

# Worksheet 11

## Lagrange's Error Estimation

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**Problem 1.** Find the 4th degree Taylor polynomial of  $\sin x$  (which we often write as  $T_4 \sin x$ ). Estimate the error  $|\sin x - T_4 \sin x|$  for  $|x| < 1$ .

**Problem 2.** Calculate the 5th degree Taylor polynomial  $T_5 e^x$  to find an approximation for  $e$ . Estimate the error of your approximation.

**Problem 3.** Recall that  $\sqrt{225} = 15$ . Calculate as many terms as you need of the Taylor polynomial for  $\sqrt{225 - x}$  to find an approximation of  $\sqrt{222}$  with an error less than  $\frac{1}{1000}$ .

**Problem 4.** A commonly used approximation is  $\sqrt{1 + x} \approx 1 + \frac{1}{2}x$  for small  $x$ . How small must  $x$  be for this approximation to be accurate to within 1% error? For simplicity, assume that  $x > 0$ .

**Problem 5.** How many terms of the Taylor series for  $\ln(1 + x)$  do you need to approximate  $\ln\left(\frac{3}{2}\right)$  with an error less than  $\frac{1}{100}$ ?

**Problem 6.** How many terms of the Taylor series for  $\ln(1 + x)$  do you need to approximate  $\ln 2$  with an error less than  $\frac{1}{100}$ ?