Section 5.6

Half-Angle Identities

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$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} \qquad \sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} \qquad \tan\frac{A}{2} = \frac{\sin A}{1+\cos A} \qquad \tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$$

Whether you will use + or − in the ±'s given above will depend on the quadrant of $\frac{A}{2}$.

Problem 1. Find the exact value of $\sin 22.5°$.

$$\sin 22.5° = \sin\frac{45°}{2} = \pm\sqrt{\frac{1-\cos 45°}{2}}$$

$$= \pm\sqrt{\frac{1-\sqrt{2}/2}{2}\cdot\frac{2}{2}}$$

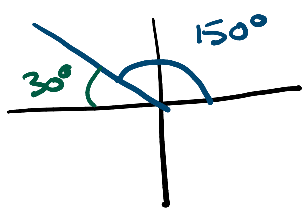
$$= \pm\sqrt{\frac{2-\sqrt{2}}{4}} = \pm\frac{\sqrt{2-\sqrt{2}}}{2}$$

22.5° in QI

So $\sin 22.5° > 0$ $\Rightarrow$ $\sin 22.5° = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

Problem 2. Find the exact value of $\tan 75°$.

$$\tan 75° = \tan\frac{150°}{2} = \frac{\sin 150°}{1+\cos 150°}$$



$$= \frac{\sin 30°}{1-\cos 30°}$$

$$= \frac{1/2}{1-\sqrt{3}/2}\times\frac{2}{2}$$

$$= \frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{2+\sqrt{3}}{4-3} = \boxed{2+\sqrt{3}}$$

Problem 3. Simplify the following expressions.

(a) $\pm\sqrt{\dfrac{1-\cos 8x}{2}}$

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}} \qquad A = 8x \text{ here.}$$

$$\sin\frac{8x}{2} = \pm\sqrt{\frac{1-\cos 8x}{2}}$$

$$\Rightarrow \pm\sqrt{\frac{1-\cos 8x}{2}} = \sin 4x.$$

(b) $\pm\sqrt{\dfrac{1-\cos 9\alpha}{1+\cos 9\alpha}}$

$$\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} \qquad A = 9\alpha \text{ here}$$

$$\tan\frac{9\alpha}{2} = \pm\sqrt{\frac{1-\cos 9\alpha}{1+\cos 9\alpha}}$$

Problem 4. Verify that the following equation is an identity: $\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2 = 1+\sin x$

$$\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2 = \underbrace{\sin^2\frac{x}{2} + \cos^2\frac{x}{2}}_{=1} + 2\cos\frac{x}{2}\sin\frac{x}{2}$$

$$= 1 + 2\cos\frac{x}{2}\sin\frac{x}{2}$$

$\sin(2A) = 2\cos A\sin A$, $A = x/2$ here.

$$= 1+\sin\left(2\cdot\frac{x}{2}\right)$$

$$= 1+\sin(x)$$

Problem 5. Verify that the following equation is an identity: $\tan^2\frac{x}{2} = \frac{\sec x+\cos x-2}{\sec x-\cos x}$

Three formulas for $\tan\frac{A}{2}$. Try one and hope for best:

$$\tan^2\frac{x}{2} = \frac{\sec x+\cos x-2}{\sec x-\cos x}$$

half angle formula →

$$\left(\frac{1-\cos x}{\sin x}\right)^2 = \frac{\frac{1}{\cos x}+\cos x-2}{\frac{1}{\cos x}-\cos x}$$

sin ? cos strategy.

$$\frac{1-2\cos x+\cos^2 x}{\sin^2 x} =$$

$$\frac{\frac{1}{\cos(x)}}{\frac{1}{\cos(x)}} \cdot \frac{1-2\cos x+\cos^2 x}{1-\cos^2 x} =$$

$$\frac{\frac{1}{\cos(x)}-2+\cos x}{\frac{1}{\cos(x)}-\cos(x)} = \frac{\frac{1}{\cos x}-2+\cos x}{\frac{1}{\cos x}-\cos x}$$ ✓