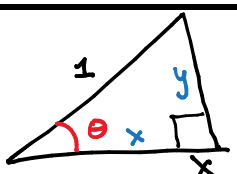
Section 1.4

Using the Definition of Trig Functions

TRIG FUNCTION DEFINITIONS



Useful: take $r=1$, $\sin\theta = \frac{y}{1}$, $\cos\theta = \frac{x}{1}$, etc...

1. Complete the following incomplete tables of values for trigonometric function values.

$y = 2/3 \rightarrow$

$\sin\theta$	$\frac{2}{3}$
$\cos\theta$	$\sqrt{5}/3$
$\tan\theta$	$\frac{2}{\sqrt{5}}$
$\csc\theta$	$3/2$
$\sec\theta$	$3/\sqrt{5}$
$\cot\theta$	$\sqrt{5}/2$

$\tan\theta = y/x$

$\Rightarrow \frac{2/3}{x} = \frac{2}{\sqrt{5}}$

$\Rightarrow x = \sqrt{5}/3$

$\sin\theta$	1
$\cos\theta$	0
$\tan\theta$	Und
$\csc\theta$	1
$\sec\theta$	Und
$\cot\theta$	0

$\leftarrow y=1,\ r=1,\ r^2=x^2+y^2$

$\Rightarrow 1^2 = x^2+1^2$

$\Rightarrow x=0!$

$\sin\theta$	$\frac{1}{2}$
$\cos\theta$	$\sqrt{3}/2$
$\tan\theta$	$1/\sqrt{3}$
$\csc\theta$	2
$\sec\theta$	$\frac{2}{\sqrt{3}}$
$\cot\theta$	$\sqrt{3}$

$y = 1/2$

$\sec\theta = \frac{r}{x}$

$\Rightarrow \frac{1}{x} = \frac{2}{\sqrt{3}}$

$\Rightarrow x = \sqrt{3}/2$

Rember, $r>0$. Always!

2. Complete the following incomplete tables of signs for trigonometric functions.

$\frac{y}{r} > 0 \rightarrow$

$\Rightarrow y > 0$

$\frac{x}{r} < 0$

$\Rightarrow x < 0$

$\sin\theta$	+
$\cos\theta$	−
$\tan\theta$	−
$\csc\theta$	+
$\sec\theta$	−
$\cot\theta$	−

$\leftarrow \frac{-}{+}$

$\sin\theta$	−
$\cos\theta$	+
$\tan\theta$	−
$\csc\theta$	−
$\sec\theta$	+
$\cot\theta$	−

$\leftarrow y<0$

$\leftarrow \frac{y}{x} < 0$

$\Rightarrow y<0$

$\Rightarrow x>0$

$\sin\theta$	+
$\cos\theta$	−
$\tan\theta$	−
$\csc\theta$	+
$\sec\theta$	−
$\cot\theta$	−

$\csc\theta = \frac{r}{y}$

$\Rightarrow \frac{r}{y} > 0.$

$\Rightarrow y > 0.$

$\cot\theta = x/y$

$\frac{+}{+}\ y>0 \Rightarrow x<0$

3. Given the following trigonometric values, determine which quadrant θ is in.

Value	Quadrant	Value	Quadrant	Value	Quadrant	Value	Quadrant
$\sin\theta > 0$ $\cos\theta < 0$	II	$\sin\theta < 0$ $\tan\theta > 0$	III	$\csc\theta < 0$ $\sec\theta < 0$	III	$\cot\theta < 0$ $\cos\theta > 0$	IV

4. Fill in the table of all the trigonometric functions given a trigonometric value and a quadrant.

$\cos^2\theta + \sin^2\theta = 1$
$\cos^2\theta + (\frac{2}{5})^2 = 1 \Rightarrow \cos\theta = \pm\frac{\sqrt{21}}{5}$

$\sin\theta = \frac{2}{5}$ with θ in quadrant II → $\cos\theta < 0$

$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{2}{5}$	$-\frac{\sqrt{21}}{5}$	$\frac{-2}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$	$\frac{5}{2}$	$\frac{-5}{\sqrt{21}} = \frac{-5\sqrt{21}}{21}$	$-\frac{\sqrt{21}}{2}$

$\sin\theta = \frac{2}{5}$ with θ in quadrant I → $\cos\theta > 0$

$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{2}{5}$	$\frac{\sqrt{21}}{5}$	$\frac{2\sqrt{21}}{21}$	$\frac{5}{2}$	$\frac{5\sqrt{21}}{21}$	$\frac{\sqrt{21}}{2}$

5. Answer the following True/False questions; most of them dealing with the possible values a trigonometric function can obtain. You should provide short justification.

a. $\cos\theta = -28$

False : $-1 \le \cos\theta \le 1$

b. $\cot\theta = -129$

True : $\cot\theta$ can be any real number.

c. $\sec\theta = 0.5$

False: $|\sec\theta| \ge 1$.

So, $\sec\theta \le -1$ or $\sec\theta \ge 1$

but nowhere inbetween.