

Worksheet 10

Taylor Polynomials

Definition: Taylor Polynomials

The **Taylor polynomial** of a function $y = f(x)$ of degree n at a point a is the polynomial:

$$T_n^a f(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x - a)^n$$

The **Taylor series** of a function $y = f(x)$ centered at $a = 0$ is the infinite sum:

$$T_\infty f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$$

Calculate the derivatives of the following functions and look for a pattern. Use this pattern to calculate the *Taylor series* for the following functions.

Problem 1. $\sin 3x$

Problem 2. $\cosh(2x)$

Recall: $\cosh(2x) = \frac{1}{2}(e^{2x} + e^{-2x})$

Problem 3. $\sqrt{1+x}$

Problem 4. $\ln \frac{1-t}{1+t}$