1.1 Comment on how much the two values differ.
The values differ by this much: 0.0000000000000000260208521396521, which is a small value, however relative to the very small numbers inputted, this is a relatively large difference.

1.2,1.3 Explain why they differ, which result do you think is more accurate

The values differ due to different floating point errors.

For the difference of squares, the squaring requires double the number of bits than the previous ones. This means when we square both values, we lose a lot of accuracy due to roundoff error (i.e. two multiplications thus approx. need 4 times the amount of bits available)

However for the other way (x-y)(x+y) we only multiply once (which only needs double the amount of available bits) , this means we do not have as bad of a truncation error, thus this way is the better way for calculation.

1.4 What is the relative error in z

0.1118215802998748

1.5 Explain what causes the relative error

When subtraction of two floats happens, catastrophic cancellation in subtraction can occur causing many of the least significant digits to be rounded out. This means when the two numbers get subtracted off each other when computing the relative error, we get cancellation of many digits, leading to large relative errors.

$$\frac{dy}{dt} + 5y = 2e^{-5t}$$

$$\lambda + 5 = 0$$
$$\lambda = -5$$
$$y_c = Ce^{-5t}$$

$$y_p = Ate^{-5t}$$
$$y_p' = Ae^{-5t} - 5Ate^{-5t}$$

$$Ae^{-5t} - 5Ate^{-5t} + 5Ate^{-5t} = 2e^{-5t}$$
$$A = 2$$

$$y = 2te^{-5t} + Ce^{-5t}$$
$$4 = 2(0) + Ce^{-5(0)}$$
$$C = 4$$

$$y = 2te^{-5t} + 4e^{-5t}$$

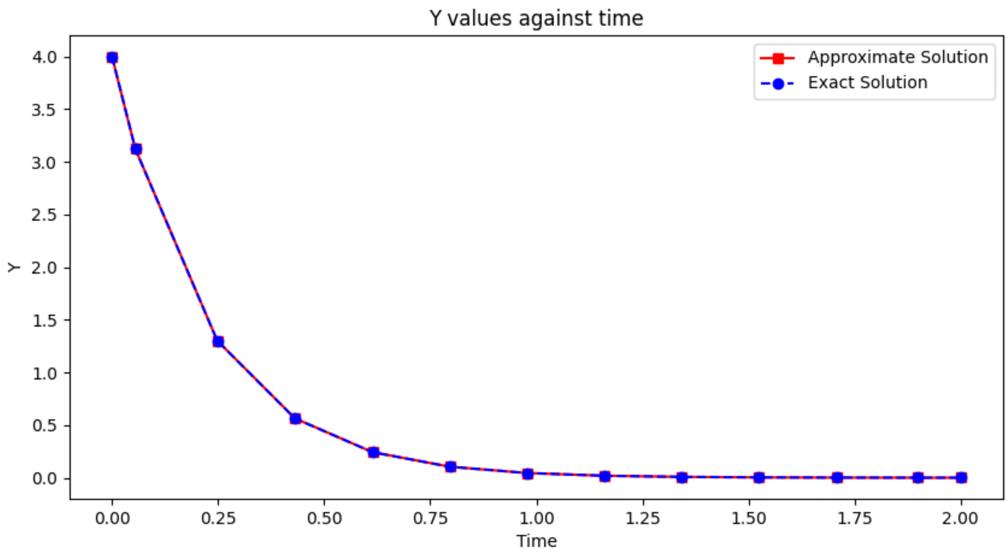
2.1 What is the value of MAE. Based on this and a visual comparison of the exact and approximate solutions, comment on how accurate this solution is.

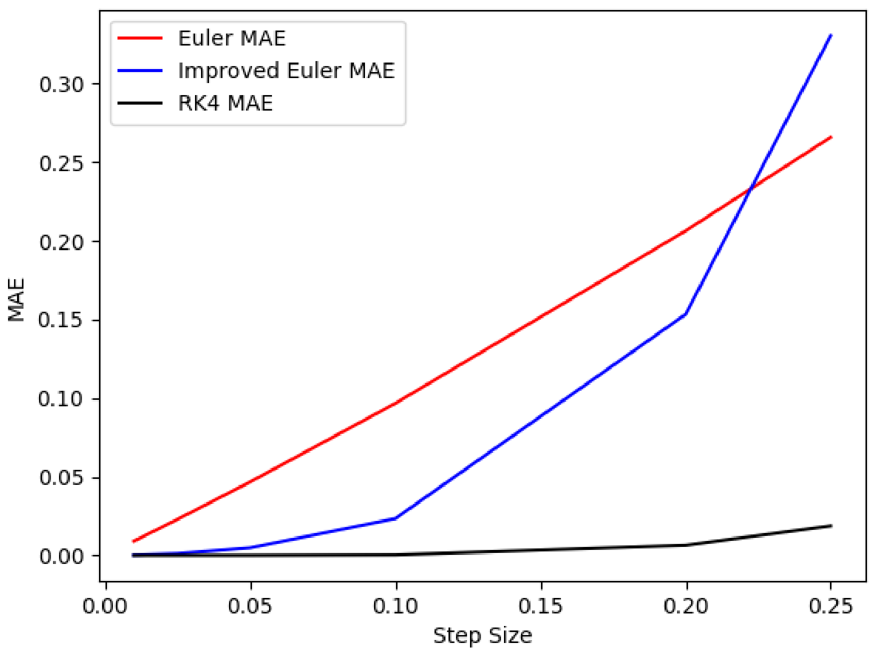
The value of MAE is 0.00019409958501967838. Based on this we can conclude that the numerical solution is very accurate, and upon looking at the graph, it is hard to distinguish the two lines apart, visually affirming the numerical MAE value.

2.2 Note that the calculation of MAE may itself be subject to numerical error, regardless of the accuracy of the approximate solution. Briefly comment on two ways that numerical error may be incurred.

We continuously subtract numbers which are very close together, this leads to catastrophic cancellation in subtraction causing many of the least significant digits to be rounded out. Over multiple sets of values, this cancellation of digits adds up and ends up creating quite a large error relative to what it should really be.

Also when we divide(i.e. multiply by an inverse) the sum of the all the differences by however many values there were passed in, it usually requires double the number of significand bits to store the result, thus we lose a lot of precision when we divide the number due to truncation error.



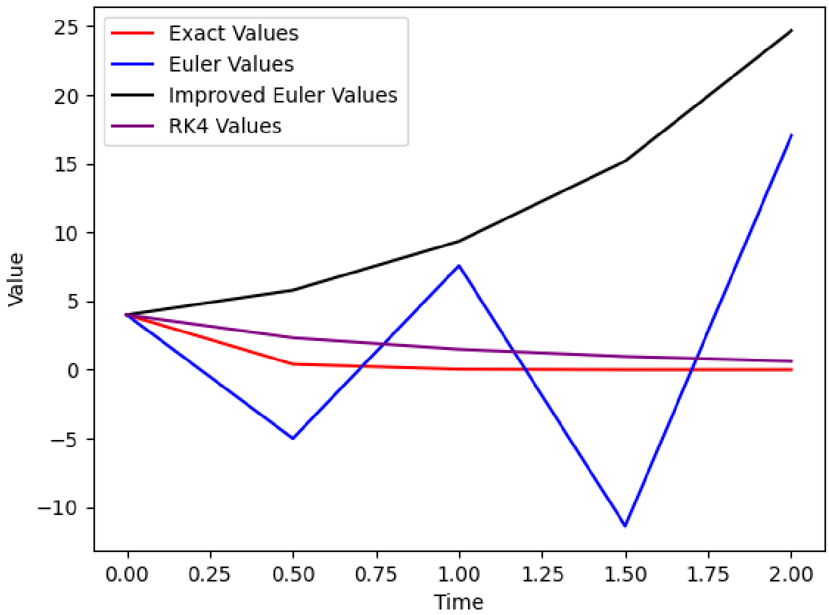


3.1 Estimate from your plot the proportionality relationship between MAE and h for the Euler method. Comment on how this compares to the expected truncation error of the method.

By looking at the plot, the relationship between MAE and h is approximately linear. This makes sense as the global truncation error for the euler method is linearly proportional to h

3.2 Comment on an advantage and a disadvantage of using the Classic RK4 method over the Euler or Improved Euler methods.

The classic RK4 method is a lot more accurate than the Euler methods as it is a fourth order method, and has a global truncation error of O(h^4) (assuming step sizes much smaller than 1). However to achieve this accuracy, it requires a lot more computation, thus one disadvantage of the RK4 method is it takes a lot longer to run in comparison to the other two methods.



3.3 Comment on the numerical stability of each method. How does this impact on the behaviour of the numerical error in each case?

Using the graph of of the values at h=0.5, the RK4 method lies close to the exact solution through the time span, thus is stable. The Euler method fluctuates around the exact solution, making it unstable for that step size and the improved Eulers method looks like it exponentially grows with time rather than converging to a value, making it unstable.

These two unstable Euler methods will cause numerical error to dominate over the solution, leading to drastically more inaccurate results. However the RK4 method is numerical stable, thus less likely to encounter large numerical error

3.4 In the context that we are unable to mathematically determine the stability condition for a given explicit RK method and ODE, briefly comment on how might we determine numerically whether or not the solution is numerically stable?

Hint: you may want to look at the MAE for h = 0.5 in addition to the previously evaluated steps.

Numerical method will tend to converge to some value given that the step size is small enough. By plotting the values of the approximate y values against time for larger and larger values of h until we find a solution which no longer converges to one value, rather diverges off. Just before this point will be the stability condition