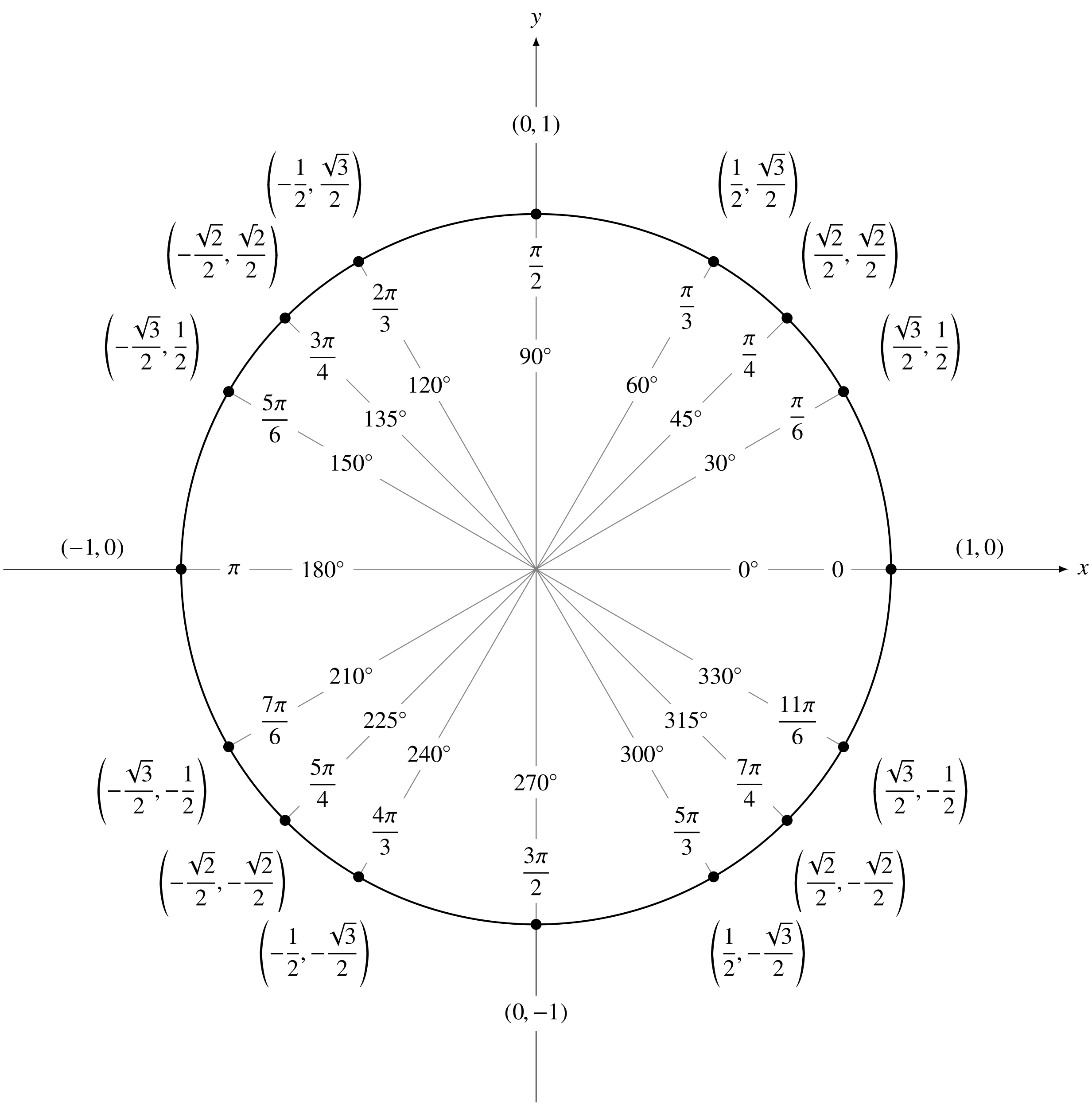
Section 4.3

The Graphs of the Other Trig Functions

You should be able to sketch all of the trigonometric functions quickly, including their period and locations of extrema and zeros, and sketch linear transformations of these functions. You will find the following diagram helpful when sketching their graphs.



Review

When graphing functions of the form:

$$y = a\sin(bx + c) \qquad \text{or} \qquad y = a\cos(bx + c)$$

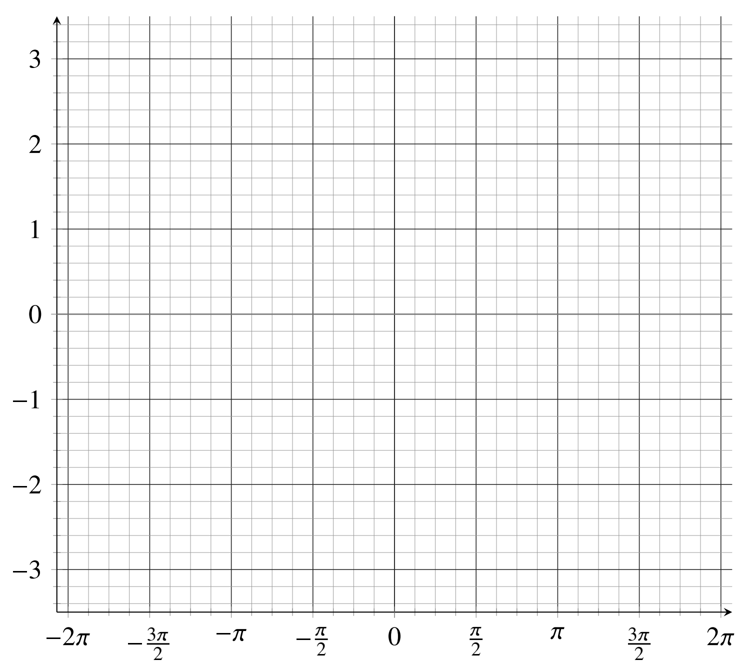
we took note of the following properties:

- **Amplitude**: In this case it is $|a|$.
- **Period**: In this case it is $\dfrac{2\pi}{b}$.
- **Phase Shift**: We took the argument and factored out the b: $bx + c = b\left(x + \frac{c}{b}\right)$. Then conclude the phase-shift is $\frac{c}{b}$.

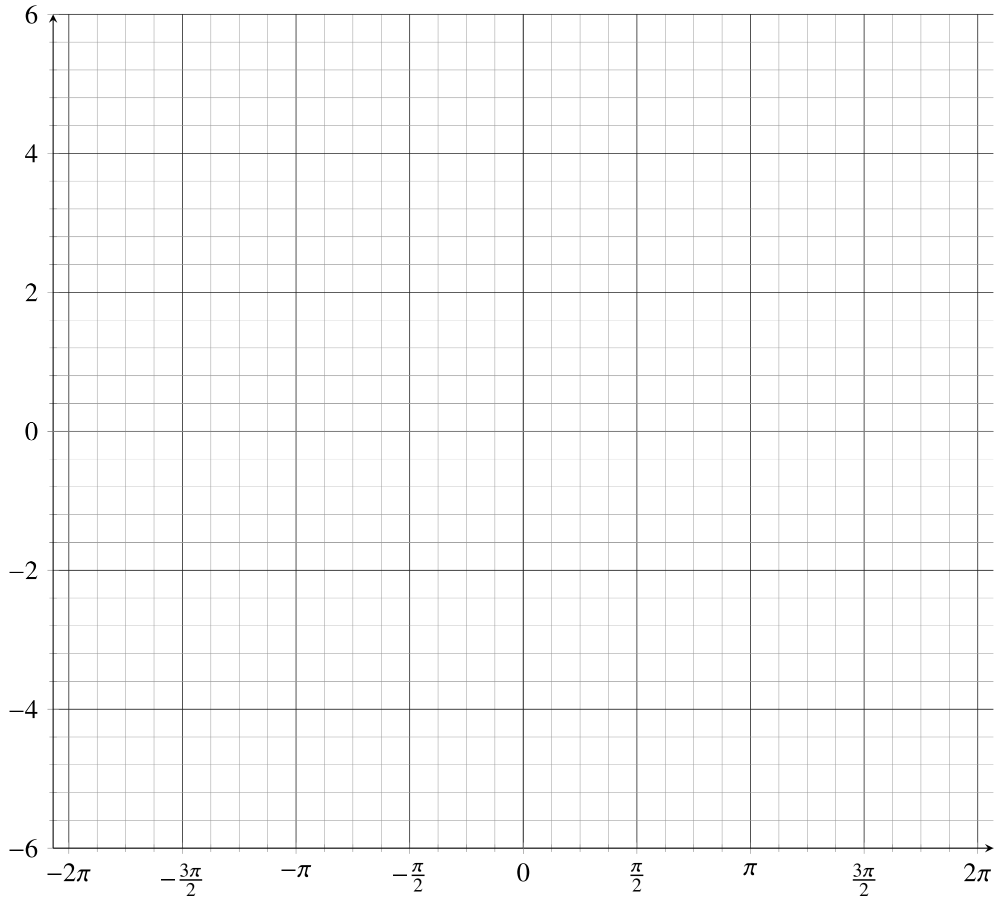
After we found these properties we were able to sketch out our trigonometric function. We generally followed the following steps:

1. Calculate how far you need to shift your graph by looking at the *phase shift*. It may be best to figure out how many divisions you need to more left or right given your phase shift.
2. Mark two points that will have a distance of your given *period*.
3. Divide your period into four equal parts. These will be where our anchor points are. They are points where calculating our trigonometric function should be easy.
4. Plot your anchor points at these four points, using the amplitude to help us figure out how tall the graph will be.

Problem 1. Graph $y = 3\sin\left(2x + \frac{2\pi}{3}\right)$.

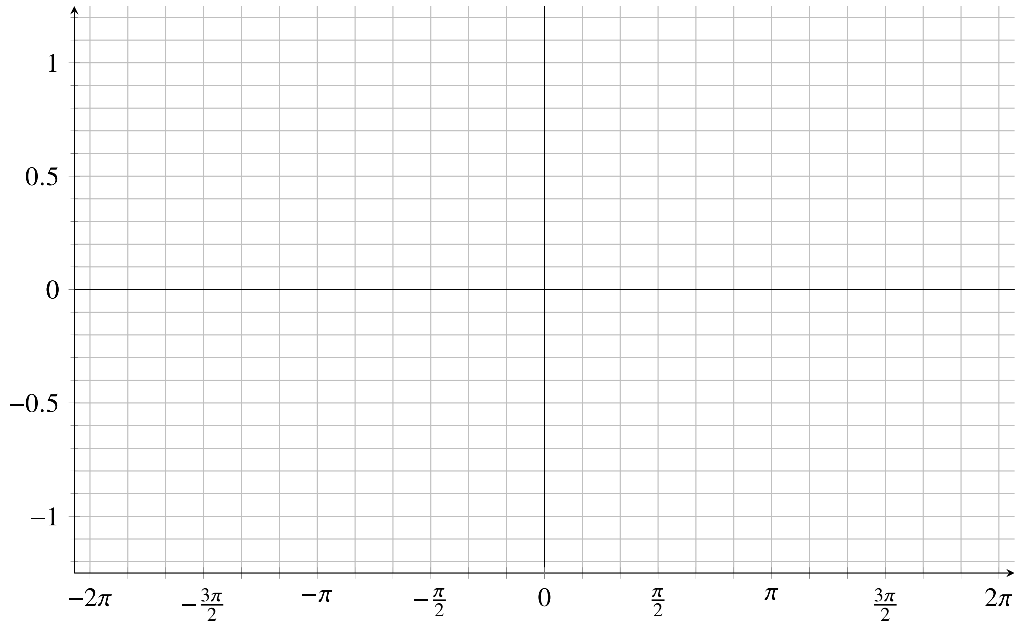


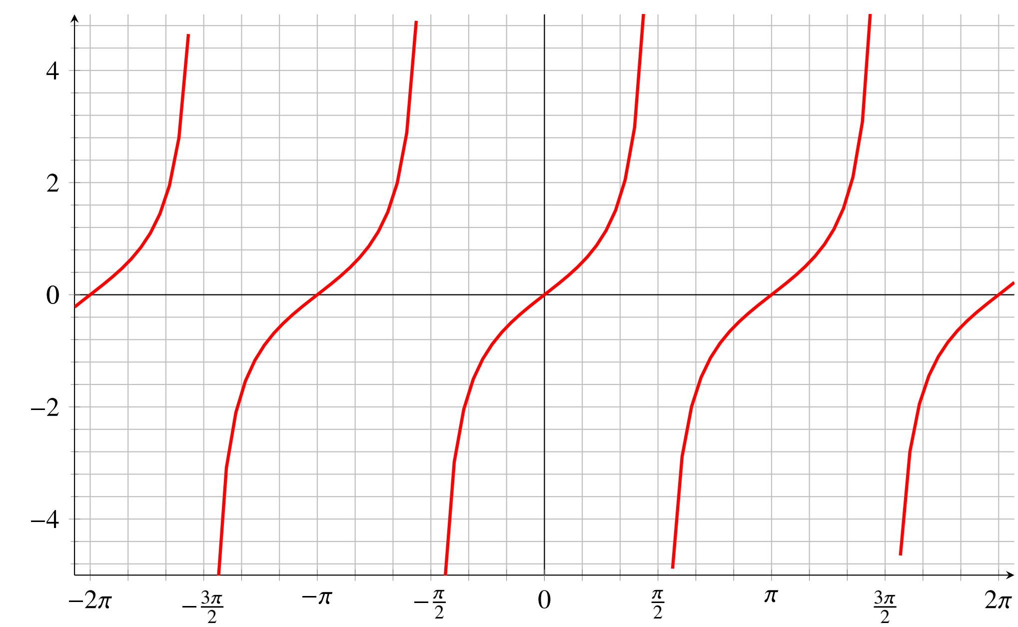
Problem 2. Graph $y = -2\cos\left(3x + \frac{\pi}{2}\right) + 1$.



Graph of the Tangent Function

x	$y = \tan x$
0	0
$\pi/6$	$\sqrt{3}/3$
$\pi/4$	1
$\pi/3$	$\sqrt{3}$
$\pi/2$	Und
$2\pi/3$	$-\sqrt{3}$
$3\pi/4$	-1
$5\pi/6$	$-\sqrt{3}/3$
π	0
$7\pi/6$	$\sqrt{3}/3$
$5\pi/4$	1
$4\pi/3$	$\sqrt{3}$
$3\pi/2$	Und
$5\pi/3$	$-\sqrt{3}$
$7\pi/4$	-1
$11\pi/6$	$-\sqrt{3}/3$



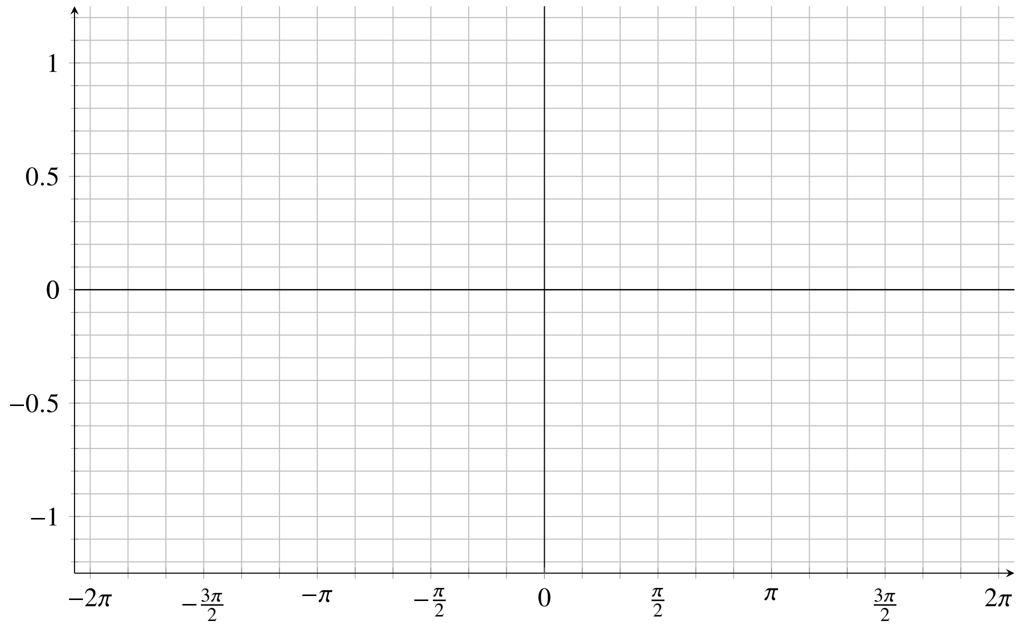


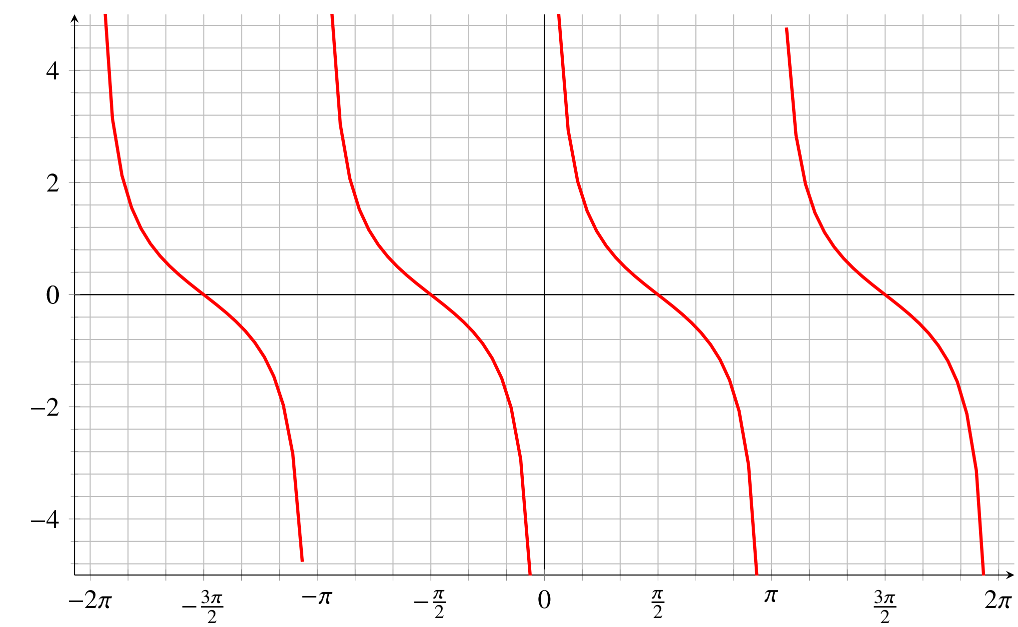
Notes:

1. The period of tan is π.
2. There are vertical asymptotes at the x-values $\dfrac{(2n+1)\pi}{2}$, where n is an integer. This is where $\cos x = 0$, and is sometimes written as $\pi/2 + n\pi$. This is because $\tan(x) = \dfrac{\sin x}{\cos x}$.
3. The x-intercepts occur at $n\pi$, where n is an integer. This is where $\sin x = 0$, since $\tan x = \dfrac{\sin x}{\cos x}$.
4. tan has no extrema since it ranges from $-\infty$ to $-\infty$, so we don't talk about the amplitude of $\tan x$.
5. Tangent is an odd function: $\tan(-x) = -\tan(x)$.

Graph of the Cotangent Function

x	$y = \cos x$
0	Und
$\pi/6$	$\sqrt{3}$
$\pi/4$	1
$\pi/3$	$\sqrt{3}/3$
$\pi/2$	0
$2\pi/3$	$-\sqrt{3}/3$
$3\pi/4$	-1
$5\pi/6$	$-\sqrt{3}$
π	Und
$7\pi/6$	$\sqrt{3}$
$5\pi/4$	1
$4\pi/3$	$\sqrt{3}/3$
$3\pi/2$	0
$5\pi/3$	$-\sqrt{3}/3$
$7\pi/4$	-1
$11\pi/6$	$-\sqrt{3}$



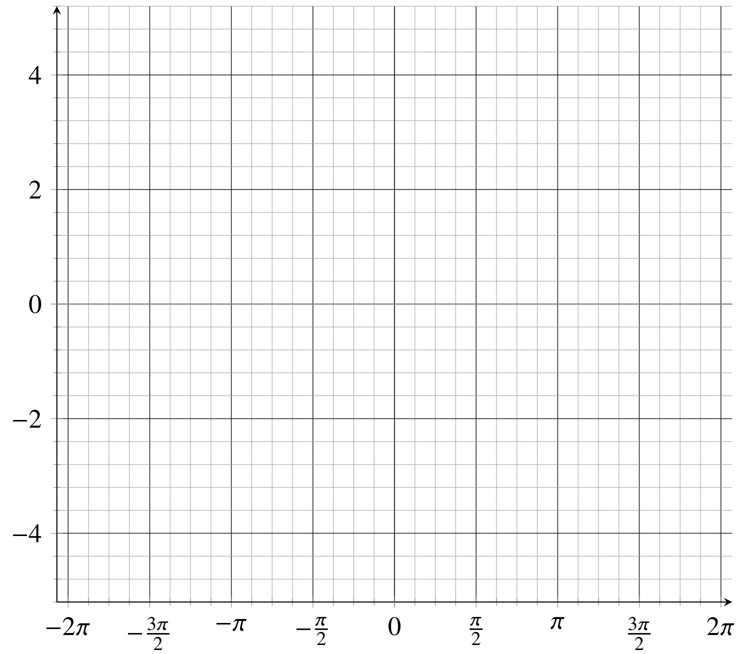


Notes:

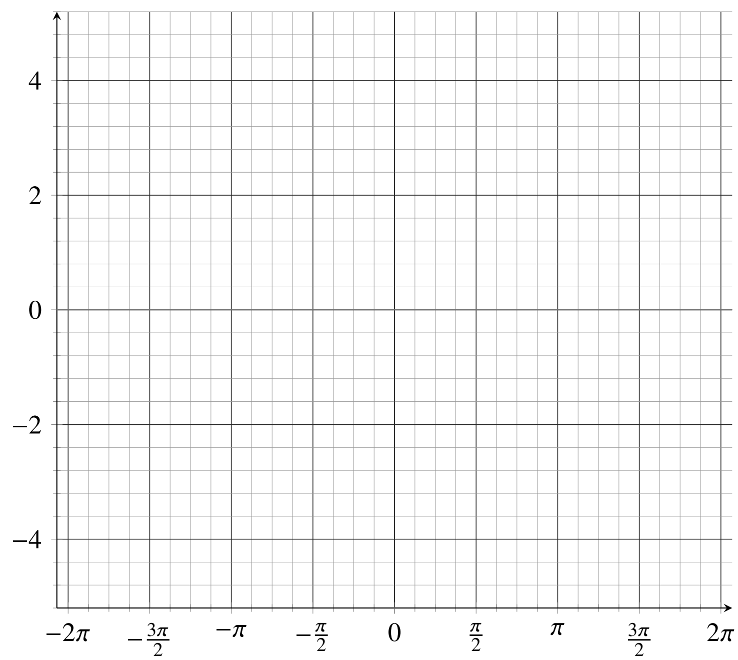
- The period of $\cot x$ is π.
- The graph has vertical asymptotes at $x = n\pi$, where n is an integer. This is where $\sin x = 0$, since $\cot x = \dfrac{\cos x}{\sin x}$.
- The x-intercepts occur at $\pi/2 + n\pi$. This is where $\cos x = 0$.
- Cotangent has no extrema since it ranges from $-\infty$ to ∞, so we don't talk about amplitudes with cotangent.
- The cotangent function is an odd function; $\cot(-x) = -\cot(x)$.

Graphing Tangent and Cotangent

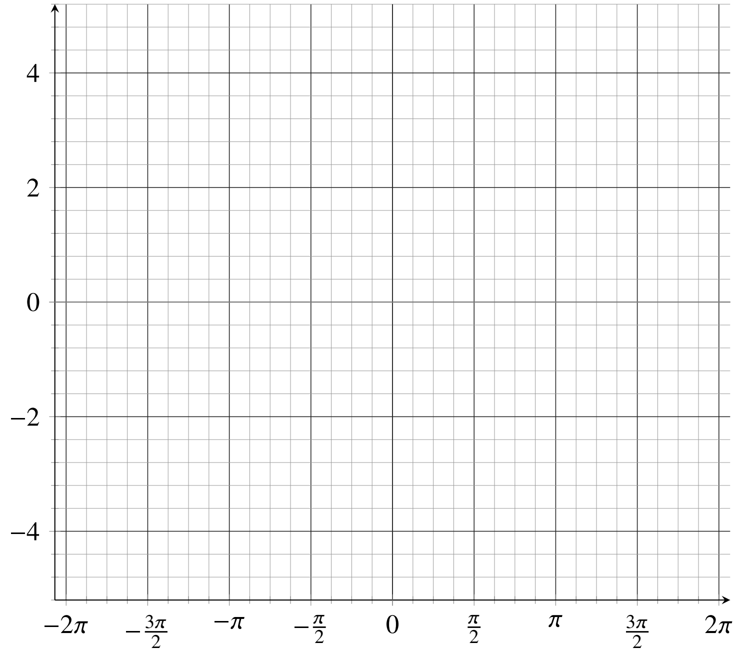
Problem 3. Graph the equation $y = \tan\left(\frac{2}{3}x\right)$.



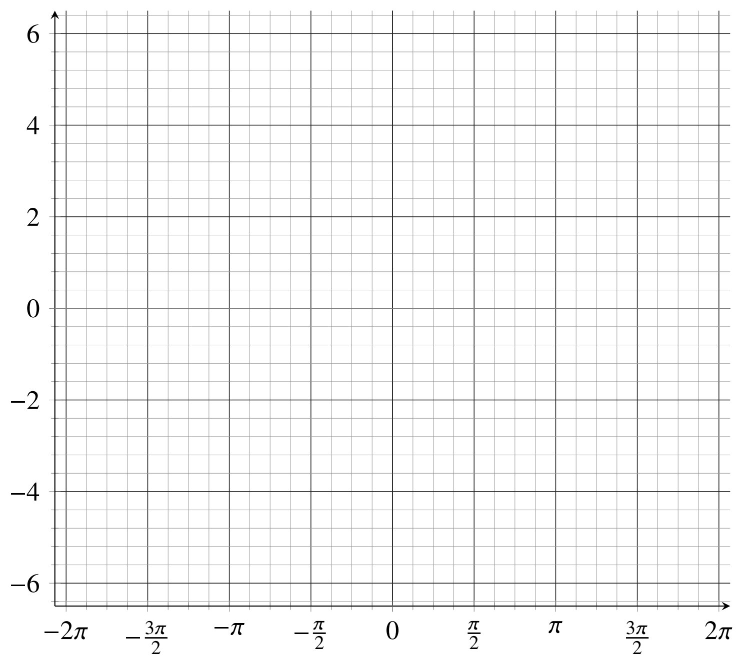
Problem 4. Graph the equation $y = -\frac{1}{2}\tan(2x)$.



Problem 5. Graph the equation $y = 3\cot\left(\frac{1}{2}x\right)$.

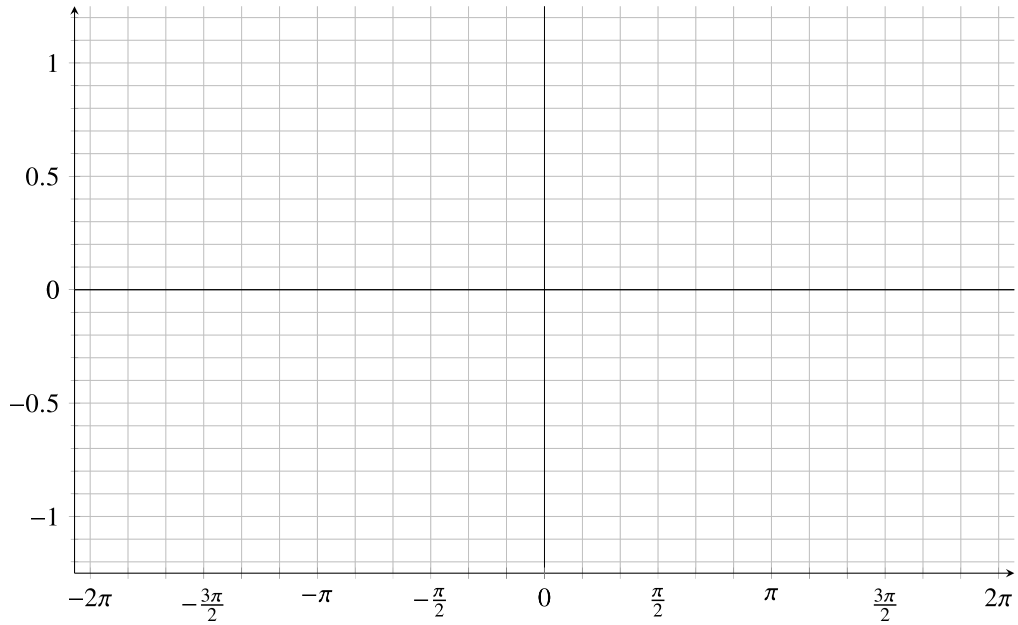


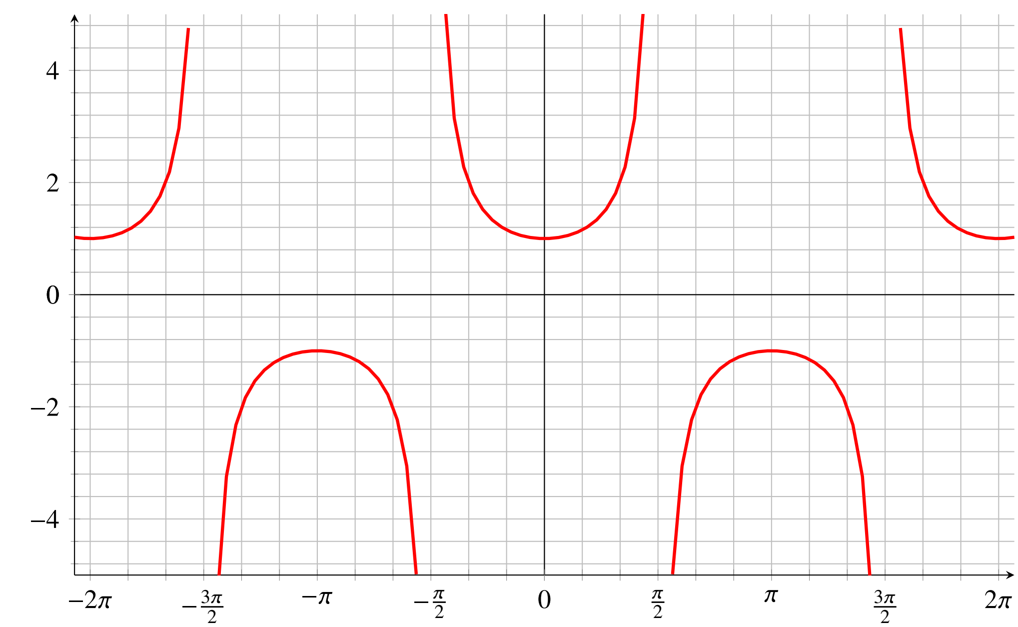
Problem 6. Graph the equation $y = 3 + \cot\left(x + \frac{\pi}{2}\right)$.



Graph of the Secant Function

x	$y = \sec x$
0	1
$\pi/6$	$2\sqrt{3}/3$
$\pi/4$	$\sqrt{2}$
$\pi/3$	2
$\pi/2$	Und
$2\pi/3$	-2
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-2\sqrt{3}/3$
π	-1
$7\pi/6$	$-2\sqrt{3}/3$
$5\pi/4$	$-\sqrt{2}$
$4\pi/3$	-2
$3\pi/2$	Und
$5\pi/3$	2
$7\pi/4$	$\sqrt{2}$
$11\pi/6$	$2\sqrt{3}/3$



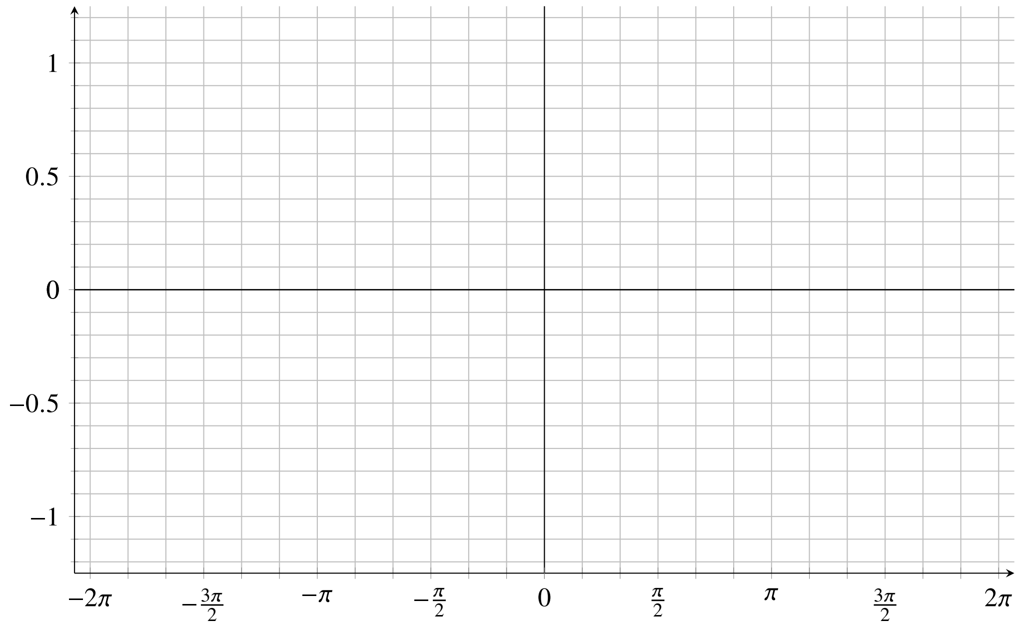


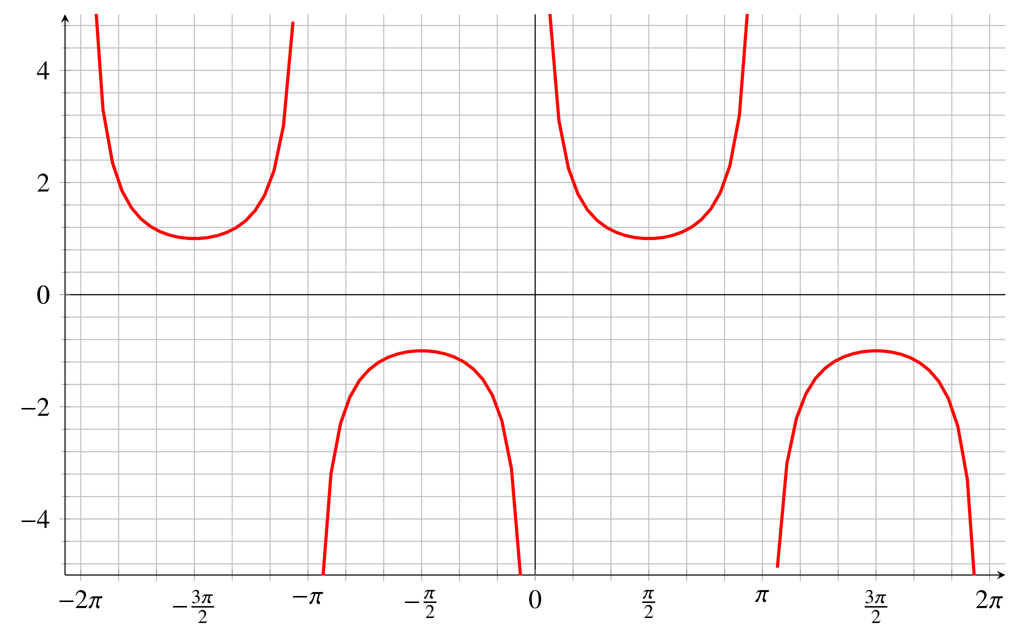
Notes:

- The period of $\sec x$ is 2π.
- The graph has vertical asymptotes at $x = \frac{\pi}{2} + n\pi$, where n is an integer. This is where $\cos x = 0$, since $\sec x = \dfrac{1}{\cos x}$.
- There are no x-intercepts. The graph never enters the $[-1, 1]$ region. The range of the graph is $(-\infty, -1] \cup [1, \infty)$.
- The secant functions is an even function; $\sec(-x) = \sec(x)$. Just like $\cos x$.

Graph of the Cosecant Function

x	$y = \csc x$
0	Und
$\pi/6$	2
$\pi/4$	$\sqrt{2}$
$\pi/3$	$2\sqrt{3}/3$
$\pi/2$	1
$2\pi/3$	$2\sqrt{3}/3$
$3\pi/4$	$\sqrt{2}$
$5\pi/6$	2
π	Und
$7\pi/6$	-2
$5\pi/4$	$-\sqrt{2}$
$4\pi/3$	$-2\sqrt{3}/3$
$3\pi/2$	-1
$5\pi/3$	$-2\sqrt{3}/3$
$7\pi/4$	$-\sqrt{2}$
$11\pi/6$	-2



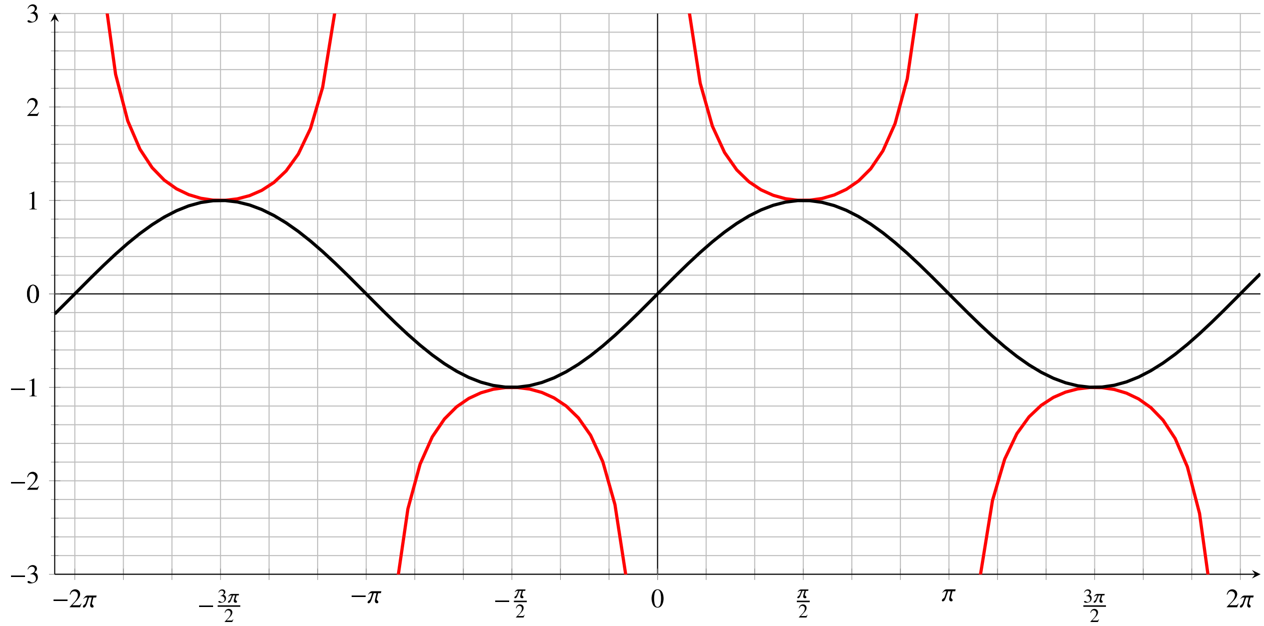


Notes:

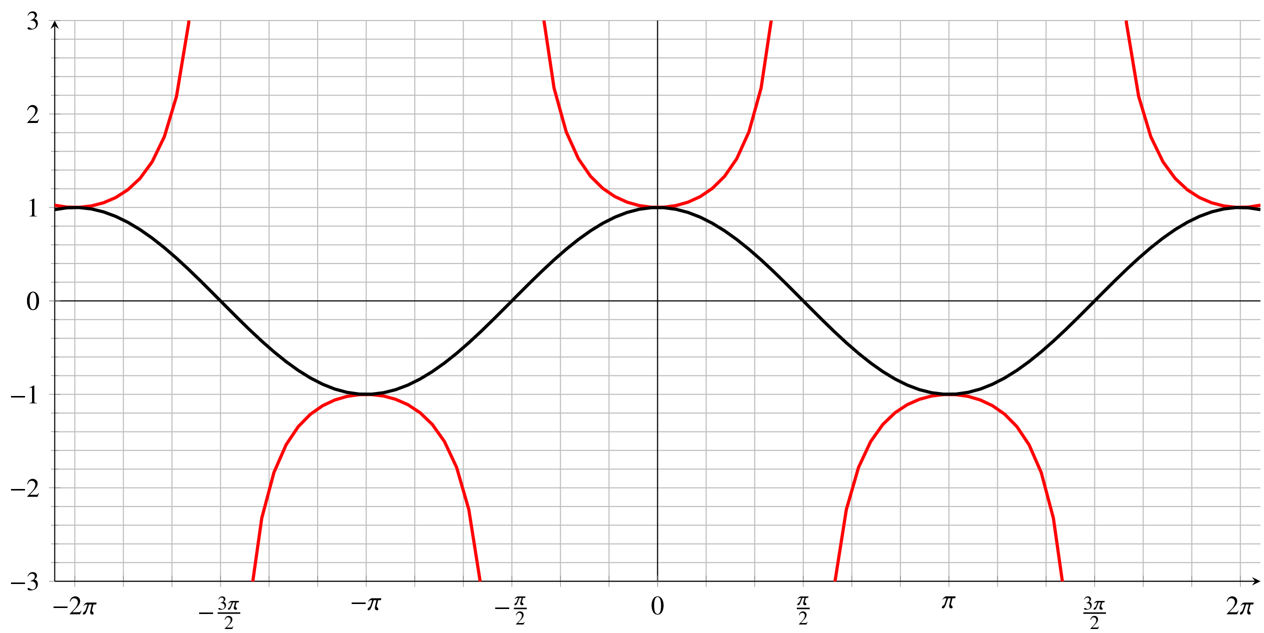
- The period of $\csc x$ is 2π.
- The graph has vertical asymptotes at $x = n\pi$, where n is an integer. This is where $\sin x = 0$, since $\csc x = \dfrac{1}{\sin x}$.
- There are no x-intercepts. The graph never enters the $[-1, 1]$ region. The range of the graph is $(-\infty, -1] \cup [1, \infty)$.
- The cosecant functions is an odd function; $\sec(-x) = -\sec(x)$. Just like $\sin x$.

Cosine vs Secant & Sine vs Cosecant

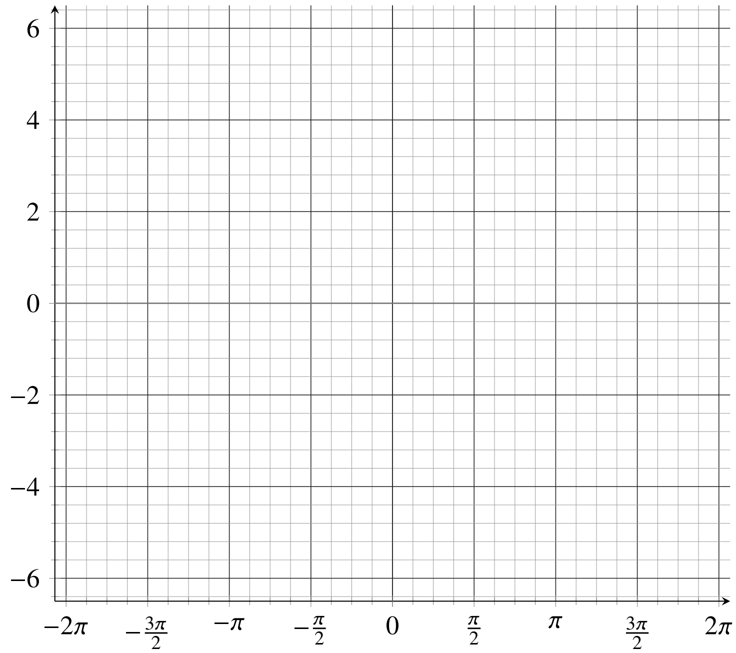
It is often helpful to think of sine when graphing cosecant. Notice that cosecant is ± 1 when sine is ± 1, and that cosecant blows up where sine is 0.



There is a similar relationship between cosine and secant. Notice that secant is ± 1 when cosine is ± 1, and that secant blows up where cosine is 0.



Problem 7. Graph the equation $y = 3\sec(2x)$.



Problem 8. Graph the equation $y = \frac{1}{2}\csc\left(x + \frac{\pi}{4}\right)$.