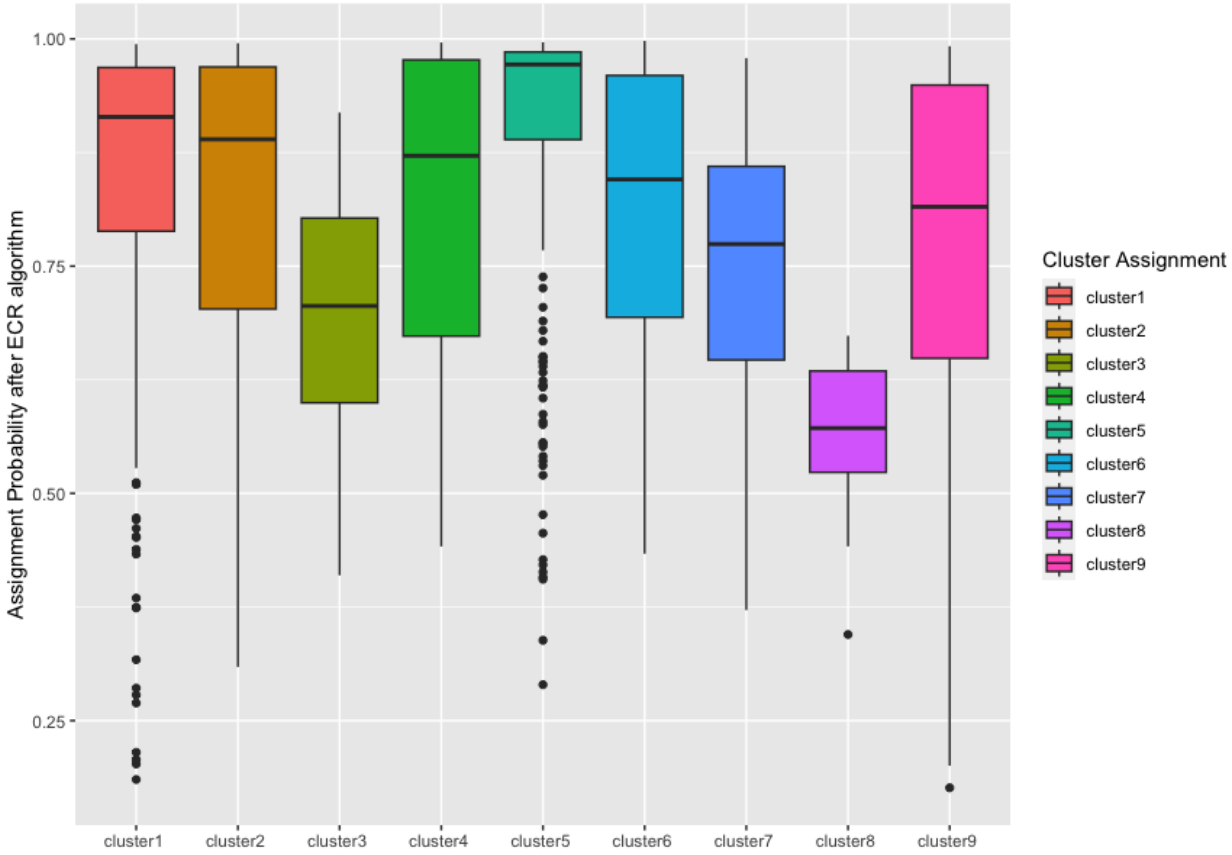
1. Heating vectors (Set 1)

ACS 2006-2010

```
* Run information:
      Number of parallel heated chains: 4
      Swap acceptance rate: 54%
      Total number of iterations: 15000
      Burn-in period: 5000
      Thinning: 10.

* Estimated posterior distribution of the number of clusters:

    8     9    10    11    12    13    14
0.156 0.428 0.256 0.100 0.043 0.013 0.004

* Most probable model: K = 9 with P(K = 9|data) = 0.428

* Estimated number of observations per cluster conditionally on K = 9 (3 label switching
algorithms):
  STEPHENS ECR ECR.ITERATIVE.1
1      324 324             324
2      167 167             167
3      161 152             161
4      145 145             145
5      209 209             209
6      135 135             135
7       73  73              73
8       22  28              22
9      242 245             242
```

Mixing weights

	Mean	SD	2.5%	97.5%
p.1	0.2043	0.0175	0.1712	0.2404
p.2	0.1054	0.0140	0.0774	0.1322
p.3	0.1126	0.0392	0.0000	0.1666
p.4	0.1003	0.0132	0.0754	0.1266
p.5	0.1380	0.0136	0.1098	0.1636
p.6	0.1005	0.0146	0.0721	0.1315
p.7	0.0530	0.0107	0.0345	0.0741
p.8	0.0284	0.0227	0.0000	0.0829
p.9	0.1573	0.0319	0.1044	0.2212



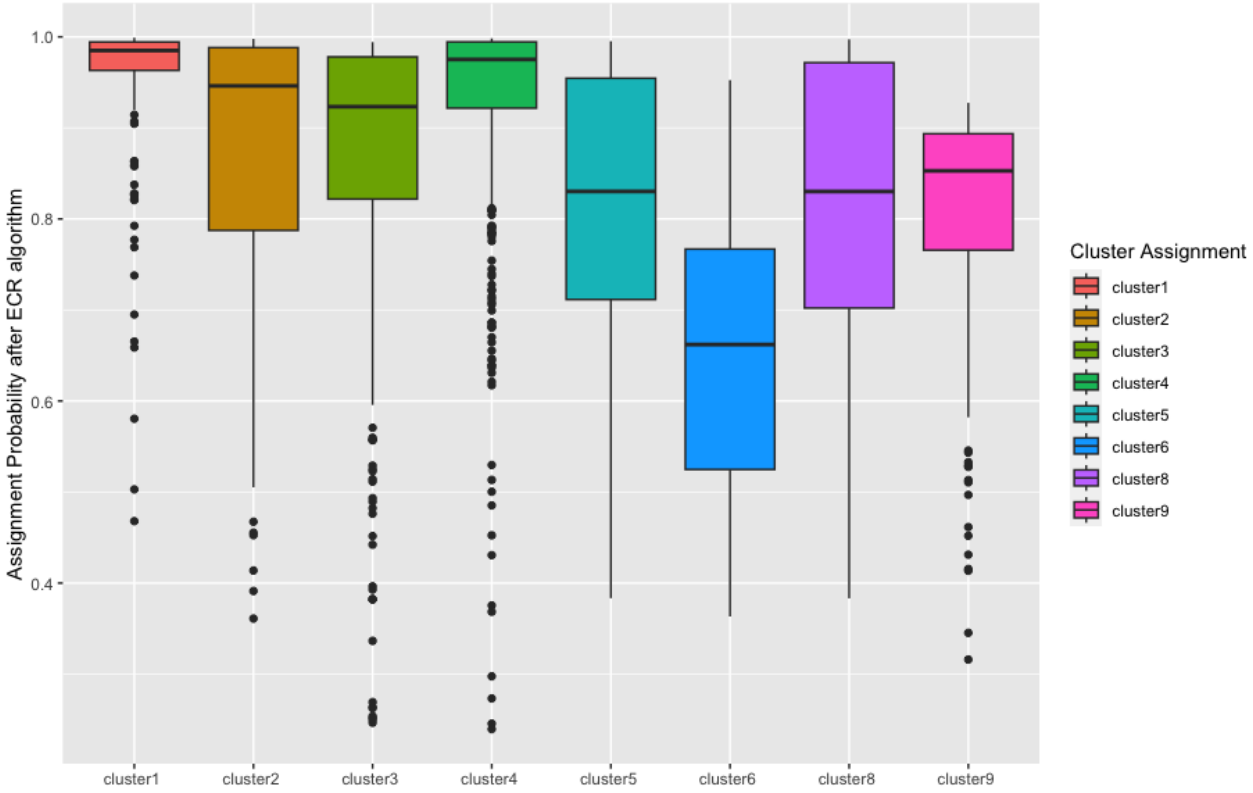
	cluster1 (N=324)	cluster2 (N=167)	cluster3 (N=152)	cluster4 (N=145)	cluster5 (N=209)	cluster6 (N=135)	cluster7 (N=73)	cluster8 (N=28)	cluster9 (N=245)
Average Assignment Probability (ECR Algorithm)									
Mean (SD)	0.841 (0.174)	0.828 (0.170)	0.704 (0.130)	0.815 (0.176)	0.891 (0.161)	0.817 (0.156)	0.744 (0.153)	0.566 (0.0796)	0.785 (0.191)
Median [Min, Max]	0.914 [0.185, 0.994]	0.890 [0.309, 0.995]	0.706 [0.410, 0.919]	0.871 [0.442, 0.996]	0.972 [0.290, 0.996]	0.845 [0.434, 0.998]	0.774 [0.372, 0.979]	0.572 [0.345, 0.673]	0.815 [0.176, 0.992]

ACS 2011-2015

```
* Run information:
      Number of parallel heated chains: 4
      Swap acceptance rate: 56.8%
      Total number of iterations: 15000
      Burn-in period: 5000
      Thinning: 10.

* Estimated posterior distribution of the number of clusters:

    7     8     9    10    11    12
0.088 0.328 0.343 0.174 0.049 0.018

* Most probable model: K = 9 with P(K = 9|data) = 0.343

* Estimated number of observations per cluster conditionally on K = 9 (3 label switching algorithms):
  STEPHENS ECR ECR.ITERATIVE.1
1      181 181             181
2      156 156             154
3      315 315             321
4      406 406             406
5      165 165             165
6       96  96              92
8       45  45              45
9      114 114             114
```

*No observations in cluster 7

	Mean	SD	2.5%	97.5%
p.1	0.1292183	0.0175580	0.1037124	0.1893165
p.2	0.1035116	0.0112210	0.0823009	0.1255278
p.3	0.1984312	0.0178423	0.1662735	0.2365589
p.4	0.2639759	0.0151615	0.2333685	0.2933483
p.5	0.1124291	0.0156044	0.0857048	0.1462296
p.6	0.0715279	0.0220546	0.0000000	0.1154052
p.7	0.0108511	0.0183613	0.0000000	0.0562619
p.8	0.0397064	0.0108644	0.0225403	0.0666677
p.9	0.0703486	0.0209471	0.0000000	0.0957974

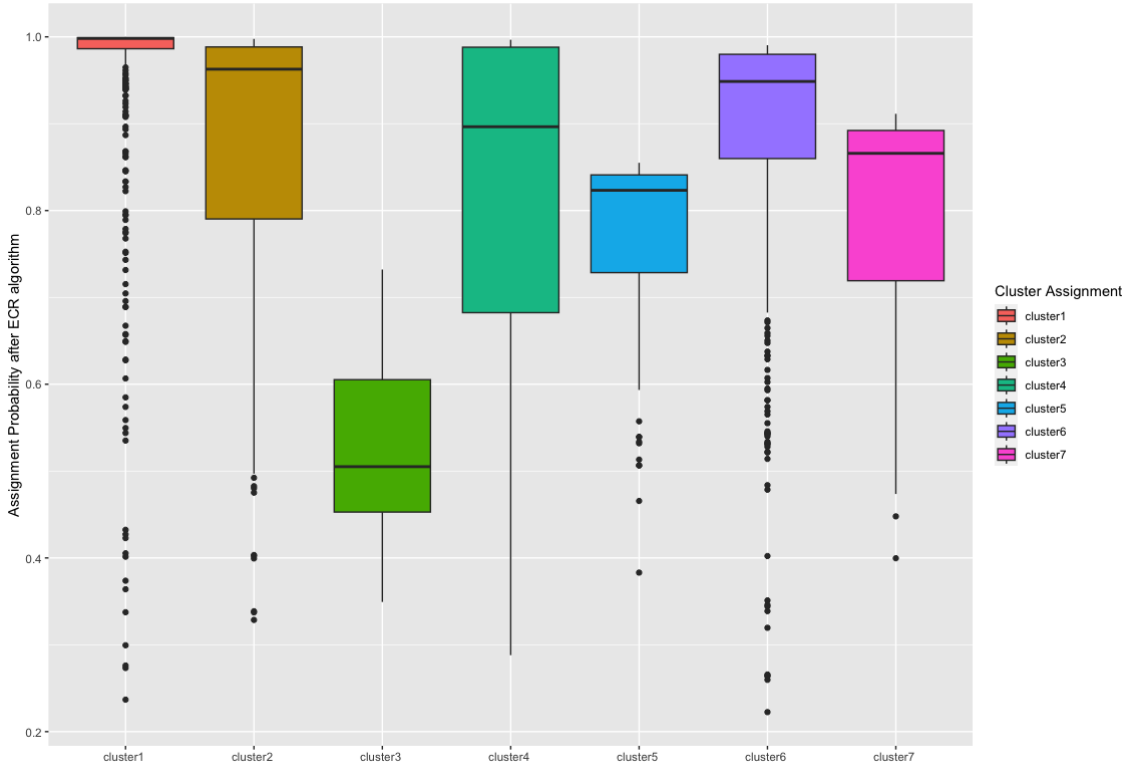


	cluster1 (N=181)	cluster2 (N=156)	cluster3 (N=315)	cluster4 (N=406)	cluster5 (N=165)	cluster6 (N=96)	cluster8 (N=45)	cluster9 (N=114)
Average Assignment Probability (ECR Algorithm)								
Mean (SD)	0.955 (0.0839)	0.866 (0.159)	0.857 (0.174)	0.924 (0.128)	0.807 (0.164)	0.657 (0.150)	0.807 (0.164)	0.794 (0.147)
Median [Min, Max]	0.985 [0.468, 0.999]	0.946 [0.361, 0.998]	0.923 [0.247, 0.994]	0.975 [0.240, 0.998]	0.830 [0.383, 0.995]	0.662 [0.363, 0.953]	0.830 [0.383, 0.997]	0.853 [0.316, 0.927]

ACS 2015-2019

```
* Run information:
      Number of parallel heated chains: 4
      Swap acceptance rate: 56.7%
      Total number of iterations: 15000
      Burn-in period: 5000
      Thinning: 10.

* Estimated posterior distribution of the number of clusters:

    6     7     8     9    10    11
0.081 0.347 0.328 0.163 0.064 0.017

* Most probable model: K = 7 with P(K = 7|data) = 0.347

* Estimated number of observations per cluster conditionally on K = 7 (3 label switching
algorithms):
  STEPHENS ECR ECR.ITERATIVE.1
1      449 448             449
2      152 154             152
3       45  39              45
4      170 171             170
5      178 167             178
6      376 379             376
7      108 120             108
```

	Mean	SD	2.5%	97.5%
p.1	0.2946242	0.0138141	0.2684068	0.3216439
p.2	0.1032780	0.0133154	0.0781711	0.1286903
p.3	0.0373058	0.0276428	0.0000000	0.0873315
p.4	0.1129444	0.0142592	0.0857370	0.1430624
p.5	0.1080820	0.0404868	0.0073321	0.1969131
p.6	0.2419764	0.0241896	0.1995722	0.2896869
p.7	0.1017891	0.0437053	0.0350052	0.2070201



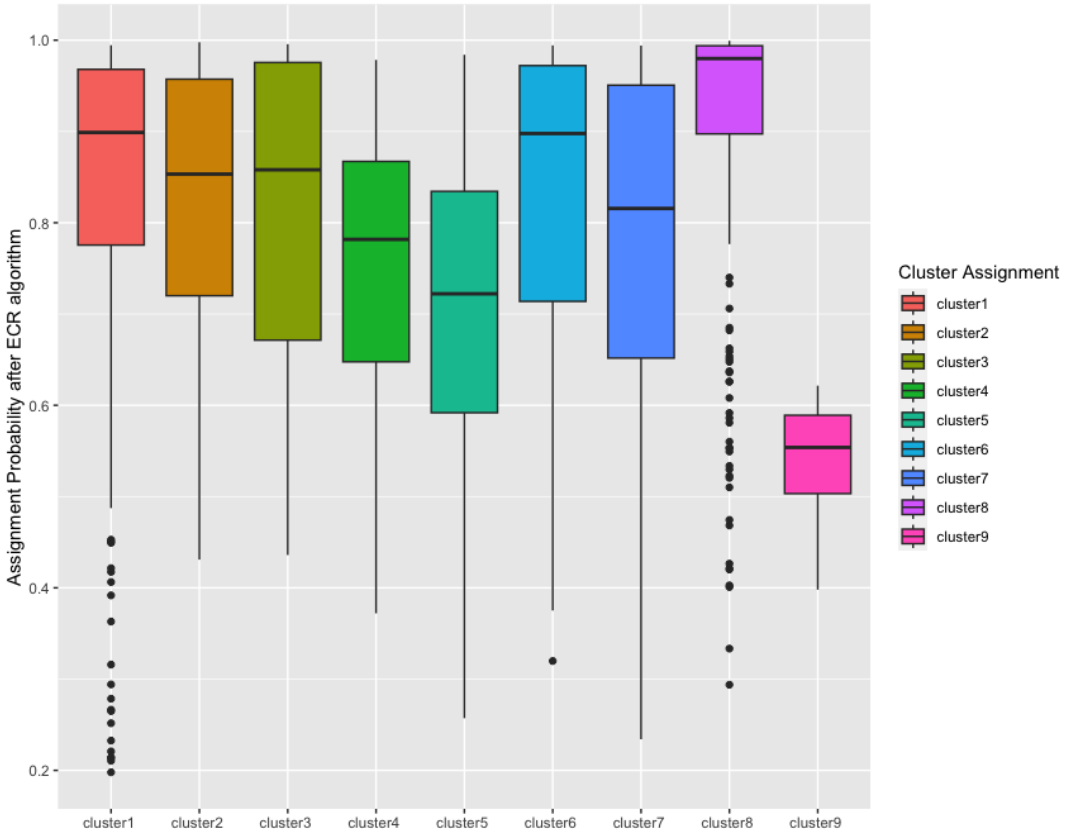
	cluster1 (N=448)	cluster2 (N=154)	cluster3 (N=39)	cluster4 (N=171)	cluster5 (N=167)	cluster6 (N=379)	cluster7 (N=120)
Average Assignment Probability (ECR Algorithm)							
Mean (SD)	0.949 (0.131)	0.856 (0.187)	0.519 (0.102)	0.822 (0.186)	0.773 (0.0984)	0.877 (0.161)	0.798 (0.131)
Median [Min, Max]	0.998 [0.237, 1.00]	0.963 [0.329, 0.998]	0.505 [0.349, 0.732]	0.896 [0.288, 0.997]	0.823 [0.383, 0.855]	0.949 [0.222, 0.990]	0.866 [0.400, 0.911]

2. Heating vectors (Set 2)

ACS 2006-2010

```
* Run information:
      Number of parallel heated chains: 4
      Swap acceptance rate: 22.2%
      Total number of iterations: 15000
      Burn-in period: 5000
      Thinning: 10.

* Estimated posterior distribution of the number of clusters:

    8     9    10    11    12    13    14    15
0.127 0.446 0.286 0.107 0.027 0.002 0.003 0.002

* Most probable model: K = 9 with P(K = 9|data) = 0.446

* Estimated number of observations per cluster conditionally on K = 9 (3 label switching
algorithms):
  STEPHENS ECR ECR.ITERATIVE.1
1      331 331             331
2      131 131             131
3      144 144             144
4       72  72              72
5      162 162             162
6      166 167             166
7      240 241             240
8      209 209             209
9       23  21              23
```

	Mean	SD	2.5%	97.5%
p.1	0.2040	0.0174	0.1630	0.2347
p.2	0.0996	0.0157	0.0714	0.1310
p.3	0.0996	0.0128	0.0756	0.1253
p.4	0.0537	0.0107	0.0345	0.0752
p.5	0.1188	0.0300	0.0596	0.1742
p.6	0.1069	0.0133	0.0808	0.1322
p.7	0.1579	0.0339	0.1027	0.2259
p.8	0.1376	0.0128	0.1122	0.1626
p.9	0.0219	0.0180	0.0000	0.0568

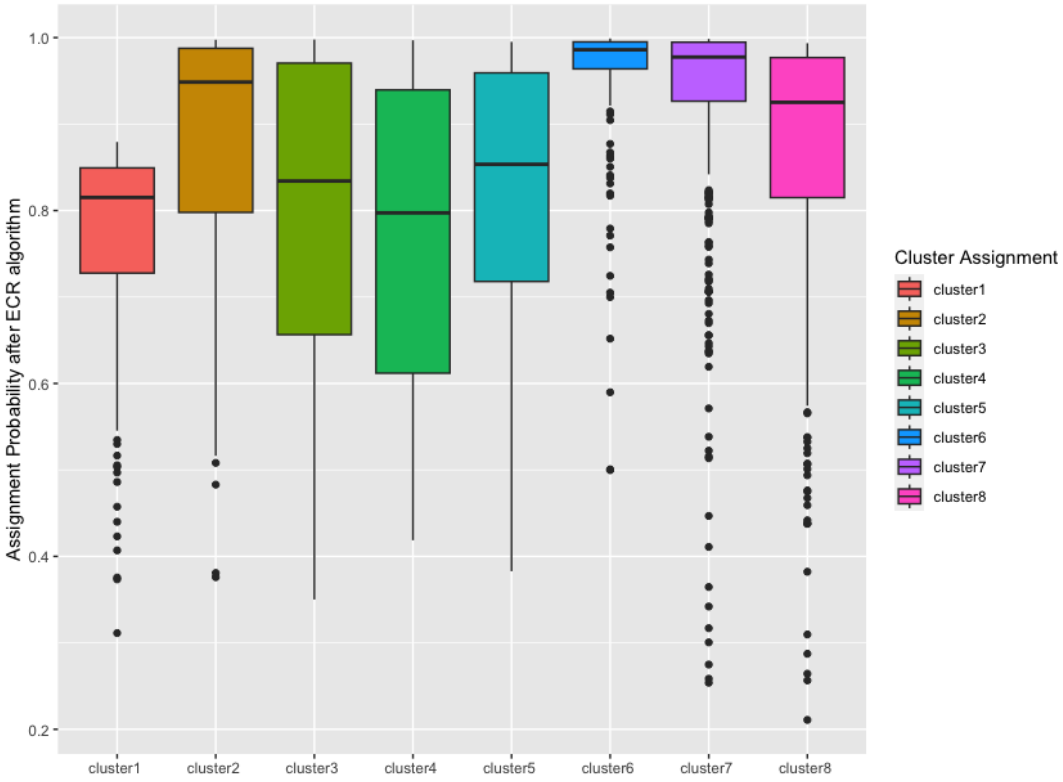


	cluster1 (N=331)	cluster2 (N=131)	cluster3 (N=144)	cluster4 (N=72)	cluster5 (N=162)	cluster6 (N=167)	cluster7 (N=241)	cluster8 (N=209)	cluster9 (N=21)
Average Assignment Probability (ECR Algorithm)									
Mean (SD)	0.831 (0.187)	0.823 (0.148)	0.811 (0.177)	0.755 (0.149)	0.717 (0.158)	0.834 (0.169)	0.793 (0.177)	0.896 (0.165)	0.536 (0.0635)
Median [Min, Max]	0.899 [0.198, 0.994]	0.853 [0.431, 0.998]	0.858 [0.436, 0.996]	0.782 [0.372, 0.978]	0.722 [0.257, 0.984]	0.898 [0.320, 0.994]	0.816 [0.234, 0.994]	0.980 [0.294, 0.999]	0.554 [0.398, 0.621]

ACS 2011-2015

```
* Run information:
      Number of parallel heated chains: 4
      Swap acceptance rate: 25.7%
      Total number of iterations: 15000
      Burn-in period: 5000
      Thinning: 10.

* Estimated posterior distribution of the number of clusters:

    7     8     9    10    11    12    13
0.074 0.432 0.255 0.152 0.065 0.020 0.002

* Most probable model: K = 8 with P(K = 8|data) = 0.432

* Estimated number of observations per cluster conditionally on K = 8 (3 label switching
algorithms):
  STEPHENS ECR ECR.ITERATIVE.1
1      113 113             113
2      154 154             154
3       46  46              46
4       91  91              91
5      169 169             169
6      181 181             181
7      407 407             407
8      317 317             317
```

	Mean	SD	2.5%	97.5%
p.1	0.0662	0.0255	0.0000	0.0956
p.2	0.1053	0.0108	0.0847	0.1276
p.3	0.0396	0.0095	0.0238	0.0591
p.4	0.0752	0.0145	0.0474	0.1031
p.5	0.1164	0.0157	0.0886	0.1499
p.6	0.1322	0.0226	0.1042	0.1943
p.7	0.2657	0.0155	0.2377	0.2990
p.8	0.1994	0.0163	0.1681	0.2313

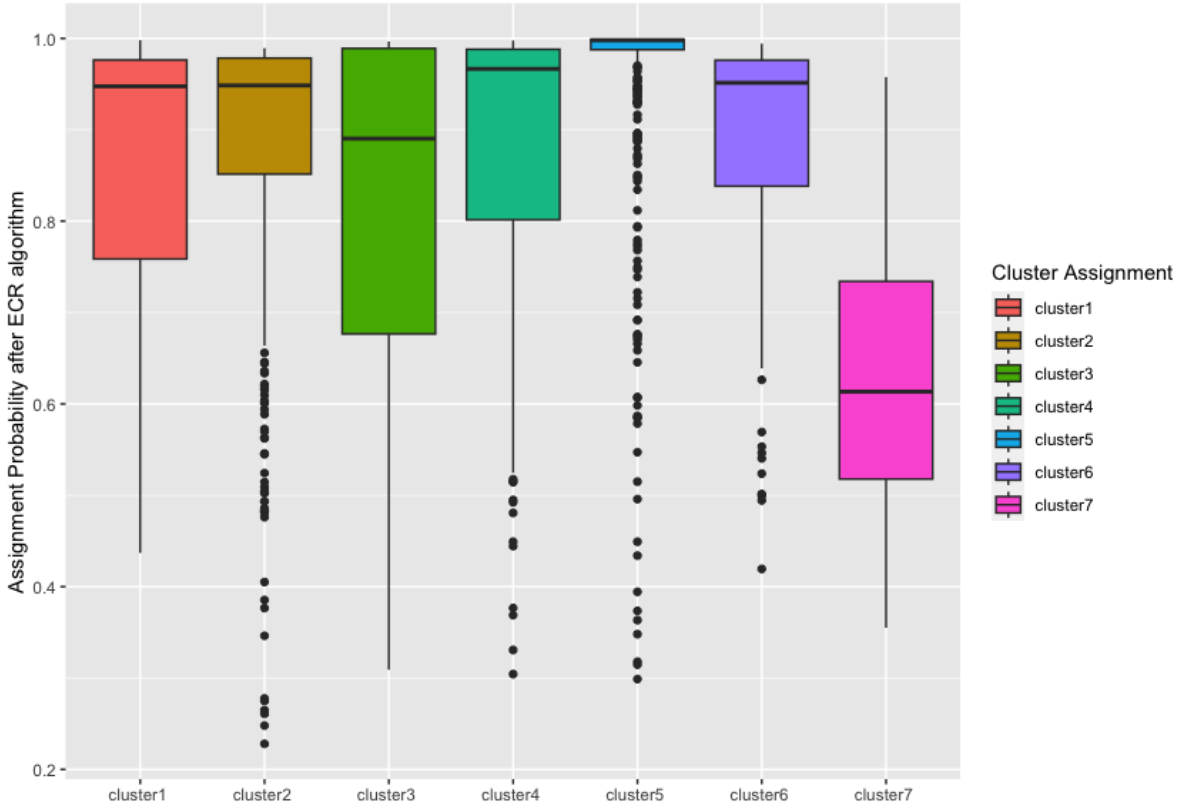


	cluster1 (N=113)	cluster2 (N=154)	cluster3 (N=46)	cluster4 (N=91)	cluster5 (N=169)	cluster6 (N=181)	cluster7 (N=407)	cluster8 (N=317)
Average Assignment Probability (ECR Algorithm)								
Mean (SD)	0.760 (0.135)	0.881 (0.145)	0.800 (0.175)	0.772 (0.173)	0.813 (0.165)	0.957 (0.0823)	0.925 (0.130)	0.863 (0.163)
Median [Min, Max]	0.815 [0.311, 0.879]	0.949 [0.376, 0.998]	0.834 [0.350, 0.998]	0.797 [0.419, 0.997]	0.853 [0.383, 0.995]	0.986 [0.500, 0.999]	0.978 [0.254, 0.998]	0.925 [0.211, 0.994]

ACS 2015-2019

```
* Run information:
      Number of parallel heated chains: 4
      Swap acceptance rate: 22.7%
      Total number of iterations: 15000
      Burn-in period: 5000
      Thinning: 10.

* Estimated posterior distribution of the number of clusters:

    7     8     9    10    11    12    13
0.366 0.354 0.205 0.058 0.010 0.004 0.003

* Most probable model: K = 7 with P(K = 7|data) = 0.366

* Estimated number of observations per cluster conditionally on K = 7 (3 label switching
algorithms):
  STEPHENS ECR ECR.ITERATIVE.1
1      122 122             122
2      371 371             371
3      176 176             176
4      147 147             147
5      445 445             445
6      166 166             166
7       51  51              51
```

	Mean	SD	2.5%	97.5%
p.1	0.0914563	0.0143482	0.0655128	0.1203082
p.2	0.2372000	0.0227225	0.1923662	0.2802821
p.3	0.1153369	0.0153521	0.0867634	0.1443147
p.4	0.1008468	0.0140745	0.0743414	0.1302943
p.5	0.2940215	0.0128921	0.2664334	0.3188759
p.6	0.1101795	0.0136717	0.0833982	0.1382458
p.7	0.0509591	0.0222961	0.0128554	0.0961418



	cluster1 (N=122)	cluster2 (N=371)	cluster3 (N=176)	cluster4 (N=147)	cluster5 (N=445)	cluster6 (N=166)	cluster7 (N=51)
Average Assignment Probability (ECR Algorithm)							
Mean (SD)	0.860 (0.161)	0.876 (0.160)	0.822 (0.184)	0.867 (0.178)	0.952 (0.123)	0.885 (0.133)	0.636 (0.156)
Median [Min, Max]	0.948 [0.437, 0.998]	0.949 [0.228, 0.989]	0.890 [0.309, 0.997]	0.967 [0.304, 0.998]	0.998 [0.299, 1.00]	0.951 [0.419, 0.994]	0.613 [0.355, 0.958]

Summary

- Found a method in Altekar(2004) to do this more efficiently by using incremental heating where the heat of the m chain is $\mathrm{h\_m} = 1/[1 + \mathrm{DeltaT} * (\mathrm{m} - 1)]$, and the parameter DeltaT is chosen s.t swaps are accepted 20%- 60% of time.

- I set DeltaT = {0.01, 0.025, 0.05, 0.1, 0.15, 0.2} → for these datasets smaller DeltaT seemed to work better (i.e., higher swap acceptance rates). The results shown above are for DeltaT = {0.01 (Set 1), 0.025 (Set 2)}

- Using Set 1 yielded swap acceptance ratios ranging from **54%-56.8%** which is pretty good. This also had shorter runtime as chains that were stuck in a local optima were able to jump to another one. However,

    - Mixing weights including zero in the credible interval
    - For ACS 2011-2015 (under Set 1), one cluster is empty.

- In terms of the above "problems", Set 2 seems to be more stable, and the swap acceptance ratios are still good.

- Remember that some census tracts ("fuzzycts") were not being classified to the cluster with highest posterior probability. You were right; it is a rounding issue! → should we communicate this the developers?

-