Section 6.1b

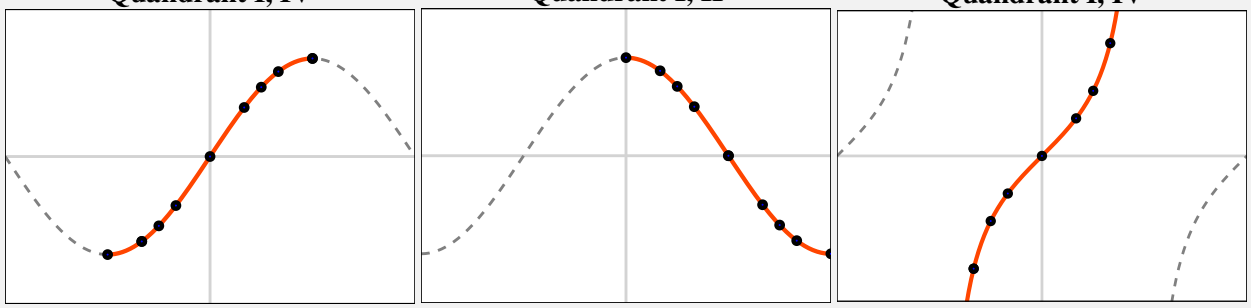
Inverse Circular Functions Continued

Other Inverse Trig Functions

In the previous worksheet we discussed the inverse trigonometric functions $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$. A brief summery of the results can be found in the following table. I find it helpful remembering the graphs we used to restrict the domains to figureout the domain and ranges of the inverse trigonometric functions.

Inverse Trig Functions

$\sin^{-1}(x) = \theta$	$\cos^{-1}(x) = \theta$	$\tan^{-1}(x) = \theta$
Domain: $-1 \le x \le 1$	**Domain:** $-1 \le x \le 1$	**Domain:** $-\infty < x < \infty$
Range: $\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$	**Range:** $0 \le \theta \le \pi$	**Range:** $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$
Quandrant I, IV	**Quandrant I, II**	**Quandrant I, IV**



Trig Functions of Inverse Trig Functions, Oh My

Problem 1. By sketching an appropriate triangle, evaluate each expression given below without using a calculator.

(a) $\cos\left(\sin^{-1}\frac{2}{3}\right)$

(b) $\sin\left(\arctan\left(-\frac{3}{4}\right)\right)$

Problem 2. It may be necessary to use trigonometric identites before proceeding. Evaluate each expression given below without using a calculator.

(a) $\sin\left(\arctan\frac{4}{3} - \arccos\frac{12}{13}\right)$

(b) $\cos\left(2\arccos\frac{3}{5}\right)$

Finding Algebraic Formulas

We have found a procedure that allows us to calculate exact values when given an inverse trig function inside of a trig function. This means that we will likely be able to calculate a general formula.

Problem 3. Write each expression as an algebraic expression in u.

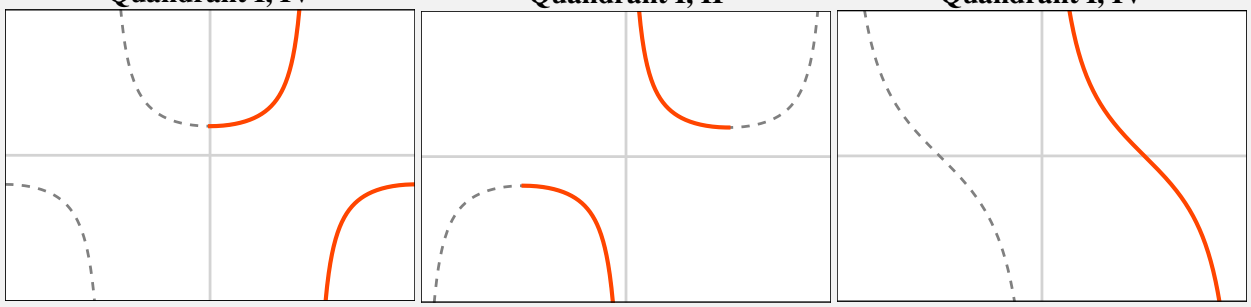
(a) $\sin\left(\cos^{-1} u\right)$

(b) $\cos\left(2\arccos\dfrac{u}{u^2+1}\right)$

The Other Inverse Trigonometric Functions

Inverse Trig Functions

$\sec^{-1}(x) = \theta$	$\csc^{-1}(x) = \theta$	$\cot^{-1}(x) = \theta$
Domain: $\lvert x\rvert \geq 1$	**Domain:** $\lvert x\rvert \geq 1$	**Domain:** $-\infty < x < \infty$
Range: $0 \leq \theta \leq \pi$, $\theta \neq \frac{\pi}{2}$	**Range:** $\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}$, $\theta \neq 0$	**Range:** $0 < \theta < \pi$
Quandrant I, IV	**Quandrant I, II**	**Quandrant I, IV**



Usually if one is trying to find the value of any of these three inverse trigonometric functions, they reduce the problem to the first three we have studied. There is one notable exception with the $\cot^{-1}$ function however.

Inverse Function Relations

$$\sec^{-1} x = \cos^{-1}\frac{1}{x} \qquad \csc^{-1} x = \sin^{-1}\frac{1}{x}$$

$$\cot^{-1} x = \begin{cases} \tan^{-1}\left(\frac{1}{x}\right) & \text{if } x > 0 \\ \tan^{-1}\left(\frac{1}{x}\right) + \pi & \text{if } x < 0 \end{cases}$$

Problem 4. Find the exact values of the following inverse trigonometric functions. Your answer should be in radians.

(a) $\sec^{-1} 2$

(b) $\csc^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$

(c) $\cot^{-1}\left(-\sqrt{3}\right)$

(d) $\cot^{-1} 0$

(e) $\sec^{-1} 0$

(f) $\cot^{-1}\sqrt{3}$

Problem 5. Use a calculator to find the following inverse trigonometric function values. Your answer should be in radians.

(a) $\csc^{-1}(1.314159)$

(b) $\cot^{-1}(1.23456)$

(c) $\cot^{-1}(-1.23456)$

(d) $\sec^{-1}(1.7779)$

Problem 6. Find the exact values of the following expressions without using a calculator.

(a) $\sin\left(2\tan^{-1}\frac{12}{5}\right)$

(b) $\csc\left(\csc^{-1}\sqrt{2}\right)$

(c) $\sin\left(2\csc^{-1}\left(-\frac{1}{2}\right)\right)$