Math 222 - Quiz 7 **Section:** ____________ **Name:** Answer Key

Problem 1 Compute the Taylor series $T_\infty f(t)$ for $f(t) = \ln(1 + 5t)$.

$f(t) = \ln(1+5t)$ $\quad f(0) = \ln(1) = 0$

$f'(t) = 5(1+5t)^{-1}$ $\quad f'(0) = 5$

$f''(t) = -5^2(1+5t)^{-2}$ $\quad f''(0) = -5^2$

$f'''(t) = 2\cdot 5^3(1+5t)^{-3}$ $\quad f'''(0) = 2\cdot 5^3$

$f^{(4)}(t) = -2\cdot 3\cdot 5^4(1+5t)^{-4}$ $\quad f^{(4)}(0) = -2\cdot 3\cdot 5^4$

$$\Rightarrow f^{(n)}(0) = (-1)^{n+1}(n-1)!\,5^n$$

$$T_\infty f(t) = 0 + 5t - 5^2t^2 + \cdots + (-1)^{n+1}\frac{\cancel{(n-1)!}\,5^n}{n\cancel{!}}x^n + \cdots$$

$$* \Rightarrow T_\infty f(t) = 5t - 5^2t^2 + \cdots + (-1)^{n+1}\frac{5^n}{n}x^n + \cdots *$$

Problem 2 Compute the 6th degree Taylor polynomial for $\sin(x)$, and use it to approximate $\sin(1)$. Show that your approximation has an error less than $\frac{1}{1000}$.

$$T_6 \sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$$

$$\Rightarrow \sin(1) \sim 1 - \frac{1}{6} + \frac{1}{120}$$

Error:

$0 < c < 1$

$$R_6 \sin(1) = \frac{-\sin(c)}{7!}(1)^7$$

$|\sin(c)| \leq 1$

$$\Rightarrow |R_6 \sin(1)| \leq \frac{1}{7!}$$

$$\frac{1}{7!} < \frac{1}{1000}$$

$5! = 120$

$6! = 720$

$7! = \underbrace{7 \times 720}_{> 1000}$

$$* \sin(1) \sim 1 - \frac{1}{6} + \frac{1}{120} \text{ w/ error } < \frac{1}{1000} * \checkmark$$