Section 3.3/3.4

The Unit Circle and Angular Speed

We were introduced to radians, an alternative measure for angles, for the last two sections and explored how we can solve problems dealing with arc lengths and areas of sectors using the definition of radians. We also learned how to convert the degrees to radians and vice versa. Today we are going to explore the relationship of the trigonometric functions and the unit circle in a little more depth and how to use this information to answer problems about objects moving in a circular path.

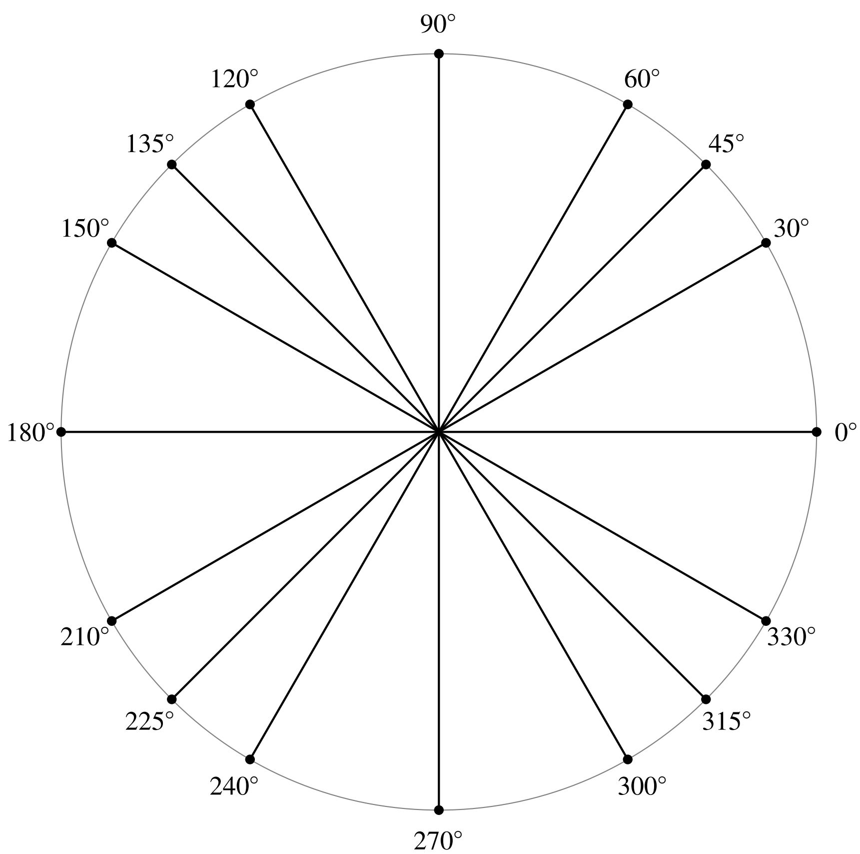
Finding Exact Values

1. Calculate the *exact* value of $\sin\left(\frac{3\pi}{4}\right)$. Avoid solving the problem by converting radians to degrees if possible.

2. Calculate the *exact* value of $\tan\left(-\frac{17\pi}{3}\right)$.

Points on a Circle

3. Convert each angle to radians, and determine the exact values for the coordinates of each point listed below. The circle is assumed to be a unit circle.



Using a Calculator

All the same tricks we learned for calculating a trigonometric function on a calculator in degrees also apply here. You *must* make sure that your calculator is in radian mode.

4. A quick trick for figuring out if you calculator is in radian mode is to calculate sin(90). If your calculator is in degree mode then you should get 1. If your calculator is in radian mode, what do you get?

5. Find an approximation to cos 1.85. Round your answer to three significant figures.

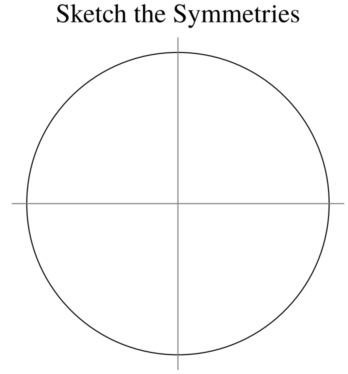
6. Find an approximation to $\cot\left(\frac{\pi}{32}\right)$. Round your answer to four significant figures.

Symmetries on the Circle

You can find trigonometric function values from known trigonometric function values by exploiting *symmetries* of the unit circle. There are three symmetries worth remembering:

1. Reflection about the **x-axis**: $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ becomes $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.
2. Reflection about the **y-axis**: $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ becomes $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.
3. Reflection about the **origin**: $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ becomes $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

Sketch the Symmetries



Problem 7. Given θ in the the first quadrant. Find formulas for angle that you will obtain for each of the three symmetries. Refer to the sketch you made illustrating the symmetries for guidance.

Inverse Trigonometric Functions

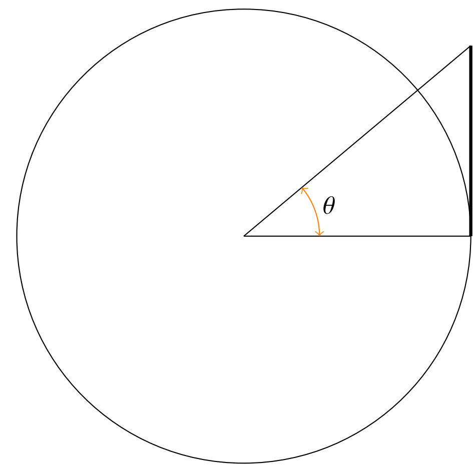
Recall that we use inverse trigonometric functions to solve for *angles* given a triangle a ratio of side lengths, the exact opposite of solving for *side lengths* given an angle like we do when using trigonometric functions.

8. Find the exact value of s in the interval $\left[\pi, \frac{3\pi}{2}\right]$ where $\tan s = 1$.

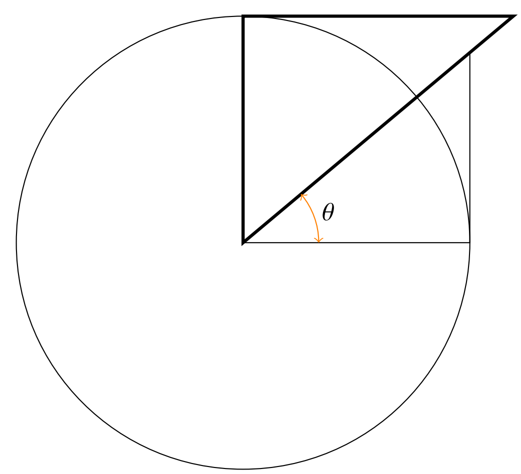
9. Find all $s \in [0, 2\pi)$ where $\cos(s) = -\frac{1}{2}$.

10. Find all $s \in [-2\pi, \pi)$ where $3\tan^2 s = 1$.

11. Show that the length of the opposite side of the triangle below is $\tan\theta$. The circle is a unit circle.



12. Show that the length of the northern edge of the bolded triangle is $\cot\theta$.



Angular Velocity

Linear speed measures how fast position (normally the position of an object) is changing. We use the formula

$$\text{speed} = \frac{\text{distance}}{\text{time}}, \qquad \text{or} \qquad v = \frac{s}{t}$$

where v is the linear speed, s is the distance, and t is the time. When describing an object moving in a circular motion, we often just describe how fast the angle is changing over time. We call this the **angular speed**, usually written as ω, and have the formula:

$$\omega = \frac{\theta}{t}, \quad \text{where } \theta \text{ is in radians.}$$

Using that $s = r\theta$ where r is the radius of a circle, and θ is the angle transversed along the circle, replacing $s = r\theta$ into our linear speed equation gives us a way to determine linear speed from angular speed and the radius of the circle the object is transversing.

$$v = \frac{s}{t} = \frac{r\theta}{t} = r\omega$$

Problem 13. A belt runs a pulley of radius 6 cm at 80 revolutions per minute.

(a) Find the angular speed of the pulley in radians per second.

(b) Find the linear speed of the belt in centimeters per second.

Problem 14. Find the angular and linear speed of a point on the earth's equator and in Madison, Wi (latitude = $43°05'$ North). The radius of the earth is about 3960 miles.

Problem 15. Geostationary satellites are always 35,786 km above the earth's surface and are stationary above a given point on the equator (ie: they have the same angular speed as the earth does). Find their linear speed.