) shows the estimated coefficients of the Phillips curve estimated using the continuous unemployment rate and a time invariant natural rate for the continuous unemployment rate. In the pre-recession period the estimated slope coefficient is -0.38, similar to the standard unemployment gap coefficient. However, the slope does not change significantly in the post recession period, again, seen by the small and statistically insignificant estimate on the $\tilde{U} \times Post$ coefficient in column (5). When using the total search rate as a measure of labor slack, the pre-recession slope is estimated at -0.67, nearly twice the size of the correlation with the standard unemployment gap in the pre-recession period. The change in the slope when using the total searcher rate is also statistically insignificant. This suggests that the flattening of the Phillips curve may be, in part, due to measurement issues in the output gap. The result stands in contrast to [Stock and Watson \(2019\)](#), who argue that the flattening of the Phillips curve is present regardless of which measure of inflation and the output gap is used.

6 Conclusion

The paper introduces a more continuous concept of labor force attachment, or unemployment intensity, by allowing individuals to differ in the labor force attachment, rather than the standard $\{0, 1\}$ approach. The American Time Use Survey is used to estimate and predict a probability of job search for each individual in the Current Population Survey starting in 1980, regardless of labor market status. The probabilities are shown to be correlated with two alternative measures of labor force attachment: job finding rates and subsequent hours worked when employed. Using the predicted job search probabilities as weights, the continuous unemployment rate and total searcher rate are constructed as weighted averages of the non-employed and total population, respectively. Adjusted labor market flows show that unemployment is a more persistent state when including an intensive margin.

The continuous unemployment rate is about 1.4 percentage points higher than the standard unemployment rate and does not display the same low frequency downward trend as the standard unemployment rate. This can be attributed to the fact that the fraction of individuals who are predicted to be searching for a job, but classified as out of the labor force, has increase gradually since 1980. The increase is largely attributed to the increasing education levels of people classified as out of the labor force. An application to the Phillips Curve shows that when using the continuous unemployment rate rather than the standard unemployment rate as a measure of the output gap, there is no post-2008 flattening.

7 Tables

Table 1: Search Effort by Labor Force Status: Extensive Margin

	Age 16+		Age 25-55	
	Daily Probability	Monthly Probability	Daily Probability	Monthly Probability
Employed	0.609 (0.022)	16.739	0.545 (0.024)	15.112
At Work	0.573 (0.022)	15.900	0.513 (0.024)	14.306
Absent	1.411 (0.161)	34.699	1.343 (0.189)	33.349
Unemployed	17.264 (0.397)	99.661	23.053 (0.591)	99.961
On Layoff	5.986 (0.759)	84.303	6.313 (0.946)	85.863
Looking	18.495 (0.432)	99.783	25.548 (0.657)	99.986
Out of the Labor Force	0.402 (0.025)	11.382	0.927 (0.072)	24.386
N	198,831	198,831	113,302	113,302

Summary statistics calculated from the pooled 2003-2018 American Time Use Survey (ATUS). The daily probability of observing search within a group is calculated as the mean across the group of a binary variable that takes on the value 1 if the individuals engaged in any positive amount of job search the day of the interview. Job search activities are coded as categories 50401-50499 in the ATUS lexicon. Standard errors given in parenthesis are calculated as $\sqrt{p(1-p)/N}$, where p is the daily probability and N is the number of observations in the group. Monthly probabilities are calculated as $1 - (1 - p)^{30}$, assuming 30 days per month and that the probability of searching for a job is the same every day.

Table 2: Search Effort by Labor Force Status: Minutes Per Day

	Age 16+		Age 25-55	
	Unconditional	Conditional	Unconditional	Conditional
Employed	0.711 (12.778)	116.787 (115.267)	0.661 (12.750)	121.284 (123.499)
At Work	0.615 (11.010)	106.818 (98.708)	0.579 (10.949)	112.871 (103.476)
Absent	3.02 (34.165)	214.411 (195.248)	2.729 (35.296)	203.194 (230.840)
Unemployed	25.213 (78.058)	146.039 (132.878)	36.208 (91.900)	157.065 (132.908)
On Layoff	8.664 (51.227)	144.747 (156.668)	9.792 (56.985)	155.103 (171.390)
Looking	27.018 (80.242)	146.085 (132.018)	40.145 (95.412)	157.137 (131.374)
Out of the Labor Force	0.529 (11.185)	131.648 (118.011)	1.263 (17.405)	136.131 (120.061)

Table 3: Search Effort by Labor Force Status: Percent of Time by Activity

	Age 16+			Age 25-55		
	E	U	O	E	U	O
Active Job Search	81.7	91.0	86.2	81.2	92.4	89.8
Interviewing	14.4	6.7	9.7	14.0	5.1	5.4
Other	3.9	2.3	4.4	4.8	2.5	4.8
N	601	1,382	203	434	982	128

Table 4: Summary Statistics

	Employed	Unemployed	Out of the Labor Force
Female	0.47	0.48	0.62
Married	0.57	0.31	0.52
Child	0.43	0.54	0.29
Age	41.27	33.49	55.38
Less than HS	0.10	0.31	0.25
High School	0.28	0.30	0.34
Some College	0.27	0.25	0.22
Collage	0.22	0.10	0.12
Advanced Degree	0.13	0.04	0.07
Sunday	0.14	0.14	0.15
Monday	0.14	0.14	0.14
Tuesday	0.15	0.14	0.14
Wednesday	0.14	0.15	0.14
Thursday	0.14	0.15	0.14
Friday	0.14	0.14	0.14
Saturday	0.14	0.14	0.14
Full Time	0.78		
Part Time	0.22		
White	0.83	0.71	0.81
Black	0.11	0.22	0.13
Other	0.06	0.07	0.06
Observations	124,631	9,045	65,155

Table 5: Logit Parameters

Parameter	Employed	Unemployed	Out of the Labor Force	Parameter	Employed	Unemployed	Out of the Labor Force
Age	0.080	0.165	0.198	High School \times Age			
Monday	0.312	0.451	0.452	Less than HS \times Age			
Saturday	-0.547	-0.934	-0.507	Some College \times Age			
Sunday	-0.581	-0.761	-0.323	Economy \times College		-3.229	0.755
Thursday	0.366	0.205	0.536	High School \times Economy			-0.321
Tuesday	0.256	0.163	0.508	Less than HS \times Economy		1.252	4.033
Wednesday	0.200	0.547	0.896	Some College \times Economy	0.318		-0.072
Economy	-0.511	-0.538	-4.610	College \times Married			
College	-0.298	-0.336	-0.466	High School \times Married			
High School	-0.826	-0.811	-1.078	Less than HS \times Married	0.021		0.269
Less than HS	-0.909	-1.310	-1.064	Some College \times Married		0.387	-0.045
Some College	-0.384	-1.029	-0.239	College \times Age ²			
Female	-0.073	-0.440	-0.225	High School \times Age ²			-0.000
Married	-0.603	-0.312	0.434	Less than HS \times Age ²			
Age ²	-0.001	-0.002	-0.003	Some College \times Age ²			
Race - Other	-0.439	-0.010	-0.085	College \times Race - Other	-0.980	-0.323	-0.490
Race - White	-0.518	-0.051	-0.760	High School \times Race - Other		-0.256	
Child	-0.089	-0.208	-0.491	Less than HS \times Race - Other			-1.866
Part Time	1.640	Not Included	Not Included	Some College \times Race - Other	-0.099	-0.134	-2.485
Female \times Age				College \times Race - White			
Female \times Economy				High School \times Race - White	-0.169		
Female \times College	0.035			Less than HS \times Race - White			
Female \times High School				Some College \times Race - White			
Female \times Less than HS				College \times Child			
Female \times Some College		0.274		High School \times Child	0.245		0.301
Female \times Married		-0.279	-1.194	Less than HS \times Child			-0.097
Female \times Age ²				Some College \times Child	0.459	0.051	0.248
Female \times Race - Other				College \times Part Time	0.370	Not Included	Not Included
Female \times Race - White			-0.438	High School \times Part Time	0.282	Not Included	Not Included
Female \times Child	-0.414		-0.165	Less than HS \times Part Time		Not Included	Not Included
Female \times Part Time	-0.684	Not Included	Not Included	Some College \times Part Time	-0.299	Not Included	Not Included
College \times Age				Constant	-5.895	-3.588	-6.250

Note: A total of 64 parameters we included in the Logit estimation for the Employed group and 58 for the Unemployed and Out of the labor force. The first 18 (19 for employed) are always included and the remaining interaction terms are chosen with a net-elastic logit with a weight of 0.95 on the LASSO penalty,. The regularization parameter is chosen using cross-validation of 10 folds.

Table 6: Percentiles of Predicted Search Effort

	Employed	Unemployed	Out of the Labor Force
5th Percentile	0.0051	0.7184	0.0000
10th Percentile	0.0180	0.8402	0.0000
25th Percentile	0.0459	0.9560	0.0012
50th Percentile	0.0924	0.9944	0.0239
75th Percentile	0.1633	0.9997	0.1261
90th Percentile	0.2668	1.0000	0.3097
95th Percentile	0.3690	1.0000	0.4695

Table 7: Correlation between Search Effort and Labor Force Attachment: 1994-2019

	Job Finding Prob.		Hours Worked		Change in Hours	
Search Probability	0.174 (0.000)	0.176 (0.000)	7.397 (0.065)	7.554 (0.065)	18.542 (0.230)	18.502 (0.229)
Mean	0.037	0.037	30.33	30.33	0.33	0.33
Month \times Year FE		✓		✓		✓
Observations	17608693	17608693	345967	345967	188130	188130
Sample	Full	Full	Nonemployed Job Finders	Nonemployed Job Finder	Employed Job Switchers	Employed Job Switchers

Table 8: Out of the Labor Force Demographics by Decade

	1980-1989	1990-1999	2000-2009	2010-2018
Men				
% of population	47.4	47.9	48.2	48.3
% of hires	47.5	47.9	48.2	48.3
Women				
% of population	52.6	52.1	51.8	51.7
% of hires	52.5	52.1	51.8	51.7
Age 16-24				
% of population	19.8	16.6	16.3	15.5
% of hires	41.1	38.5	36.8	34.3
Age 25-55				
% of population	54.1	58.2	56.7	52.3
% of hires	41.1	43.3	43.8	42.0
Age 56+				
% of population	26.1	25.2	27.1	32.2
% of hires	17.7	18.1	19.3	23.8
White				
% of population	86.3	84.1	81.9	79.0
% of hires	85.6	82.1	79.2	76.5
Black				
% of population	11.0	11.7	11.9	12.4
% of hires	11.3	12.8	13.6	13.9
Other				
% of population	2.7	4.2	6.2	8.6
% of hires	3.1	5.1	7.1	9.6
Less than HS				
% of population	23.4	20.8	16.8	13.3
% of hires	25.7	21.6	17.8	12.9
High School				
% of population	8.9	35.0	32.4	30.5
% of hires	9.6	7.9	7.3	6.9
Some College				
% of population	48.6	23.9	26.3	27.4
% of hires	48.2	55.4	56.9	58.1
College				
% of population	3.9	13.5	16.4	18.6
% of hires	4.5	4.5	5.4	6.0
Advanced Degree				
% of population	15.4	6.8	8.1	10.1
% of hires	12.1	10.5	12.6	16.0

Table 9: Correlations with Inflation and Nominal Wage Growth

	1980Q1 - 2019Q4					
	Average Hourly Earnings			Consumer Price Index		
	(1)	(2)	(3)	(4)	(5)	(6)
$U\text{-Gap}$	-0.072** (0.033)			-0.309* (0.177)		
$U\text{-Gap} \times \text{Post}$	0.087* (0.051)			0.755* (0.398)		
\tilde{U}		-0.070*** (0.023)			-0.382** (0.154)	
$\tilde{U} \times \text{Post}$		0.011 (0.029)			0.063 (0.445)	
S			-0.071*** (0.021)			-0.667*** (0.199)
$S \times \text{Post}$			0.014 (0.028)			0.357 (0.459)
$\bar{\pi}_{t-1}^{PCE}$	0.205*** (0.024)	0.078*** (0.027)	0.046** (0.023)			
$\bar{\pi}_{t-1}^{PCE} \times \text{Post}$	0.034 (0.072)	-0.072* (0.037)	-0.041 (0.035)			
$\bar{\pi}_{t-1}^{CPI}$				0.971*** (0.047)	0.622*** (0.133)	0.557*** (0.118)
$\bar{\pi}_{t-1}^{CPI} \times \text{Post}$				-0.441 (0.292)	-0.876** (0.404)	-0.810** (0.400)
Intercept		0.008*** (0.002)	0.014*** (0.003)		0.046*** (0.013)	0.124*** (0.032)
Post		0.002 (0.002)	0.001 (0.005)		0.035 (0.047)	-0.019 (0.085)

Notes: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are heteroskedastic and autocorrelation consistent. $U\text{-Gap}$ refers to the difference of the standard unemployment rate and the Congressional Budget Office's natural rate of unemployment (NRU). $\bar{\pi}_{t-1}^x$ is the four quarter average of the year-over-year inflation rate measured by the Consumer Price Index ($x=CPI$) or the Personal Consumption Expenditure Index ($x=PCE$). Post is an indicator variable that takes on the value 1 after 2007Q2.

8 Figures

Figure 1: Receiver Operating Characteristic Curve

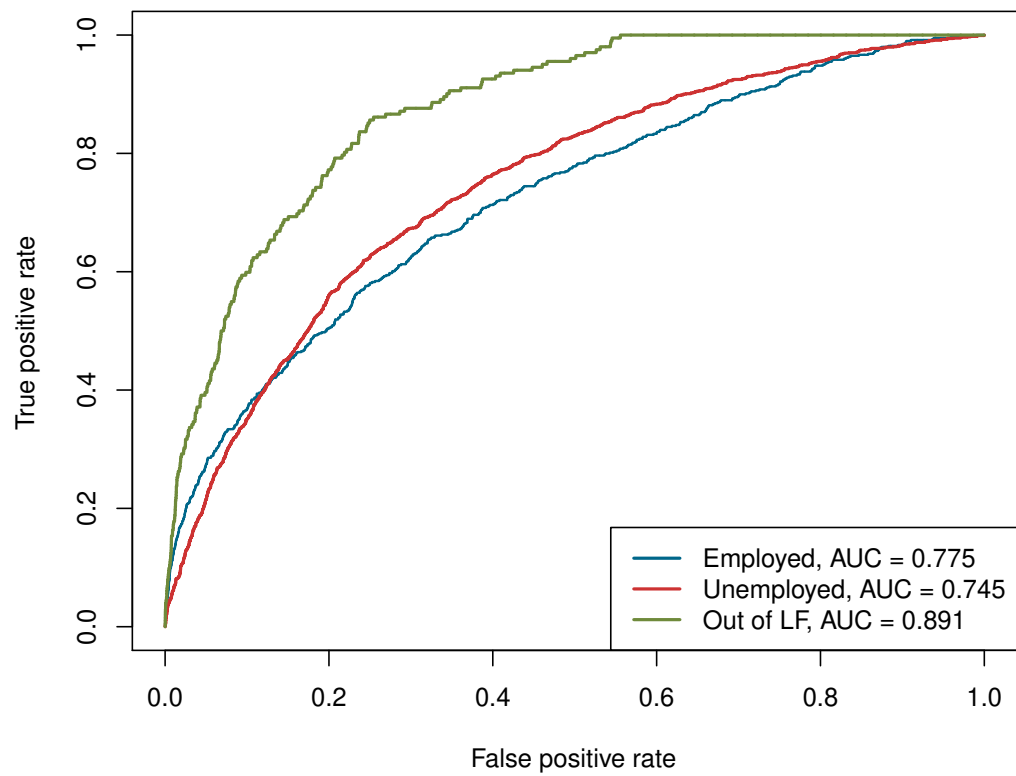
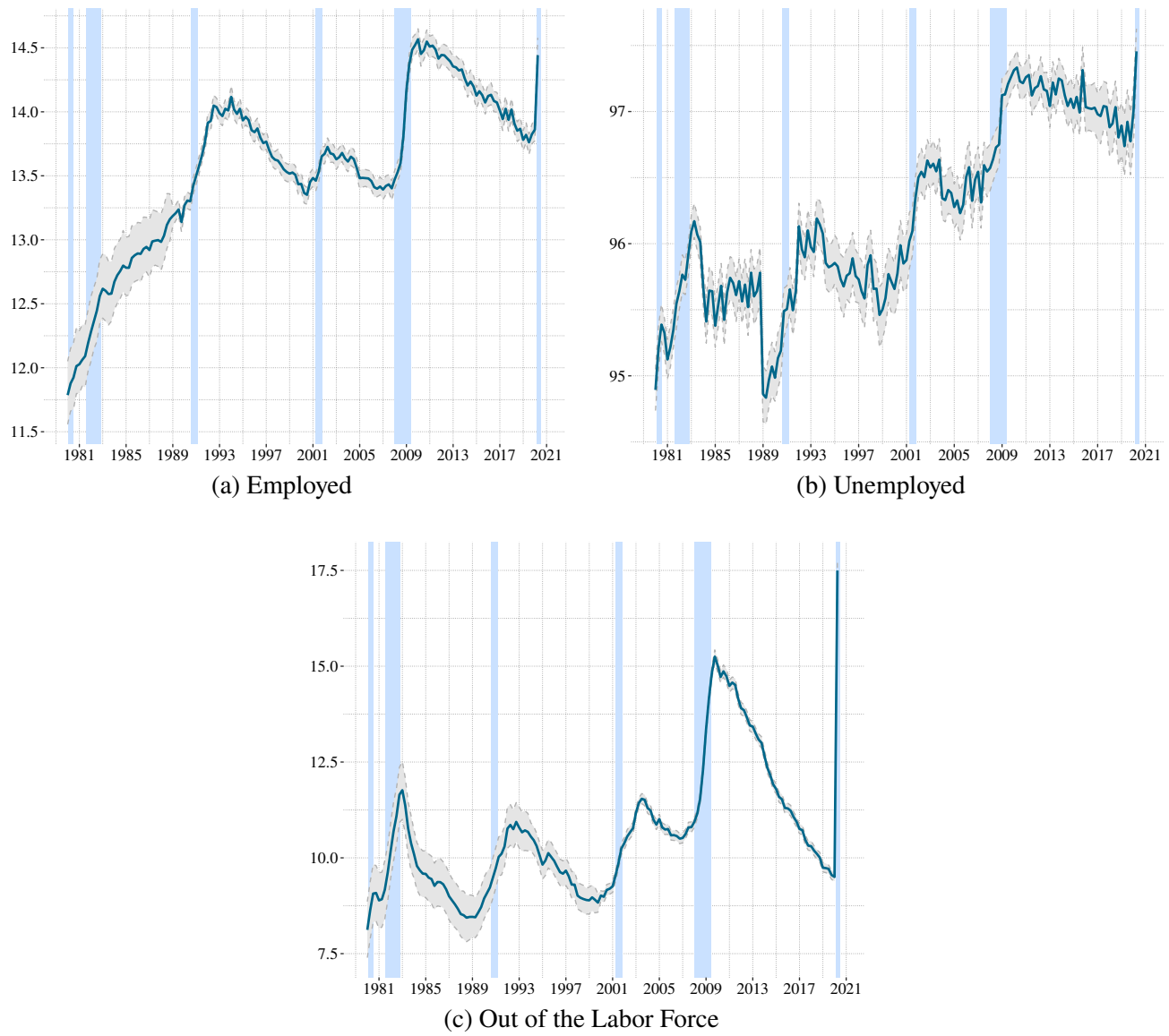
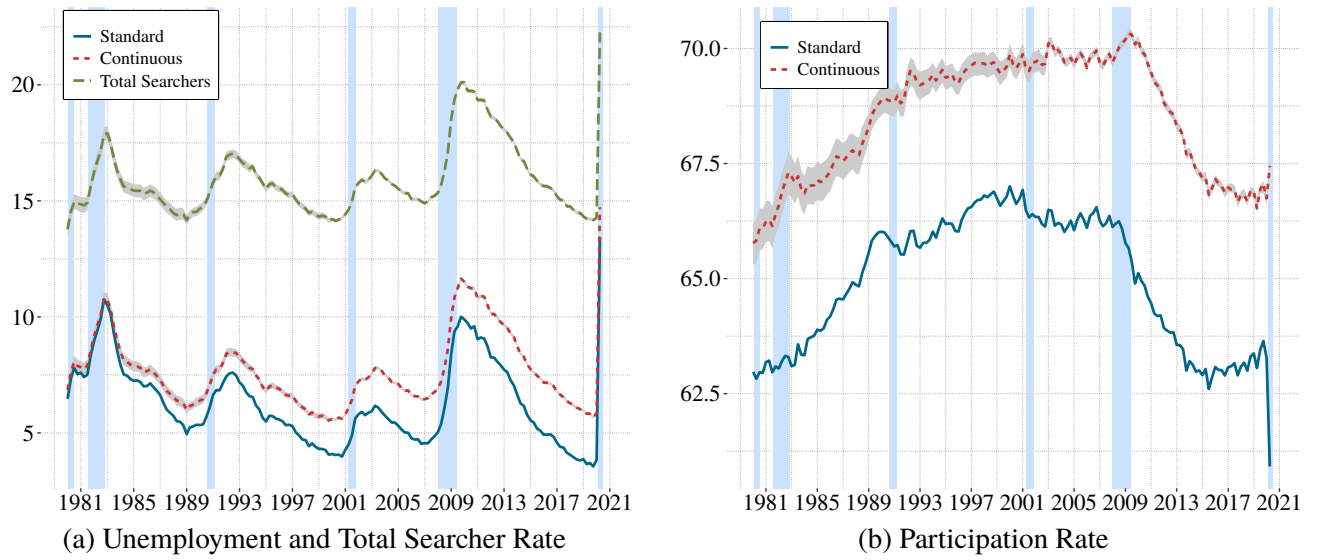


Figure 2: Fraction of Job Searchers



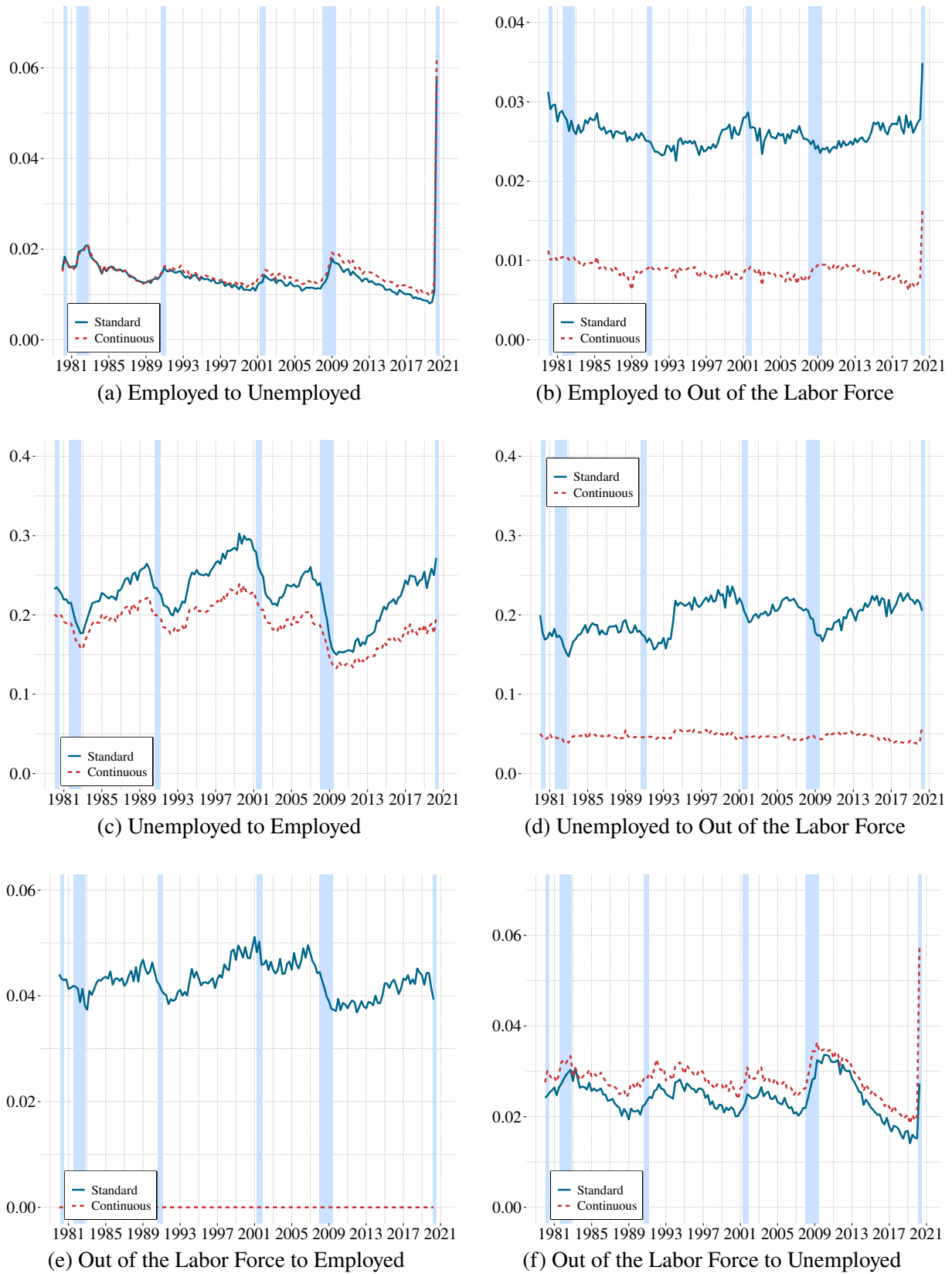
Note: The figure plots the quarterly fraction of job searchers among the employed, unemployed, out of the labor force and total population. The grey bands are 99% confidence intervals obtained from 1000 bootstrapped samples.

Figure 3: Unemployment, Total Searcher and Participation Rate



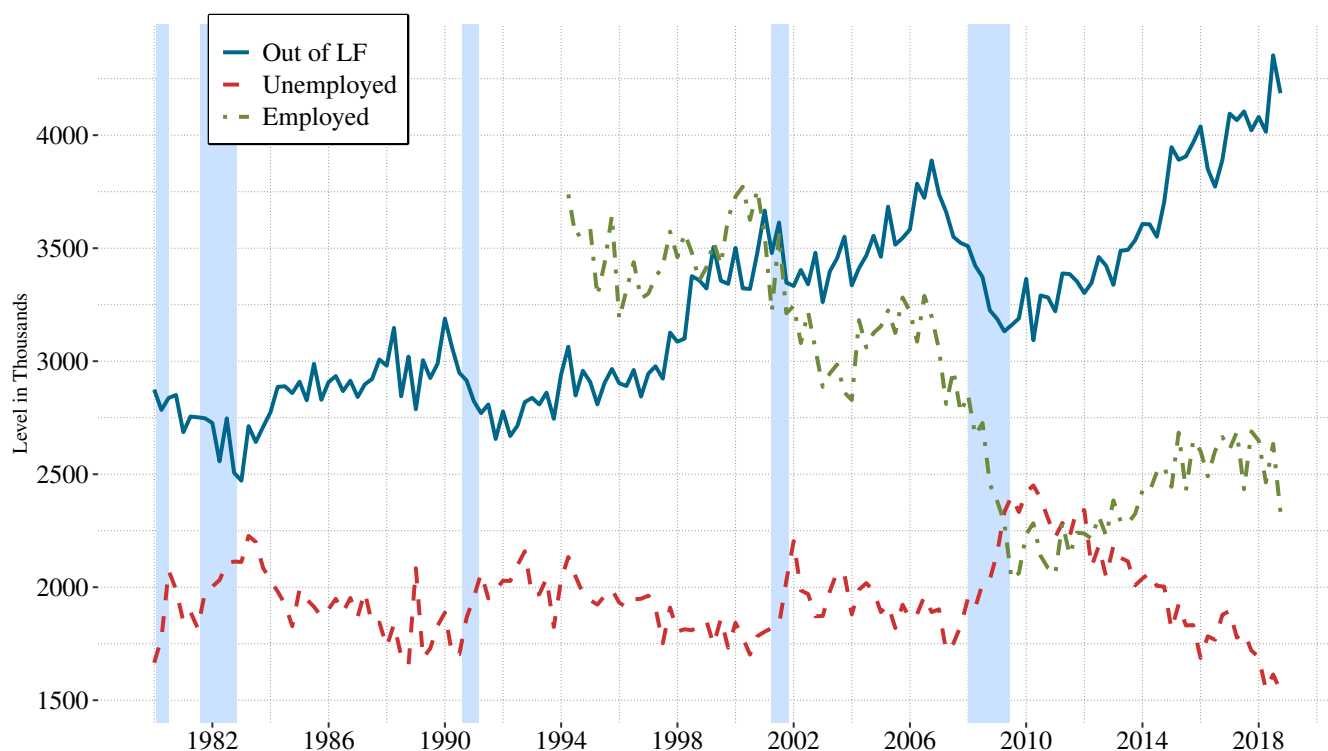
Note: The figure plots the quarterly standard and continuous unemployment rate and participation rate. The grey bands are 99% confidence intervals obtained from 1000 bootstrapped samples.

Figure 4: Labor Market Flows



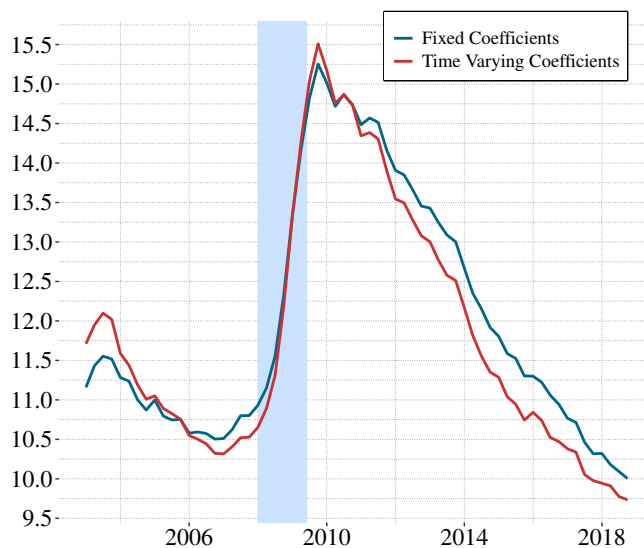
Note: The figure plots the seasonally adjusted quarterly standard and continuous flows across labor market states.

Figure 5: Total Hires by Labor Market Status



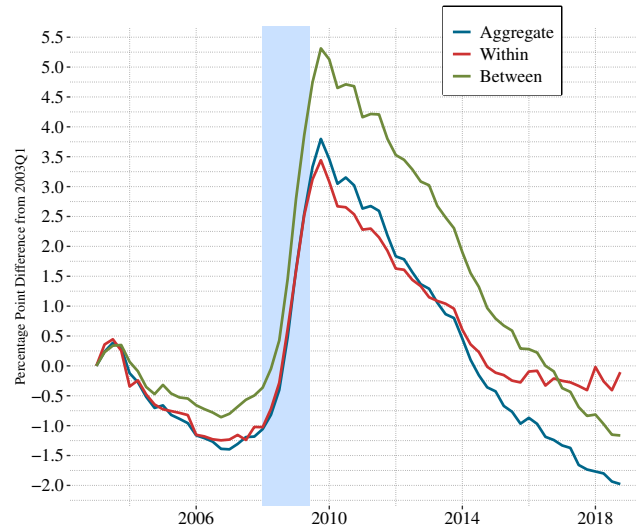
Note: The figure plots the number of new hires from each labor market state in thousands.

Figure 6: Fraction of Out of the Labor Force Job Searchers

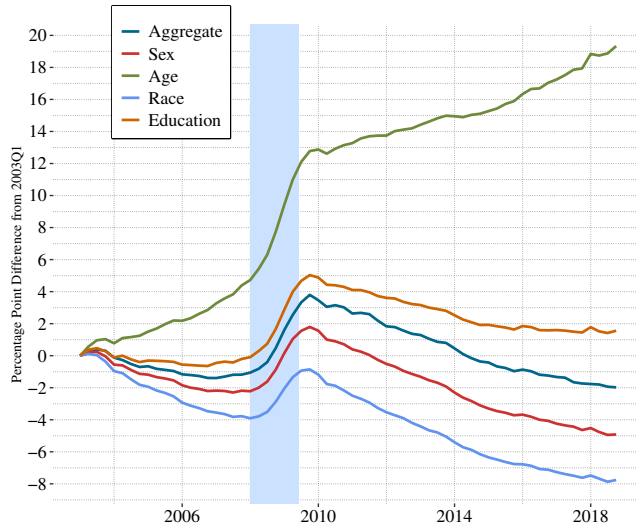


Note: The figure plots the seasonally adjusted and quarterly average of the fraction of the out of the labor force population that is predicted to be searching for a job when demographic effects are constant over time and allowed to vary over time. The “Fixed coefficients” series is the same as what is plotted in [Figure 2](#) panel (c).

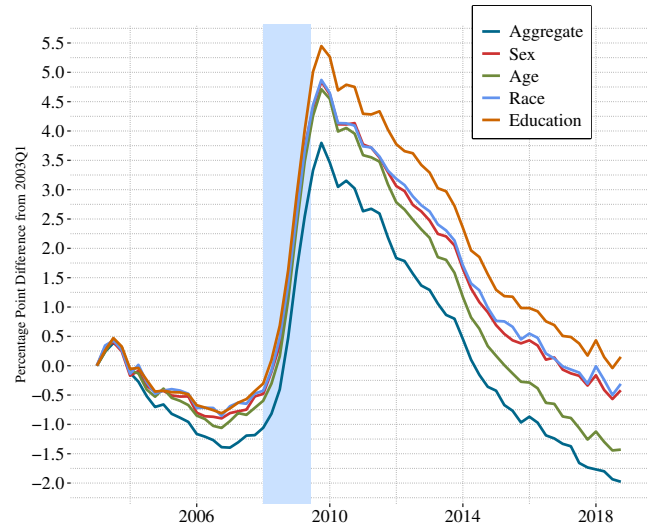
Figure 7: Decomposition Within and Between Demographic groups



(a) Aggregate Within and Between

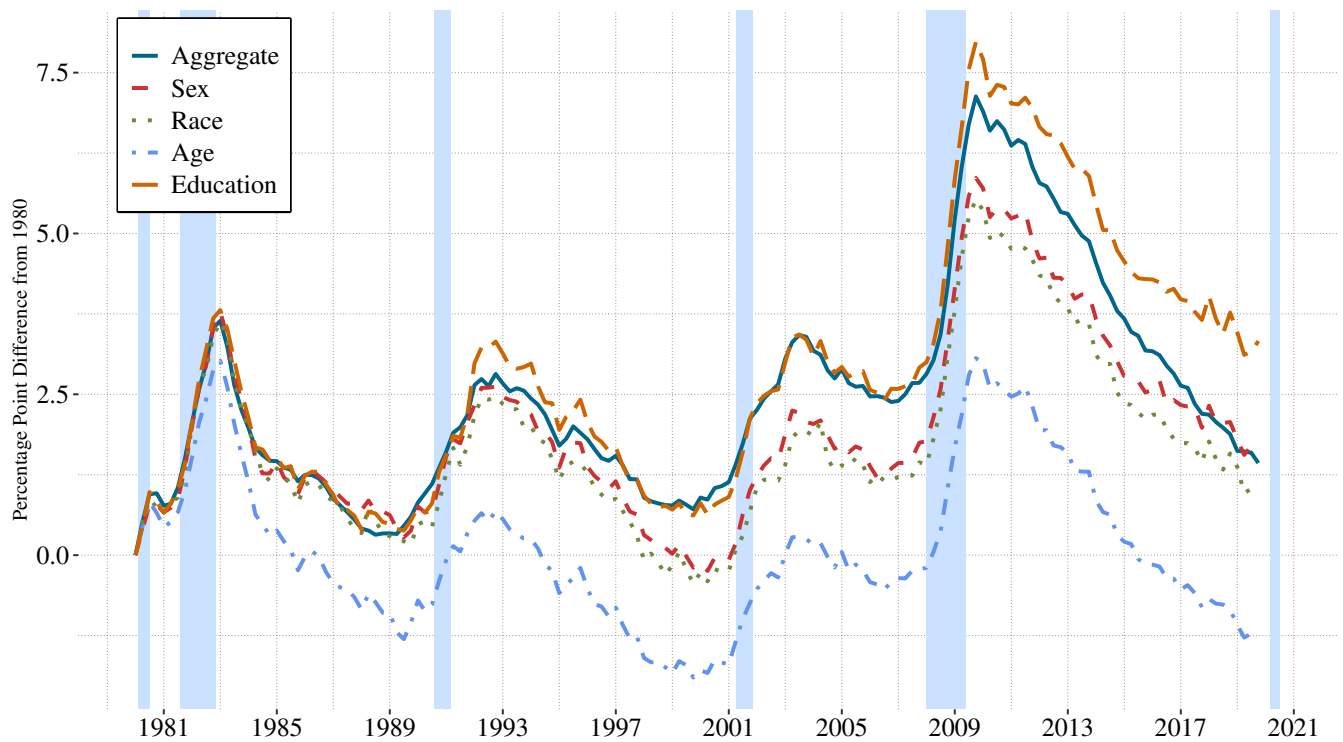


(b) Within



(c) Between

Figure 8: Fraction of Out of the Labor Force Searching for a Job



Note: The figure plots the fraction of out of the labor force that is searching for a job ("Aggregate") along with the four counterfactual series if the fraction of each demographic group remained at its 1980's values. Each series is graphed as a percentage point difference from the 1980 value.

References

- Abowd, John M. and Arnold Zellner**, “Estimating Gross Labor-Force Flows,” *Journal of Business and Economic Statistics*, 1985, 3 (3), 254–283.
- Ahn, Hie and James Hamilton**, “Measuring Labor-Force Participation and the Incidence and Duration of Unemployment,” *Finance and Economics Discussion Series*, 05 2019, 2019.
- Ahn, Hie Joo and Ling Shao**, “Precautionary On-the-Job Search Over the Business Cycle,” Working Paper, FEDS Working Paper No. 2017-025 2017.
- Ball, Laurence and Sandeep Mazumder**, “Inflation Dynamics and the Great Recession,” *Brookings Papers on Economic Activity*, 2011, 42 (1 (Spring), 337–405.
- Barnichon, Regis and Geert Mesters**, “On the Demographic Adjustment of Unemployment,” *The Review of Economics and Statistics*, 2018, 100 (2), 219–231.
- Blanchard, Olivier**, “The Phillips Curve: Back to the ’60s?,” *American Economic Review*, May 2016, 106 (5), 31–34.
- Card, David**, “Origins of the Unemployment Rate: The Lasting Legacy of Measurement without Theory,” *The American Economic Review*, 2011, 101 (3), 552–557.
- Coibion, Olivier and Yuriy Gorodnichenko**, “Is the Phillips Curve Alive and Well after All? Inflation Expectations and the Missing Disinflation,” *American Economic Journal: Macroeconomics*, January 2015, 7 (1), 197–232.
- Crump, Richard K, Stefano Eusepi, Marc Giannoni, and Ayegül ahin**, “A Unified Approach to Measuring u^* ,” Working Paper 25930, National Bureau of Economic Research June 2019.
- DeLoach, Stephen and Mark Kurt**, “Discouraging Workers: Estimating the Impacts of Macroeconomic Shocks on the Search Intensity of the Unemployed,” *Journal of Labor Research*, December 2013, 34 (4), 433–454.
- Elsby, Michael W.L., Bart Hobijn, and Aysegül Sahin**, “On the importance of the participation margin for labor market fluctuations,” *Journal of Monetary Economics*, 2015, 72, 64 – 82.
- Faberman, Jason, Andreas Müller, Aysegul Sahin, and Giorgio Topa**, “The Shadow Margins of Labor Market Slack,” Technical Report, Working Paper 2019.

- Fallick, Bruce C. and Charles A. Fleischman**, “Employer-to-employer flows in the U.S. labor market: the complete picture of gross worker flows,” Technical Report 2004.
- Feng, Shuaizhang and Yingyao Hu**, “Misclassification Errors and the Underestimation of the US Unemployment Rate,” *The American Economic Review*, 2013, *103* (2), 1054–1070.
- Flaim, Paul O.**, “The effect of demographic changes on the Nation’s unemployment rate,” *Monthly Labor Review*, 1979, *102* (3), 13–23.
- Flinn, Christopher J. and James J. Heckman**, “Are Unemployment and Out of the Labor Force Behaviorally Distinct Labor Force States?,” *Journal of Labor Economics*, 1983, *1* (1), 28–42.
- Fujita, Shigeru, Giuseppe Moscarini, and Fabien Postel-Vinay**, “Measuring Employer-to-Employer Reallocation,” Technical Report, Working Paper 2019.
- Galí, Jordi and Luca Gambetti**, “Has the U.S. Wage Phillips Curve Flattened? A Semi-Structural Exploration,” NBER Working Papers 25476, National Bureau of Economic Research, Inc January 2019.
- Garibaldi, Pietro and Etienne Wasmer**, “Equilibrium Search Unemployment, Endogenous Participation, and Labor Market Flows,” *Journal of the European Economic Association*, 2005, *3* (4), 851–882.
- Gomme, Paul and Damba Lkhagvasuren**, “Worker search effort as an amplification mechanism,” *Journal of Monetary Economics*, 2015, *75* (C), 106–122.
- Halpern-Manners, Andrew and John Warren**, “Panel Conditioning in Longitudinal Studies: Evidence From Labor Force Items in the Current Population Survey,” *Demography*, November 2012, *49* (4), 1499–1519.
- Hornstein, Andreas, Marianna Kudlyak, and Fabian Lange**, “Measuring Resource Utilization in the Labor Market,” *Economic Quarterly*, 2014, (1Q), 1–21.
- Jaeger, David A.**, “Reconciling the Old and New Census Bureau Education Questions: Recommendations for Researchers,” *Journal of Business & Economic Statistics*, 1997, *15* (3), 300–309.
- Jones, Jerry, Michael Joyce, and Jonathan Thomas**, “Non-Employment and Labour Availability,” 2003.

- Krueger, Alan B., Alexandre Mas, and Xiaotong Niu**, “The Evolution of Rotation Group Bias: Will the Real Unemployment Rate Please Stand Up?,” *The Review of Economics and Statistics*, 2017, 99 (2), 258–264.
- Krusell, Per, Toshihiko Mukoyama, Richard Rogerson, and Aysegül Sahin**, “Gross Worker Flows over the Business Cycle,” *American Economic Review*, November 2017, 107 (11), 3447–76.
- , —, —, and **Aysegül Sahin**, “Aggregate implications of indivisible labor, incomplete markets, and labor market frictions,” *Journal of Monetary Economics*, July 2008, 55 (5), 961–979.
- , —, —, and —, “Aggregate labor market outcomes: The roles of choice and chance,” *Quantitative Economics*, 2010, 1 (1), 97–127.
- , —, —, and —, “A three state model of worker flows in general equilibrium,” *Journal of Economic Theory*, 2011, 146 (3), 1107 – 1133. Incompleteness and Uncertainty in Economics.
- Kudlyak, Marianna and Jason Faberman**, “The Intensity of Job Search and Search Duration,” 2014 Meeting Papers 306, Society for Economic Dynamics 2014.
- McLeay, Michael and Silvana Tenreyro**, “Optimal Inflation and the Identification of the Phillips Curve,” Working Paper 25892, National Bureau of Economic Research May 2019.
- Mukoyama, Toshihiko, Christina Patterson, and Aysegül Sahin**, “Job Search Behavior over the Business Cycle,” *American Economic Journal: Macroeconomics*, January 2018, 10 (1), 190–215.
- Perry, George L.**, “Changing Labor Markets and Inflation,” *Brookings Papers on Economic Activity*, 1970, 1 (3), 411–448.
- Poterba, James M and Lawrence H Summers**, “Reporting Errors and Labor Market Dynamics,” *Econometrica*, November 1986, 54 (6), 1319–1338.
- Schweitzer, Mark E.**, “Ready, willing, and able? measuring labour availability in the UK,” Technical Report 2003.
- Shimer, Robert**, “The Impact of Young Workers on the Aggregate Labor Market,” *The Quarterly Journal of Economics*, 2001, 116 (3), 969–1007.
- , “Search Intensity,” Mimeo 2004.
- , “Reassessing the Ins and Outs of Unemployment,” *Review of Economic Dynamics*, April 2012, 15 (2), 127–148.

A Appendix

A.1 CPS Data Cleaning and Aggregate Series Creation

The main data used to calculate the number of individuals and flows across the labor market come from the basic monthly files of the Current Population Survey from January 1979 to December 2018. Over this period several changes to the demographic variables used to predict the search probability occurred. First, whether or not a child was present in the home was not asked prior to 1984 and between January 1994 to October 1999. For the years prior to 1984, the indicator for if a child was present is replaced with the sample mean in 1984. During the months between January 1994 and October 1999, the indicator is replaced with the average of the sample average in 1993 and 2000. Second, the education variable changed from a continuous measure to a discrete degree based measure in 1992. The education variable is made consistent using the method described in Jaeger (1997).

After predicting and aggregating the series of total employed and out of the labor force that are searching for a job, each series has a small discrete jump at different dates. The employed searchers series as a discrete jump in January 1989. The jump is removed from the series by adding a constant term to the series pre January 1989 by a constant factor such that the ratio of January 1989 to December 1988 is equal to the average January/December ratio over the whole sample. Similarly the out of the labor force series is adjusted in January 2003 and January 1989. The OLF search series is adjusted in December 1999, January 1994, February 1994, February and January 1989. Each series is then seasonally adjusted using the X-13ARIMA-SEATS seasonal adjustment program provided by the Census Bureau, available at: <https://www.census.gov/srd/www/x13as/>.

Flows of the labor market are calculated using the matched basic monthly files of the Current Population Survey. The matching and flow calculation files are taken from Robert Shimer and modified; the original programs are available at: <https://sites.google.com/site/robertshimer/research/flows>. The flows are first seasonally adjusted using X-13. Next the flows are corrected for margin error similar to Elsby et al. (2015) (described in detail below) and finally the flows are adjusted for time aggregation as proposed by Shimer (2012).

A.2 Margin Error Adjustment

The margin error adjustment is similar to [Elsby et al. \(2015\)](#). First define new stocks of unemployed and out of the labor force as:

$$\tilde{U}_t = U_t^S + O_t^S$$

$$\tilde{O}_t = O_t - O_t^S$$

. Then let $S_t = [E_t \ \tilde{U}_t]$ be the vector containing the number of employed in unemployed and let $\Delta S_t = [E_t \ \tilde{U}_t]' - [E_{t-1} \ \tilde{U}_{t-1}]'$ be the change in the current state vector. The change in the current state vector can be written as:

$$\Delta S_t = \begin{bmatrix} -E_{t-1} & -E_{t-1} & \tilde{U}_{t-1} & 0 & 0 \\ E_{t-1} & 0 & -\tilde{U}_{t-1} & -\tilde{U}_{t-1} & \tilde{O}_{t-1} \end{bmatrix} \times \begin{bmatrix} p_{E\tilde{U}} \\ p_{E\tilde{O}} \\ p_{\tilde{U}E} \\ p_{\tilde{U}\tilde{O}} \\ p_{\tilde{O}\tilde{U}} \end{bmatrix}$$

$$\Delta S_t = \mathbf{X}_{t-1} \mathbf{p}$$

Where p_{jk} is probability that an individual transitions from labor market state j to k . Notice that here, individuals can not directly transition from out of the labor force to employment. The estimated vector of transition probabilities, denoted $\hat{\mathbf{p}}$, has a covariance matrix proportional to the matrix that is consistently estimated using:

$$\mathbf{W} = \begin{bmatrix} \frac{\hat{p}_{E\tilde{U}}(1-\hat{p}_{E\tilde{U}})}{E_{t-1}} & -\frac{\hat{p}_{E\tilde{U}}\hat{p}_{E\tilde{O}}}{E_{t-1}} & 0 & 0 & 0 \\ -\frac{\hat{p}_{E\tilde{U}}\hat{p}_{E\tilde{O}}}{E_{t-1}} & \frac{\hat{p}_{E\tilde{O}}(1-\hat{p}_{E\tilde{O}})}{E_{t-1}} & 0 & 0 & 0 \\ 0 & 0 & \frac{\hat{p}_{\tilde{U}E}(1-\hat{p}_{\tilde{U}E})}{\tilde{U}_{t-1}} & -\frac{\hat{p}_{\tilde{U}E}\hat{p}_{\tilde{U}\tilde{O}}}{\tilde{U}_{t-1}} & 0 \\ 0 & 0 & -\frac{\hat{p}_{\tilde{U}E}\hat{p}_{\tilde{U}\tilde{O}}}{\tilde{U}_{t-1}} & \frac{\hat{p}_{\tilde{U}\tilde{O}}(1-\hat{p}_{\tilde{U}\tilde{O}})}{\tilde{U}_{t-1}} & 0 \\ 0 & 0 & 0 & 0 & \frac{\hat{p}_{\tilde{O}\tilde{U}}(1-\hat{p}_{\tilde{O}\tilde{U}})}{\tilde{O}_{t-1}} \end{bmatrix}$$

The vector of transition probabilities, \mathbf{p} , is chosen to minimize the weighted least square to the estimated transition probabilities and restricted to match the observed changes in labor market states.

That is:

$$\mathbf{p} = \text{argmin } (\mathbf{p} - \hat{\mathbf{p}})' \mathbf{W}^{-1} (\mathbf{p} - \hat{\mathbf{p}}) \text{ s.t. } \Delta S_t = \mathbf{X}_{t-1} \mathbf{p}$$

The Lagrangian is

$$\mathcal{L} = (\mathbf{p} - \hat{\mathbf{p}})' \mathbf{W}^{-1} (\mathbf{p} - \hat{\mathbf{p}}) - 2\mu [\Delta \mathbf{S}_t - \mathbf{X}_{t-1} \mathbf{p}]$$

where μ is the vector of Lagrange multipliers. The solution is,

$$\begin{bmatrix} \mathbf{p} \\ \mu \end{bmatrix} = \begin{bmatrix} \mathbf{W} & \mathbf{X}_{t-1}' \\ \mathbf{X}_{t-1} & 0 \end{bmatrix}^{-1} \times \begin{bmatrix} \mathbf{W} \hat{\mathbf{p}} \\ \Delta \mathbf{S}_t \end{bmatrix}.$$

Since all objects on the right-hand side are known, the above equation gives the solution to the margin error adjusted probabilities \mathbf{p} .