Worksheet 11

Lagrange's Error Estimation

Differential Equations

Problem 1. Find a solution to the initial value problem: $\frac{dy}{dx} = \left(1 + y^2\right) e^x \qquad y(0) = 0$

Problem 2. Find a solution to the initial value problem: $\frac{dy}{dx} = \left(y^2 - 1\right) \cos(x) \qquad y(0) = 1$

Problem 3. Find a solution to the initial value problem: $x\frac{dy}{dx} + 2y = -\frac{\sin(x)}{x} \qquad y\left(\frac{\pi}{2}\right) = 1$

Problem 4. Find the general solution to the differential equation: $\dfrac{1}{2x}\dfrac{dy}{dx} = y + e^{x^2}$.

Problem 5. Find a particular solution to the initial value problem: $\dfrac{1+x^3}{3x^2}\dfrac{dy}{dx} = 1 - y(x)$.

Problem 6. Find the exact solution to the initial value problem $\dfrac{dy}{dx} = -2xy$ where $y(0) = 1$. Then use Euler's method with step size $\Delta x = 0.1$ to estimate $y(0.3)$.

Problem 7. A 100 litre vat of water begins with an algae concentration of 1,000 organisms per litre. Suppose that the algae naturally reproduce at a rate of five percent per minute and die at a rate of four percent per minute. If the vat is being drained at a rate of one litre per minute, what will the algae concentration be ten minutes from now? You should assume that the algae are uniformly distributed in the vat. Remember to define your variables with units.

Problem 8. A tank with 200 gallons of brine solution contains 40 lbs of salt. A concentration of 2 lb/gal is pumped in at a rate of 4 gal/min. The concentration leaving the tank is pumped out at a rate of 4 gal/min. Find a differential equation describing the amount of salt in the tank at time t.

Problem 9. The rate at which a certain drug is eliminated from the bloodstream is proportional to the amount of the drug in the bloodstream. A patient now has 45 mg of the drug in his bloodstream. The drug is being administered to the patient intravenously at a constant rate of 5 milligrams per hour. Write a differential equation modeling the situation.

Problem 10. In the absence of predators the population of mosquitoes in a certain area would increase at a rate proportional to its current population and its population would double every three weeks. There are 250,000 mosquitoes in the area initially when a flock of birds arrives that eats 80,000 mosquitoes per week. How many mosquitoes remain after two weeks?

Taylor Polynomials/Series

Problem 11. For $f(x) = x^4 + x + 1$, find the degree three Taylor polynomial of $f(x)$ around 1. That is, find $T_3^1 f(x)$.

Problem 12. Hadrubal has designed a rocket. While proving mathematically that it won't ex