

Section 5.2

Verifying Identities

Fundamental Identities

Reciprocal Identities

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1 \quad \tan^2(\theta) + 1 = \sec^2(\theta) \quad 1 + \cot^2(\theta) = \csc^2(\theta)$$

Negative Angle Identities

$$\sin(-\theta) = -\sin(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

$$\sec(-\theta) = \sec(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\cot(-\theta) = -\cot(\theta)$$

$$\csc(-\theta) = -\csc(\theta)$$

Complementary Identities

$$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right) \quad \cot(\theta) = \tan\left(\frac{\pi}{2} - \theta\right) \quad \csc(\theta) = \sec\left(\frac{\pi}{2} - \theta\right)$$

VERIFYING IDENTITIES

Problem 1. Verify that the following equation is an identity.

$$\sec \theta (\sin \theta + \cos \theta) = 1 + \tan \theta$$

Problem 2. Verify that the following equation is an identity.

$$\frac{\tan^2 t}{\sec^2 t} = (1 + \cos t)(1 - \cos t)$$

Problem 3. Verify that the following equation is an identity.

$$\frac{\sec x + \tan x}{\sin x} = \frac{\csc x}{\sec x - \tan x}$$

Problem 4. Verify that the following equation is an identity.

$$\frac{1 + \sin x}{\cos x} = \frac{\cos x}{1 - \sin x}$$

Problem 5. Verify $\cot \theta + \tan \theta = \sec \theta \csc \theta$ is an identity.

Problem 6. Verify the following equation is an identity.

$$\frac{\cot \alpha + 1}{\cot \alpha - 1} = \frac{1 + \tan \alpha}{1 - \tan \alpha}$$