Exam III

Problem 1. Convert −735° to radians. Your answer should be exact.

$$-735^\circ \times \frac{\pi}{180^\circ} = -\frac{735}{180}\pi = \boxed{-\frac{49}{12}\pi}$$

Problem 2. Convert $\frac{17\pi}{20}$ to degrees.

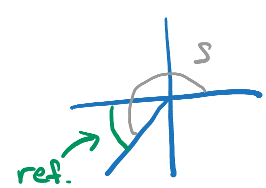
$$\frac{17\pi}{20} \times \frac{180^\circ}{\pi} = \frac{17\times 180^\circ}{20} = 17\times 9^\circ = \boxed{153^\circ}$$

Problem 3. Find the exact value of s in the interval $\left[\pi, \frac{3\pi}{2}\right)$ where $\sec s = -\sqrt{2}$.

Known: $\cos(45^\circ) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\Rightarrow \sec\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}.$

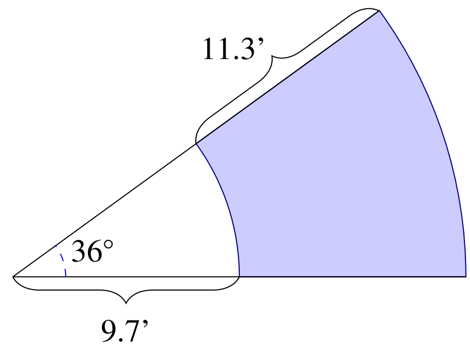
$\left[\pi, \frac{3\pi}{2}\right) \leftarrow$ s is in quadrant III.



Need reference angle $\frac{\pi}{4}$.

$s - \frac{\pi}{4} = \pi \Rightarrow \boxed{s = \frac{5\pi}{4}}$

Problem 4. Find the **area** and **perimeter** of the annulus sector shown in the figure below.



Perimeter:

Two striaght line segments: 11.3' each.

Inner arc:

$$s = r\theta \qquad r = 9.7'$$

$$\theta = 36^\circ \times \frac{\pi}{180^\circ} = \frac{36}{180}\pi = \frac{\pi}{5}$$

$$s = 9.7' \times \frac{\pi}{5} \approx 6.1'$$

Outter arc:

$$r = 11.3' + 9.7' = 21'$$

$$s = 21' \times \frac{\pi}{5} \approx 13.2'$$

Perimeter: $13.2' + 6.1' + 2 \times 11.3' = \boxed{41.9'}$

Area:

Large Sector:

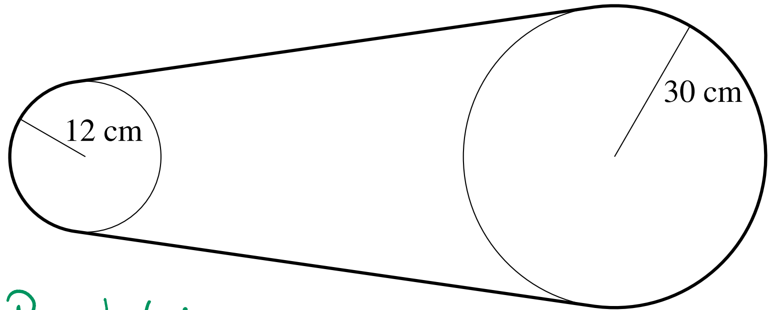
$$\text{Area} = \frac{1}{2}\theta r^2 = \frac{1}{2}\frac{\pi}{5}(21')^2 \approx 138.5 \text{ sq. ft.}$$

Small Sector:

$$\text{Area} = \frac{1}{2}\theta r^2 = \frac{1}{2}\frac{\pi}{5}(9.7')^2 \approx 29.6 \text{ sq ft}$$

$$\text{Area} = 138.5 \text{ sq ft} - 29.6 \text{ sq ft} = \boxed{108.9 \text{ sq ft}}$$

Problem 5. In a transmission system two gears are connected as shown in the figure below so that both gears will rotate with the same linear speed. Suppose that the gear on the right rotates at 3200 RPM. Calculate how many RPMs the left gear will rotate.



Angular Speed of Right:

$$\frac{3200\ \text{ROT}}{\text{MIN}} \times \frac{2\pi\ \text{RAD}}{\text{ROT}} = 6400\pi\ \frac{\text{RAD}}{\text{MIN}}$$

Linear speed of Right:

$$V = r\omega \Rightarrow v = 30 \times 6400\pi\ \frac{\text{CM}}{\text{MIN}}$$

Right & Left have same Lin. Speed

$$\Rightarrow 12\text{cm} \cdot \omega = 30 \cdot 6400\pi\ \frac{\text{CM}}{\text{MIN}}$$

$$\Rightarrow \omega = \frac{30 \cdot 6400\pi}{12}\ \frac{\text{RAD}}{\text{MIN}} = 16000\pi\ \frac{\text{RAD}}{\text{MIN}}$$

$$\Rightarrow 16000\cancel{\pi}\ \frac{\text{RAD}}{\text{MIN}} \times \frac{1\ \text{ROT}}{2\cancel{\pi}\ \text{RAD}} = \boxed{8000\ \text{RPM}}$$

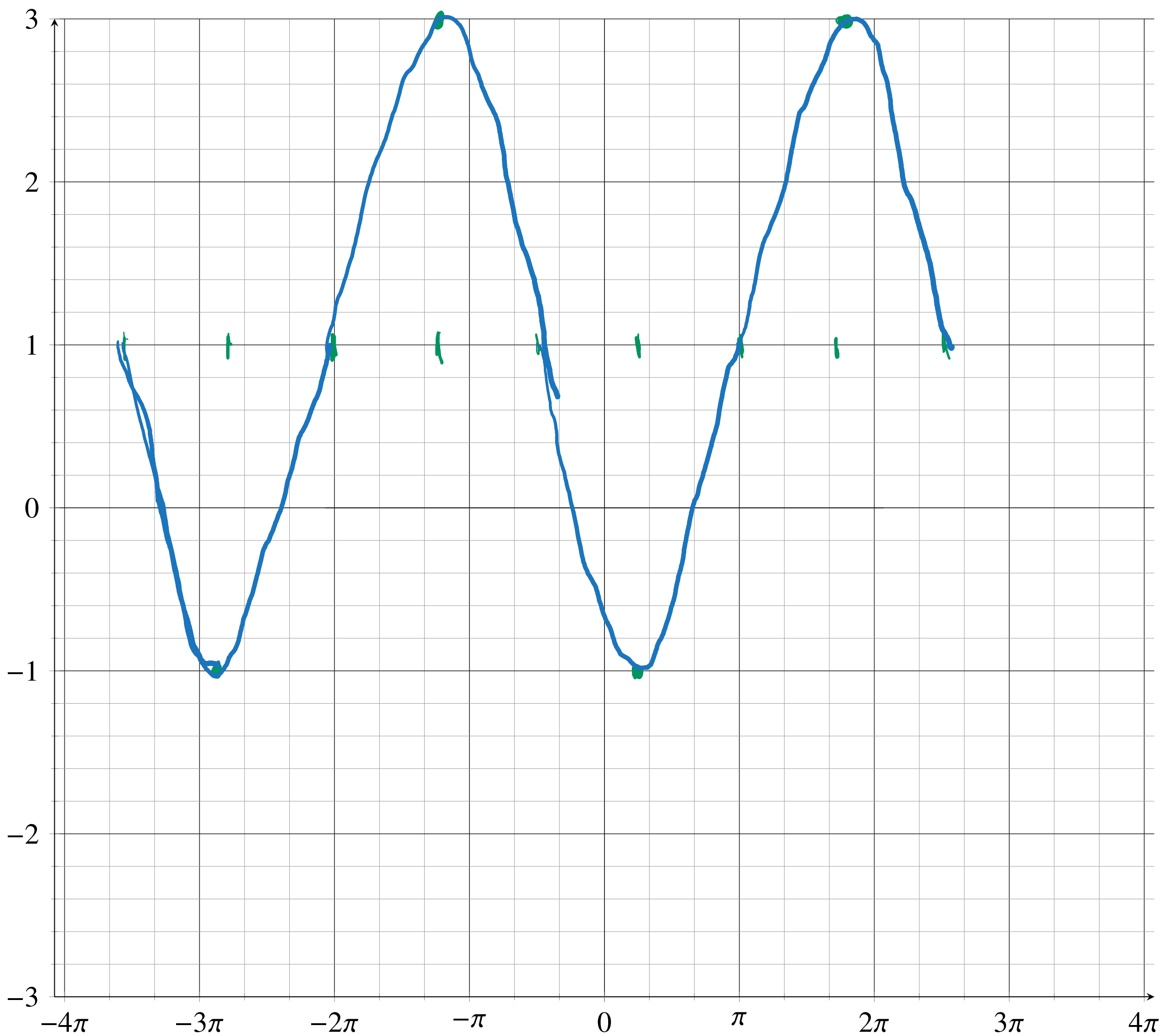
Problem 6. Sketch the graph of the function for $y = 1 - 2\sin\left(\frac{2x}{3} + \frac{\pi}{3}\right)$ over *two* periods.

Amplitude $= |-2| = 2.$

Period $= \dfrac{2\pi}{2/3} = 3\pi$

$$\frac{2}{3}x + \frac{\pi}{3} = \frac{2}{3}\left(x + \frac{\pi}{2}\right)$$

Phase-Shift $= \pi/2$ Left.



Problem 7. Suppose that a mass is attached to the end of a spring and pulled down 3 cm from its point of rest. The mass begins oscillating and a camera measures the frequency of the oscillation at 47 Hz.

(a) Calculate the amplitude and period of the oscillation.

Amplitude = 3 cm.

$$\text{Period} = \frac{1}{\text{freq}} = \frac{1}{47} \text{ secs.}$$

(b) Write out an equation that models the oscillating motion.

$$y = a\cos(bt)$$

$$\text{model Period} = \frac{2\pi}{b}.$$

$$\Rightarrow \frac{2\pi}{b} = \frac{1}{47} \Rightarrow b = 47 \times 2\pi = 96\pi.$$

$$\boxed{y = -3\cos(96\pi t).}$$

(c) At what position, relative to the resting point of the spring, will the mass be after one minutes has elapsed?

t is in seconds. 1 min = 60 secs.

$$y = -3\cos(96\pi \times 60)$$

$$\boxed{\Rightarrow y = -3 \text{ cm.}}$$