Worksheet 5

Partial Fractions

Whenever we want to integrate a rational function (that is one of the form $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials), we need to make sure that the polynomial in the numerator is **smaller** than the one in the denominator. We do this with **polynomial division**.

USING POLYNOMIAL DIVISION

1. $\displaystyle\int \frac{x^3+x^2+x+2}{x^2+1}$

CANONICAL EXAMPLES

When working with rational functions you will run into three canonical examples, two of which require you to brush up on your **completing the square** skills. The canonical examples are of the form:

$$\int \frac{1}{(x-a)^2+b^2}\,dx, \qquad \int \frac{1}{(x-a)^n}\,dx, \qquad \int \frac{x}{(x-a)^2+b^2}\,dx$$

There are a few more slightly more complicated cases, but knowing how to handle these 3 are absolutely critical. Just keep in mind when you see these guys you **don't** want to use partial fractions.

2. $\displaystyle\int \frac{1}{x^2-2x+1}\,dx$

3. $\displaystyle\int \frac{1}{x^2-2x+1}\,dx$

4. $\displaystyle\int \frac{1}{2x^2+4x+10}\,dx$

5. $\displaystyle\int \frac{x}{x^2+2x+2}\,dx$

6. $\displaystyle\int \frac{x+1}{x^2-2x+1}\,dx$

PARTIAL FRACTION DECOMPOSITION

Remember these general steps for rational functions:

1. Perform polynomial division, if necessary
2. Factor the denominator (the quadratic formula may be helpful)
3. Perform partial fractions decomposition [PFD] (only if you have multiple factors)
4. Solve for unknowns in PFD using either Heaviside or solving a system of equations.

6. $\displaystyle\int \frac{x^5}{x^4-1}\,dx$

7. $\displaystyle\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$

8. $\displaystyle\int \frac{w-1}{w^2+w}\,dx$

9. $\displaystyle\int \frac{x}{x^4 - 2x^2 + 1}\,dx$

10. $\displaystyle\int \frac{x}{x^3 + 2x^2 + 2x}\,dx$