1. (intbyparts:arcsine)
   Compute $\int \arcsin(x)dx$.

   Solution:

$$\begin{aligned}\int \arcsin(x)dx &= \int \underbrace{\arcsin(x)}_{\text{F(x)}} \underbrace{dx}_{\text{G'(x)dx}} \\ &= \underbrace{x}_{\text{G(x)}} \underbrace{\arcsin(x)}_{\text{F(x)}} - \int \underbrace{x}_{\text{G(x)}} \underbrace{\frac{1}{\sqrt{1-x^2}}dx}_{\text{F'(x)dx}} \\ &= x \arcsin(x) + \sqrt{1-x^2} + C\end{aligned}$$

2. (intbyparts:ln)
   Compute $\int \ln(x)dx$

   Solution:

$$\begin{aligned}\int \ln(x)dx &= \int \underbrace{\ln(x)}_{\text{F(x)}} \underbrace{dx}_{\text{G'(x)dx}} \\ &= \underbrace{x}_{\text{G(x)}} \underbrace{\ln(x)}_{\text{F(x)}} - \int \underbrace{x}_{\text{G(x)}} \underbrace{\frac{1}{x}dx}_{\text{F'(x)dx}} \\ &= x \ln(x) - x + C\end{aligned}$$

3. (intbyparts:arccos)
   Compute $\int \arccos(x)dx$.

   Solution:

$$\begin{aligned}\int \arccos(x)dx &= \int \underbrace{\arccos(x)}_{\text{F(x)}} \underbrace{dx}_{\text{G'(x)dx}} \\ &= \underbrace{x}_{\text{G(x)}} \underbrace{\arccos(x)}_{\text{F(x)}} - \int \underbrace{x}_{\text{G(x)}} \underbrace{\frac{-1}{\sqrt{1-x^2}}dx}_{\text{F'(x)dx}} \\ &= x \arccos(x) - \sqrt{1-x^2} + C\end{aligned}$$

4. (intbyparts:arctan)
   Compute $\int \arctan(x)dx$.

Solution:

$$\begin{aligned}
\int \arctan(x)dx &= \int \underbrace{\arctan(x)}_{\text{F(x)}} \underbrace{dx}_{\text{G'(x)dx}} \\
&= \underbrace{x}_{\text{G(x)}} \underbrace{\arctan(x)}_{\text{F(x)}} - \int \underbrace{x}_{\text{G(x)}} \underbrace{\frac{1}{1+x^2} dx}_{\text{F'(x)dx}} \\
&= x \arcsin(x) - \frac{1}{2} \ln |1 + x^2| + C
\end{aligned}$$

5. (intbyparts:sec3)
   Compute $\int \sec^3(x)dx$.

   Solution:

$$\begin{aligned}
\int sec^3(x)dx &= \int \underbrace{\sec(x)}_{\text{F(x)}} \underbrace{sec^2(x)dx}_{\text{G'(x)dx}} \\
&= \underbrace{sec(x)}_{\text{F(x)}} \underbrace{\tan(x)}_{\text{G(x)}} - \int \underbrace{\tan(x)}_{\text{G(x)}} \underbrace{\tan(x)\sec(x)dx}_{\text{F'(x)dx}} \\
&= \sec(x)\tan(x) - \int \tan^2(x)\sec(x)dx \\
&= \sec(x)\tan(x) - \int \big(\sec^2(x) - 1\big)\sec(x)dx \\
2\int \sec^3(x)dx &= \sec(x)\tan(x) - \int \sec(x)dx \\
&= \sec(x)\tan(x) - \ln|\sec(x) + \tan(x)| + C \\
\int \sec^3(x) &= \frac{1}{2}\sec(x)\tan(x) - \frac{1}{2}\ln|\sec(x) + \tan(x)| + C
\end{aligned}$$

6. (intbyparts:xnlog)
   Let $n \neq -1$ and compute $\int x^n \ln(x)dx$.

Solution:

$$\begin{aligned}\int x^n \ln(x)dx &= \int \underbrace{\ln(x)}_{\text{F(x)}} \underbrace{x^n dx}_{\text{G'(x)dx}} \\ &= \underbrace{\frac{1}{n+1}x^{n+1}}_{\text{G(x)}} \underbrace{\ln(x)}_{\text{F(x)}} - \int \underbrace{\frac{1}{n+1}x^{n+1}}_{\text{G(x)}} \underbrace{\frac{1}{x}dx}_{\text{F'(x)dx}} \\ &= \frac{1}{n+1}x^{n+1}\ln(x) - \frac{1}{(n+1)^2}x^{n+1} + C\end{aligned}$$

7. (intbyparts:gamma)
   For $x > 0$, call $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$. Show that $\Gamma(x+1) = x\Gamma(x)$.

   Solution:

$$\begin{aligned}\Gamma(x+1) &= \int_0^\infty \underbrace{t^x}_{\text{F(x)}} \underbrace{e^{-t}dt}_{\text{G'(x)dx}} \\ &= \underbrace{t^x}_{\text{F(x)}} \underbrace{-e^{-t}}_{\text{G(x)}} |_0^\infty + \int_0^\infty \underbrace{xt^{x-1}}_{\text{F'(x)}} \underbrace{e^{-t}dt}_{\text{G'(x)dx}} \\ &= x\Gamma(x)\end{aligned}$$

8. (intbyparts:taylorfo)
   Suppose that h is twice continuously differentiable. Use integration by parts and the fundamental theorem of calculus to show that

$$h(x) = h(0) + h^{'}(0)x + \int_0^x (x-t)h^{''}(t)dt$$

   Solution:

$$\begin{aligned}h(x) &= h(0) + \int_0^x \underbrace{h^{'}(t)}_{\text{F(t)}} \underbrace{dt}_{\text{G'(t)dt}} \\ &= h(0) + \underbrace{h^{'}(x)}_{\text{F(x)}} \underbrace{x}_{\text{G(x)}} - \int_0^x th^{''}(t)dt \\ &= h(0) + x\left(h^{'}(0) + \int_0^x h^{''}(t)dt\right) - \int_0^x th^{''}(t)dt \\ &= h(0) + h^{'}(0)x + \int_0^x (x-t)h^{''}(t)dt\end{aligned}$$

9. (intbyparts:expasinb)
   Compute $\int e^{ax}\sin(bx)dx$ where $a, b \neq 0$.

   Solution:

$$\begin{aligned}
\int e^{ax}\sin(bx)dx &= \int \underbrace{e^{ax}}_{\text{F(x)}}\underbrace{\sin(bx)dx}_{\text{G'(x)dx}} \\
&= \underbrace{e^{ax}}_{\text{F(x)}}\underbrace{\frac{-1}{b}\cos(bx)}_{\text{G(x)}} - \int \underbrace{\frac{-1}{b}\cos(bx)}_{\text{G(x)}}\underbrace{ae^{ax}dx}_{\text{F'(x)dx}} \\
&= \frac{-1}{b}e^{ax}\cos(b(x)) + \frac{a}{b}\int \underbrace{e^{ax}}_{\text{F(x)}}\underbrace{\cos(bx)dx}_{\text{G'(x)dx}} \\
&= -\frac{1}{b}e^{ax}\cos(b(x)) + \frac{a}{b}\left[\underbrace{e^{ax}}_{\text{F(x)}}\underbrace{\frac{1}{b}\sin(bx)}_{\text{G(x)}} - \int \underbrace{\frac{1}{b}\sin(bx)}_{\text{G(x)}}\underbrace{ae^{ax}dx}_{\text{F'(x)dx}}\right] \\
&= -\frac{1}{b}e^{ax}\cos(bx) + \frac{a}{b^2}e^{ax}\sin(bx) - \left(\frac{a}{b}\right)^2\int e^{ax}\sin(bx)dx \\
\left(1+\left(\frac{a}{b}\right)^2\right)\int e^{ax}\sin(bx)dx &= -\frac{1}{b}e^{ax}\cos(bx) + \frac{a}{b^2}e^{ax}\sin(bx) \\
\int e^{ax}\sin(bx)dx &= \frac{1}{1+\left(\frac{a}{b}\right)^2}\left(\frac{a}{b^2}e^{ax}\sin(bx) - \frac{1}{b}e^{ax}\cos(bx)\right)
\end{aligned}$$

10. (intbyparts:expacosb)
    Compute $\int e^{ax}\cos(bx)dx$ where $a, b \neq= 0$.

Solution:

$$\begin{aligned}
\int e^{ax}\cos(bx)dx &= \int \underbrace{e^{ax}}_{\text{F(x)}}\underbrace{\cos(bx)dx}_{\text{G'(x)dx}} \\
&= \underbrace{e^{ax}}_{\text{F(x)}}\underbrace{\frac{1}{b}\sin(bx)}_{\text{G(x)}} - \int \underbrace{\frac{1}{b}\sin(bx)}_{\text{G(x)}}\underbrace{ae^{ax}dx}_{\text{F'(x)dx}} \\
&= \frac{1}{b}e^{ax}\sin(b(x)) - \frac{a}{b}\int \underbrace{e^{ax}}_{\text{F(x)}}\underbrace{\sin(bx)dx}_{\text{G'(x)dx}} \\
&= \frac{1}{b}e^{ax}\sin(b(x)) - \frac{a}{b}\left[\underbrace{e^{ax}}_{\text{F(x)}}\underbrace{\frac{-1}{b}\cos(bx)}_{\text{G(x)}} - \int \underbrace{\frac{-1}{b}\cos(bx)}_{\text{G(x)}}\underbrace{ae^{ax}dx}_{\text{F'(x)dx}}\right] \\
&= \frac{1}{b}e^{ax}\sin(bx) + \frac{a}{b^2}e^{ax}\cos(bx) - \left(\frac{a}{b}\right)^2\int e^{ax}\cos(bx)dx \\
\left(1+\left(\frac{a}{b}\right)^2\right)\int e^{ax}\cos(bx)dx &= \frac{1}{b}e^{ax}\sin(bx) + \frac{a}{b^2}e^{ax}\cos(bx) \\
\int e^{ax}\cos(bx)dx &= \frac{1}{1+\left(\frac{a}{b}\right)^2}\left(\frac{a}{b^2}e^{ax}\sin(bx) + \frac{1}{b}e^{ax}\cos(bx)\right)
\end{aligned}$$

11. (intbyparts:xmlnn)
Assuming that $m \neq -1$, show that

$$\int x^m (\ln(x))^n \, dx = \frac{1}{m+1}x^{m+1}(\ln(x))^n - \frac{n}{m+1}\int x^m (\ln(x))^{n-1}\, dx$$

Solution:

$$\begin{aligned}
\int x^m (\ln(x))^n \, dx &= \int \underbrace{(\ln(x))^n}_{\text{F(x)}}\underbrace{x^m dx}_{\text{G'(x)dx}} \\
&= \underbrace{\frac{1}{m+1}x^{m+1}}_{\text{G(x)}}\underbrace{(\ln(x))^n}_{\text{F(x)}} - \int \underbrace{\frac{1}{m+1}x^{m+1}}_{\text{G(x)}}\underbrace{n\frac{1}{x}(\ln(x))^{n-1}\,dx}_{\text{F'(x)dx}} \\
&= \frac{1}{m+1}x^{m+1}(\ln(x))^n - \frac{n}{m+1}\int x^m (\ln(x))^{n-1}\,dx
\end{aligned}$$

12. (intbyparts:xln)
Compute $\int x\ln(x)dx$.

Solution:

$$\begin{aligned}
\int x\ln(x)dx &= \int \underbrace{\ln(x)}_{\text{F(x)}}\ \underbrace{xdx}_{\text{G'(x)dx}} \\
&= \underbrace{\frac{x^2}{2}}_{\text{G(x)}}\underbrace{\ln(x)}_{\text{F(x)}} - \int \underbrace{\frac{x^2}{2}}_{\text{G(x)}}\ \underbrace{\frac{1}{x}dx}_{\text{F'(x)dx}} \\
&= \frac{1}{2}x^2\ln(x) - \frac{1}{2}\int xdx \\
&= \frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + C
\end{aligned}$$

13. (intbyparts:xex)
    Compute $\int xe^x dx$.

    Solution:

$$\begin{aligned}
\int xe^x dx &= \int \underbrace{x}_{\text{F(x)}}\ \underbrace{e^x dx}_{\text{G'(x) dx}} \\
&= \underbrace{x}_{\text{F(x)}}\underbrace{e^x}_{\text{G(x)}} - \int \underbrace{e^x}_{\text{G(x)}}\ \underbrace{dx}_{\text{F'(x)dx}} \\
&= xe^x - e^x + C
\end{aligned}$$

14. (intbyparts:exsin)
    Compute $\int e^x \sin(x)dx$.

Solution:

$$\begin{aligned}
\int e^x \sin(x)dx &= \int \underbrace{e^x}_{\text{F(x)}} \underbrace{\sin(x)dx}_{\text{G'(x)dx}} \\
&= \underbrace{e^x}_{\text{F(x)}} \underbrace{(-\cos(x))}_{\text{G(x)}} - \int \underbrace{-\cos(x)}_{\text{G(x)}} \underbrace{e^x dx}_{\text{F'(x)dx}} + C \\
&= -e^x \cos(x) + \int e^x \cos(x)dx + C \\
&= -e^x \cos(x) + \int \underbrace{e^x}_{\text{F(x)}} \underbrace{\cos(x)dx}_{\text{G'(x)dx}} + C \\
&= -e^x \cos(x) + \left[\underbrace{e^x}_{\text{F(x)}} \underbrace{\sin(x)}_{\text{G(x)}} - \int \underbrace{\sin(x)}_{\text{G(x)}} \underbrace{e^x dx}_{\text{F'(x)dx}} \right] + C \\
&= e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x)dx + C \\
2\int e^x \sin(x)dx &= e^x \left(\sin(x) - \cos(x)\right) + C \\
&= e^x \left(\sin(x) - \cos(x)\right) + C
\end{aligned}$$

15. (intbyparts:definite1)
Compute $\int_0^1 \ln(2t+1)dt$.

Solution:

$$\begin{aligned}
\int_0^1 \ln(2t+1)dt &= \frac{1}{2}\int_{t=0}^{t=1} \underbrace{\ln(u)}_{\text{F(u)}} \underbrace{du}_{\text{G'(u)du}} \qquad\qquad u = 2t+1 \quad \frac{1}{2}du = dt \\
&= \underbrace{u}_{\text{G(u)}} \underbrace{\ln(u)}_{\text{F(u)}} \Big|_{t=0}^{t=1} - \int_{t=0}^{t=1} \underbrace{u}_{\text{G(u)}} \underbrace{\frac{1}{u}du}_{\text{F'(u)du}}
\end{aligned}$$