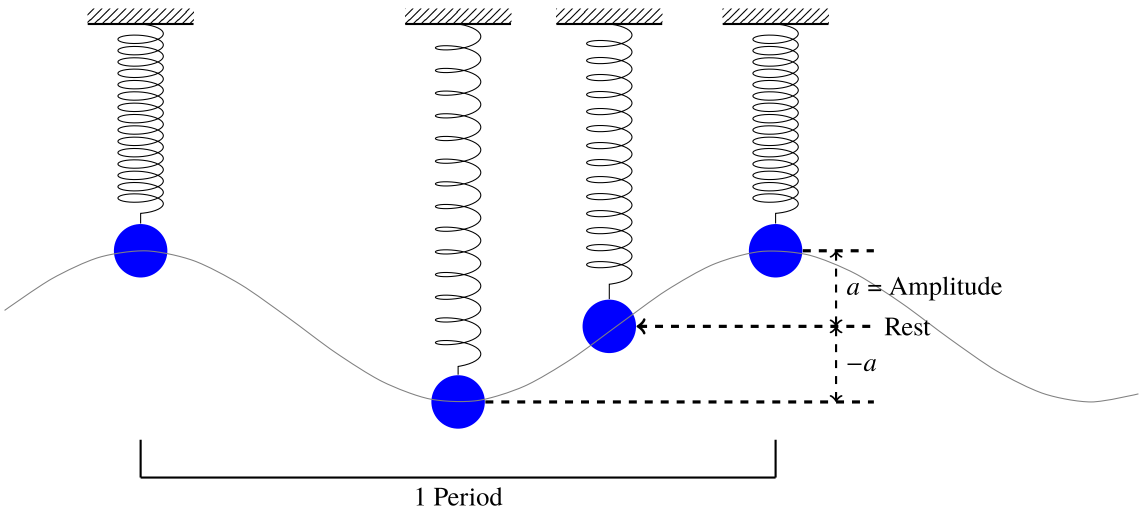
Section 4.5/5.1

Harmonic Motions and Fundamental Trigonometric Identities

Simple Harmonic Motions

Simple harmonic motion occurs when an object is moving under the influence of a force whose strength is proportional to the distance the object has moved from its rest position, and that always points toward the rest position. Mathematically, we can write this out as a formula $F = -kx$. As we increase the distance an object moves from the position of rest, x, then the greater the force, F. The negative sign says that the force of the spring will point in the opposite direction of the displacement of the mass from the point of rest.



- When the mass is not moving the spring is in a state of *rest*.
- If the mass is pulled down or pushed up from this rest position by a distance of a, it will bounce up and down after it is released.
- The amplitude of oscillatory motion of the mass bouncing up and down will be $|a|$.
- The time it takes for one full oscillation is called the **period**. The period is the time for one cycle, and is often labeled as T. T is usually measured in seconds.
- The frequency is how many complete oscillations, or cycles, occur per unit time. That is how frequent and oscillation occurs. Frequency of often labeld as f and is often measured in cycles per second. This unit of measurement, cycles per second, is called Hertz.
- The reciprocal of frequency is seconds per cycle which is the period.
- The angular frequency of the oscillation is frequently labeled ω, and is used to convert from the "regular" frequency measured in cycles per second to a measure in radians per second so that we can model the oscillation using sine or cosine as a function of time.
- The bouncing would continue forever if it weren't for friction, air drag, etc.

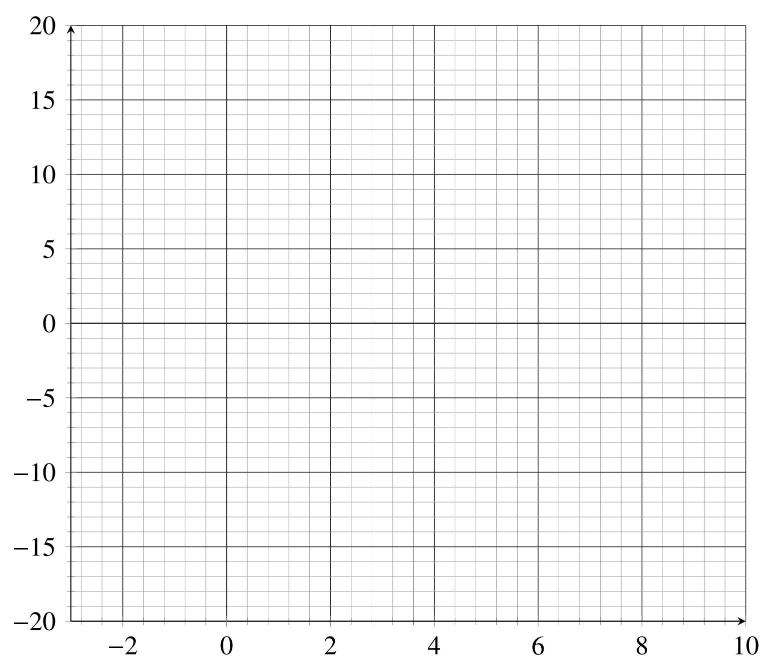
Problem 1. Supposed that an object is attached to a coiled spring. It is puled down a distance of 16 cm form its equilibrium position and then released. The time it takes for one complete oscillation is 6 seconds.

(a) Calculate the amplitude, period, and frequency for which the object oscillates.

(b) Give an equation that models the position of the object at time t.

(c) Determine the position of the object after 1 minute.

(d) Graph the position of the object over time for two periods.

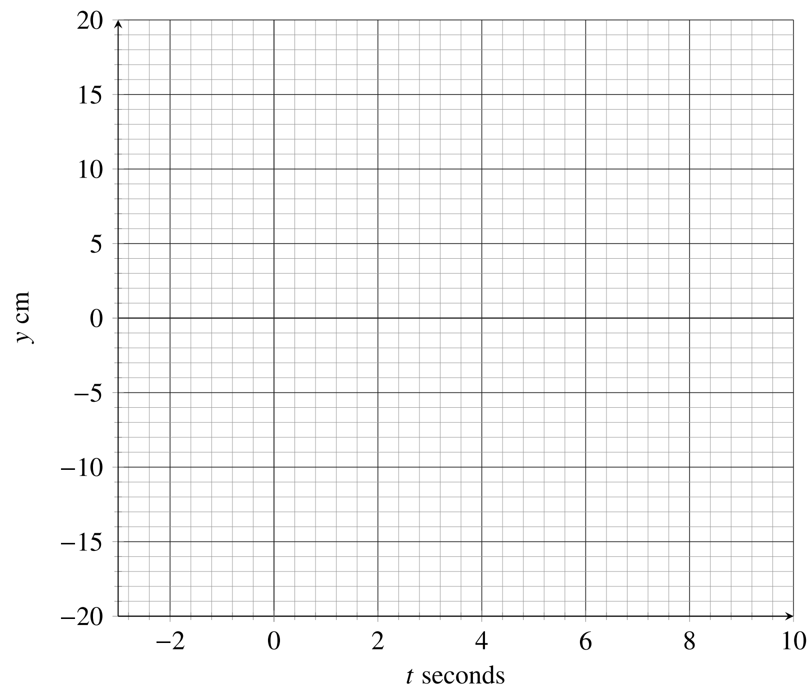


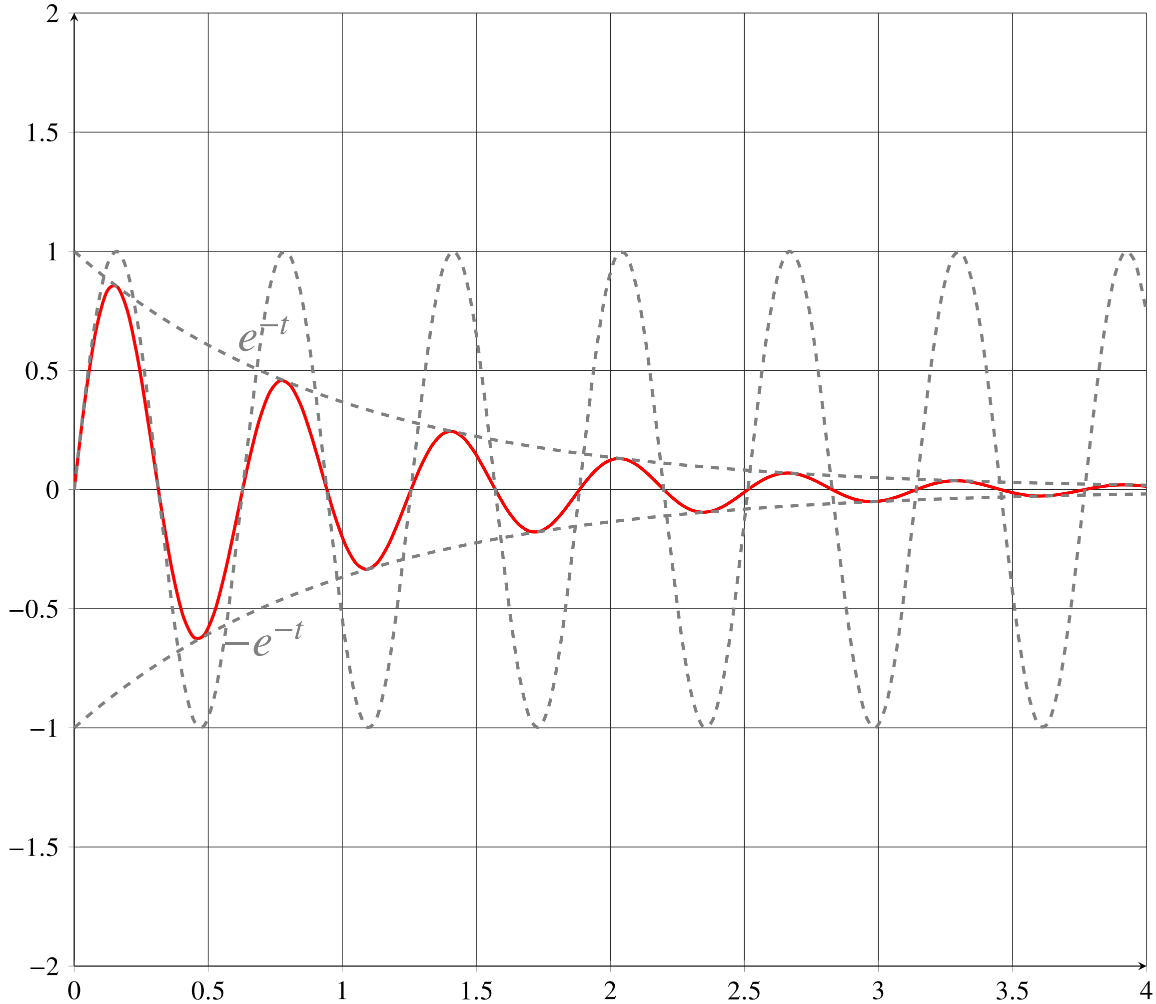
Problem 2. A weight attached to a spring is pulled down 2 in. below the equilibrium position.

(a) Assuming that the frequency is $\frac{6}{\pi}$ cycles per sec, determine a model (equation) that gives the postion of the weight at time t seconds.

(b) What is the frequency?

(c) Graph the position of the object over time for two periods. Label the graph with the units you are using.





2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
0
0.5
1
1.5
2
2.5
3
3.5
4
e^{-t}
$-e^{-t}$

Funamental Identities

Reciprocoal Identities

$$\cot\theta = \frac{1}{\tan\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \tan^2(\theta) + 1 = \sec^2(\theta) \qquad 1 + \cot^2(\theta) = \csc^2(\theta)$$

Negative Angle Identities

$$\sin(-\theta) = -\sin(\theta) \qquad \tan(-\theta) = -\tan(\theta) \qquad \sec(-\theta) = \sec(\theta)$$
$$\cos(-\theta) = \cos(\theta) \qquad \cot(-\theta) = -\cot(\theta) \qquad \csc(-\theta) = -\csc(\theta)$$

Complementary Identities

$$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right) \qquad \cot(\theta) = \tan\left(\frac{\pi}{2} - \theta\right) \qquad \csc(\theta) = \sec\left(\frac{\pi}{2} - \theta\right)$$

3. Given $\cos\theta = \frac{5}{8}$ and θ is in quadrant IV, find $\sin\theta$ and $\tan\theta$.

Writing a Trig Function in Terms of Another

4. Write $\tan x$ in terms of $\cos x$.

5. Write $\cot x$ in terms of $\sin x$.

6. Write $\sec x$ in terms of $\sin x$.

Rewriting in Terms of Sine and Cosine

Write each expression in terms of sine and cosine, and simplify so that no quotients appear in the final expression, and so that all your functions are of θ only. Ie, don't leave $-\theta$ in any of your arguments.

7. $\dfrac{1+\tan^2\theta}{1-\sec^2\theta}$

8. $(1+\cos(-\theta))\,(1+\sin(\theta))$

9. $\dfrac{1-\sin^2(-\theta)}{1+\cot^2(-\theta)}$