Worksheet 15
Limits of Sequences

Problem 1. Let $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Find $\vec{a}^{/\!/}$ and $\vec{a}^{\perp}$ such that $\vec{a} = \vec{a}^{/\!/} + \vec{a}^{\perp}$, where $\vec{a}^{/\!/}$ is parallel to $\vec{b}$ and $\vec{a}^{\perp}$ is perpendicular to $\vec{b}$.

Problem 2. Let $\vec{a} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Find $\vec{a}^{/\!/}$ and $\vec{a}^{\perp}$ where $\vec{a} = \vec{a}^{/\!/} + \vec{a}^{\perp}$ as described in the previous problem.

Problem 3. Consider the line defined by the equation $2x + 3y = 5$.

(a) Write out a parametric equation for the line.

(b) Write out $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ as $\vec{a} = \vec{a}^{/\!/} + \vec{a}^{\perp}$, where $\vec{a}^{/\!/}$ is parallel to the line, and $\vec{a}^{\perp}$ is perpendicular to the line.

Problem 4. Consider the plane containing the points $(1, 0, 0)$, $(1, 0, 1)$, and $(0, 1, 0)$.

(a) Write out an equation for the plane.

(b) Which of the following points are on the plane: $(1, 0, -3)$, $(-1, 2, 1)$, and $(2, 0, 1)$?

(c) Consider the line through the points $(1, 2, 1)$ and $(0, 1, 1)$. Where does this line intersect the plane?

(d) What is the distance of the point $(1, 2, 1)$ from the plane?

Problem 5. Simplify the following expressions:

(a) $\vec{\imath} \times (\vec{\imath} - \vec{\jmath})$

(e) $\vec{\imath} \cdot (\vec{\imath} - \vec{\jmath})$

(b) $(\vec{a} + \vec{b}) \times (\vec{a} + \vec{b})$

(f) $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$

(c) $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

(g) $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$

(d) $\vec{a} \cdot \left[\vec{b} \times \left(\vec{a} + \vec{b} \right) \right]$

(h) $\left\| \vec{a} + 2\vec{b} \right\|^2$

Problem 6. Determine if the following lines intersect or not. If so, determine their point of intersection and find the angle between the two lines:

$$\vec{l}(t) = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix},$$

$$\vec{m}(s) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + s \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}.$$