Math 222 - Quiz 3 **Section:** ________ **Name:** Answer.

Problem 1. Evaluate the indefinite integral $\int \frac{1}{(1+x^2)(1+x)}\,dx$.

Via partial fraction decomposition

$$\frac{1}{(1+x^2)(1+x)} = \frac{Ax+B}{1+x^2} + \frac{C}{1+x}$$

$$\Rightarrow +A = \tfrac{1}{2},\quad B = \tfrac{1}{2},\quad C = \tfrac{1}{2}$$

So $$\int \frac{1}{(1+x^2)(1+x)}\,dx = \int \frac{-\frac{1}{2}x + \frac{1}{2}}{1+x^2} + \frac{1/2}{1+x}\,dx$$

$$= \underbrace{-\frac{1}{2}\int \frac{x}{1+x^2}\,dx}_{\substack{u\text{-sub: } u = 1+x^2 \\ du = 2x\,dx}} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x}\,dx.$$

$$= -\frac{1}{2}\int \frac{\frac{1}{2}du}{u} + \frac{1}{2}\arctan(x) + \frac{1}{2}\ln|1+x| + C$$

$$= -\frac{1}{4}\ln|1+x^2| + \frac{1}{2}\arctan x + \frac{1}{2}\ln|1+x| + C$$

Problem 2. Evaluate the definite integral $\int_0^1 \sqrt{2-t^2}\,dt$. It may help to remember that $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

Sol^n#1. Inverse trig sub: $t = \sqrt{2}\sin\theta$, $dt = \sqrt{2}\cos\theta\,d\theta$. | $t=0 \leftrightarrow \theta = 0$. $t = 1 \leftrightarrow \frac{1}{\sqrt{2}} = \sin\theta \Rightarrow \theta = \frac{\pi}{4}$.

$$\int_{t=0}^{1} \sqrt{2-t^2}\,dt = \int_{\theta=0}^{\pi/4} \sqrt{2}\left(\sqrt{1-\sin^2\theta}\right)\sqrt{2}\cos\theta\,d\theta$$

$$= 2\int_0^{\pi/4} \cos^2\theta\,d\theta$$

$$= \int_0^{\pi/4} 1 + \cos(2\theta)\,d\theta$$

$$= \left[\theta + \frac{\sin(2\theta)}{2}\right]\Bigg|_{\theta=0}^{\pi/4}$$

$$= \frac{\pi}{4} + \frac{1}{2}\left[\sin\left(\frac{\pi}{2}\right) - \cancel{\sin(0)}\right]$$

$$= \frac{\pi}{4} + \frac{1}{2}.$$

Sol^{n} #2 (to problem 2).

If treated as indefinite integral first, then

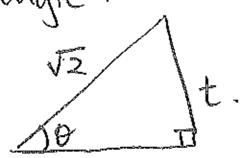
$$\int \sqrt{2-t^2}\, dt = \theta + \frac{\sin(2\theta)}{2} + C$$

$$= \theta + \frac{\cancel{2}}{\cancel{2}} \sin\theta\cos\theta + C.$$

Our initial substitution $t = \sqrt{2}\sin\theta$ has the corresponding right-angle triangle:

$\frac{t}{\sqrt{2}} = \sin\theta$

$= \frac{\text{opp}}{\text{hyp}}.$



$\Rightarrow$ adj. side is of length $\sqrt{2-t^2}$.

Thus, $\sin\theta = \frac{t}{\sqrt{2}}$, $\cos\theta = \frac{\sqrt{2-t^2}}{\sqrt{2}}$, and $\theta = \arcsin\left(\frac{t}{\sqrt{2}}\right)$.

$$\int_0^1 \sqrt{2-t^2}\, dt = \left[\theta + \sin\theta\cos\theta\right]\Bigg|_{t=0}^{t=1}$$

$$= \left[\arcsin\frac{t}{\sqrt{2}} + \left(\frac{t}{\sqrt{2}}\right)\left(\frac{\sqrt{2-t^2}}{\sqrt{2}}\right)\right]\Bigg|_{t=0}^{1}$$

$$= \arcsin\left(\frac{1}{\sqrt{2}}\right) - \cancel{\arcsin(0)}^{\,0} + \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2-1}}{\sqrt{2}}\right) - \cancel{\frac{0}{\sqrt{2}}\left(\frac{\sqrt{2-0}}{\sqrt{2}}\right)}^{\,0}$$

$$= \arcsin\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}$$

$$= \frac{\pi}{4} + \frac{1}{2}.$$