Math 222 - Quiz 6 **Section:** ___________ **Name:** ____________________________

Problem 1 The natural logarithm satisfies the initial value problem $y'(x) = \frac{1}{x}$, where $y(1) = 0$. Use Euler's method with a step-size of $h = \frac{1}{3}$ to approximate $\ln 2$.

$x_0 = 1 \quad y_0 = 0$

$y(4/3) \rightsquigarrow y_1 = y_0 + m\Delta x, \quad m = y'(x_0) = \frac{1}{1}.$

$= 0 + \frac{1}{3} = 1/3.$

$y(5/3) \rightarrow y_2 = y_1 + m \cdot \Delta x, \quad m = y'(x_1) = \frac{1}{4/3} = \frac{3}{4}$

$= 1/3 + 3/4 \cdot \frac{1}{3} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

$y(2) \rightarrow y_3 = y_2 + m\Delta x, \quad m = y'(x_2) = \frac{1}{5/3} = \frac{3}{5}$

$= \frac{7}{12} + \frac{3}{5} \cdot \frac{1}{3} = \boxed{\frac{7}{12} + \frac{1}{5}}$

$\Rightarrow \ln(2) \sim \frac{7}{12} + \frac{1}{5}$

Problem 2 We start with a full 10,000 gallon vat containing a solution of 3% acid. There is a pipe brining in a solution of 5% acid at a rate of 10 gallons per minute, and another pipe removing the mixed solution from the vat at a rate of 15 gallons per minute. Write out a differential equation, with any necessary initial conditions, that describes the total amount of acid (in gallons) in the vat at any given time t (in minutes). **You do not need to solve this differential equation**

$A(t)$ - acid in vat (in gals) @ time t (min).

Ic $\rightarrow$ $\boxed{A(0) = 10{,}000 \times 0.03 = 300 \text{ gal}}$

$A'(t) = \{\text{Rate in}\} - \{\text{Rate out}\}$

$10 \cdot 0.05 \ \frac{\text{gal}}{\text{min}}$

$\{\text{concentration}\}\{\text{Rate}\}$

$15 \ \frac{\text{gal}}{\text{min}}$

$\text{Concentration} = \frac{\text{Acid}}{\text{Volume}} = \frac{A(t)}{10000 - 5t}$

$\Rightarrow \boxed{A'(t) = 0.5 - 15\frac{A(t)}{10000 - 5t}}$