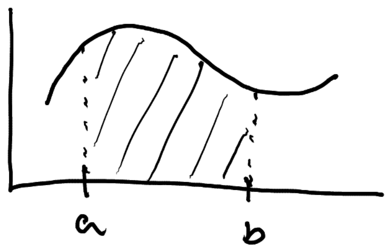
Quick Integral Review: Definite vs Indefinite

Definite

$\int_a^b f(x)\,dx$ = Area



Always a number!

Indefinite

$\int f(x)\,dx$

Anti-derivative

Always a function!

$\int f(x)\,dx$ is a func. $F(x)$ where:

$$\frac{d}{dx}F(x) = f(x).$$

Fundamental Theorem of Calculus

Connects the ideas of definite and indef. integrals! To solve definite integral, you can instead solve indefinite integrals.

$$\underbrace{\int_a^b f(x)\,dx}_{\text{Definite Int.}} = F(b) - F(a)$$

Indefinite Integral of f(x).

Caution: Always be sure to check which variable you are integrating with:

$$\int \tfrac{1}{2}x^2 \sin(t^2)\,dt = \tfrac{1}{2}x^2 \int \sin(t^2)\,dt$$

$\boxed{x \text{ is constant w/r to } t}$
so pull x out of integral like you do w/ 1/2.

Trig Identities:

Need to Memorize

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$
$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$

On test, forget which one has $\pm$? Plug in $x=0$!

Trig Integrals

$\int \cos^m(x) \sin^k(x)\,dx$ & similar.

Method 1 Look for odd powers of sin or cos: and use $\cos^2 + \sin^2 = 1$ to convert to easy substitution problem:

Ex

$$\int \cos^3 x\,dx = \int \cos^2(x)\cos(x)\,dx$$
$$= \int (1 - \sin^2(x))\cos(x)\,dx$$

Let $u = \sin(x)$, so $du = \cos(x)\,dx$, get:

$$= \int (1 - u^2)\,du$$
$$= u - \tfrac{1}{3}u^3 + C = \boxed{\sin(x) - \tfrac{1}{3}\sin^3(x) + C}$$

Ex $\int \cos^5(x)\,dx = \int \cos^4(x)\cos(x)\,dx$

$$= \int (1 - \sin^2(x))^2 \cos(x)\,dx$$

$\left.\begin{array}{l} u = \sin(x) \\ du = \cos(x)dx \end{array}\right\}$ $= \int (1 - u^2)^2\,du = \cdots$ etc.

$\boxed{\text{Ex}}$

$$\int \sin^5(x)\cos^2(x)\,dx = \int \sin(x)\sin^4(x)\cos^2(x)\,dx$$

$$= \int \sin(x)\left(1-\cos^2(x)\right)^2\cos^2(x)\,dx$$

$u = \cos(x)$

$du = -\sin(x)$

$$= \boxed{-}\int (1-u^2)^2u^2\,du = \cdots \text{ etc.}$$

Method 2 Even powers?? Use reduction identities to get smaller powers!

$$\cos^2(x) = \tfrac{1}{2}\left(1+\cos(2x)\right)$$

$$\sin^2(x) = \tfrac{1}{2}\left(1-\cos(2x)\right)$$

$\boxed{\text{Ex}}$

$$\int \sin^2(x)\,dx = \int \tfrac{1}{2}\left(1-\cos(2x)\right)dx$$

$$= \int \tfrac{1}{2} - \tfrac{1}{2}\cos(2x)\,dx$$

$$\boxed{= \tfrac{1}{2}x - \tfrac{1}{4}\sin(2x) + C}$$

$\boxed{\text{Ex}}$ May need to use multiple times!!

$$\int \sin^4(x)\,dx = \int \left(\tfrac{1}{2} - \tfrac{1}{2}\cos(2x)\right)^2dx$$

$$= \int \tfrac{1}{4} - \tfrac{1}{2}\cos(2x) + \tfrac{1}{4}\cos^2(2x)\,dx$$

$$= \tfrac{1}{4}x - \tfrac{1}{4}\sin(2x) + \tfrac{1}{4}\int \cos^2(2x)\,dx$$

Use again for last guy:

$$\int \cos^2(2x)\,dx = \int \tfrac{1}{2}(1+\cos(4x))\,dx$$

$$= \int \tfrac{1}{2} + \tfrac{1}{2}\cos(4x)\,dx$$

$$= \tfrac{1}{2}x + \tfrac{1}{8}\sin(4x) + C$$

Plug back in:

$$\int \sin^4(x)\,dx =$$

$$\tfrac{1}{4}x - \tfrac{1}{4}\sin(2x) + \tfrac{1}{4}\left(\tfrac{1}{2}x + \tfrac{1}{8}\sin(4x)\right) + C$$

$$= \tfrac{1}{4}x + \tfrac{1}{8}x - \tfrac{1}{4}\sin(2x) + \tfrac{1}{32}\sin(4x) + C$$

$$\boxed{= \tfrac{3}{8}x - \tfrac{1}{4}\sin(2x) + \tfrac{1}{32}\sin(4x) + C}$$

Method 3 Arguments don't match, like:

$$\int \sin(2x)\cos(3x)\,dx$$

Use trig formulas to seperate trig functions!

$$\sin A \sin B = \tfrac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos A \cos B = \tfrac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A \cos B = \tfrac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

Note: You probably don't need to memorize these for a test, they should be given. But know how to use them!!

$\boxed{\text{Ex}}$ $\int \sin(2x)\cos(3x)\,dx$

Here we use:

$$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$A = 2x, \quad B = 3x$$

Get:

$$= \int \frac{1}{2}\left[\sin(5x) - \sin(-x)\right] dx$$

$$\boxed{= \frac{-1}{10}\cos(5x) - \frac{1}{2}\cos(x) + C}$$