Section 3.2

Area of Circle Sectors

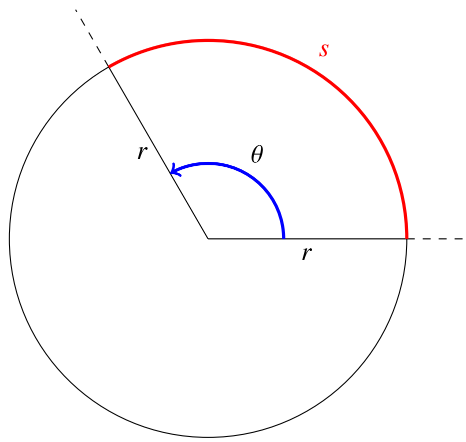
Review

We defined the radian measure as the ratio between the arc length of a circle the angle intercepts and the raidus of the circle.

Definition: Radians

The angle intersects a circle of radius r. From this we can measure the arc length s, and then calculate θ in radians using the formula

$$\theta = \frac{s}{r} \qquad \text{or} \qquad s = r\theta.$$



There are a few remarks involving the formulas describing the radian measure of an angle.

1. Most people find it easier to *think* about radians when the radius has unit length, ie: $r = 1$. In this case, we have $\theta = \frac{s}{1} = s$. That is, the angle θ in radians *is* the length of the arc length intercepted by the cirlce.

2. The arc length s and radisu r both have a dimension of length. So when we divide $\theta = \frac{s}{r}$ the dimensions cancel. This says that θ is dimensionless. So when I say θ *is* the arc length (when $r = 1$) I mean that they have the same number, but θ does not carry the dimension that s has.

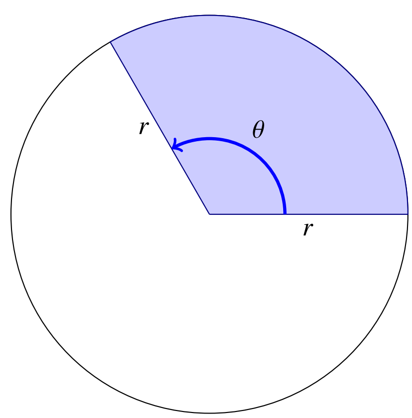
Area of Circle Sectors

Let us recall the formula to calculate the area of a circle sector given the angle in *radians*.

Formula: Area of Sector

The area of a circle sector of radius r with angle θ is given by

$$\textbf{Area} = \theta r^2$$

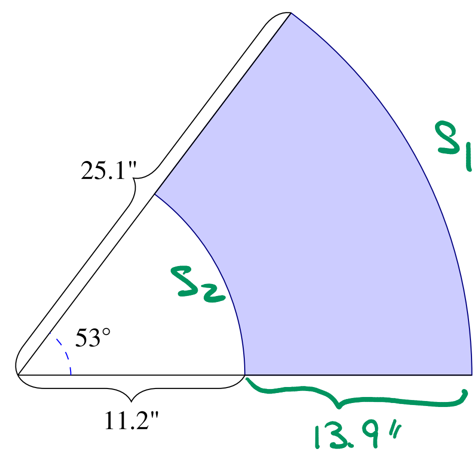


Mnemonic: Remembering the arc length and sector area formulas

Circumference of a circle $C = 2\pi r$	Area of a circle $\text{Area} = \pi r^2$
Arc length of an arc $s = \theta r$	Area of a sector $\text{Area} = \frac{1}{2}\theta r^2$

Obtain either equation by taking the familiar formulas and replace $2\pi = \theta$. ie: $\pi = \dfrac{\theta}{2}$

Problem 1. Calculate the area and perimeter of the shaded annular region below.



$53° \times \frac{\pi}{180°} = 0.925$ Rad.

$s_1 = r\,\theta = 25.1 \times 0.925$

$\approx 23.2''$

$s_2 = r\,\theta = 11.2 \times 0.925$

$\approx 10.36''$

Perimeter: $13.9'' + 13.9'' + 23.2'' + 10.36'' = \boxed{61.36''}$

Area of Large Sector:

$\text{Area} = \frac{1}{2}\theta r^2 = \frac{1}{2}\cdot 0.925 \cdot (25.1)^2$

$= 291$ sq in.

Smaller Sector:

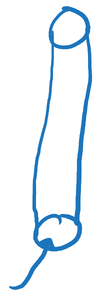
$\text{Area} = \frac{1}{2}\theta r^2 = \frac{1}{2}\, 0.925 \cdot (11.2)^2 = 58$ sq in.

Shaded Area $= 291 - 58 = \boxed{233 \text{ sq in}}$

More Applications

Problem 2. When cutting aluminum with a bandsaw, it is recommended that the blade speed be set to 750 feet per minute. At how many RPM should a bandsaw be run to cut aluminum if the wheel of the bandsaw has a 2.5 foot diameter?

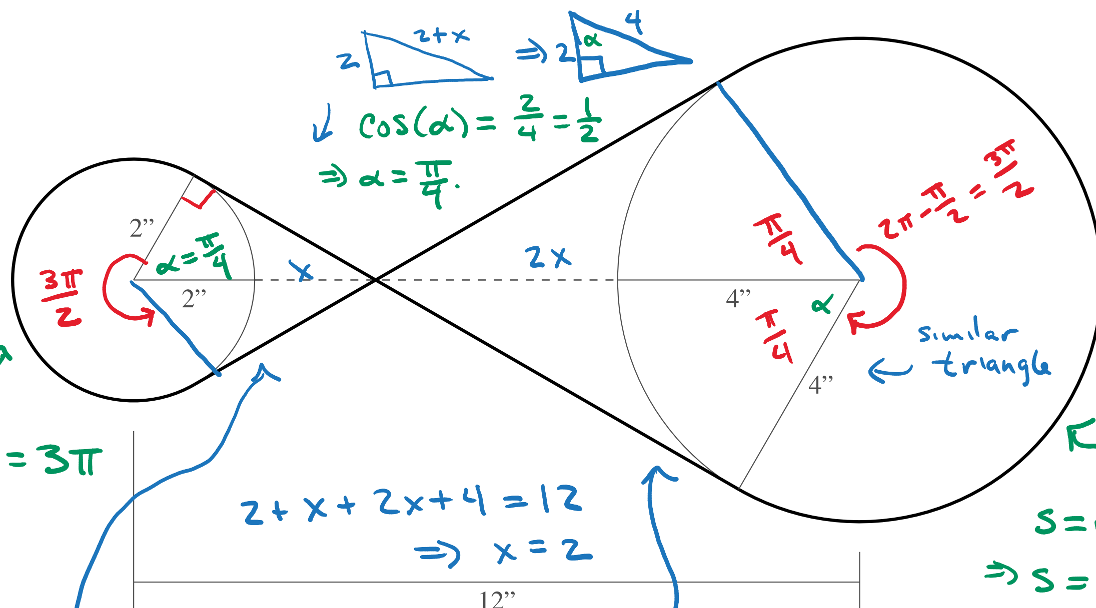
$$750\ \frac{\text{ft}}{\text{MIN}} \times \frac{1\ \text{Rot}}{2.5\pi\ \text{ft}} = \frac{750}{2.5\pi}\ \text{RPM}.$$



2.5 ft diameter

↑ Circumference $= 2.5\pi$ ft

Problem 3. Find the length of a cross belt needed to connect two circular pulleys, one with radius 2" and the other with radius 4", whose centers are separated by 12". See the figure below. First see if you can find any similar triangles.



$\cos(\alpha) = \frac{2}{4} = \frac{1}{2}$

$\Rightarrow \alpha = \frac{\pi}{4}.$

$s = 2 \cdot \frac{3\pi}{2} = 3\pi$

$2 + x + 2x + 4 = 12$

$\Rightarrow x = 2$

$s = r\theta$

$\Rightarrow s = 4 \cdot \frac{3\pi}{2} = 6\pi$

Pythagoreans:

$2^2 + x^2 = 4^2$

$\Rightarrow x^2 = 16 - 4 = 12$

$\Rightarrow x = \sqrt{12}$

similar triangle double smaller one

so: $2\sqrt{12}$

Adding everything together:

$$\underset{\substack{\uparrow \\ \text{small gear}}}{3\pi} + \underbrace{\sqrt{12} + \sqrt{12}}_{\text{small triangles}} + \underset{\substack{\uparrow \\ \text{large gear}}}{6\pi} + \underbrace{2\sqrt{12} + 2\sqrt{12}}_{\text{large triangles}} = 9\pi + 6\sqrt{12}$$

$$\boxed{= 9\pi + 12\sqrt{3}}$$