
Worksheet 12

Little Oh and Taylor Series Manipulations

Problem 1. Show that $\sqrt{1+x^2} + \sqrt{1-x^2} = 2 + o(x^3)$.

Problem 2. For which k is it true that $\sqrt{1+3x^4} = 1 + o(x^k)$?

Problem 3. Calculate $T_3 \cos(x)$, and estimate the error $|\cos(x) - T_3 \cos(x)|$. Use this to show that the following is true for any x :

$$1 - \frac{x^2}{2} - \frac{x^4}{24} \leq \cos(x) \leq 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

Problem 4. Calculate $T_{11}f(x)$ for $f(x) = x^2 \sin(4x^3)$, and use this to find the derivatives $f^{(8)}(0)$, $f^{(10)}(0)$, and $f^{(11)}(0)$.

Problem 5. Calculate $T_8f(x)$ for $f(x) = (1 + x^2)e^{-x^4}$, and use this to find the derivatives $f^{(6)}(0)$, $f^{(7)}(0)$, and $f^{(8)}(0)$.

Problem 6. Calculate $T_4f(x)$ for $f(x) = \sin(3x)e^{x^2}$ without calculating any derivatives.