Worksheet 9

Differential Equations II

First Order Linear Differential Equations

Problem 1. Find the **general solution** to the differential equation: $x\dfrac{dy}{dx} - y = 2x\ln x$.

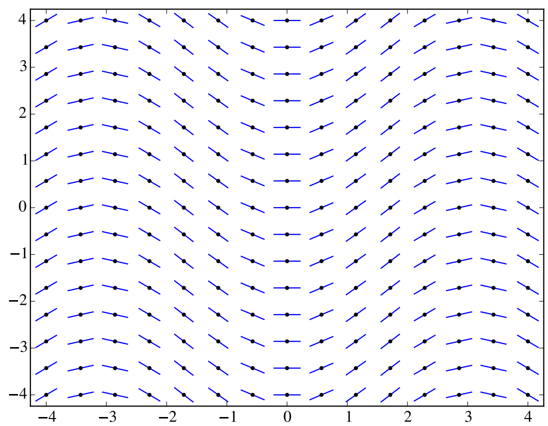
Problem 2. Find the **general solution** to the differential equation: $x\dfrac{dy}{dx} + 3y = \dfrac{\sin x}{x^2}$.

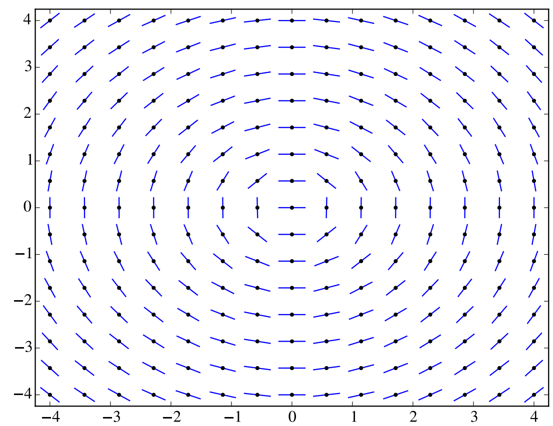
Problem 3. Find a solution to the **initial value problem**: $\dfrac{dy}{dx} + xy = x, \qquad y(0) = -6.$

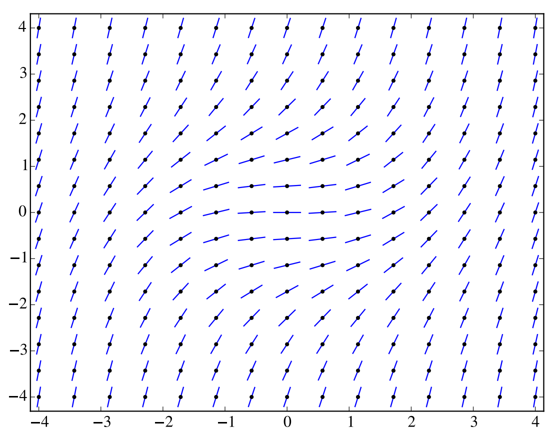
Problem 4. Find a solution to the **initial value problem**: $\dfrac{dy}{dx} = (y-1)\dfrac{1}{x}, \qquad y(-1) = 0.$

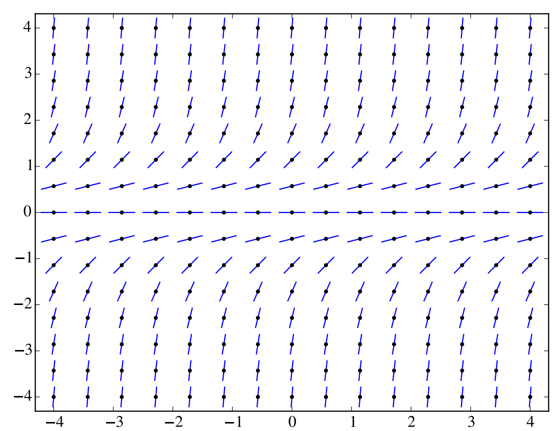
Problem 5. Match the following differential equations with their corresponding direction fields.

(a) $\dfrac{dy}{dx} = \dfrac{-x}{y}$ (b) $\dfrac{dy}{dx} = y^2$ (c) $\dfrac{dy}{dx} = \sin x$ (d) $\dfrac{dy}{dx} = x^2 + y^2$









Euler's Method

Problem 6. Find an exact solution of the following initial value problem, then use Euler's method with step size $\Delta x = 0.1$ to estimate $y(0.2)$.

$$\begin{aligned} \frac{dy}{dx} &= 2xy + x \\ y(0) &= 0 \end{aligned}$$

Problem 7. Use Euler's method with a step size of $\Delta t = 0.1$ to approximate the solution to $y'(t) = e^{-t^2}$, with $y(0) = 0$ to estimate $y(0.3)$.

Word Problems

Problem 8. A tank starts with 100 liters of water and 1,000 bacteria in it. For now we assume the bacteria do not reproduce. Let $B(t)$ be the number of bacteria in the tank as a function of time, where t is in hours. For each of the situations below, write down a first order differential equation satisfied by $B(t)$, of the form $B'(t) = f(t, B)$. You do not need to solve it.

(a) A little goblin is pouring bacteria into the tank at a rate of 2015 bacteria per hour.

(b) Like part (a), but we are also draining the tank at a rate of 3 L/hr.

(c) Like part (b), but now the bacteria are reproducing. This is a strain of bacteria which, if left alone, will double its population every hour.

Problem 9. Retaw is a mysterious living liquid; it grows at a rate of 5% of its volume per hour. A scientist has a tank initially holding y_0 gallons of retaw and removes retaw from the tank continuously at the rate of 3 gallons per hour.

(a) Find a differential equation for the number $y(t)$ of gallons of retaw in the tank at time t.

(b) Solve this equation for y as a function of t. (The initial volume y_0 will appear in our answer.)

(c) What is $\lim_{t\to\infty} y(t)$ if $y_0 = 100$?

(d) What should the value of y_0 be so that $y(t)$ remains constant as the scientist drains retaw.