Section 5.2

Verifying Identities

Funamental Identities

Reciprocoal Identities

$$\cot\theta = \frac{1}{\tan\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \tan^2(\theta) + 1 = \sec^2(\theta) \qquad 1 + \cot^2(\theta) = \csc^2(\theta)$$

Negative Angle Identities

$$\sin(-\theta) = -\sin(\theta) \qquad \tan(-\theta) = -\tan(\theta) \qquad \sec(-\theta) = \sec(\theta)$$
$$\cos(-\theta) = \cos(\theta) \qquad \cot(-\theta) = -\cot(\theta) \qquad \csc(-\theta) = -\csc(\theta)$$

Complementary Identities

$$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right) \qquad \cot(\theta) = \tan\left(\frac{\pi}{2} - \theta\right) \qquad \csc(\theta) = \sec\left(\frac{\pi}{2} - \theta\right)$$

Verifying Identities

Problem 1. Verify that the following equation is an identitiy.

$$\sec\theta(\sin\theta + \cos\theta) = 1 + \tan\theta$$

$\sec\theta(\sin\theta+\cos\theta) = \sec\theta\sin\theta + \sec\theta\cos\theta$

$= \frac{1}{\cos\theta}\sin\theta + \frac{1}{\cos\theta}\cos\theta$

$= \tan\theta + 1$ ✓

Problem 2. Verify that the following equation is an identity.

$$\frac{\tan^2 t}{\sec^2 t} = (1+\cos t)(1-\cos t)$$

Work LHS & RHS at same time!

diff of squares!

$\frac{\tan^2 t}{\sec^2 t} = (1+\cos t)(1-\cos t)$

$\cos^2 t \cdot \frac{\sin^2 t}{\cos^2 t} = 1-\cos^2 t$ ← Pythagorean theorem

$\sin^2 t = \sin^2 t$ ✓

Problem 3. Verify that the following equation is an identity.

$$\frac{\sec x + \tan x}{\sin x} = \frac{\csc x}{\sec x - \tan x}$$

Pattern:
$(\sec x - \tan x)(\sec x + \tan x)$
$= \sec^2 x - \tan^2 x$
$= 1$

RHS

$$\frac{\csc x}{\sec x - \tan x} \cdot \frac{\sec x + \tan x}{\sec x + \tan x} = \frac{\csc x (\sec x + \tan x)}{\sec^2 x - \tan^2 x} \leftarrow 1$$

$$= \csc x (\sec x + \tan x)$$

$$= \frac{1}{\sin x} (\sec x + \tan x)$$

$$= \frac{\sec x + \tan x}{\sin x} \checkmark$$

LHS

Problem 4. Verify that the following equation is an identity.

$$\frac{1 + \sin x}{\cos x} = \frac{\cos x}{1 - \sin x}$$

Pattern!
$(1 - \sin x)(1 + \sin x)$
$= 1 - \sin^2 x = \cos^2 x.$

$$\frac{\cos x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} = \frac{\cos x (1 + \sin x)}{1 - \sin^2 x}$$

LHS

$$= \frac{\cos x (1 + \sin x)}{\cos^2 x}$$

$$= \frac{1 + \sin x}{\cos x} \checkmark$$

RHS

Problem 5. Verify $\cot\theta + \tan\theta = \sec\theta\csc\theta$ is an identity.

LHS

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{\sin^2\theta}{\sin\theta\cos\theta}$$

Find common denominator

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

$\cos^2\theta + \sin^2\theta = 1$ by Pythagorean

$$= \frac{1}{\sin\theta\cos\theta}$$

$\frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta}$

$$= \csc\theta\sec\theta \checkmark$$

RHS

Problem 6. Verify the following equation is an identity.

$$\frac{\cot\alpha + 1}{\cot\alpha - 1} = \frac{1 + \tan\alpha}{1 - \tan\alpha}$$

Work both LHS & RHS

Rewrite as sin/cos strategy

$$\frac{\sin\alpha}{\sin\alpha} \times \frac{\frac{\cos\alpha}{\sin\alpha} + 1}{\frac{\cos\alpha}{\sin\alpha} - 1} = \frac{1 + \frac{\sin\alpha}{\cos\alpha}}{1 - \frac{\sin\alpha}{\cos\alpha}} \times \frac{\cos\alpha}{\cos\alpha}$$

Get rid of fractions

$$\frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} \checkmark$$

SAME!