Worksheet 2

Trig Integrals

The following problems deal with integrals of the form

$$\int \cos^m(x)\sin^n(x)\,dx$$

EITHER m OR n IS ODD

1. $\int \sin^2(x)\cos(x)\,dx$

2. $\int \cos^3(x)\,dx$

3. $\int \sin^5(x)\cos^2(x)\,dx$

EVEN POWERS

Use the **reduction identities** (also call the **half-angle** formula) to get smaller powers.

$$\cos^2(x) = \frac{1}{2}\big(1+\cos(2x)\big), \qquad \sin^2(x) = \frac{1}{2}\big(1-\cos(2x)\big)$$

4. $\displaystyle\int \sin^2(x)\,dx$

5. $\displaystyle\int \sin^4(x)\,dx$

6. $\displaystyle\int \sin^2(x)\cos^2(x)\,dx$

MIS-MATCHED ARGUMENTS

These problems deal with trig integrals where the arguments of the functions don't match, and therefore invalidating all of the previous techniques discussed. Here we will use the **separation identities** to break the trig functions up:

$$\sin A \sin B = \frac{1}{2}\big[\cos(A-B) - \cos(A+B)\big], \qquad \cos A \cos B = \frac{1}{2}\big[cos(A-B) + \cos(A+B)\big]$$

$$\sin A \cos B = \frac{1}{2}\big[\sin(A+B) + \sin(A-B)\big]$$

7. $\displaystyle\int \sin(2x)\cos(3x)\,dx$

8. $\displaystyle\int \cos^2(x)\sin(2x)\,dx.$

(These arguments are mismatched, but you might find this one isn't as hard as it might appear)