## Worksheet 12

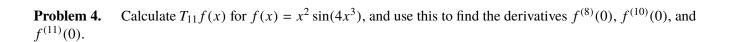
Little Oh and Taylor Series Manipulations

**Problem 1.** Show that  $\sqrt{1+x^2} + \sqrt{1-x^2} = 2 + o(x^3)$ .

**Problem 2.** For which k is it true that  $\sqrt{1+3x^4} = 1 + o(x^k)$ ?

**Problem 3.** Calculate  $T_3 \cos(x)$ , and estimate the error  $|\cos(x) - T_3 \cos(x)|$ . Use this to show that the following is true for any x:

$$1 - \frac{x^2}{2} - \frac{x^4}{24} \le \cos(x) \le 1 - \frac{x^2}{2} + \frac{x^4}{24}$$



**Problem 5.** Calculate  $T_8 f(x)$  for  $f(x) = (1 + x^2)e^{-x^4}$ , and use this to find the derivatives  $f^{(6)}(0)$ ,  $f^{(7)}(0)$ , and  $f^{(8)}(0)$ .

**Problem 6.** Calculate  $T_4 f(x)$  for  $f(x) = \sin(3x)e^{x^2}$  without calculating any derivatives.