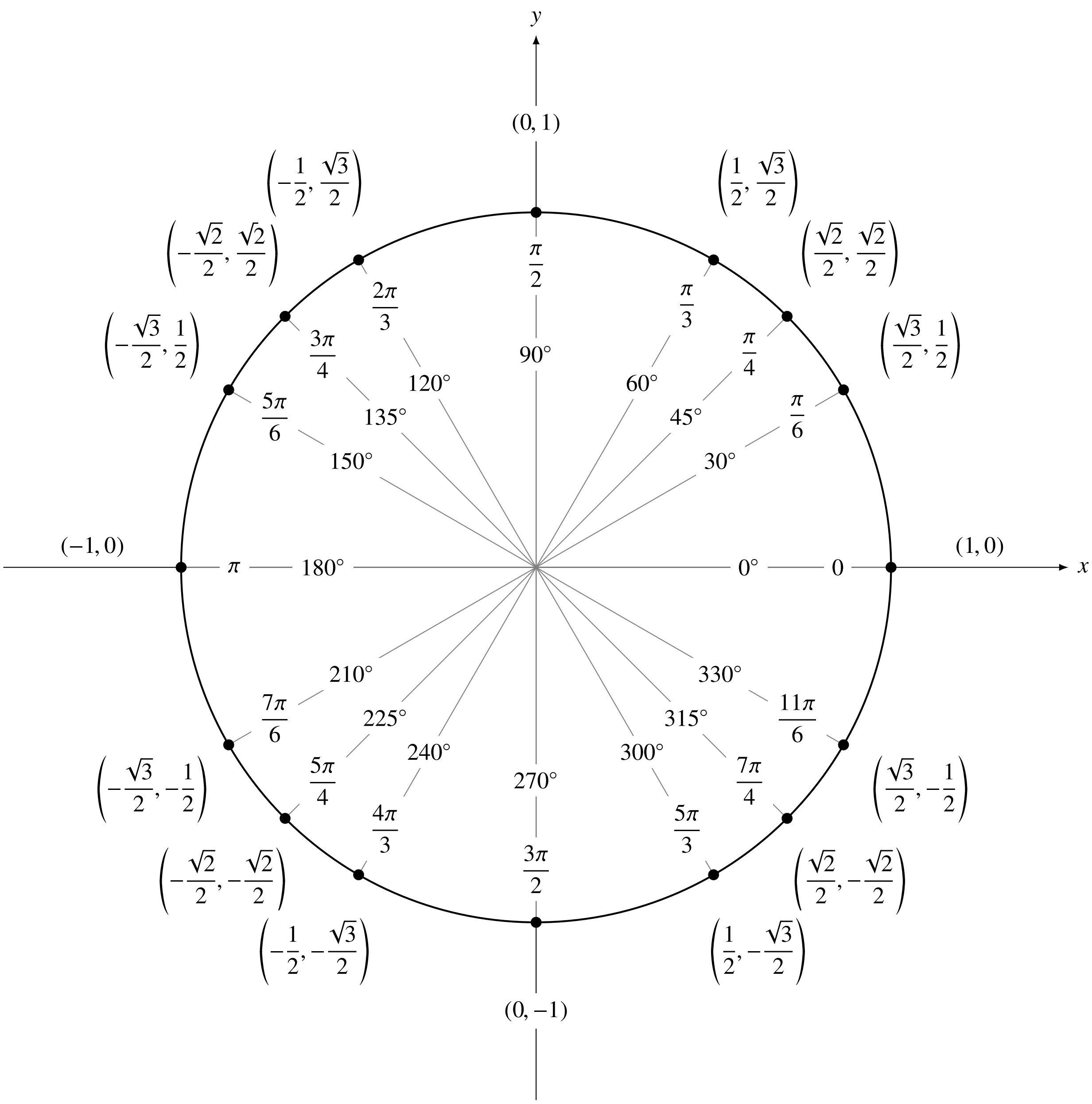
Section 4.1
The Graphs of Sine and Cosine

A **periodic function** f is a function such that

$$f(x) = f(x + np)$$

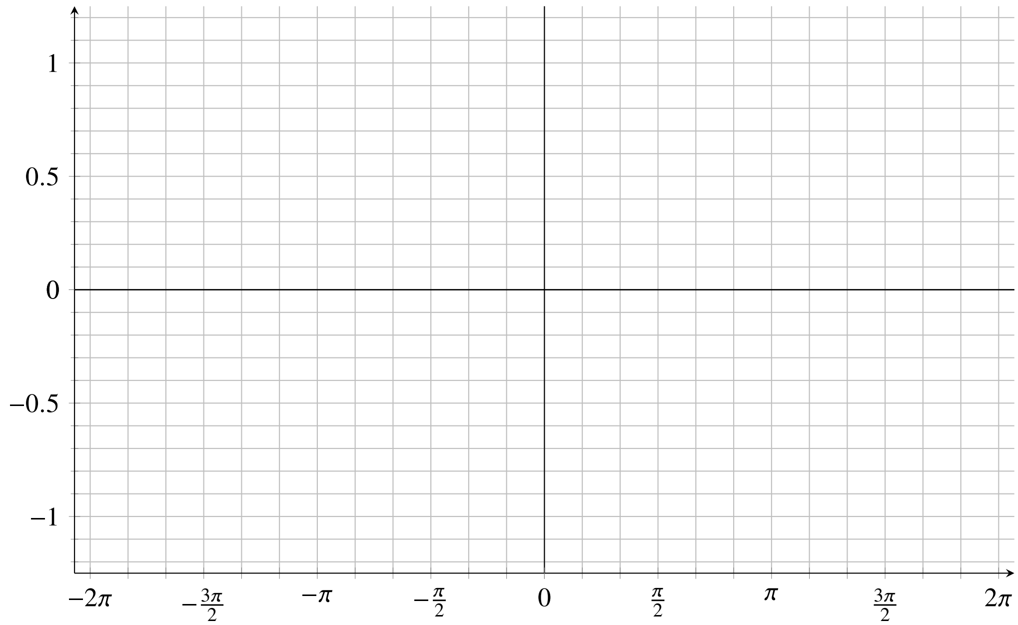
for every integer n and some positive real p. The smallest possible p is called the **period** of the function.

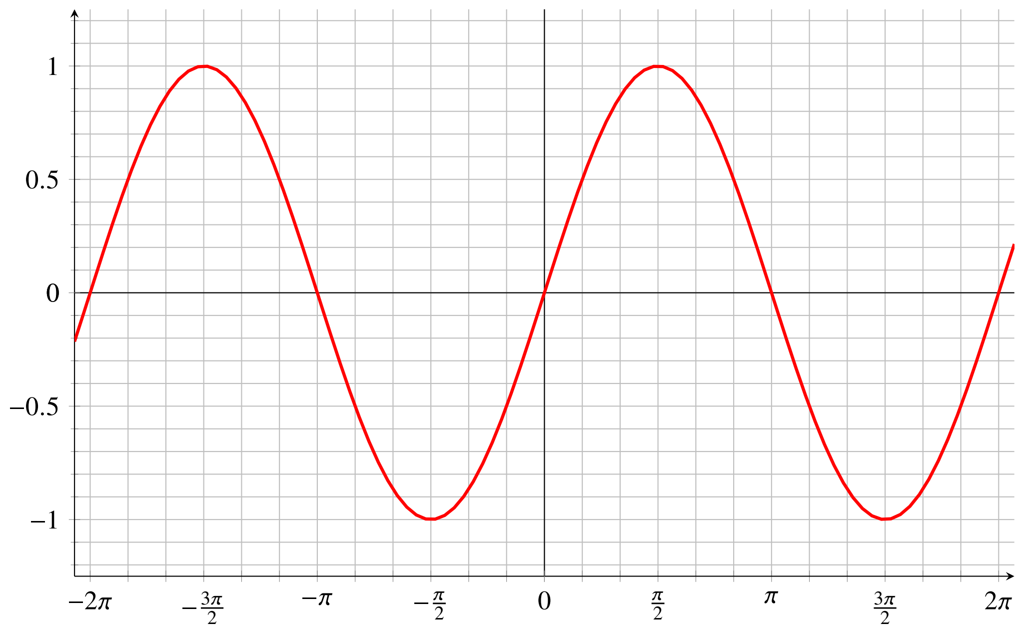
While trying to graph the trigonometric functions, we will refer to the known values shown below:



Graph of the Sine Function

x	$y = \sin x$
0	
$\pi/6$	
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	0
$7\pi/6$	$-1/2$
$5\pi/4$	$-\sqrt{2}/2$
$4\pi/3$	$-\sqrt{3}/2$
$3\pi/2$	-1
$5\pi/3$	$-\sqrt{3}/2$
$7\pi/4$	$-\sqrt{2}/2$
$11\pi/6$	$-1/2$



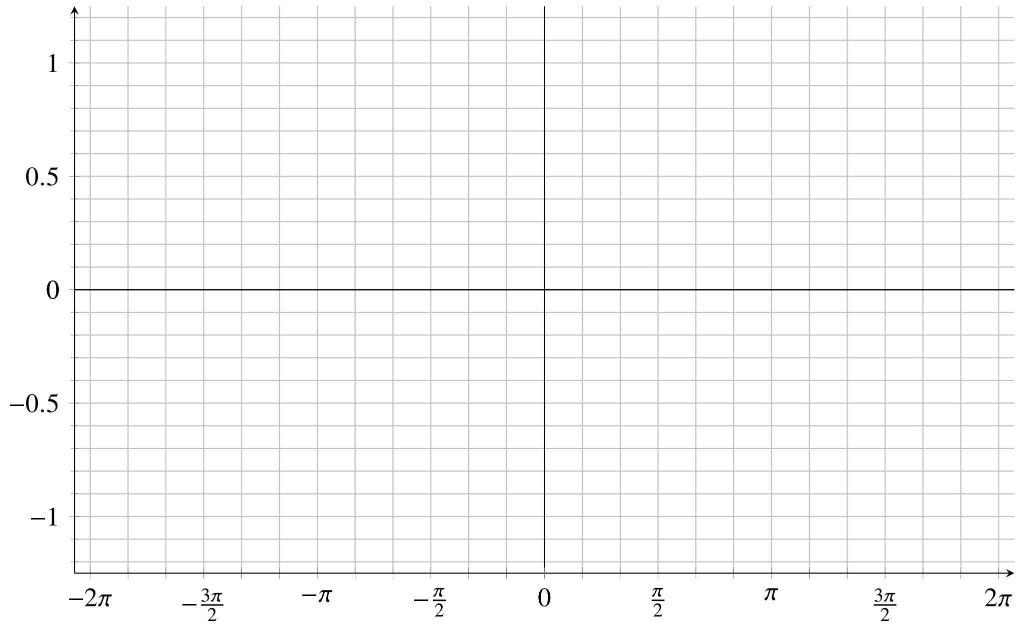


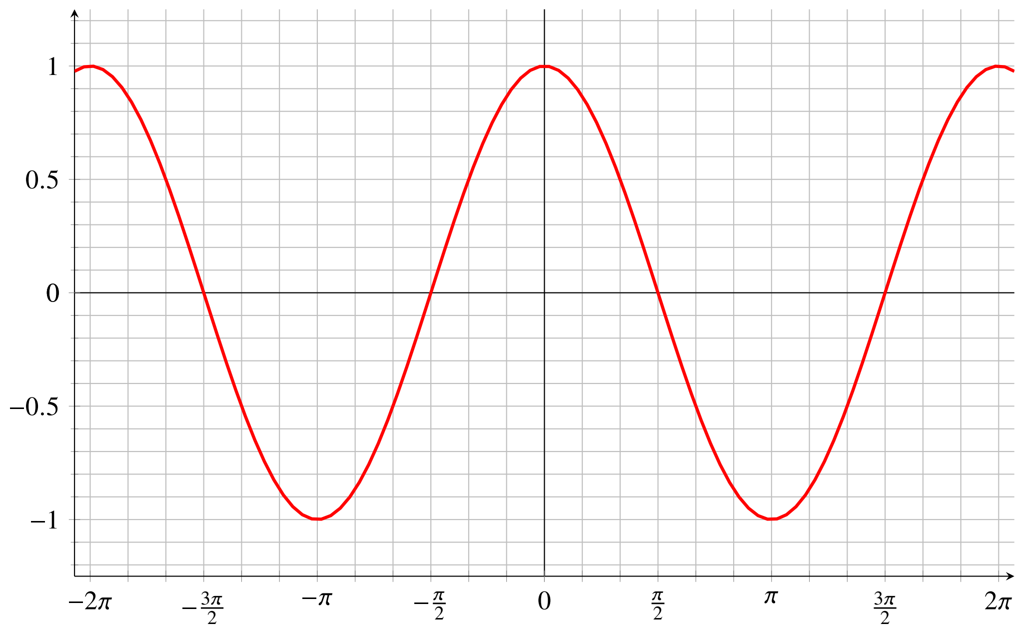
Notes:

1. The period of 2π.
2. The x-intercepts occur at $n\pi$, where n is any integer.
3. $\sin(x) = 1$ at $x = {}^{pi}/_{2} + 2n\pi$, where n is any integer.
4. $\sin(x) = -1$ at $x = {}^{-pi}/_{2} + 2n\pi$ where n is any integer.
5. The sine function is an odd function. That is: $\sin(-x) = -\sin(x)$.

Graph of the Cosine Function

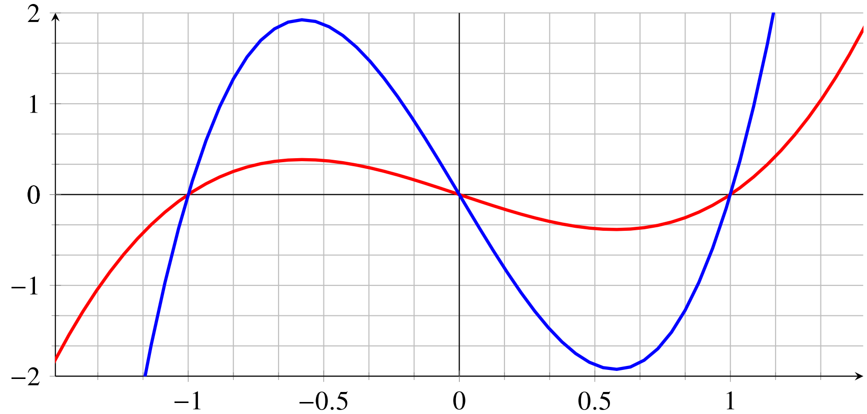
x	$y = \cos x$
0	
$\pi/6$	
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	-1
$7\pi/6$	$-\sqrt{3}/2$
$5\pi/4$	$-\sqrt{2}/2$
$4\pi/3$	$-1/2$
$3\pi/2$	0
$5\pi/3$	$1/2$
$7\pi/4$	$\sqrt{2}/2$
$11\pi/6$	$\sqrt{3}/2$





Vertically Scaled Sines and Cosines

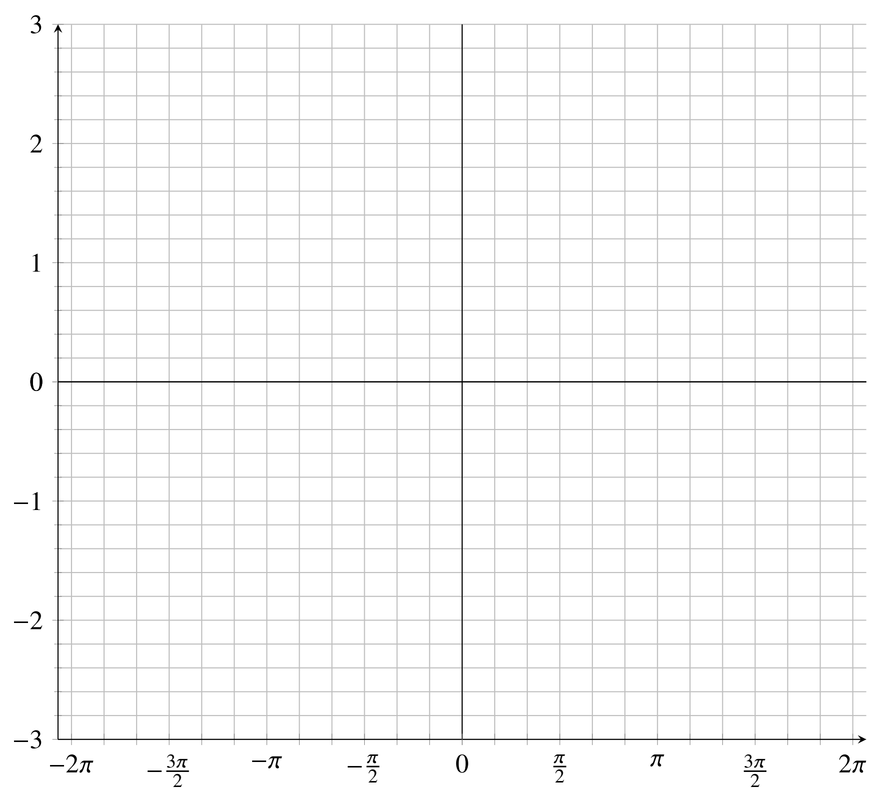
Recall that the graph of $y = a \cdot f(x)$ is the same as the graph of $y = f(x)$ that has been stretched vertically by a factor of a. See for example, the following diagram which shows two graphs, one of $y = f(x)$ where $f(x) = x^3 - x$ and another where $y = 5 \cdot f(x)$.



The graph of $y = a\sin(x)$ and $y = a\cos(x)$ with $a \neq 0$ will have the same shape as their orignal graph without the a, except with a range of $[-|a|, |a|]$. The **amplitude** of is $|a|$.

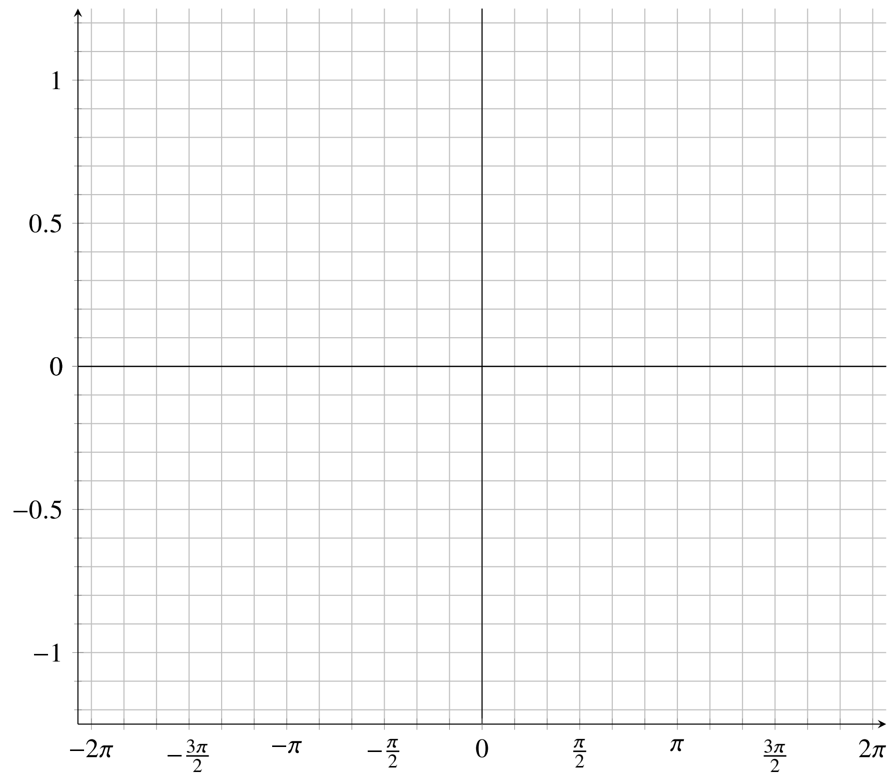
Problem 1. Graph the equation $y = 2\sin(x)$.

x	$y = 2\sin(x)$



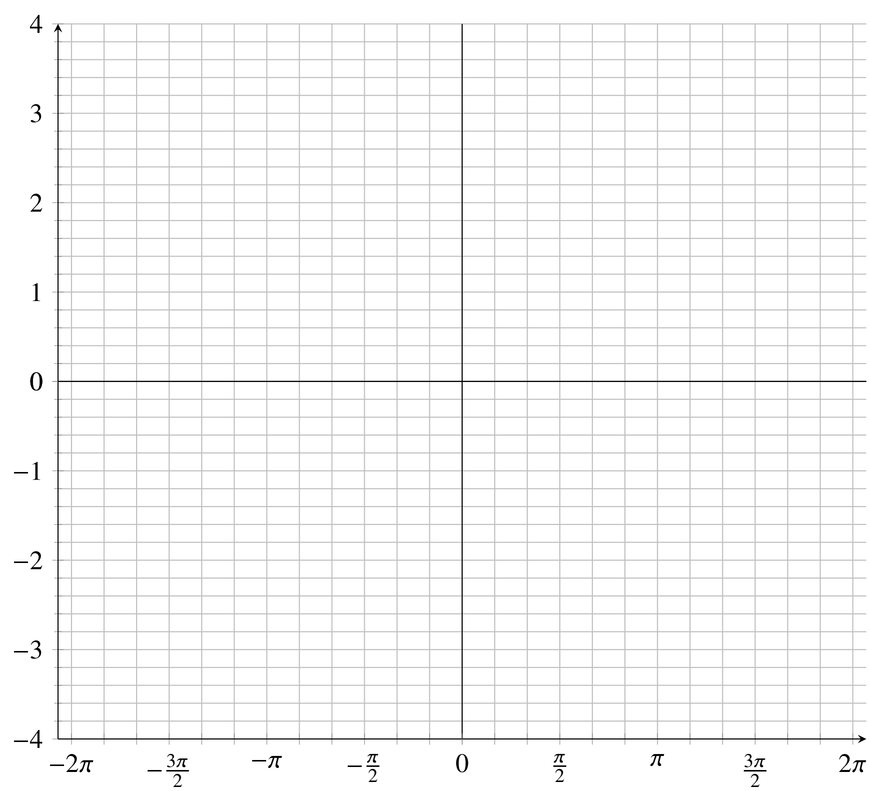
Problem 2. Graph the equation $y = -\frac{1}{2}\sin(x)$.

x	$y = -\frac{1}{2}\sin(x)$



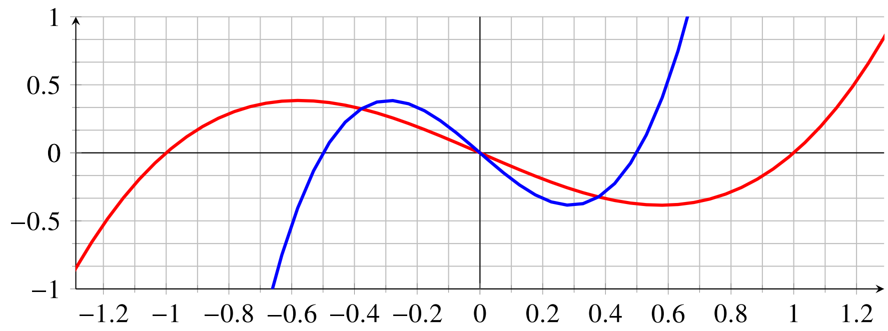
Problem 3. Graph the equation $y = 3.5\cos(x)$.

x	$y = 3.5\cos(x)$



Horizontally Scaled Sines and Cosines

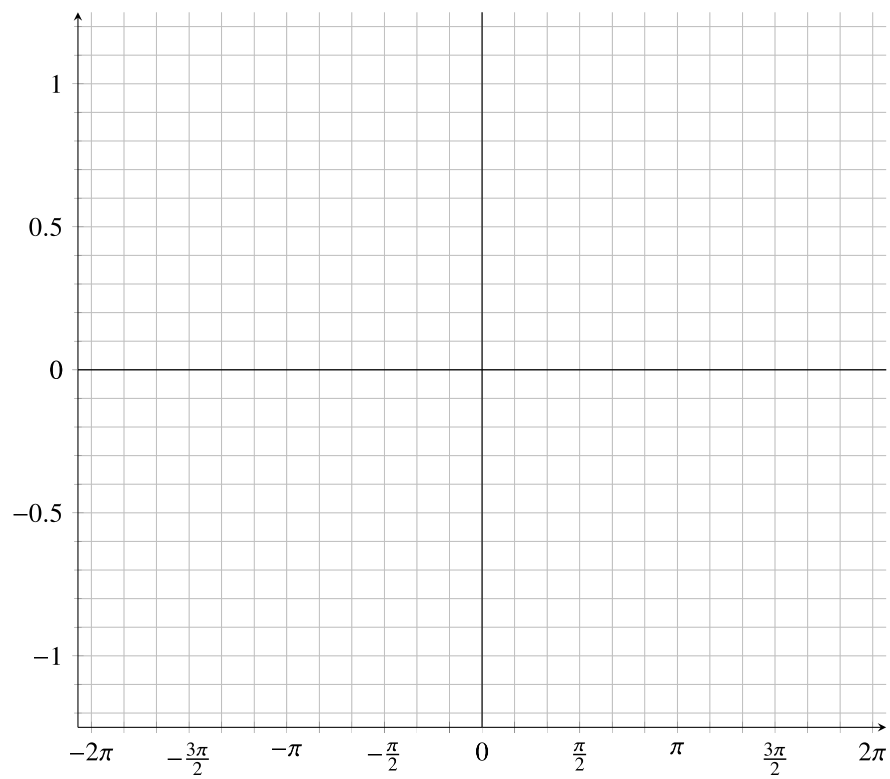
The graph of $y = f(bx)$ is the same as the graph of $y = f(x)$ that has been *compressed* (the opposite of stretched!) horizontally by a factor of b. See for example, the following figure which shows two graphs, one of $y = f(x)$ where $f(x) = x^3 - x$ and another where $y = f(2x)$.



The graph of $y = \sin(bx)$ and $y = \cos(bx)$ with $b \neq 0$ will have the same shape as the graph as their orignal, except with a new **period** of $\frac{2\pi}{b}$. Compressing by a factor of b compresses the period also by a factor of b.

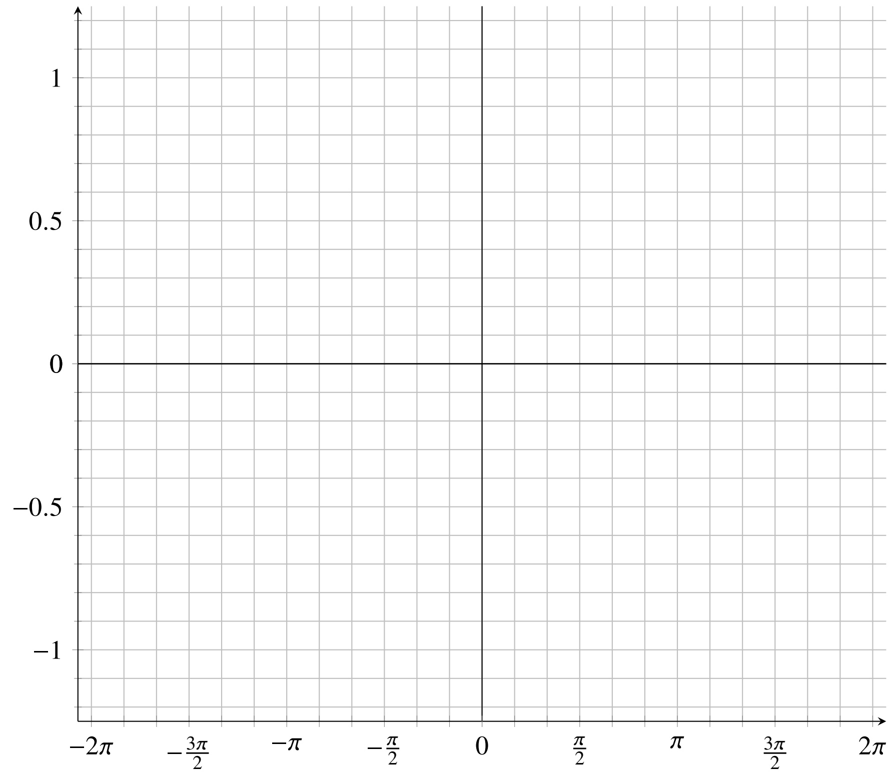
Problem 4. Graph the equation $y = \sin(2x)$.

x	$2x$	$y = \sin(2x)$



Problem 5. Graph the equation $y = \cos(3x)$.

x	$3x$	$y = \cos(3x)$

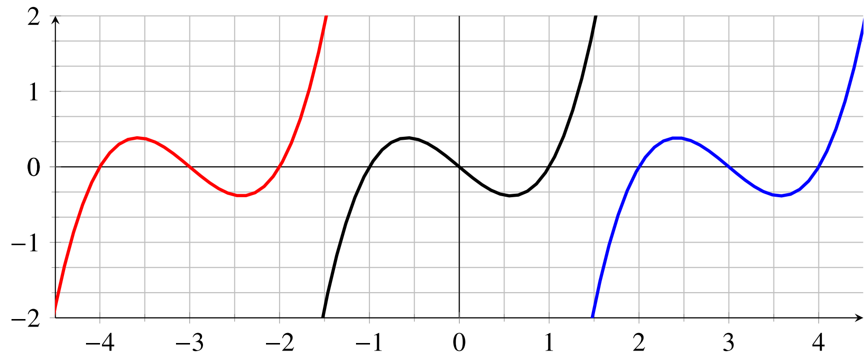


Horizontal Translations of Sine and Cosine

Recall that adding or subtracting a constant to the independent variable in a function shifts the graph left or right (depending whether you are adding or subtracting), which is called a horizontal translation.

- The graph of $y = f(x - d)$, assuming $d > 0$, shifts the graph of $y = f(x)$ to the **right** by d units.
- The graph of $y = f(x + d)$, assuming $d < 0$, shifts the graph of $y = f(x)$ to the **left** by d units.

See for example, the following figure which shows three graphs, one of $y = f(x)$ where $f(x) = x^3 - x$, another where $y = f(x - 2)$, and another where $f(x + 2)$.



The same result applies to the graphs of the trigonometric functions, with two new vocabulary words:

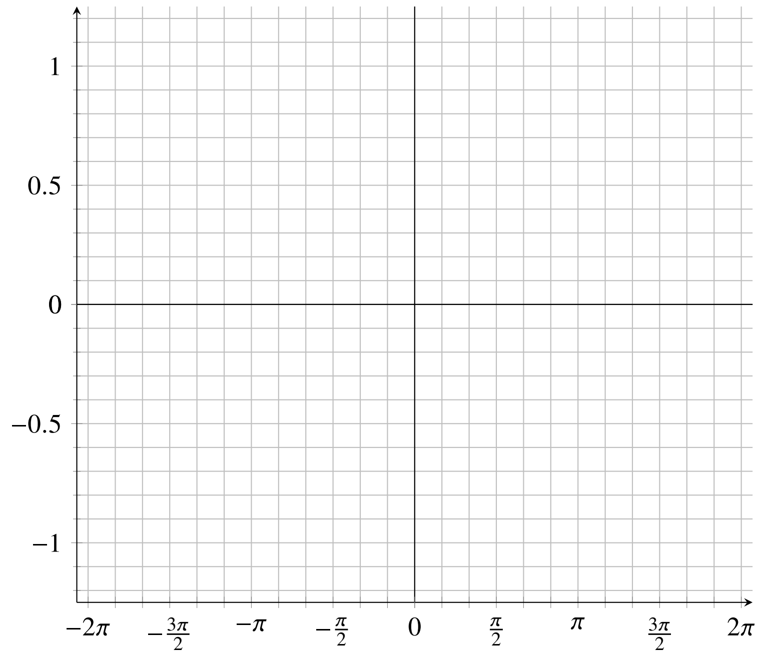
- A horinztal translation is called a **phase shift**.
- The expression $x - d$ that you plug into the trig function is called the **argument**.

To graph a horizontally translated trigonometric function, you can use one of two methods:

- Draw the trig function shifted left or right by the given phase shift,
- Make a table of values where you list the key values for the argument $x - d$, and then work backwards to find the x value that produces those key values.

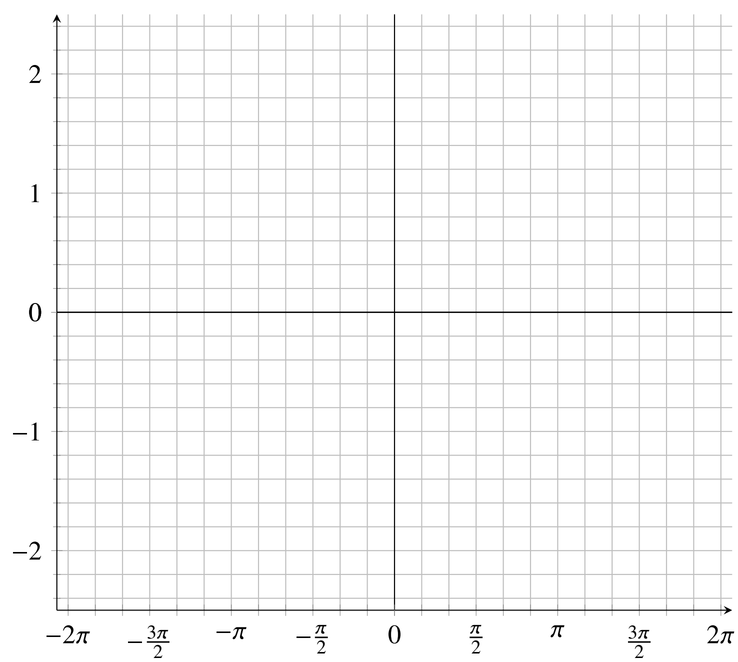
Problem 6. Graph the function $y = \sin\left(x + \frac{3\pi}{4}\right)$ over two periods.

x	$x + \frac{3\pi}{4}$	$y = \sin\left(x + \frac{3\pi}{4}\right)$



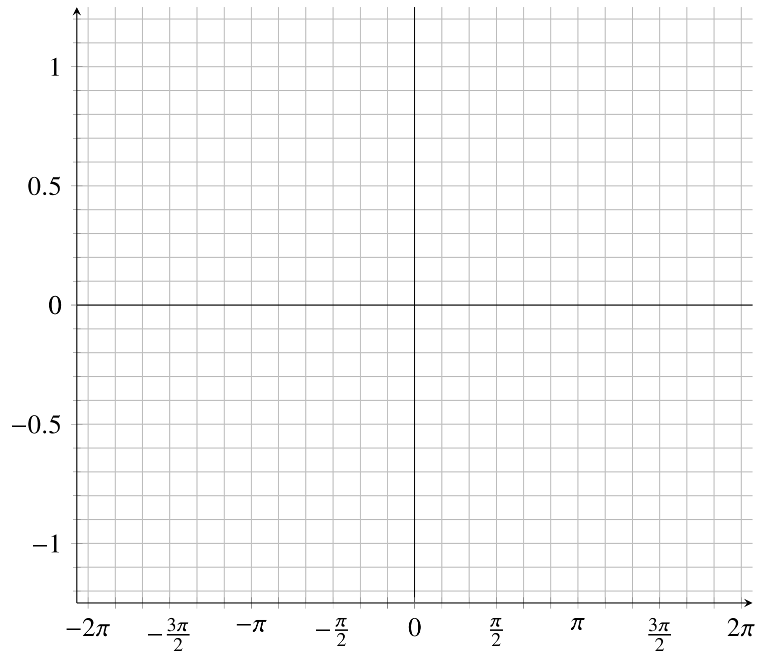
Problem 7. Graph the function $y = -2\cos\left(x - \frac{\pi}{3}\right)$.

x	$x - \frac{\pi}{3}$	$y = -2\cos\left(x - \frac{\pi}{3}\right)$



Problem 8. Graph the function $y = \frac{2}{3}\cos(2x - \pi)$. Note that you have to factor a 2 out of both terms in the argument to get the correct phase shift.

x	$2x - \pi$	$y = \frac{2}{3}\cos(2x - \pi)$

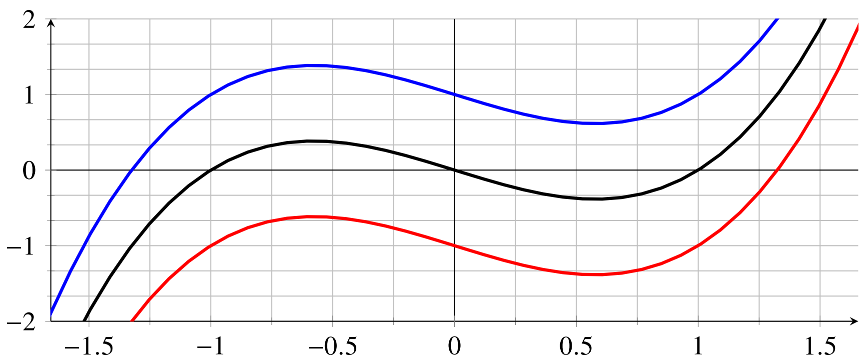


Vertical Translations of Sine and Cosine

Recall from algebra that adding or subtracting a constant to a function shifts the graph up or down, which is called a vertical translation.

- The graph of $y = f(x) + c$, assuming $c > 0$, shifts the graph of $y = f(x)$ up by c units.
- The graph of $y = f(x) - c$, assuming $c < 0$, shifts the graph of $y = f(x)$ down by c units.

See for example, the following figure which shows three graphs, one of $y = f(x)$ where $f(x) = x^3 - x$, another where $y = f(x) + 1$, and another where $f(x) - 1$.

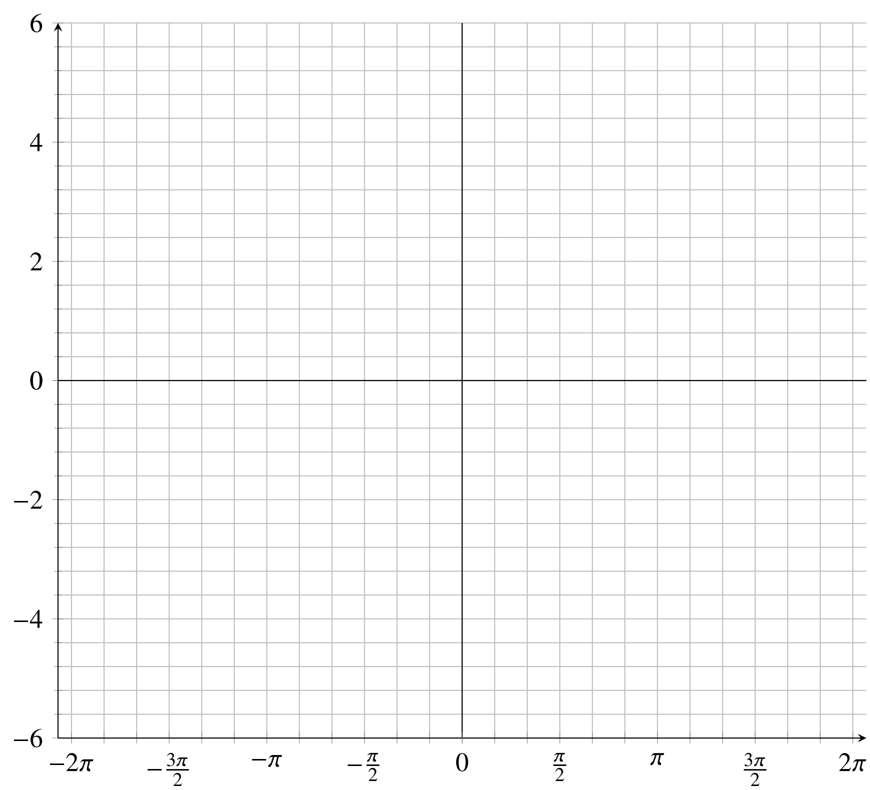


Like before you can approach this problem two different ways:

- Draw that trig function shifted up or down
- Make a table of values where you compute values for key values of the argument of the trig function.

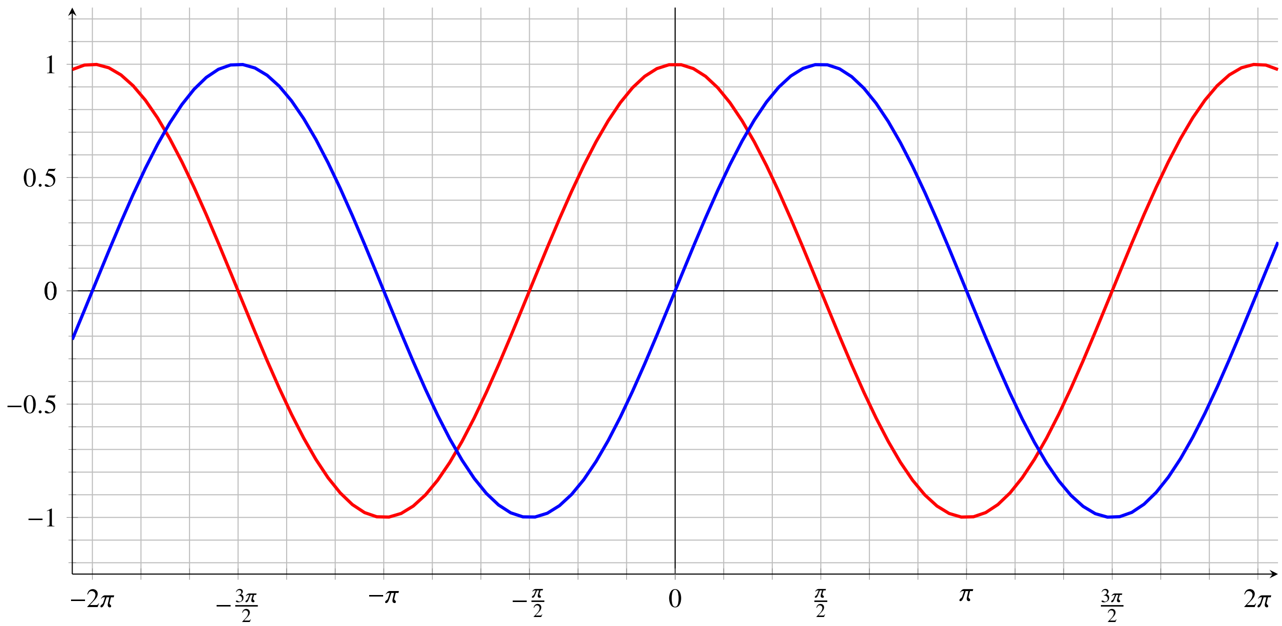
Problem 9. Graph $y = -2 + 3\cos(2x)$ over two periods.

x	$y = -2 + 3\cos(2x)$



Sine vs Cosine

Here are both trigonometric functions graphed side-by-side:



Problem 10. From the graph above, it appears that the $\sin(x)$ graph is the same as the $\cos(x)$ graph, but shifted. Can you explain this?