Worksheet 14

Limits of Sequences

Computing Limits

We will start with what should be some familiar concepts from Calculus I for computing limits.

Problem 1. $\displaystyle\lim_{n\to\infty} \frac{n^2 - n + 1}{n + 2 - 2n^2}$

Problem 2. $\displaystyle\lim_{n\to\infty} 2^{-n} n!$

Problem 3. $\displaystyle\lim_{n\to\infty} \frac{n^6}{n! + n}$

Using the Definition

In this section we will cover a few basic techniques for proving the convergence of sequence. Proving the convergence gives us *quantitative* information about the convergence of a sequence. This can be important if instead of asking *If n gets large enough, does my sequence get close to L?* you instead ask *How far must I go in my sequence to approximate L within my given error tolerance?*. Quite often you might find youself needing to stay within a given error tolerance, and going over the proof can tell you how to stay within that error tolerance.

Problem 4. Prove that $\lim_{n\to+\infty} \frac{1}{n^2} = 0$.

Problem 5. Prove that $\lim_{n\to+\infty} \frac{1}{n^2+n} = 0$.

Problem 6. Prove that $\lim_{n\to+\infty} \frac{n^2-1}{n^2+2n} = 1$.

Problem 7. Prove that $\lim_{n\to+\infty} \sqrt{n-1} - \sqrt{n} = 0$.

Problem 8. Prove that $\lim_{n\to\infty} n^5 \left(\frac{9}{10}\right)^n = 0$.

Additional Practice

Problem 9. Prove that $\lim_{n\to+\infty} \frac{2n+5}{3n^3-2n+1} = 0$

Problem 10. Prove that $\lim_{n\to+\infty} \frac{n^3+2}{3n^3-2n+1} = \frac{1}{3}$.

Problem 11. Prove that $\lim_{n\to+\infty} (n^2+n+1)\left(\frac{1}{2}\right)^{2n} = 0$.