Name: Answer Key

Exam III

PRACTICE

1. Convert $140°$ to radians. Give an exact value if possible.

$$140° \times \frac{\pi}{180°} = \frac{140}{180}\pi = \frac{7}{9}\pi$$

2. Convert $-\frac{11\pi}{23}$ to degrees.

$$-\frac{11\pi}{23} \times \frac{180°}{\pi} = -\frac{11 \cdot 180\pi°}{23\pi} = \frac{1980°}{23}$$

3. Find the exact value of s in $\left[\frac{\pi}{2}, \pi\right]$ where $\tan s = -\sqrt{3}$.

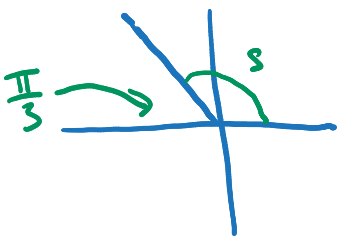
First we need to find x in quadrant I where $\tan x = \sqrt{3}$.

Reference:

	sin	cos		tan
0	0	1	⇒	0
$\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	⇒	$1/\sqrt{3}$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	⇒	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	⇒	$\sqrt{3}$
$\pi/2$				

$\tan(\pi/3) = \sqrt{3}$.

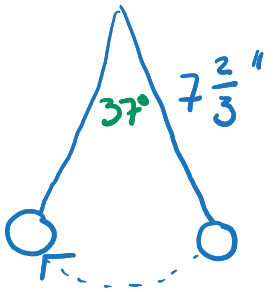
Now need s in quad II w/ Ref. angle $\pi/3$.



$s = \pi - \pi/3$

$$s = \frac{2\pi}{3}$$

4. A clock has a pendulum of $7\frac{2}{3}$ inches long. If it swings through an angle of 37°, how far does the bottom of the bob travel in one swing? Include a sketch with your answer.



arclength

must be in radians!

$$s = r\theta$$

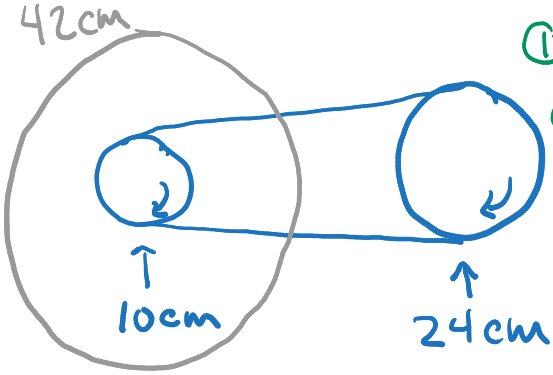
$$\theta = 37° \times \frac{\pi}{180°} = \frac{37}{180}\pi$$

$$r = 7\frac{2}{3}" = \frac{23}{3}"$$

$$s = \frac{23}{3} \cdot \frac{37}{180}\pi" = \frac{851}{540}\pi"$$

$$\Rightarrow \quad s \approx 4.95"$$

5. Janet is pedaling up a mountain trail. She is turning the front crank at a constant rate of 64 RPM. The gear on the front crank has a diameter of 24 cm while the gear on the back has a diameter of 10 cm. If her back wheel has a diameter of 42 cm, how fast is she riding in kilometers per hour? Include a sketch with your answer.



① * Crank has same linear speed as rear gear *

② * Wheel has same angular speed as rear gear *

① Find lin. speed of crank.

Crank speed → $v_c = r\omega_c$

$$\omega_c = \frac{64\ \text{Rot}}{1\ \text{MIN}} \times \frac{2\pi\ \text{Rad}}{1\ \text{Rot}} = 128\pi\ \frac{\text{RAD}}{\text{MIN}}$$

$$\Rightarrow v_c = 12 \times 128\pi\ \text{CM/MIN} = 1536\pi\ \frac{\text{CM}}{\text{MIN}}$$

② Find angular speed of rear gear:

$$v_r = v_c = 1536\pi\ \frac{\text{CM}}{\text{MIN}}$$

$$\Rightarrow 5\text{cm}\,\omega_r = 1536\pi\ \frac{\text{CM}}{\text{MIN}}$$

$$\Rightarrow \omega_r = \frac{1536\pi}{5}\ \frac{\text{Rad}}{\text{MIN}}$$

angular speed of rear gear

③ Find linear speed of wheel. same as in ②

$$v_w = 24\text{cm}\,\omega$$

$$\Rightarrow v_w = 24 \times \frac{1536\pi}{5}\ \frac{\text{CM}}{\text{MIN}} \leftarrow 23{,}162\ \frac{\text{CM}}{\text{MIN}}$$

$$v_w = 23162\ \frac{\text{CM}}{\text{MIN}} \times \frac{60\text{MIN}}{1\text{hr}} \times \frac{1\text{km}}{100{,}000\text{cm}} \approx \boxed{13.9\ \frac{\text{KM}}{\text{hr}}}$$

6. Sketch the function $y = 2 - \frac{1}{2}\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$ over *two* periods.

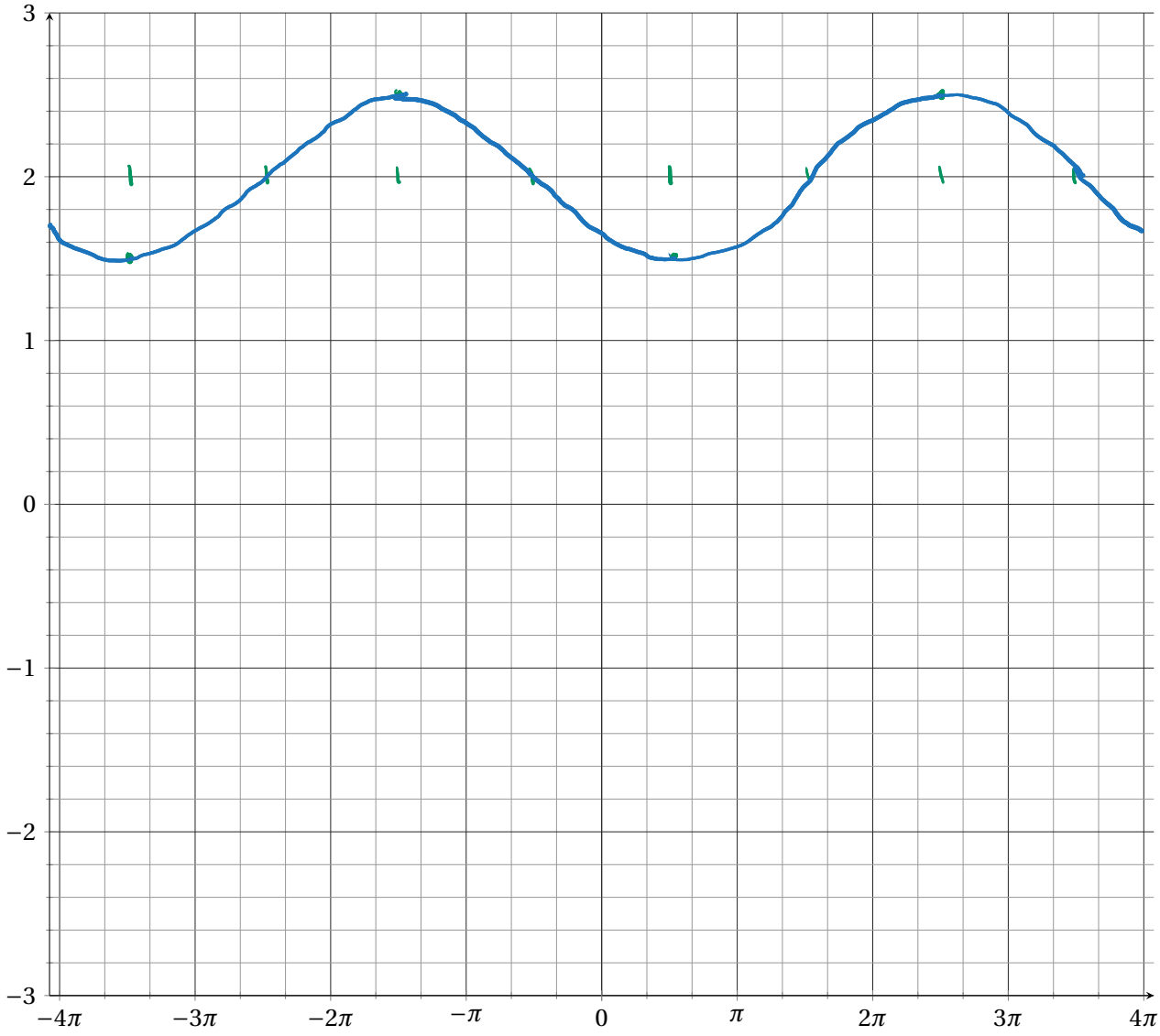
$\cos\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$

Amplitude $= \left|-\frac{1}{2}\right| = \frac{1}{2}$

Period $= \frac{2\pi}{1/2} = 4\pi$

Phase Shift $= -\frac{\pi}{2}$ ← so $\pi/2$ units right

Vertical Shift $= 2$.



7. A weight is attached to a coiled spring. It is pulled down a distance of 2 inches and released. The time for the weight to complete one oscillation is 2.5 seconds.

Why so much whitespace?

Good grief

a) Write out the **amplitude**, **period**, and **frequency** for oscillating weight.

Amplitude = 2"

Period = 2.5 sec

Frequency = $\frac{1}{2.5}$ Hz = $\frac{2}{5}$ Hz = 0.4 Hz

b) Give an equation that models the position of the weight at time t.

$y = a\cos(bt)$

Period = $\frac{2\pi}{b} \Rightarrow 2.5 = \frac{2\pi}{b} \Rightarrow b = \frac{2\pi}{2.5}$

$\Rightarrow b = \frac{4\pi}{5}$

$\boxed{y = -2\cos\left(\frac{4\pi}{5}t\right)}$

c) Use the equation to determine the position of the weight at $t = 3$ seconds.

$y = -2\cos\left(\frac{4\pi}{5} \cdot 3\right)$

$\Rightarrow y \approx -0.618$" ← don't forget to use radians!