Math 222 - Quiz 5 **Section:** _____ **Name:** _____

Problem 1 Solve the initial value problem $y' = x(y - y^2)$ with $y(0) = 1/2$.

separable Diff Eq.

$$\Rightarrow \int \frac{1}{y-y^2}\,dy = \int x\,dx$$

$$\frac{1}{2}x^2 + C$$

$$\frac{1}{y-y^2} = \frac{A}{y} + \frac{B}{1-y}$$

$$\Rightarrow \quad 1 = A(1-y) + By$$

$$\Rightarrow \quad B - A = 0 \Rightarrow B = 1$$

$$A = 1$$

$$\Rightarrow \frac{1}{y-y^2} = \frac{1}{y} + \frac{1}{1-y}$$

$$\int \frac{1}{y-y^2}\,dy = \int \frac{1}{y} + \frac{1}{1-y}\,dy$$

$$= \ln|y| - \ln|1-y|$$

$$= \ln\left|\frac{y}{1-y}\right|$$

$$\Rightarrow \ln\left|\frac{y}{1-y}\right| = \frac{1}{2}x^2 + C$$

$$\Rightarrow \frac{y}{1-y} = Ce^{\frac{1}{2}x^2}$$

$$\Rightarrow y = \frac{Ce^{1/2x^2}}{1+Ce^{1/2x^2}}$$

$y(0) = 1/2$

$$\Rightarrow \frac{1}{2} = \frac{C}{1+C}$$

$$\Rightarrow C = 1$$

$$\Rightarrow \boxed{y = \frac{e^{1/2x^2}}{1+e^{1/2x^2}}}$$

Problem 2 Find the **general solution** to the differential equation $x\frac{dy}{dx} - y = 2x\ln x$.

1st order linear

Standard Form:

$$\frac{dy}{dx} - \frac{y}{x} = 2\ln x$$

$$\Rightarrow m(x) = -\frac{1}{x} \Rightarrow IF = e^{-\int \frac{1}{x}} = e^{-\ln|x|} = \boxed{\frac{1}{x}}$$

$$\Rightarrow \left(\frac{1}{x}y\right)' = 2\frac{\ln x}{x}$$

$$\Rightarrow \frac{1}{x}y = 2\int \frac{\ln x}{x}\,dx + C$$

$u = \ln x, \; du = \frac{1}{x}dx$

$$\Rightarrow \frac{1}{x}y = (\ln x)^2 + C$$

$$\Rightarrow \boxed{y = x(\ln x)^2 + Cx}$$