Worksheet 4

Reduction Formulas

You often find that you just need to apply integration by parts a lot; I really mean a lot, like if you want to solve $\int \sin^{64}(x)\,dx$. So to simplify doing the same thing over and over again, we write down a formula for the reduction of the integral. These are called **reduction formulas**; computers love them.

USING REDUCTION FORMULAS

1. Using the reduction formula $\int x^n e^x\,dx = x^n e^x - n\int x^{n-1}e^x\,dx$, compute $\int x^4 e^x\,dx$.

2. Compute $\int_0^{\pi} \cos(x)\,dx$ and $\int_0^{\pi} x^2\cos(x)\,dx$. Then use the reduction formula:

$$\int x^n \cos(x)\,dx = x^n \sin(x) + nx^{n-1}\cos(x) - n(n-1)\int x^{n-2}\cos(x)\,dx$$

to compute $\int_0^{\pi} x^4\cos(x)\,dx$.

3. Use the reduction formula $\int \tan^n(x)\,dx = \dfrac{\tan^{n-1}(x)}{n-1} - \int \tan^{n-2}(x)\,dx$ to calculate $\int_0^{\pi/4} \tan^5(x)\,dx$.

DERIVING REDUCTION FORMULAS

4. Show $\int x^n e^x\,dx = x^n e^x - n\int x^{n-1}e^x\,dx$

5. Show $\tan^n(x)\,dx = \dfrac{\tan^{n-1}(x)}{n-1} - \int \tan^{n-2}(x)\,dx$.
(must of the time, deriving a reduction formula comes down to IBP, but not always).