1. (trigint:cos2)
   Compute $\int \cos^2(x)dx$.

   Solution:

$$\begin{aligned}\int \cos^2(x)dx &= \frac{1}{2}\int 1+\cos(2x)dx \\ &= \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2x)dx \\ &= \frac{1}{2}x + \frac{1}{4}\sin(2x) + C\end{aligned}$$

2. (trigint:sin2)
   Compute $\int \sin^2(x)dx$.

   Solution:

$$\begin{aligned}\int \sin^2(x)dx &= \frac{1}{2}\int 1-\cos(2x)dx \\ &= \frac{1}{2}\int dx - \frac{1}{2}\int \cos(2x)dx \\ &= \frac{1}{2}x - \frac{1}{4}\sin(2x) + C\end{aligned}$$

3. (trigint:cos2sin1)
   Compute $\int \cos^2(x)\sin(x)dx$.

   Solution:

$$\begin{aligned}\int \cos^2(x)\sin(x)dx &= -\int u^2du \qquad\qquad u=\cos(x)\ \ du=-\sin(x)dx \\ &= -\frac{u^3}{3} + C \\ &= -\frac{\cos^3(x)}{3} + C\end{aligned}$$

4. (trigint:cos1sin2)
   Compute $\int \cos(x)\sin^2(x)dx$.

   Solution:

$$\begin{aligned}\int \cos(x)\sin^2(x)dx &= \int u^2du \qquad\qquad u=\sin(x)\ \ du=\cos(x)dx \\ &= \frac{u^3}{3} + C \\ &= \frac{\sin^3(x)}{3} + C\end{aligned}$$

5. (trigint:cos3)
   Compute $\int \cos^3(x)dx$.

   Solution:

$$
\begin{aligned}
\int \cos^3(x)dx &= \int \cos^2(x)\cos(x)dx \\
&= \int (1-\sin^2(x))\cos(x)dx \quad u = \sin(x) \;\; du = \cos(x)dx \\
&= \int (1-u^2)du \\
&= u - \frac{u^3}{3} + C \\
&= \sin(x) - \frac{\sin^3(x)}{3} + C
\end{aligned}
$$

6. (trigint:sin3)
   Compute $\int \sin^3(x)dx$.

   Solution:

$$
\begin{aligned}
\int \sin^3(x)dx &= \int \sin^2(x)\sin(x)dx \\
&= \int (1-\cos^2(x))\sin(x)dx \quad u = \cos(x) \;\; du = -\sin(x)dx \\
&= -\int (1-u^2)du \\
&= -u + \frac{u^3}{3} + C \\
&= -\cos(x) + \frac{\cos^3(x)}{3} + C
\end{aligned}
$$

7. (trigint:sin4)
   Compute $\int \sin^4(x)dx$.

Solution:

$$\begin{aligned}
\int \sin^4(x)dx &= \int \left(\sin^2(x)\right)^2 dx \\
&= \frac{1}{4}\int (1-\cos(2x))^2\,dx \\
&= \frac{1}{4}\int 1-2\cos(2x)+\cos^2(2x)dx \\
&= \frac{1}{4}\int 1-2\cos(2x)+\frac{1}{2}\left(1+\cos(4x)\right)dx \\
&= \frac{1}{4}x-\frac{1}{4}\sin(2x)+\frac{1}{8}x+\frac{1}{32}\sin(4x)+C \\
&= \frac{3}{8}x-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)+C
\end{aligned}$$

8. (trigint:cos4)
   Compute $\int \cos^4(x)dx$.

   Solution:

$$\begin{aligned}
\int \cos^4(x)dx &= \int \left(\cos^2(x)\right)^2 dx \\
&= \frac{1}{4}\int (1+\cos(2x))^2\,dx \\
&= \frac{1}{4}\int 1+2\cos(2x)+\cos^2(2x)dx \\
&= \frac{1}{4}\int 1+2\cos(2x)+\frac{1}{2}\left(1+\cos(4x)\right)dx \\
&= \frac{1}{4}x+\frac{1}{4}\sin(2x)+\frac{1}{8}x+\frac{1}{32}\sin(4x)+C \\
&= \frac{3}{8}x+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)+C
\end{aligned}$$

9. (trigint:cos2sin2)
   Compute $\int \cos^2(x)\sin^2(x)dx$.

Solution:

$$\begin{aligned}
\int \cos^2(x)\sin^2(x)dx &= \int (\cos(x)\sin(x))^2\, dx \\
&= \int \left(\frac{1}{2}\sin(2x)\right)^2 dx \\
&= \frac{1}{4}\int \sin^2(2x)dx \\
&= \frac{1}{8}\int 1-\cos(4x)dx \\
&= \frac{1}{8}x - \frac{1}{32}\sin(4x) + C
\end{aligned}$$

10. (trigint:cos1sin3)
    Compute $\int \cos(x)\sin^3(x)dx$.

    Solution:

$$\begin{aligned}
\int \cos(x)\sin^3(x)dx &= \int \cos(x)\sin^2(x)\sin(x)dx \\
&= \int \cos(x)\left(1-\cos^2(x)\right)\sin(x)dx \quad u=\cos(x)\ du=-\sin(x)dx \\
&= -\int u(1-u^2)du \\
&= -\frac{u^2}{2} + \frac{u^4}{4} + C \\
&= -\frac{\cos^2(x)}{2} + \frac{\cos^4(x)}{4} + C
\end{aligned}$$

11. (trigint:cos3sin1)
    Compute $\int \cos^3(x)\sin(x)dx$.

Solution:

$$\begin{aligned}
\int \cos^3(x)\sin(x)dx &= \int \sin(x)\cos^2(x)\cos(x)dx \\
&= \int \sin(x)\left(1-\sin^2(x)\right)\cos(x)dx \quad u=\sin(x)\ du=\cos(x)dx \\
&= \int u(1-u^2)du \\
&= \frac{u^2}{2}-\frac{u^4}{4}+C \\
&= \frac{\sin^2(x)}{2}-\frac{\sin^4(x)}{4}+C
\end{aligned}$$

12. (trigint:cos5)
Compute $\int \cos^5(x)dx$.

Solution:

$$\begin{aligned}
\int \cos^5(x)dx &= \int \cos^4(x)\cos(x)dx \\
&= \int \left(\cos^2(x)\right)^2\cos(x)dx \\
&= \int \left(1-\sin^2(x)\right)^2\cos(x)dx \qquad u=\sin(x)\ du=\cos(x)dx \\
&= \int \left(1-u^2\right)^2 du \\
&= \int 1-2u^2+u^4du \\
&= u-\frac{2}{3}u^3+\frac{1}{5}u^5+C \\
&= \sin(x)-\frac{2}{3}\sin^3(x)+\frac{1}{5}\sin^5(x)+C
\end{aligned}$$

13. (trigint:sin5)
Compute $\int \sin^5(x)dx$.

Solution:

$$\begin{aligned}
\int \sin^5(x)dx &= \int \sin^4(x)\sin(x)dx \\
&= \int \left(\sin^2(x)\right)^2 \sin(x)dx \\
&= \int \left(1-\cos^2(x)\right)^2 \sin(x)dx \qquad\qquad u=\cos(x)\ du=-\sin(x)dx \\
&= -\int \left(1-u^2\right)^2 du \\
&= -\int 1-2u^2+u^4du \\
&= -u+\frac{2}{3}u^3-\frac{1}{5}u^5+C \\
&= -\cos(x)+\frac{2}{3}\cos^3(x)-\frac{1}{5}\cos^5(x)+C
\end{aligned}$$

14. (trigint:cos1sin4)
Compute $\int \cos(x)\sin^4(x)dx$

Solution:

$$\begin{aligned}
\int \cos(x)\sin^4(x)dx &= \int u^4du \qquad\qquad u=\sin(x)\ du=\cos(x)dx \\
&= \frac{1}{5}u^5+C \\
&= \frac{1}{5}\sin^5(x)+C
\end{aligned}$$

15. (trigint:cos4sin1)
Compute $\int \cos^4(x)\sin(x)dx$

Solution:

$$\begin{aligned}
\int \cos^4(x)\sin(x)dx &= -\int u^4du \qquad\qquad u=\cos(x)\ du=-\sin(x)dx \\
&= -\frac{1}{5}u^5+C \\
&= -\frac{1}{5}\cos^5(x)+C
\end{aligned}$$

16. (trigint:cos3sin2)
Compute $\int \cos^3(x)\sin^2(x)dx$.

Solution:

$$\begin{aligned}
\int \cos^3(x)\sin^2(x)dx &= \int \sin^2(x)\cos^2(x)\cos(x)dx \\
&= \int \sin^2(x)\left(1-\sin^2(x)\right)\cos(x)dx \quad u=\sin(x)\ du=\cos(x)dx \\
&= \int u^2\left(1-u^2\right)du \\
&= \frac{u^3}{3}-\frac{u^5}{5}+C \\
&= \frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}+C
\end{aligned}$$

17. (trigint:cos2sin3)
Compute $\int \cos^2(x)\sin^3(x)dx$.

Solution:

$$\begin{aligned}
\int \cos^2(x)\sin^3(x)dx &= \int \cos^2(x)\sin^2(x)\sin(x)dx \\
&= \int \cos^2(x)\left(1-\cos^2(x)\right)\sin(x)dx \quad u=\cos(x)\ du=-\sin(x)dx \\
&= -\int u^2\left(1-u^2\right)du \\
&= -\frac{u^3}{3}+\frac{u^5}{5}+C \\
&= -\frac{\cos^3(x)}{3}+\frac{\cos^5(x)}{5}+C
\end{aligned}$$

18. (trigint: sin1sin2)
Compute $\int \sin(x)\sin(2x)dx$.

Solution:

$$\begin{aligned}
\int \sin(x)\sin(2x)dx &= \int \frac{1}{2}\left[\cos(x)-\cos(3x)\right]dx \\
&= \frac{1}{2}\left[\sin(x)-\frac{1}{3}\sin(3x)\right]+C
\end{aligned}$$

19. (trigint:sinnsinm)
Compute $\int \sin(nx)\sin(mx)dx$ where not both of $n, m = 0$. Remember not to divide by zero.

Solution:

$$\int \sin(nx)\sin(mx)dx$$

$$= \int \frac{1}{2}\int \left[\cos\left((n-m)x\right) - \cos\left((n+m)x\right)\right] dx$$

$$= \begin{cases} \frac{1}{2}\left[\frac{1}{n-m}\sin\left((n-m)x\right) - \frac{1}{n+m}\sin\left((n+m)x\right)\right] & \text{if } (n-m),(n+m) \neq 0 \\ \frac{1}{2}\left[x - \frac{1}{n+m}\sin\left((n+m)x\right)\right] & \text{if } n-m=0 \\ \frac{1}{2}\left[\frac{1}{n-m}\sin\left((n-m)x\right) - x\right] & \text{if } n+m=0 \end{cases}$$

20. (trigint:(cos+sin)2)
Compute $\int (\cos(x)+\sin(x))^2\, dx$.

Solution:

$$\begin{aligned} \int (\cos(x)+\sin(x))^2\, dx &= \int \cos^2(x) + 2\sin(x)\cos(x) + \sin^2(x)dx \\ &= \int 1 + \sin(2x)dx \\ &= x - \frac{1}{2}\cos(2x) + C \end{aligned}$$

21. (trigint:sec1)
Compute $\int \sec(x)dx$ with the substitution $u = \sec(x)+\tan(x)$.

Solution:

$$\begin{aligned} \int \sec(x)dx &= \int \sec(x)\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}dx & u &= \sec(x)+\tan(x) \\ &= \int \frac{du}{u} & du &= (\sec(x)\tan(x)+\sec^2(x))dx \\ &= \ln|u| + C \\ &= \ln|\sec(x)+\tan(x)| + C \end{aligned}$$

22. (trigint:sec1tan1)
Compute $\int \sec(x)\tan(x)dx$.

Solution:

$$\begin{aligned} \int \sec(x)\tan(x)dx &= \int du \qquad u = \sec(x) \quad du = \sec(x)\tan(x)dx \\ &= u + C \\ &= \sec(x) + C \end{aligned}$$

23. (trigint:sec2tan1)
Compute $\int \sec^2(x)\tan(x)dx$.

Solution:

$$\begin{aligned}
\int \sec^2(x)\tan(x)dx &= \int u du & u &= \sec(x) \\
&= \frac{u^2}{2} + C & du &= \sec(x)\tan(x)dx \\
&= \frac{1}{2}\sec^2(x) + C
\end{aligned}$$

24. (trigint:sec3)
Compute $\int \sec^3(x)dx$.

Solution:

$$\begin{aligned}
\int \sec^3(x)dx &= \int \underbrace{\sec(x)}_{\text{F(x)}}\underbrace{\sec^2(x)}_{\text{G'(x)}} dx \\
&= \underbrace{\sec(x)}_{\text{F(x)}}\underbrace{\tan(x)}_{\text{G(x)}} - \int \underbrace{\sec(x)\tan(x)}_{\text{F'(x)}}\underbrace{\tan(x)}_{\text{G(x)}} dx \\
&= \sec(x)\tan(x) - \int \sec(x)(\sec^2(x) - 1)dx
\end{aligned}$$

where we have used that $\tan^2(x) = \sec^2(x) - 1$ in the last line. Recognizing that $\int \sec^3(x)dx$ appears on both sides of this equality, we obtain

$$\begin{aligned}
2\int \sec^3(x)dx &= \sec(x)\tan(x) + \int \sec(x)dx \\
\int \sec^3(x)dx &= \frac{1}{2}\left(\sec(x)\tan(x) + \ln|\sec(x) + \tan(x)|\right) + C
\end{aligned}$$