

1. (vectorprob:nonsense)

Let $\vec{\mathbf{a}} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$, $\vec{\mathbf{b}} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$ and $\vec{\mathbf{c}} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$. Which of the following expressions are nonsense? Evaluate the sensible ones.

- (a) $3\vec{\mathbf{a}} + \vec{\mathbf{b}}$
- (b) $\vec{\mathbf{a}} + \vec{\mathbf{c}}$
- (c) $\vec{\mathbf{a}} \cdot \vec{\mathbf{c}}$
- (d) $\vec{\mathbf{a}} - 2\vec{\mathbf{b}}$
- (e) $t\vec{\mathbf{a}}$ where t is a real number.
- (f) $\vec{\mathbf{a}}\vec{\mathbf{b}}$
- (g) $\vec{\mathbf{a}} + 5$

2. (vectorprob:basislinear)

Let $\vec{\mathbf{a}} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and $\vec{\mathbf{b}} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Find s and t so that $\begin{pmatrix} 3 \\ 5 \end{pmatrix} = s\vec{\mathbf{a}} + t\vec{\mathbf{b}}$.

3. (vectorprob:orbits)

Planet X rotates around a super-massive black hole, and a satellite rotates around planet X. The black hole is a lot more massive than planet X, which is a lot more massive than the satellite, so their orbits are nearly circular. Planet X orbits the black hole at a constant radius of 10^9 miles and goes around twice in a year. The satellite orbits planet X in the same plane, at a constant radius of 10^3 miles, and goes around 100 times in a year. Taking the center of the black hole as the origin, parametrize the position $\vec{\mathbf{p}}(t)$ of the satellite.

4. (vectorprob:projquiz1)

Let $\vec{\mathbf{a}} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\vec{\mathbf{b}} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Find $\vec{\mathbf{a}}^{\parallel}$ and $\vec{\mathbf{a}}^{\perp}$ so that $\vec{\mathbf{a}} = \vec{\mathbf{a}}^{\parallel} + \vec{\mathbf{a}}^{\perp}$, where $\vec{\mathbf{a}}^{\parallel}$ is parallel to $\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}^{\perp}$ is perpendicular to $\vec{\mathbf{b}}$.

5. (vectorprob:projquiz2)

Let $\vec{\mathbf{a}} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$ and $\vec{\mathbf{b}} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Find $\vec{\mathbf{a}}^{\parallel}$ and $\vec{\mathbf{a}}^{\perp}$ so that $\vec{\mathbf{a}} = \vec{\mathbf{a}}^{\parallel} + \vec{\mathbf{a}}^{\perp}$, where $\vec{\mathbf{a}}^{\parallel}$ is parallel to $\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}^{\perp}$ is perpendicular to $\vec{\mathbf{b}}$.

6. (vectorprob:proj1)

Let $\vec{\mathbf{a}} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ and $\vec{\mathbf{b}} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$. Find $\vec{\mathbf{a}}^{\parallel}$ and $\vec{\mathbf{a}}^{\perp}$ so that $\vec{\mathbf{a}} = \vec{\mathbf{a}}^{\parallel} + \vec{\mathbf{a}}^{\perp}$, where $\vec{\mathbf{a}}^{\parallel}$ is parallel to $\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}^{\perp}$ is perpendicular to $\vec{\mathbf{b}}$.

7. (vectorprob:param1)

Find a parametric equation for the line that passes through the points $A = (1, 0, 2)$ and $B = (3, 1, 4)$.

8. (vectorprob:word1)

Suppose that a merchant sells three types of goods in quantities q_1 , q_2 , and q_3 and that the merchant sells these goods at prices p_1 , p_2 , and p_3 dollars per unit respectively. Suppose further that it costs the merchant c_i dollars to make one unit of the i^{th} good. If

$$\vec{\mathbf{q}} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \quad \vec{\mathbf{p}} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \quad \vec{\mathbf{c}} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

then what is the significance of the quantity

$$\vec{\mathbf{q}} \cdot (\vec{\mathbf{p}} - \vec{\mathbf{c}})?$$

Describe in words why we the merchant cares if this quantity is positive or negative.

9. (vectorprob:plane1)

Does the plane containing the points $A = (1, 0, 0)$, $B = (0, 1, 0)$, and $C = (0, 0, 1)$ also contain the point $(1, 1, 1)$?