Section 3.1

The Radian Measure and Arclength

1. State the radian measure of all multiples of 30° and 45° going around a circle using the notion of these angles as a fraction of a full circle.

30° is $\frac{30}{360}$ th of circle $\Rightarrow$ 30° is $\frac{1}{12}$th of circle

45° is $\frac{45}{360}$ of circle $\Rightarrow$ 45° is $\frac{1}{8}$th of circle

60° is twice 30° $\Rightarrow$ 60° is $\frac{1}{6}$th of circle

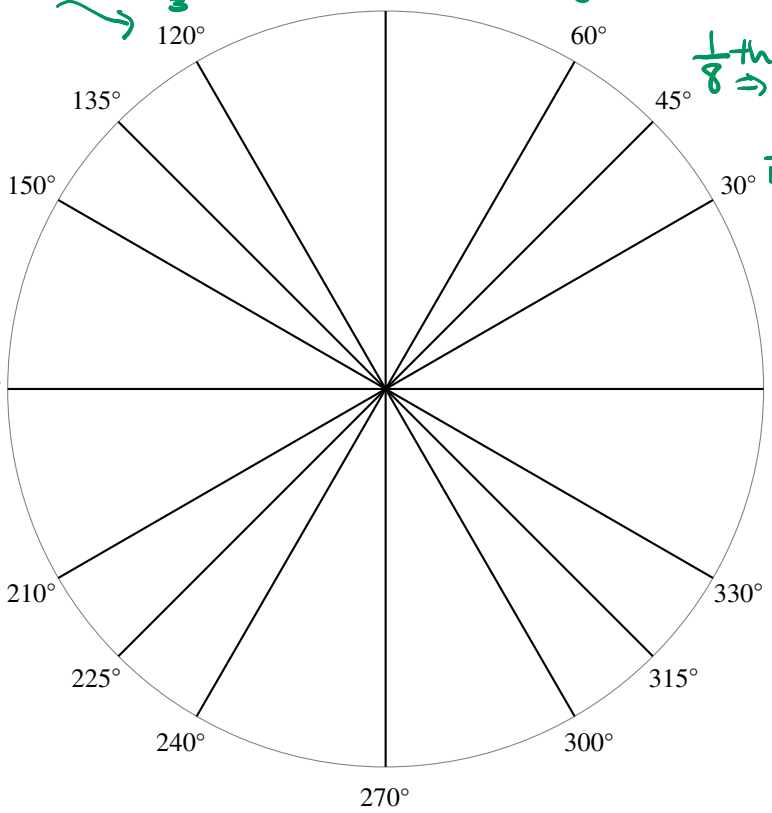
$\frac{1}{4}$th circ $\Rightarrow \frac{1}{4} \cdot 2\pi = \frac{\pi}{2}$ rad

$\frac{1}{3}$rd circ $\Rightarrow \frac{2\pi}{3}$ rad

$\frac{1}{6}$th circle $\Rightarrow \frac{1}{6} 2\pi = \frac{\pi}{3}$ radian

$\frac{1}{8}$th circ $\Rightarrow \frac{1}{8} \cdot 2\pi = \frac{\pi}{4}$ radian

$\frac{1}{12}$th circ $\Rightarrow \frac{1}{12} \cdot 2\pi = \frac{\pi}{6}$ rad



$\frac{1}{2}$ circ $\Rightarrow \frac{1}{2} 2\pi = \pi$ rad ...

0° 0th circ $\Rightarrow 0 \cdot 2\pi = 0$ rad

or 1 circ $\Rightarrow 1 \cdot 2\pi = 2\pi$ rad

$\frac{3}{4}$ circ $\Rightarrow \frac{3}{4} \cdot 2\pi = \frac{3\pi}{2}$ rad.

Converting Between Measures

2. Convert $108°$ to radians. $108° \times \frac{\pi}{180°} = \frac{108}{180}\pi = \frac{3\pi}{5}$

3. Convert $-457.89°$ to radians. $-457.89° \times \frac{\pi}{180°} \approx -7.991$

4. Convert $\frac{11\pi}{12}$ to degrees. $\frac{11\pi}{12} \times \frac{180°}{\pi} = 165°$

5. Convert $-\frac{7\pi}{6}$ to degrees. $-\frac{7\pi}{6} \times \frac{180°}{\pi} = -210°$

6. Convert 15.292 to degrees. $15.299 \times \frac{180°}{\pi} \approx 876.57°$

7. Find the value of $\cos\left(\frac{5\pi}{6}\right)$. 0.866

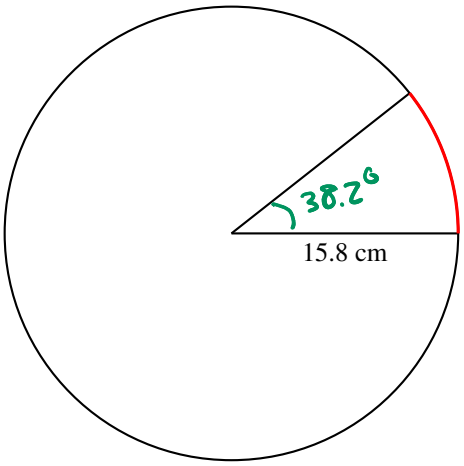
8. Find the value of $\sin\left(\frac{\pi}{4}\right)$. $\frac{\pi}{4} = 45° \Rightarrow \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

9. Find the value of $\cot\left(-\frac{2\pi}{3}\right)$.

$$= \frac{1}{\tan\left(-\frac{2\pi}{3}\right)} \approx \frac{1}{1.732} \approx 0.577$$

ARC LENGTH

Problem 10. Find the arc length intercepted by a central angle of 38.2° in a circle of radius 15.8 cm.

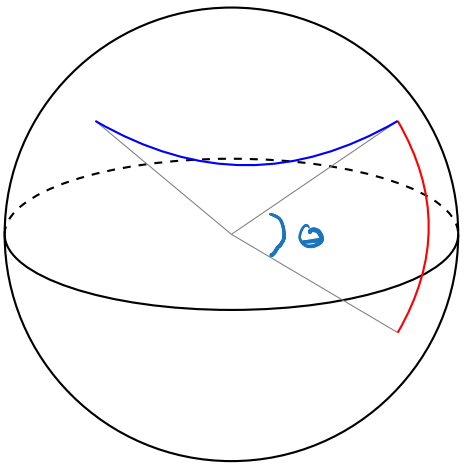


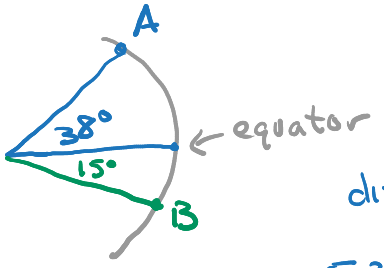
Arc Length: $S = r\theta$ ← must be in radians!

$38.2° = 38.2 \times \frac{\pi}{180°} \approx 0.67$ radians.

$S = 15.8 \times 0.67 \approx \boxed{10.586 \text{ cm}}$

Problem 11. Suppose that city A has a latitude of 38°N and city B, which is due south, has a latitude of 15°S, what is the distance between the cities given that the radius of the Earth is 3959 mi? Could you answer the same question, but for a city C that is measured 38° west of city A?





Angle from A to B:

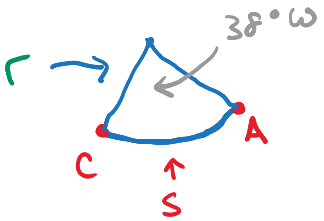
$38° + 15° = 53°$.

distance = arc length

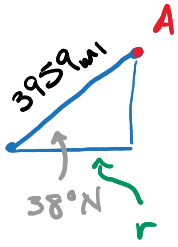
$53° \times \frac{\pi}{180°} \approx 0.925$ Rad.

$S = 3959 \times 0.925 \approx \boxed{3662 \text{ mi}}$

Top View:



Side View:



$r = 3959 \cos(38°)$

$r \approx 3274$ mi

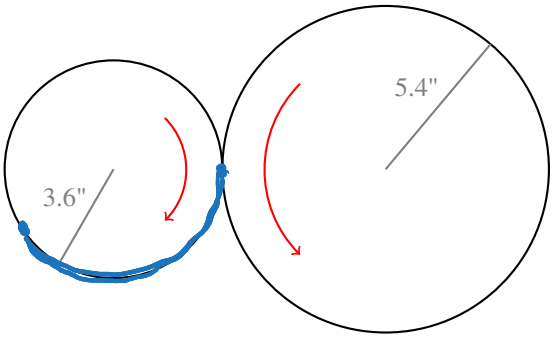
$S = r\theta$

$38° \times \frac{\pi}{180°} \approx 0.663$ RAD.

$S = 3274 \times 0.663$

$\Rightarrow \boxed{S \approx 2171 \text{ mi}}$

Problem 12. A small gear of radius 3.6 inches drives a larger gear of radius 5.4 inches. Find the angle the larger gear rotates if the smaller drive gear rotates 150°.



Point on small gear travels:

$$s = r\theta$$

$$150^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{6}$$

$$s = 3.6 \times \frac{5\pi}{6} \approx 9.42''$$

Distance travel on larger gear must also be 9.42":

$$9.42'' = 5.4''\,\theta$$

$$\Rightarrow \theta = \frac{9.42''}{5.4''} \approx 1.74 \text{ Radians}$$

$$\Rightarrow \theta = 1.74 \times \frac{180^\circ}{\pi} = \boxed{99.9^\circ}$$