
Section 5.6

Half-Angle Identities

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$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$$

$$\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A}$$

Whether you will use + or – in the \pm 's given above will depend on the quadrant of $\frac{A}{2}$.

Problem 1. Find the exact value of $\sin 22.5^\circ$.

Problem 2. Find the exact value of $\tan 75^\circ$.

Problem 3. Simplify the following expressions.

(a) $\pm\sqrt{\frac{1 - \cos 8x}{2}}$

(b) $\pm\sqrt{\frac{1 - \cos 9\alpha}{1 + \cos 9\alpha}}$

Problem 4. Verify that the following equation is an identity: $\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2 = 1 + \sin x$

Problem 5. Verify that the following equation is an identity: $\tan^2 \frac{x}{2} = \frac{\sec x + \cos x - 2}{\sec x - \cos x}$