

**Problem 1** Calculate  $\int_0^{\pi} \cos^5(2t) dt$ .

$$\begin{aligned}
 \cos^5(2t) &= \cos^4(2t) \cos(2t) \\
 &= (1 - \sin^2(2t))^2 \cos(2t) \\
 \Rightarrow \int_0^{\pi} \cos^5(2t) dt &= \int_0^{\pi} (1 - \sin^2(2t))^2 \cos(2t) dt \\
 u &= \sin(2t) \Rightarrow du = 2 \cos(2t) dt \\
 &\Rightarrow \frac{1}{2} du = \cos(2t) dt \\
 &= \frac{1}{2} \int (1 - u^2)^2 du \\
 &= \frac{1}{2} u - \frac{1}{3} u^3 + \frac{1}{10} u^5 \\
 &= \frac{1}{2} \sin(2t) - \frac{1}{3} \sin^3(2t) + \frac{1}{10} \sin^5(2t) \Big|_0^{\pi} \\
 &= 0 - 0 + 0 = \boxed{0}
 \end{aligned}$$

**Problem 2** Calculate  $\int (1 + \sin(2t))^2 dt$ .

$$\begin{aligned}
 &= \int 1 + 2\sin(2t) + \sin^2(2t) dt \\
 &= t - \cos(2t) + \int \sin^2(2t) dt \\
 \sin^2(2t) &= \frac{1}{2} [1 - \cos(4t)] \\
 \Rightarrow \int \sin^2(2t) dt &= \frac{1}{2} \int [1 - \cos(4t)] dt \\
 &= \frac{1}{2} t - \frac{1}{8} \sin(4t) + C \\
 &= t - \cos(2t) + \frac{1}{2} t - \frac{1}{8} \sin(4t) + C \\
 &= \boxed{\frac{3}{2} t - \cos(2t) - \frac{1}{8} \sin(4t) + C}
 \end{aligned}$$