Math 222 - Quiz 4 **Section:** ___________ **Name:** ___________________________

Problem 1 Calculate $\int \frac{x}{x^2+2x+5}\,dx$.

Try $u = x^2+2x+5$

$du = (2x+2)\,dx$

$$\int \frac{x}{x^2+2x+5}\,dx = \frac{1}{2}\int \frac{2x \overset{+2-2}{}}{x^2+2x+5}\,dx = \frac{1}{2}\int \frac{\overbrace{2x+2}^{du}}{\underbrace{x^2+2x+5}_{u}}\,dx - \frac{1}{2}\int \frac{2}{x^2+2x+5}\,dx$$

$$= \frac{1}{2}\ln|x^2+2x+5| - \cancel{\frac{2}{2}}^{1}\int \frac{1}{x^2+2x+5}\,dx$$

$$\int \frac{1}{x^2+2x+5}\,dx = \int \frac{1}{(x+1)^2+4}\,dx = \left\{\begin{array}{l} u = x+1 \\ du = dx \end{array}\right\} = \int \frac{1}{u^2+4}\,du$$

$$= \frac{1}{4}\int \frac{1}{\left(\frac{u}{2}\right)^2+1}\,du = \left\{\begin{array}{l} v = \frac{u}{2} \\ dv = \frac{1}{2}du \end{array}\right\} = \frac{2}{4}\int \frac{1}{v^2+1}\,dv$$

$$= \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + C$$

$$\boxed{\frac{1}{2}\ln|x^2+2x+5| - \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + C}$$

Problem 2 Calculate $\int_1^\infty \frac{1}{x^2+x}\,dx$. It may help to recall $\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$ for when calculating the limit.

$$\int_1^\infty \frac{1}{x^2+x}\,dx = \lim_{m\to+\infty} \int_1^m \frac{1}{x(x+1)}\,dx$$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} \Rightarrow 1 = A(x+1) + Bx$$

$$\Rightarrow \left\{\begin{array}{l} A+B=0 \\ A=1 \end{array}\right. \Rightarrow B=-1$$

$$\Rightarrow \frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$$

$$\Rightarrow \lim_{m\to+\infty} \int_1^m \frac{1}{x} - \frac{1}{x+1}\,dx = \lim_{m\to\infty} \ln|x| - \ln|x+1| \Big|_1^m$$

$$= \lim_{m\to\infty} \ln m - \ln(m+1) - \ln 1 + \ln(2)$$

$$= \lim_{m\to+\infty} \underbrace{\ln\left(\frac{m}{m+1}\right)}_{\to 0} + \ln(2) = \boxed{\ln(2)}$$