Lab 4

Connor Brown

11:59PM March 9, 2019

Note: the content of this lab is on the midterm exam (March 5) even though the lab itself is due after the midterm exam.

We now move on to simple linear modeling using the ordinary least squares algorithm.

Let's quickly recreate the sample data set from practice lecture 7:

```
n = 20
x = runif(n)
    [1] 0.88378238 0.36972179 0.07176939 0.95750058 0.05950548 0.78023393
##
   [7] 0.91358867 0.52847378 0.84638218 0.29880908 0.32159353 0.35077350
## [13] 0.26775541 0.60411849 0.89746416 0.19399546 0.83562078 0.61667994
## [19] 0.06928716 0.49461156
beta 0 = 3
beta_1 = -2
y = beta_0 + beta_1 * x + rnorm(n, mean = 0, sd = 0.33)
    [1] 1.6006477 2.2147082 2.2873566 0.9165754 2.9481029 1.9761075 1.0689102
   [8] 1.3580338 1.2244187 2.3488318 1.9057780 2.8132407 2.8485919 2.0720249
## [15] 1.4316933 3.2499953 2.1253546 2.1143346 2.9718556 2.2344761
Solve for the least squares line by computing b_0 and b_1 without using the functions mean, cor, cov, var, sd
but instead computing it from the x and y quantities manually using base function such as sum and other
```

basic operators. See the class notes.

```
b_1 = (sum(x*y) - n * (sum(x) / n) * (sum(y) / n)) / (sum(x^2) - n*((sum(x) / n)^2))
b_1
## [1] -1.776447
b_0 = (sum(y) / n) - (b_1*(sum(x)/n))
b_0
```

[1] 3.0059

Verify your computations are correct using the lm function in R:

```
lm_mod = lm(y~x)
lm_mod
##
## Call:
## lm(formula = y \sim x)
## Coefficients:
## (Intercept)
                            х
         3.006
##
                      -1.776
```

6. We are now going to repeat one of the first linear model building exercises in history — that of Sir Francis Galton in 1886. First load up package HistData.

```
pacman::p_load(HistData)
```

In it, there is a dataset called Galton. Load it up.

Galton

```
##
      parent child
## 1
        70.5 61.7
## 2
        68.5 61.7
## 3
        65.5 61.7
## 4
        64.5 61.7
## 5
        64.0 61.7
        67.5 62.2
## 6
## 7
        67.5 62.2
## 8
        67.5 62.2
## 9
        66.5 62.2
## 10
        66.5 62.2
        66.5 62.2
## 11
## 12
        64.5
              62.2
        70.5 63.2
## 13
## 14
        69.5 63.2
## 15
        68.5 63.2
## 16
        68.5
              63.2
## 17
        68.5 63.2
## 18
        68.5 63.2
## 19
        68.5 63.2
        68.5 63.2
## 20
## 21
        68.5 63.2
## 22
        67.5 63.2
## 23
        67.5
              63.2
        67.5 63.2
## 24
## 25
        67.5 63.2
## 26
        67.5 63.2
## 27
        66.5
              63.2
        66.5 63.2
## 28
## 29
         66.5 63.2
## 30
        65.5
              63.2
## 31
        65.5
              63.2
## 32
        65.5 63.2
## 33
        65.5 63.2
## 34
        65.5 63.2
## 35
        65.5
              63.2
## 36
        65.5 63.2
```

```
## 37
         65.5
               63.2
         65.5
## 38
               63.2
         64.5
               63.2
## 39
## 40
         64.5
               63.2
## 41
         64.5
               63.2
## 42
         64.5
               63.2
## 43
         64.0
               63.2
               63.2
## 44
         64.0
## 45
         69.5
               64.2
## 46
         69.5
               64.2
## 47
         69.5
               64.2
         69.5
               64.2
## 48
## 49
         69.5
               64.2
         69.5
               64.2
## 50
## 51
         69.5
               64.2
## 52
         69.5
               64.2
## 53
         69.5
               64.2
         69.5
               64.2
## 54
## 55
         69.5
               64.2
         69.5
               64.2
## 56
         69.5
## 57
               64.2
## 58
         69.5
               64.2
         69.5
               64.2
## 59
## 60
         69.5
               64.2
         68.5 64.2
## 61
## 62
         68.5
               64.2
## 63
         68.5
               64.2
## 64
         68.5
               64.2
         68.5
## 65
               64.2
## 66
         68.5
               64.2
## 67
         68.5
               64.2
## 68
         68.5
               64.2
## 69
         68.5
               64.2
## 70
         68.5
               64.2
## 71
         68.5
               64.2
## 72
         67.5
               64.2
## 73
         67.5
               64.2
## 74
         67.5
               64.2
               64.2
## 75
         67.5
## 76
         67.5 64.2
## 77
         67.5
               64.2
         67.5
               64.2
## 78
## 79
         67.5
               64.2
## 80
         67.5
               64.2
## 81
         67.5
               64.2
         67.5
               64.2
## 82
## 83
         67.5
               64.2
## 84
         67.5
               64.2
## 85
               64.2
         67.5
## 86
         66.5
               64.2
         66.5
               64.2
## 87
## 88
         66.5
               64.2
               64.2
## 89
         66.5
## 90
         66.5 64.2
```

```
65.5 64.2
## 91
         65.5
               64.2
## 92
         65.5
               64.2
## 93
## 94
         65.5
               64.2
## 95
         65.5
               64.2
## 96
         64.5
               64.2
## 97
         64.5
               64.2
         64.5
               64.2
## 98
## 99
         64.5
               64.2
## 100
         64.0
               64.2
## 101
         64.0
               64.2
         64.0
               64.2
## 102
## 103
         64.0
               64.2
               65.2
## 104
         71.5
## 105
         70.5
               65.2
## 106
         69.5
               65.2
## 107
         69.5
               65.2
         69.5
               65.2
## 108
## 109
         69.5
               65.2
               65.2
## 110
         68.5
## 111
         68.5
               65.2
## 112
         68.5
               65.2
         68.5
               65.2
## 113
## 114
         68.5
               65.2
## 115
         68.5 65.2
## 116
         68.5
               65.2
## 117
         68.5
               65.2
## 118
         68.5
               65.2
## 119
         68.5
               65.2
## 120
         68.5
               65.2
## 121
         68.5
               65.2
## 122
         68.5
               65.2
## 123
         68.5
               65.2
## 124
         68.5
               65.2
         68.5
               65.2
## 125
         67.5
## 126
               65.2
## 127
         67.5
               65.2
## 128
         67.5
               65.2
## 129
         67.5
               65.2
## 130
         67.5
               65.2
## 131
         67.5
               65.2
## 132
         67.5
               65.2
## 133
         67.5
               65.2
## 134
         67.5
               65.2
## 135
         67.5
               65.2
## 136
         67.5
               65.2
## 137
         67.5
               65.2
## 138
         67.5
               65.2
               65.2
## 139
         67.5
         67.5
               65.2
## 140
         66.5
## 141
               65.2
## 142
         66.5
               65.2
## 143
         65.5
               65.2
## 144
         65.5 65.2
```

```
65.5 65.2
## 145
## 146
         65.5
               65.2
         65.5
               65.2
## 147
## 148
         65.5
               65.2
## 149
         65.5
               65.2
## 150
         64.5
               65.2
## 151
         64.0
               65.2
         71.5
               66.2
## 152
## 153
         71.5
               66.2
## 154
         71.5
               66.2
## 155
         70.5
               66.2
         69.5
               66.2
## 156
## 157
         69.5
               66.2
## 158
         69.5
               66.2
## 159
         69.5
               66.2
## 160
         69.5
               66.2
## 161
         69.5
               66.2
         69.5
               66.2
## 162
## 163
         69.5
               66.2
               66.2
## 164
         69.5
## 165
         69.5
               66.2
## 166
         69.5
               66.2
         69.5
               66.2
## 167
## 168
         69.5
               66.2
## 169
         69.5
               66.2
## 170
         69.5
               66.2
## 171
         69.5
               66.2
## 172
         69.5
               66.2
## 173
         68.5
               66.2
## 174
         68.5
               66.2
## 175
         68.5
               66.2
## 176
         68.5
               66.2
## 177
         68.5
               66.2
## 178
         68.5
               66.2
## 179
         68.5
               66.2
         68.5
## 180
               66.2
## 181
         68.5
               66.2
## 182
         68.5
               66.2
## 183
         68.5
               66.2
## 184
         68.5
               66.2
## 185
         68.5
               66.2
## 186
         68.5
               66.2
## 187
         68.5
               66.2
## 188
         68.5
               66.2
## 189
         68.5
               66.2
## 190
         68.5
               66.2
## 191
         68.5
               66.2
## 192
         68.5
               66.2
## 193
         68.5
               66.2
         68.5
               66.2
## 194
         68.5
## 195
               66.2
## 196
         68.5
               66.2
## 197
         68.5
               66.2
## 198
         67.5
               66.2
```

```
67.5
               66.2
## 199
## 200
         67.5
               66.2
         67.5
               66.2
## 201
## 202
         67.5
               66.2
## 203
         67.5
               66.2
## 204
         67.5
               66.2
## 205
         67.5
               66.2
## 206
         67.5
               66.2
## 207
         67.5
               66.2
## 208
         67.5
               66.2
## 209
         67.5
               66.2
         67.5
               66.2
## 210
## 211
         67.5
               66.2
## 212
         67.5
               66.2
## 213
         67.5
               66.2
## 214
         67.5
               66.2
## 215
         67.5
               66.2
## 216
         67.5
               66.2
## 217
         67.5
               66.2
## 218
         67.5
               66.2
## 219
         67.5
               66.2
## 220
         67.5
               66.2
## 221
         67.5
               66.2
## 222
         67.5
               66.2
## 223
         67.5
               66.2
## 224
         67.5
               66.2
## 225
         67.5
               66.2
## 226
         67.5
               66.2
## 227
         67.5
               66.2
## 228
         67.5
               66.2
## 229
         67.5
               66.2
         67.5
## 230
               66.2
## 231
         67.5
               66.2
## 232
         67.5
               66.2
## 233
         67.5
               66.2
         66.5
## 234
               66.2
## 235
         66.5
               66.2
## 236
         66.5
               66.2
## 237
         66.5
               66.2
## 238
         66.5
               66.2
## 239
         66.5
               66.2
## 240
         66.5
               66.2
## 241
         66.5
               66.2
## 242
         66.5
               66.2
## 243
         66.5
               66.2
## 244
         66.5
               66.2
## 245
         66.5
               66.2
## 246
         66.5
               66.2
## 247
         66.5
               66.2
## 248
               66.2
         66.5
         66.5
## 249
               66.2
## 250
         66.5
               66.2
## 251
         65.5
               66.2
## 252
         65.5
               66.2
```

```
## 253
         65.5
               66.2
## 254
         65.5
               66.2
## 255
               66.2
         65.5
## 256
         65.5
               66.2
## 257
         65.5
               66.2
## 258
         65.5
               66.2
## 259
         65.5
               66.2
         65.5
               66.2
## 260
## 261
         65.5
               66.2
## 262
         64.5
               66.2
## 263
         64.5
               66.2
         64.5
## 264
               66.2
## 265
         64.5
               66.2
## 266
         64.5
               66.2
## 267
         64.0
               66.2
## 268
         64.0
               66.2
## 269
         71.5
               67.2
## 270
         71.5
               67.2
## 271
         71.5
               67.2
## 272
         71.5
               67.2
## 273
         70.5
               67.2
## 274
         70.5
               67.2
         70.5
               67.2
## 275
## 276
         69.5
               67.2
         69.5 67.2
## 277
## 278
         69.5
               67.2
## 279
         69.5
               67.2
## 280
         69.5
               67.2
## 281
         69.5
               67.2
## 282
         69.5
               67.2
## 283
         69.5
               67.2
## 284
         69.5
               67.2
## 285
         69.5
               67.2
## 286
         69.5
               67.2
## 287
         69.5
               67.2
## 288
         69.5
               67.2
## 289
         69.5
               67.2
## 290
         69.5
               67.2
## 291
         69.5
               67.2
## 292
         69.5 67.2
## 293
         69.5
               67.2
## 294
         69.5
               67.2
## 295
         69.5
               67.2
## 296
         69.5
               67.2
## 297
         69.5
               67.2
## 298
         69.5
               67.2
## 299
         69.5
               67.2
## 300
         69.5
               67.2
## 301
         69.5
               67.2
         69.5
               67.2
## 302
         68.5
## 303
               67.2
## 304
         68.5
               67.2
## 305
         68.5
               67.2
## 306
         68.5 67.2
```

```
## 307
         68.5 67.2
## 308
         68.5
               67.2
               67.2
## 309
         68.5
## 310
         68.5
               67.2
## 311
         68.5
               67.2
## 312
         68.5
               67.2
## 313
         68.5
               67.2
         68.5
               67.2
## 314
## 315
         68.5
               67.2
## 316
         68.5
               67.2
## 317
         68.5
               67.2
## 318
         68.5
               67.2
## 319
         68.5
               67.2
## 320
         68.5
               67.2
## 321
         68.5
               67.2
## 322
         68.5
               67.2
## 323
         68.5
               67.2
## 324
         68.5
               67.2
## 325
         68.5
               67.2
## 326
         68.5
               67.2
## 327
         68.5
               67.2
## 328
         68.5
               67.2
## 329
         68.5
               67.2
## 330
         68.5
               67.2
## 331
         68.5 67.2
## 332
         68.5 67.2
## 333
         68.5
               67.2
## 334
         67.5
               67.2
## 335
         67.5
               67.2
## 336
         67.5
               67.2
## 337
         67.5
               67.2
         67.5
## 338
               67.2
## 339
         67.5
               67.2
## 340
         67.5
               67.2
## 341
         67.5
               67.2
         67.5
## 342
               67.2
## 343
         67.5
               67.2
## 344
         67.5
               67.2
## 345
         67.5
               67.2
## 346
         67.5 67.2
## 347
         67.5
               67.2
## 348
         67.5
               67.2
## 349
         67.5
               67.2
## 350
         67.5
               67.2
## 351
         67.5
               67.2
         67.5
               67.2
## 352
## 353
         67.5
               67.2
## 354
         67.5
               67.2
## 355
         67.5
               67.2
## 356
         67.5
               67.2
         67.5
## 357
               67.2
## 358
         67.5
               67.2
## 359
         67.5
               67.2
## 360
         67.5 67.2
```

```
67.5 67.2
## 361
## 362
         67.5
               67.2
         67.5
               67.2
## 363
## 364
         67.5
               67.2
## 365
         67.5
               67.2
## 366
         67.5
               67.2
## 367
         67.5
               67.2
         67.5
               67.2
## 368
## 369
         67.5
               67.2
## 370
         67.5
               67.2
## 371
         67.5
               67.2
         66.5
               67.2
## 372
## 373
         66.5
               67.2
## 374
         66.5
               67.2
## 375
         66.5
               67.2
## 376
         66.5
               67.2
## 377
         66.5
               67.2
         66.5
## 378
               67.2
## 379
         66.5
               67.2
## 380
         66.5
               67.2
## 381
         66.5
               67.2
## 382
         66.5
               67.2
               67.2
## 383
         66.5
## 384
         66.5
               67.2
## 385
         66.5
               67.2
## 386
         66.5
               67.2
## 387
         66.5
               67.2
## 388
         66.5
               67.2
## 389
         65.5
               67.2
## 390
         65.5
               67.2
## 391
         65.5
               67.2
               67.2
## 392
         65.5
## 393
         65.5
               67.2
## 394
         65.5
               67.2
## 395
         65.5
               67.2
         65.5
## 396
               67.2
## 397
         65.5
               67.2
## 398
         65.5
               67.2
## 399
         65.5
               67.2
## 400
         64.5
               67.2
## 401
         64.5
               67.2
## 402
         64.5
               67.2
## 403
         64.5
               67.2
## 404
         64.5
               67.2
## 405
         64.0
               67.2
         64.0
               67.2
## 406
## 407
         72.5
               68.2
## 408
         71.5
               68.2
## 409
         71.5
               68.2
## 410
         71.5
               68.2
         70.5
## 411
               68.2
## 412
         70.5
               68.2
## 413
         70.5
               68.2
## 414
         70.5
               68.2
```

```
70.5 68.2
## 415
## 416
         70.5
                68.2
## 417
         70.5
                68.2
## 418
         70.5
                68.2
## 419
         70.5
                68.2
## 420
         70.5
                68.2
## 421
         70.5
                68.2
## 422
         70.5
                68.2
## 423
         69.5
                68.2
## 424
         69.5
                68.2
## 425
         69.5
                68.2
## 426
         69.5
                68.2
## 427
         69.5
                68.2
## 428
         69.5
                68.2
## 429
         69.5
                68.2
## 430
         69.5
                68.2
## 431
         69.5
                68.2
         69.5
## 432
                68.2
## 433
         69.5
                68.2
                68.2
## 434
         69.5
## 435
         69.5
                68.2
## 436
         69.5
                68.2
         69.5
                68.2
## 437
## 438
         69.5
                68.2
## 439
         69.5
                68.2
## 440
         69.5
                68.2
## 441
         69.5
                68.2
## 442
         69.5
                68.2
## 443
         68.5
                68.2
## 444
         68.5
                68.2
## 445
         68.5
                68.2
## 446
         68.5
                68.2
## 447
         68.5
                68.2
## 448
         68.5
                68.2
                68.2
## 449
         68.5
         68.5
## 450
                68.2
## 451
         68.5
                68.2
## 452
         68.5
                68.2
## 453
         68.5
                68.2
## 454
         68.5
                68.2
## 455
         68.5
                68.2
## 456
         68.5
                68.2
## 457
         68.5
                68.2
## 458
         68.5
                68.2
## 459
         68.5
                68.2
         68.5
                68.2
## 460
## 461
         68.5
                68.2
## 462
         68.5
                68.2
                68.2
## 463
         68.5
         68.5
                68.2
## 464
         68.5
## 465
                68.2
## 466
         68.5
                68.2
## 467
         68.5
                68.2
## 468
         68.5
                68.2
```

```
## 469
         68.5
                68.2
## 470
         68.5
                68.2
         68.5
                68.2
## 471
## 472
         68.5
                68.2
## 473
         68.5
                68.2
## 474
         68.5
                68.2
## 475
         68.5
                68.2
## 476
         68.5
                68.2
## 477
         67.5
                68.2
## 478
         67.5
                68.2
## 479
         67.5
                68.2
         67.5
                68.2
## 480
## 481
         67.5
                68.2
         67.5
## 482
                68.2
## 483
         67.5
                68.2
## 484
         67.5
                68.2
## 485
         67.5
                68.2
         67.5
                68.2
## 486
## 487
         67.5
                68.2
                68.2
## 488
         67.5
## 489
         67.5
                68.2
## 490
         67.5
                68.2
         67.5
                68.2
## 491
## 492
         67.5
                68.2
         67.5
## 493
                68.2
## 494
         67.5
                68.2
## 495
         67.5
                68.2
## 496
         67.5
                68.2
## 497
         67.5
                68.2
## 498
         67.5
                68.2
## 499
         67.5
                68.2
         67.5
## 500
                68.2
## 501
         67.5
                68.2
## 502
         67.5
                68.2
## 503
         67.5
                68.2
         67.5
## 504
                68.2
## 505
         66.5
                68.2
## 506
         66.5
                68.2
## 507
         66.5
                68.2
## 508
         66.5
                68.2
## 509
         66.5
                68.2
## 510
         66.5
                68.2
## 511
         66.5
                68.2
## 512
         66.5
                68.2
## 513
         66.5
                68.2
## 514
         66.5
                68.2
## 515
         66.5
                68.2
## 516
         66.5
                68.2
                68.2
## 517
         66.5
## 518
         66.5
                68.2
         65.5
## 519
                68.2
## 520
         65.5
                68.2
## 521
         65.5
                68.2
## 522
         65.5
                68.2
```

```
## 523
         65.5
               68.2
## 524
         65.5
               68.2
## 525
         65.5
               68.2
## 526
         64.0
               68.2
## 527
         72.5
               69.2
## 528
         72.5
               69.2
## 529
         71.5
               69.2
               69.2
## 530
         71.5
## 531
         71.5
               69.2
               69.2
## 532
         71.5
## 533
         71.5
               69.2
         70.5
               69.2
## 534
## 535
         70.5
               69.2
         70.5
               69.2
## 536
## 537
         70.5
               69.2
## 538
         70.5
               69.2
## 539
         70.5
               69.2
## 540
         70.5
               69.2
## 541
         70.5
               69.2
## 542
         70.5
               69.2
## 543
         70.5
               69.2
## 544
         70.5
               69.2
         70.5
               69.2
## 545
## 546
         70.5
               69.2
         70.5
               69.2
## 547
## 548
         70.5
               69.2
## 549
         70.5
               69.2
## 550
         70.5
               69.2
         70.5
               69.2
## 551
## 552
         69.5
               69.2
         69.5
               69.2
## 553
## 554
         69.5
               69.2
## 555
         69.5
               69.2
## 556
         69.5
               69.2
         69.5
               69.2
## 557
         69.5
               69.2
## 558
## 559
         69.5
               69.2
## 560
         69.5
               69.2
               69.2
## 561
         69.5
## 562
         69.5
               69.2
## 563
         69.5
               69.2
         69.5
## 564
               69.2
## 565
         69.5
               69.2
## 566
         69.5
               69.2
## 567
         69.5
               69.2
         69.5
               69.2
## 568
## 569
         69.5
               69.2
## 570
         69.5
               69.2
               69.2
## 571
         69.5
## 572
         69.5
               69.2
## 573
         69.5
               69.2
## 574
         69.5
               69.2
               69.2
## 575
         69.5
## 576
         69.5
               69.2
```

```
69.5
               69.2
## 577
## 578
         69.5
                69.2
         69.5
                69.2
## 579
## 580
         69.5
                69.2
## 581
         69.5
                69.2
## 582
         69.5
                69.2
## 583
         69.5
                69.2
         69.5
                69.2
## 584
## 585
         68.5
                69.2
## 586
         68.5
                69.2
## 587
         68.5
                69.2
         68.5
                69.2
## 588
## 589
         68.5
                69.2
## 590
         68.5
                69.2
## 591
         68.5
                69.2
## 592
         68.5
                69.2
## 593
         68.5
                69.2
## 594
         68.5
                69.2
## 595
         68.5
                69.2
## 596
                69.2
         68.5
## 597
         68.5
                69.2
## 598
         68.5
                69.2
         68.5
                69.2
## 599
## 600
         68.5
                69.2
         68.5
## 601
                69.2
## 602
         68.5
                69.2
## 603
         68.5
                69.2
## 604
         68.5
                69.2
## 605
         68.5
                69.2
## 606
         68.5
                69.2
## 607
         68.5
                69.2
## 608
         68.5
                69.2
## 609
         68.5
                69.2
## 610
         68.5
                69.2
         68.5
                69.2
## 611
         68.5
## 612
                69.2
## 613
         68.5
                69.2
## 614
         68.5
                69.2
## 615
         68.5
                69.2
## 616
         68.5
                69.2
## 617
         68.5
                69.2
## 618
         68.5
                69.2
## 619
         68.5
                69.2
## 620
         68.5
                69.2
## 621
         68.5
                69.2
## 622
         68.5
                69.2
## 623
         68.5
                69.2
## 624
         68.5
                69.2
## 625
         68.5
                69.2
## 626
         68.5
                69.2
         68.5
## 627
                69.2
## 628
         68.5
                69.2
## 629
         68.5
                69.2
## 630
         68.5
               69.2
```

```
## 631
         68.5
               69.2
## 632
         68.5
               69.2
         67.5
               69.2
## 633
## 634
         67.5
               69.2
## 635
         67.5
               69.2
## 636
         67.5
               69.2
## 637
         67.5
               69.2
               69.2
## 638
         67.5
## 639
         67.5
               69.2
## 640
         67.5
               69.2
## 641
         67.5
               69.2
## 642
         67.5
               69.2
## 643
         67.5
               69.2
         67.5
               69.2
## 644
## 645
         67.5
               69.2
## 646
         67.5
               69.2
## 647
         67.5
               69.2
         67.5
               69.2
## 648
## 649
         67.5
               69.2
## 650
         67.5
               69.2
         67.5
## 651
               69.2
## 652
         67.5
               69.2
         67.5
               69.2
## 653
## 654
         67.5
               69.2
         67.5
               69.2
## 655
## 656
         67.5
               69.2
## 657
         67.5
               69.2
## 658
         67.5
               69.2
## 659
         67.5
               69.2
## 660
         67.5
               69.2
               69.2
## 661
         67.5
## 662
         67.5
               69.2
## 663
         67.5
               69.2
## 664
         67.5
               69.2
         67.5
               69.2
## 665
         67.5
## 666
               69.2
## 667
         67.5
               69.2
## 668
         67.5
               69.2
               69.2
## 669
         67.5
## 670
         67.5
               69.2
## 671
         66.5
               69.2
## 672
         66.5
               69.2
## 673
         66.5
               69.2
## 674
         66.5
               69.2
## 675
         66.5
               69.2
## 676
         66.5
               69.2
## 677
         66.5
               69.2
## 678
         66.5
               69.2
               69.2
## 679
         66.5
## 680
         66.5
               69.2
         66.5
## 681
               69.2
## 682
         66.5
               69.2
## 683
         66.5
               69.2
## 684
         65.5
               69.2
```

```
## 685
         65.5 69.2
## 686
         65.5
               69.2
         65.5
               69.2
## 687
## 688
         65.5
               69.2
## 689
         65.5
               69.2
## 690
         65.5
               69.2
## 691
         64.5
               69.2
               69.2
## 692
         64.5
## 693
         64.0
               69.2
## 694
         72.5
               70.2
## 695
         71.5
               70.2
## 696
         71.5
               70.2
## 697
         71.5
               70.2
               70.2
## 698
         71.5
## 699
         71.5
               70.2
## 700
         71.5
               70.2
## 701
         71.5
               70.2
## 702
         71.5
               70.2
               70.2
## 703
         71.5
## 704
         71.5
               70.2
         70.5
## 705
               70.2
## 706
         70.5
               70.2
## 707
         70.5
               70.2
## 708
         70.5
               70.2
         70.5 70.2
## 709
## 710
         70.5
               70.2
## 711
         70.5
               70.2
## 712
         70.5
               70.2
## 713
         70.5
               70.2
## 714
         70.5
               70.2
## 715
         70.5
               70.2
## 716
         70.5
               70.2
## 717
         70.5
               70.2
         70.5
               70.2
## 718
## 719
         69.5
               70.2
         69.5
## 720
               70.2
## 721
         69.5
               70.2
## 722
         69.5
               70.2
         69.5
               70.2
## 723
## 724
         69.5
               70.2
## 725
         69.5
               70.2
         69.5
               70.2
## 726
## 727
         69.5
               70.2
## 728
         69.5
               70.2
## 729
         69.5
               70.2
               70.2
## 730
         69.5
## 731
         69.5
               70.2
## 732
         69.5
               70.2
## 733
               70.2
         69.5
## 734
         69.5
               70.2
         69.5
## 735
               70.2
         69.5
## 736
               70.2
               70.2
## 737
         69.5
## 738
         69.5 70.2
```

```
## 739
         69.5
               70.2
         69.5
## 740
               70.2
         69.5
               70.2
## 741
## 742
         69.5
               70.2
## 743
         69.5
               70.2
## 744
         68.5
               70.2
## 745
         68.5
               70.2
               70.2
## 746
         68.5
## 747
         68.5
               70.2
         68.5
## 748
               70.2
## 749
         68.5
               70.2
## 750
         68.5
               70.2
## 751
         68.5
               70.2
               70.2
## 752
         68.5
## 753
         68.5
               70.2
## 754
         68.5
               70.2
## 755
         68.5
               70.2
## 756
         68.5
               70.2
               70.2
## 757
         68.5
## 758
         68.5
               70.2
## 759
         68.5
               70.2
## 760
         68.5
               70.2
## 761
         68.5
               70.2
## 762
         68.5
               70.2
         68.5
## 763
               70.2
## 764
         68.5
               70.2
## 765
         67.5
               70.2
## 766
         67.5
               70.2
## 767
         67.5
               70.2
## 768
         67.5
               70.2
         67.5
               70.2
## 769
## 770
         67.5
               70.2
## 771
         67.5
               70.2
## 772
         67.5
               70.2
## 773
         67.5
               70.2
         67.5
## 774
               70.2
## 775
         67.5
               70.2
## 776
         67.5
               70.2
         67.5
## 777
               70.2
## 778
         67.5
               70.2
## 779
         67.5
               70.2
               70.2
## 780
         67.5
## 781
         67.5
               70.2
## 782
         67.5
               70.2
## 783
         67.5
               70.2
## 784
         66.5
               70.2
## 785
         66.5
               70.2
               70.2
## 786
         66.5
## 787
               70.2
         66.5
## 788
         65.5
               70.2
         65.5
## 789
               70.2
## 790
         65.5
               70.2
               70.2
## 791
         65.5
## 792
         65.5 70.2
```

```
## 793
         72.5 71.2
         72.5 71.2
## 794
## 795
         71.5 71.2
## 796
         71.5 71.2
         71.5
## 797
               71.2
## 798
         71.5
               71.2
## 799
         70.5
               71.2
         70.5
               71.2
## 800
## 801
         70.5
               71.2
## 802
         70.5 71.2
## 803
         70.5
              71.2
         70.5
               71.2
## 804
## 805
         70.5
               71.2
               71.2
         69.5
## 806
## 807
         69.5
               71.2
## 808
         69.5
               71.2
## 809
         69.5 71.2
         69.5
              71.2
## 810
         69.5 71.2
## 811
## 812
         69.5
               71.2
## 813
         69.5
              71.2
## 814
         69.5
               71.2
## 815
         69.5 71.2
## 816
         69.5
               71.2
         69.5 71.2
## 817
## 818
         69.5 71.2
## 819
         69.5
              71.2
## 820
         69.5
               71.2
## 821
         69.5
               71.2
## 822
         69.5
               71.2
         69.5
               71.2
## 823
              71.2
## 824
         69.5
## 825
         69.5
              71.2
## 826
         68.5
              71.2
         68.5
               71.2
## 827
         68.5
## 828
               71.2
## 829
         68.5
               71.2
## 830
         68.5
               71.2
         68.5
               71.2
## 831
## 832
         68.5 71.2
## 833
         68.5 71.2
         68.5 71.2
## 834
## 835
         68.5
               71.2
## 836
         68.5
               71.2
## 837
         68.5
               71.2
         68.5
               71.2
## 838
## 839
         68.5
               71.2
## 840
         68.5 71.2
         68.5 71.2
## 841
## 842
         68.5
               71.2
         68.5
## 843
               71.2
## 844
         67.5
              71.2
         67.5 71.2
## 845
## 846
         67.5 71.2
```

```
## 847
         67.5 71.2
         67.5
## 848
               71.2
## 849
         67.5
               71.2
## 850
         67.5 71.2
## 851
         67.5
               71.2
## 852
         67.5
               71.2
## 853
         67.5
               71.2
               71.2
         67.5
## 854
## 855
         65.5
               71.2
         65.5
## 856
               71.2
## 857
         73.0
               72.2
         72.5
               72.2
## 858
## 859
         72.5
               72.2
         72.5
               72.2
## 860
## 861
         72.5
               72.2
         72.5
               72.2
## 862
## 863
         72.5
               72.2
         72.5
               72.2
## 864
              72.2
## 865
         71.5
         71.5
               72.2
## 866
         71.5
## 867
               72.2
## 868
         71.5
               72.2
         71.5
               72.2
## 869
               72.2
## 870
         71.5
         71.5 72.2
## 871
## 872
         71.5 72.2
## 873
         71.5
               72.2
## 874
         70.5
               72.2
         70.5
## 875
               72.2
## 876
         70.5
               72.2
         70.5
               72.2
## 877
## 878
         69.5
               72.2
## 879
         69.5
               72.2
               72.2
## 880
         69.5
         69.5
               72.2
## 881
         69.5
               72.2
## 882
## 883
         69.5
               72.2
## 884
         69.5
               72.2
         69.5
               72.2
## 885
## 886
         69.5 72.2
## 887
         69.5
              72.2
         69.5 72.2
## 888
## 889
         68.5
               72.2
## 890
         68.5
               72.2
## 891
         68.5
               72.2
               72.2
## 892
         68.5
## 893
         67.5
               72.2
## 894
         67.5 72.2
         67.5
               72.2
## 895
## 896
         67.5
               72.2
         65.5
## 897
               72.2
## 898
         73.0
               73.2
         73.0 73.2
## 899
## 900
         73.0 73.2
```

```
## 901
         72.5
               73.2
## 902
         72.5
               73.2
               73.2
## 903
         71.5
               73.2
## 904
         71.5
## 905
         70.5
               73.2
## 906
         70.5
               73.2
## 907
         70.5
               73.2
               73.2
## 908
         69.5
## 909
         69.5
               73.2
         69.5
## 910
               73.2
## 911
         69.5
               73.2
## 912
         68.5
               73.2
## 913
         68.5
               73.2
         68.5
## 914
               73.2
## 915
         72.5
               73.7
## 916
         72.5
               73.7
## 917
         72.5
               73.7
## 918
         72.5
               73.7
## 919
         71.5
               73.7
## 920
         71.5
               73.7
## 921
         70.5
               73.7
## 922
         70.5
               73.7
## 923
         70.5
               73.7
## 924
         69.5
               73.7
## 925
         69.5 73.7
         69.5 73.7
## 926
## 927
         69.5
               73.7
## 928
         69.5
               73.7
```

You now should have a data frame in your workspace called \mathtt{Galton} . Summarize this data frame and write a few sentences about what you see. Make sure you report n, p and a bit about what the columns represent and how the data was measured. See the help file $\mathtt{?Galton}$.

summary(Galton)

```
##
        parent
                          child
##
    Min.
            :64.00
                     Min.
                             :61.70
##
    1st Qu.:67.50
                     1st Qu.:66.20
##
    Median :68.50
                     Median :68.20
##
    Mean
            :68.31
                     Mean
                             :68.09
                     3rd Qu.:70.20
    3rd Qu.:69.50
##
##
    Max.
            :73.00
                     Max.
                             :73.70
n = nrow(Galton)
n
## [1] 928
p = ncol(Galton)
p
## [1] 2
```

TO-DO

#The height summaries for the parents column and child column appear very similar.

 $\#Both\ columns\ have\ comparable\ heights\ for\ Q1-Q3,\ median\ and\ mean.\ n\ is\ 928\ and\ p\ is\ 2.$

#The parents column represents the mean of the mother and father's height.

Find the average height (include both parents and children in this computation).

```
avg_height = ((2*sum(Galton$parent)) + sum(Galton$child)) / (3*n)
avg_height
```

[1] 68.23495

If you were to use the null model, what would the RMSE be of this model be?

```
y_hat = sum(Galton$child) / n
y_hat = rep(y_hat, n)

Galton$child
```

```
##
##
##
##
```

```
## [687] 69.2 69.2 69.2 69.2 69.2 69.2 69.2 70.2 70.2 70.2 70.2 70.2 70.2 70.2
## [925] 73.7 73.7 73.7 73.7
```

y_hat

```
##
     [1] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
##
    [8] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
    [15] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
##
    [22] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
##
    [29] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
##
    [36] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
##
    [43] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
##
   [50] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
    [57] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
    [64] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
##
##
    [71] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
##
    [78] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
   [85] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
##
    [92] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
   [99] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
##
## [106] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [113] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [120] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [127] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [134] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [141] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [148] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [155] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [162] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [169] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [176] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [183] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
```

```
## [190] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [197] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [204] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [211] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [218] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [225] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [232] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [239] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [246] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [253] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [260] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [267] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [274] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [281] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [288] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [295] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [302] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [309] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [316] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [323] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [330] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [337] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [344] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [351] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [358] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [365] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [372] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [379] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [386] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [393] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [400] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [407] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [414] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [421] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [428] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [435] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [442] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [449] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [456] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [463] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [470] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [477] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [484] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [491] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [498] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [505] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [512] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [519] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [526] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [533] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [540] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [547] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [554] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [561] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
```

```
## [568] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [575] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [582] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [589] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [596] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [603] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [610] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [617] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [624] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [631] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [638] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [645] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [652] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [659] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [666] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [673] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [680] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [687] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [694] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [701] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [708] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [715] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [722] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [729] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [736] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [743] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [750] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [757] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [764] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [771] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [778] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [785] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [792] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [799] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [806] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [813] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [820] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [827] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [834] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [841] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [848] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [855] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [862] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [869] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [876] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [883] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [890] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [897] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
  [904] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [911] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [918] 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847 68.08847
## [925] 68.08847 68.08847 68.08847 68.08847
```

```
SSE = sum((Galton$child - y_hat)^2)
SSE

## [1] 5877.207

MSE = (1 / (n-2)) * SSE

MSE

## [1] 6.346875

RMSE = sqrt(MSE)
RMSE
```

[1] 2.519301

Note that in Math 241 you learned that the sample average is an estimate of the "mean", the population expected value of height. We will call the average the "mean" going forward since it is probably correct to the nearest tenth of an inch with this amount of data.

Run a linear model attempting to explain the childrens' height using the parents' height. Use 1m and use the R formula notation. Compute and report b_0 , b_1 , RMSE and R^2 . Use the correct units to report these quantities.

Galton

```
##
       parent child
## 1
         70.5
               61.7
## 2
         68.5
               61.7
## 3
         65.5
               61.7
         64.5
## 4
               61.7
## 5
         64.0
               61.7
## 6
         67.5
               62.2
## 7
         67.5
               62.2
## 8
         67.5
               62.2
## 9
         66.5
               62.2
## 10
         66.5
               62.2
## 11
         66.5
               62.2
## 12
         64.5
               62.2
## 13
         70.5
               63.2
               63.2
## 14
         69.5
## 15
         68.5
               63.2
## 16
         68.5
               63.2
         68.5
               63.2
## 17
         68.5
##
  18
               63.2
##
  19
         68.5
               63.2
         68.5
## 20
               63.2
## 21
         68.5
               63.2
## 22
         67.5
               63.2
## 23
         67.5
               63.2
## 24
         67.5
               63.2
## 25
         67.5
               63.2
## 26
         67.5
               63.2
## 27
         66.5
               63.2
## 28
         66.5
               63.2
## 29
               63.2
         66.5
## 30
         65.5
               63.2
## 31
         65.5
               63.2
## 32
         65.5 63.2
```

```
65.5
                63.2
## 33
         65.5
                63.2
## 34
         65.5
                63.2
## 35
## 36
         65.5
                63.2
## 37
         65.5
                63.2
## 38
         65.5
                63.2
## 39
         64.5
                63.2
         64.5
## 40
                63.2
## 41
         64.5
                63.2
                63.2
## 42
         64.5
## 43
         64.0
                63.2
         64.0
                63.2
## 44
## 45
         69.5
                64.2
         69.5
                64.2
## 46
## 47
         69.5
                64.2
## 48
         69.5
                64.2
## 49
         69.5
                64.2
## 50
         69.5
                64.2
                64.2
## 51
         69.5
         69.5
                64.2
## 52
         69.5
## 53
                64.2
## 54
         69.5
                64.2
         69.5
                64.2
## 55
                64.2
## 56
         69.5
         69.5
               64.2
## 57
## 58
         69.5
                64.2
## 59
         69.5
                64.2
## 60
         69.5
                64.2
         68.5
                64.2
## 61
## 62
         68.5
                64.2
         68.5
                64.2
## 63
## 64
         68.5
                64.2
## 65
         68.5
                64.2
                64.2
## 66
         68.5
         68.5
                64.2
## 67
         68.5
                64.2
## 68
## 69
         68.5
                64.2
## 70
         68.5
                64.2
                64.2
## 71
         68.5
## 72
         67.5
                64.2
## 73
         67.5
                64.2
         67.5
                64.2
## 74
## 75
         67.5
                64.2
## 76
         67.5
                64.2
## 77
         67.5
                64.2
         67.5
                64.2
## 78
## 79
         67.5
                64.2
## 80
         67.5
                64.2
         67.5
                64.2
## 81
## 82
         67.5
                64.2
         67.5
                64.2
## 83
         67.5
## 84
                64.2
         67.5
                64.2
## 85
## 86
         66.5 64.2
```

```
66.5 64.2
## 87
         66.5
               64.2
## 88
         66.5
               64.2
## 89
## 90
         66.5
               64.2
## 91
         65.5
               64.2
## 92
         65.5
               64.2
## 93
         65.5
               64.2
               64.2
         65.5
## 94
## 95
         65.5
               64.2
               64.2
## 96
         64.5
## 97
         64.5
               64.2
         64.5
               64.2
## 98
## 99
         64.5
               64.2
               64.2
## 100
         64.0
         64.0
## 101
               64.2
## 102
         64.0
               64.2
## 103
         64.0
               64.2
         71.5
               65.2
## 104
## 105
         70.5
               65.2
         69.5
               65.2
## 106
## 107
         69.5
               65.2
## 108
         69.5
               65.2
         69.5
               65.2
## 109
## 110
         68.5
               65.2
         68.5
## 111
               65.2
## 112
         68.5
               65.2
## 113
         68.5
               65.2
## 114
         68.5
               65.2
## 115
         68.5
               65.2
## 116
         68.5
               65.2
## 117
         68.5
               65.2
## 118
         68.5
               65.2
## 119
         68.5
               65.2
## 120
         68.5
               65.2
         68.5
               65.2
## 121
         68.5
## 122
               65.2
## 123
         68.5
               65.2
## 124
         68.5
               65.2
               65.2
## 125
         68.5
## 126
         67.5
               65.2
## 127
         67.5
               65.2
## 128
         67.5
               65.2
## 129
         67.5
               65.2
## 130
         67.5
               65.2
## 131
         67.5
               65.2
         67.5
               65.2
## 132
## 133
         67.5
               65.2
## 134
         67.5
               65.2
               65.2
## 135
         67.5
## 136
         67.5
               65.2
         67.5
## 137
               65.2
         67.5
## 138
               65.2
               65.2
## 139
         67.5
## 140
         67.5 65.2
```

```
66.5
               65.2
## 141
## 142
         66.5
               65.2
         65.5
               65.2
## 143
## 144
         65.5
               65.2
## 145
         65.5
               65.2
## 146
         65.5
               65.2
## 147
         65.5
               65.2
## 148
         65.5
               65.2
## 149
         65.5
               65.2
## 150
         64.5
               65.2
## 151
         64.0
               65.2
         71.5
               66.2
## 152
## 153
         71.5
               66.2
         71.5
               66.2
## 154
## 155
         70.5
               66.2
## 156
         69.5
               66.2
## 157
         69.5
               66.2
         69.5
               66.2
## 158
## 159
         69.5
               66.2
               66.2
## 160
         69.5
## 161
         69.5
               66.2
## 162
         69.5
               66.2
         69.5
               66.2
## 163
## 164
         69.5
               66.2
         69.5
## 165
               66.2
## 166
         69.5
               66.2
## 167
         69.5
               66.2
## 168
         69.5
               66.2
## 169
         69.5
               66.2
## 170
         69.5
               66.2
               66.2
## 171
         69.5
## 172
         69.5
               66.2
## 173
         68.5
               66.2
## 174
         68.5
               66.2
         68.5
               66.2
## 175
         68.5
               66.2
## 176
## 177
         68.5
               66.2
## 178
         68.5
               66.2
               66.2
## 179
         68.5
## 180
         68.5
               66.2
## 181
         68.5
               66.2
## 182
         68.5
               66.2
## 183
         68.5
               66.2
## 184
         68.5
               66.2
## 185
         68.5
               66.2
         68.5
               66.2
## 186
## 187
         68.5
               66.2
## 188
         68.5
               66.2
               66.2
## 189
         68.5
## 190
         68.5
               66.2
         68.5
## 191
               66.2
## 192
         68.5
               66.2
## 193
         68.5
               66.2
## 194
         68.5
               66.2
```

```
68.5
               66.2
## 195
## 196
         68.5
               66.2
               66.2
## 197
         68.5
## 198
         67.5
               66.2
## 199
         67.5
               66.2
## 200
         67.5
               66.2
## 201
         67.5
               66.2
## 202
         67.5
               66.2
## 203
         67.5
               66.2
## 204
         67.5
               66.2
## 205
         67.5
               66.2
## 206
         67.5
               66.2
## 207
         67.5
               66.2
         67.5
## 208
               66.2
## 209
         67.5
               66.2
## 210
         67.5
               66.2
## 211
         67.5
               66.2
## 212
         67.5
               66.2
## 213
         67.5
               66.2
## 214
               66.2
         67.5
## 215
         67.5
               66.2
## 216
         67.5
               66.2
## 217
         67.5
               66.2
## 218
         67.5
               66.2
         67.5
               66.2
## 219
## 220
         67.5
               66.2
## 221
         67.5
               66.2
## 222
         67.5
               66.2
## 223
         67.5
               66.2
## 224
         67.5
               66.2
         67.5
## 225
               66.2
## 226
         67.5
               66.2
## 227
         67.5
               66.2
## 228
         67.5
               66.2
## 229
         67.5
               66.2
         67.5
## 230
               66.2
## 231
         67.5
               66.2
## 232
         67.5
               66.2
## 233
         67.5
               66.2
## 234
         66.5
               66.2
## 235
         66.5
               66.2
## 236
         66.5
               66.2
## 237
         66.5
               66.2
## 238
         66.5
               66.2
## 239
         66.5
               66.2
## 240
         66.5
               66.2
## 241
         66.5
               66.2
## 242
         66.5
               66.2
## 243
         66.5
               66.2
## 244
         66.5
               66.2
         66.5
## 245
               66.2
## 246
         66.5
               66.2
## 247
         66.5
               66.2
## 248
         66.5
               66.2
```

```
## 249
         66.5
               66.2
## 250
         66.5
               66.2
               66.2
## 251
         65.5
## 252
         65.5
               66.2
## 253
         65.5
               66.2
## 254
         65.5
               66.2
## 255
         65.5
               66.2
## 256
         65.5
               66.2
## 257
         65.5
               66.2
## 258
         65.5
               66.2
## 259
         65.5
               66.2
               66.2
## 260
         65.5
## 261
         65.5
               66.2
## 262
         64.5
               66.2
## 263
         64.5
               66.2
## 264
         64.5
               66.2
## 265
         64.5
               66.2
         64.5
               66.2
## 266
## 267
         64.0
               66.2
## 268
               66.2
         64.0
## 269
         71.5
               67.2
## 270
         71.5
               67.2
         71.5
               67.2
## 271
## 272
         71.5
               67.2
         70.5
## 273
               67.2
## 274
         70.5
               67.2
## 275
         70.5
               67.2
## 276
         69.5
               67.2
## 277
         69.5
               67.2
## 278
         69.5
               67.2
               67.2
## 279
         69.5
## 280
         69.5
               67.2
## 281
         69.5
               67.2
## 282
         69.5
               67.2
         69.5
               67.2
## 283
         69.5
## 284
               67.2
## 285
         69.5
               67.2
## 286
         69.5
               67.2
## 287
         69.5
               67.2
## 288
         69.5
               67.2
## 289
         69.5
               67.2
## 290
         69.5
               67.2
## 291
         69.5
               67.2
## 292
         69.5
               67.2
## 293
         69.5
               67.2
## 294
         69.5
               67.2
## 295
         69.5
               67.2
## 296
         69.5
               67.2
## 297
               67.2
         69.5
## 298
         69.5
               67.2
         69.5
## 299
               67.2
## 300
         69.5
               67.2
## 301
         69.5
               67.2
## 302
         69.5 67.2
```

```
## 303
         68.5
               67.2
## 304
         68.5
               67.2
## 305
               67.2
         68.5
## 306
         68.5
               67.2
## 307
         68.5
               67.2
## 308
         68.5
               67.2
## 309
         68.5
               67.2
         68.5
               67.2
## 310
## 311
         68.5
               67.2
## 312
         68.5
               67.2
## 313
         68.5
               67.2
               67.2
## 314
         68.5
## 315
         68.5
               67.2
## 316
         68.5
               67.2
## 317
         68.5
               67.2
## 318
         68.5
               67.2
## 319
         68.5
               67.2
         68.5
## 320
               67.2
## 321
         68.5
               67.2
## 322
               67.2
         68.5
## 323
         68.5
               67.2
## 324
         68.5
               67.2
## 325
               67.2
         68.5
## 326
         68.5
               67.2
## 327
         68.5
               67.2
## 328
         68.5
               67.2
## 329
         68.5
               67.2
## 330
         68.5
               67.2
## 331
         68.5
               67.2
## 332
         68.5
               67.2
## 333
         68.5
               67.2
         67.5
               67.2
## 334
## 335
         67.5
               67.2
## 336
         67.5
               67.2
         67.5
               67.2
## 337
         67.5
## 338
               67.2
## 339
         67.5
               67.2
## 340
         67.5
               67.2
## 341
         67.5
               67.2
## 342
         67.5
               67.2
## 343
         67.5
               67.2
## 344
         67.5
               67.2
## 345
         67.5
               67.2
## 346
         67.5
               67.2
## 347
         67.5
               67.2
         67.5
               67.2
## 348
## 349
         67.5
               67.2
## 350
         67.5
               67.2
## 351
         67.5
               67.2
## 352
         67.5
               67.2
         67.5
## 353
               67.2
## 354
         67.5
               67.2
## 355
         67.5
               67.2
## 356
         67.5 67.2
```

```
67.5 67.2
## 357
## 358
         67.5
               67.2
## 359
         67.5
               67.2
## 360
         67.5
               67.2
## 361
         67.5
               67.2
## 362
         67.5
               67.2
## 363
         67.5
               67.2
         67.5
               67.2
## 364
## 365
         67.5
               67.2
## 366
         67.5
               67.2
## 367
         67.5
               67.2
         67.5
               67.2
## 368
## 369
         67.5
               67.2
## 370
         67.5
               67.2
## 371
         67.5
               67.2
## 372
         66.5
               67.2
## 373
         66.5
               67.2
## 374
         66.5
               67.2
## 375
         66.5
               67.2
## 376
               67.2
         66.5
## 377
         66.5
               67.2
## 378
         66.5
               67.2
         66.5
               67.2
## 379
## 380
         66.5
               67.2
## 381
         66.5
               67.2
## 382
         66.5
               67.2
## 383
         66.5
               67.2
## 384
         66.5
               67.2
## 385
         66.5
               67.2
## 386
         66.5
               67.2
## 387
         66.5
               67.2
## 388
         66.5
               67.2
## 389
         65.5
               67.2
## 390
         65.5
               67.2
         65.5
               67.2
## 391
         65.5
## 392
               67.2
## 393
         65.5
               67.2
## 394
         65.5
               67.2
## 395
         65.5
               67.2
## 396
         65.5
               67.2
## 397
         65.5
               67.2
## 398
         65.5
               67.2
## 399
         65.5
               67.2
## 400
         64.5
               67.2
## 401
         64.5
               67.2
         64.5
               67.2
## 402
## 403
         64.5
               67.2
## 404
         64.5
               67.2
## 405
         64.0
               67.2
## 406
         64.0
               67.2
         72.5
## 407
               68.2
## 408
         71.5
               68.2
## 409
         71.5
               68.2
## 410
         71.5
               68.2
```

```
70.5 68.2
## 411
## 412
         70.5
                68.2
         70.5
                68.2
## 413
## 414
         70.5
                68.2
## 415
         70.5
                68.2
## 416
         70.5
                68.2
## 417
         70.5
                68.2
## 418
         70.5
                68.2
## 419
         70.5
                68.2
## 420
         70.5
                68.2
## 421
         70.5
                68.2
## 422
         70.5
                68.2
## 423
         69.5
                68.2
         69.5
                68.2
## 424
## 425
         69.5
                68.2
## 426
         69.5
                68.2
## 427
         69.5
                68.2
         69.5
                68.2
## 428
## 429
         69.5
                68.2
                68.2
## 430
         69.5
## 431
         69.5
                68.2
## 432
         69.5
                68.2
         69.5
                68.2
## 433
## 434
         69.5
                68.2
## 435
         69.5
                68.2
## 436
         69.5
                68.2
## 437
         69.5
                68.2
## 438
         69.5
                68.2
## 439
         69.5
                68.2
## 440
         69.5
                68.2
## 441
         69.5
                68.2
## 442
         69.5
                68.2
## 443
         68.5
                68.2
## 444
         68.5
                68.2
         68.5
                68.2
## 445
         68.5
## 446
                68.2
## 447
         68.5
                68.2
## 448
         68.5
                68.2
                68.2
## 449
         68.5
## 450
         68.5
                68.2
## 451
         68.5
                68.2
## 452
         68.5
                68.2
## 453
         68.5
                68.2
## 454
         68.5
                68.2
## 455
         68.5
                68.2
## 456
         68.5
                68.2
## 457
         68.5
                68.2
## 458
         68.5
                68.2
                68.2
## 459
         68.5
         68.5
                68.2
## 460
         68.5
## 461
                68.2
## 462
         68.5
                68.2
## 463
         68.5
                68.2
## 464
         68.5
                68.2
```

```
## 465
         68.5
                68.2
## 466
         68.5
                68.2
                68.2
## 467
         68.5
         68.5
                68.2
## 468
## 469
         68.5
                68.2
## 470
         68.5
                68.2
## 471
         68.5
                68.2
## 472
         68.5
                68.2
## 473
         68.5
                68.2
## 474
         68.5
                68.2
## 475
         68.5
                68.2
## 476
                68.2
         68.5
## 477
         67.5
                68.2
## 478
         67.5
                68.2
## 479
         67.5
                68.2
## 480
         67.5
                68.2
## 481
         67.5
                68.2
         67.5
                68.2
## 482
## 483
         67.5
                68.2
                68.2
## 484
         67.5
## 485
         67.5
                68.2
## 486
         67.5
                68.2
         67.5
                68.2
## 487
## 488
         67.5
                68.2
## 489
         67.5
                68.2
## 490
         67.5
                68.2
## 491
         67.5
                68.2
## 492
         67.5
                68.2
## 493
         67.5
                68.2
## 494
         67.5
                68.2
## 495
         67.5
                68.2
         67.5
## 496
                68.2
## 497
         67.5
                68.2
## 498
         67.5
                68.2
         67.5
                68.2
## 499
         67.5
## 500
                68.2
## 501
         67.5
                68.2
## 502
         67.5
                68.2
                68.2
## 503
         67.5
## 504
         67.5
                68.2
## 505
         66.5
                68.2
## 506
         66.5
                68.2
## 507
         66.5
                68.2
## 508
         66.5
                68.2
## 509
         66.5
                68.2
## 510
         66.5
                68.2
## 511
         66.5
                68.2
## 512
         66.5
                68.2
                68.2
## 513
         66.5
## 514
         66.5
                68.2
         66.5
## 515
                68.2
## 516
         66.5
                68.2
## 517
         66.5
                68.2
## 518
         66.5
                68.2
```

```
## 519
         65.5
               68.2
## 520
         65.5
               68.2
         65.5
               68.2
## 521
## 522
         65.5
               68.2
## 523
         65.5
               68.2
## 524
         65.5
               68.2
## 525
         65.5
               68.2
               68.2
## 526
         64.0
## 527
         72.5
               69.2
               69.2
## 528
         72.5
## 529
         71.5
               69.2
         71.5
               69.2
## 530
## 531
         71.5
               69.2
         71.5
               69.2
## 532
## 533
         71.5
               69.2
## 534
         70.5
               69.2
## 535
         70.5
               69.2
## 536
         70.5
               69.2
## 537
         70.5
               69.2
## 538
         70.5
               69.2
## 539
         70.5
               69.2
## 540
         70.5
               69.2
         70.5
               69.2
## 541
## 542
         70.5
               69.2
         70.5
               69.2
## 543
## 544
         70.5
               69.2
## 545
         70.5
               69.2
## 546
         70.5
               69.2
               69.2
## 547
         70.5
## 548
         70.5
               69.2
         70.5
               69.2
## 549
## 550
         70.5
               69.2
## 551
         70.5
               69.2
## 552
         69.5
               69.2
## 553
         69.5
               69.2
         69.5
               69.2
## 554
## 555
         69.5
               69.2
## 556
         69.5
               69.2
               69.2
## 557
         69.5
## 558
         69.5
               69.2
## 559
         69.5
               69.2
         69.5
## 560
               69.2
## 561
         69.5
               69.2
## 562
         69.5
               69.2
## 563
         69.5
               69.2
         69.5
               69.2
## 564
## 565
         69.5
               69.2
## 566
         69.5
               69.2
               69.2
## 567
         69.5
## 568
         69.5
               69.2
         69.5
## 569
               69.2
## 570
         69.5
               69.2
               69.2
## 571
         69.5
## 572
         69.5
               69.2
```

```
69.5
                69.2
## 573
## 574
         69.5
                69.2
## 575
         69.5
                69.2
## 576
         69.5
                69.2
## 577
         69.5
                69.2
## 578
         69.5
                69.2
## 579
         69.5
                69.2
## 580
         69.5
                69.2
## 581
         69.5
                69.2
## 582
         69.5
                69.2
## 583
         69.5
                69.2
         69.5
                69.2
## 584
## 585
         68.5
                69.2
## 586
         68.5
                69.2
## 587
         68.5
                69.2
## 588
         68.5
                69.2
## 589
         68.5
                69.2
## 590
         68.5
                69.2
## 591
         68.5
                69.2
## 592
                69.2
         68.5
## 593
         68.5
                69.2
## 594
         68.5
                69.2
         68.5
                69.2
## 595
## 596
         68.5
                69.2
         68.5
## 597
                69.2
## 598
         68.5
                69.2
## 599
         68.5
                69.2
## 600
         68.5
                69.2
## 601
         68.5
                69.2
## 602
         68.5
                69.2
## 603
         68.5
                69.2
## 604
         68.5
                69.2
## 605
         68.5
                69.2
## 606
         68.5
                69.2
         68.5
                69.2
## 607
         68.5
## 608
                69.2
## 609
         68.5
                69.2
## 610
         68.5
                69.2
                69.2
## 611
         68.5
## 612
         68.5
                69.2
## 613
         68.5
                69.2
## 614
         68.5
                69.2
## 615
         68.5
                69.2
## 616
         68.5
                69.2
## 617
         68.5
                69.2
## 618
         68.5
                69.2
## 619
         68.5
                69.2
## 620
         68.5
                69.2
## 621
         68.5
                69.2
## 622
         68.5
                69.2
         68.5
## 623
                69.2
## 624
         68.5
                69.2
## 625
         68.5
                69.2
## 626
         68.5
               69.2
```

```
## 627
         68.5
                69.2
## 628
         68.5
                69.2
## 629
         68.5
                69.2
## 630
         68.5
                69.2
## 631
         68.5
                69.2
## 632
         68.5
                69.2
## 633
         67.5
                69.2
## 634
         67.5
                69.2
## 635
         67.5
                69.2
## 636
         67.5
                69.2
## 637
         67.5
                69.2
## 638
         67.5
                69.2
##
  639
         67.5
                69.2
## 640
         67.5
                69.2
## 641
         67.5
                69.2
## 642
         67.5
                69.2
## 643
         67.5
                69.2
         67.5
                69.2
## 644
## 645
         67.5
                69.2
## 646
                69.2
         67.5
## 647
         67.5
                69.2
## 648
         67.5
                69.2
         67.5
                69.2
## 649
## 650
         67.5
                69.2
         67.5
## 651
                69.2
## 652
         67.5
                69.2
## 653
         67.5
                69.2
## 654
         67.5
                69.2
## 655
         67.5
                69.2
## 656
         67.5
                69.2
## 657
         67.5
                69.2
## 658
         67.5
                69.2
## 659
         67.5
                69.2
## 660
         67.5
                69.2
         67.5
                69.2
## 661
         67.5
## 662
                69.2
## 663
         67.5
                69.2
## 664
         67.5
                69.2
                69.2
## 665
         67.5
## 666
         67.5
                69.2
## 667
         67.5
                69.2
## 668
         67.5
                69.2
## 669
         67.5
                69.2
## 670
         67.5
                69.2
## 671
         66.5
                69.2
## 672
         66.5
                69.2
## 673
         66.5
                69.2
## 674
         66.5
                69.2
## 675
         66.5
                69.2
## 676
         66.5
                69.2
         66.5
## 677
                69.2
## 678
         66.5
                69.2
## 679
         66.5
                69.2
## 680
         66.5
               69.2
```

```
## 681
         66.5 69.2
## 682
         66.5
               69.2
         66.5
               69.2
## 683
## 684
         65.5
               69.2
## 685
         65.5
               69.2
## 686
         65.5
               69.2
## 687
         65.5
               69.2
               69.2
## 688
         65.5
## 689
         65.5
               69.2
               69.2
## 690
         65.5
## 691
         64.5
               69.2
## 692
         64.5
               69.2
## 693
         64.0
               69.2
               70.2
## 694
         72.5
## 695
         71.5
               70.2
               70.2
## 696
         71.5
## 697
         71.5
               70.2
         71.5
               70.2
## 698
               70.2
## 699
         71.5
## 700
         71.5
               70.2
         71.5
## 701
               70.2
## 702
         71.5
               70.2
## 703
         71.5
               70.2
## 704
         71.5
               70.2
         70.5 70.2
## 705
## 706
         70.5
               70.2
## 707
         70.5
               70.2
## 708
         70.5
               70.2
## 709
         70.5
               70.2
## 710
         70.5
               70.2
         70.5
               70.2
## 711
## 712
         70.5
               70.2
## 713
         70.5
               70.2
         70.5
               70.2
## 714
## 715
         70.5
               70.2
         70.5
               70.2
## 716
## 717
         70.5
               70.2
## 718
         70.5
               70.2
         69.5
               70.2
## 719
## 720
         69.5 70.2
## 721
         69.5
               70.2
         69.5
               70.2
## 722
## 723
         69.5
               70.2
## 724
         69.5
               70.2
## 725
         69.5
               70.2
               70.2
## 726
         69.5
## 727
         69.5
               70.2
## 728
         69.5
               70.2
## 729
               70.2
         69.5
## 730
         69.5
               70.2
         69.5
               70.2
## 731
         69.5
## 732
               70.2
               70.2
## 733
         69.5
## 734
         69.5 70.2
```

```
## 735
         69.5
               70.2
         69.5
## 736
                70.2
         69.5
                70.2
## 737
## 738
         69.5
                70.2
## 739
         69.5
                70.2
## 740
         69.5
                70.2
## 741
         69.5
                70.2
                70.2
## 742
         69.5
## 743
         69.5
                70.2
         68.5
## 744
                70.2
## 745
         68.5
                70.2
## 746
         68.5
                70.2
## 747
         68.5
                70.2
                70.2
## 748
         68.5
## 749
         68.5
                70.2
                70.2
## 750
         68.5
## 751
         68.5
                70.2
## 752
         68.5
                70.2
                70.2
## 753
         68.5
## 754
         68.5
                70.2
## 755
         68.5
                70.2
## 756
         68.5
                70.2
## 757
         68.5
                70.2
## 758
         68.5
                70.2
         68.5
               70.2
## 759
## 760
         68.5
               70.2
## 761
         68.5
                70.2
##
  762
         68.5
                70.2
         68.5
## 763
                70.2
## 764
         68.5
                70.2
         67.5
                70.2
## 765
## 766
         67.5
                70.2
## 767
         67.5
                70.2
                70.2
## 768
         67.5
## 769
         67.5
                70.2
         67.5
                70.2
## 770
## 771
         67.5
                70.2
## 772
         67.5
                70.2
         67.5
                70.2
## 773
## 774
         67.5
               70.2
## 775
         67.5
               70.2
                70.2
## 776
         67.5
## 777
         67.5
                70.2
## 778
         67.5
                70.2
## 779
         67.5
                70.2
## 780
         67.5
                70.2
## 781
         67.5
                70.2
## 782
         67.5
                70.2
## 783
                70.2
         67.5
## 784
         66.5
                70.2
         66.5
                70.2
## 785
## 786
         66.5
                70.2
                70.2
## 787
         66.5
## 788
         65.5
               70.2
```

```
## 789
         65.5 70.2
         65.5
## 790
               70.2
         65.5
               70.2
## 791
## 792
         65.5
               70.2
## 793
         72.5
               71.2
## 794
         72.5
               71.2
## 795
         71.5
               71.2
               71.2
## 796
         71.5
## 797
         71.5
               71.2
## 798
         71.5
               71.2
## 799
         70.5
               71.2
         70.5
               71.2
## 800
## 801
         70.5
               71.2
               71.2
         70.5
## 802
## 803
         70.5
               71.2
## 804
         70.5
               71.2
## 805
         70.5
               71.2
         69.5
               71.2
## 806
## 807
         69.5
               71.2
         69.5
               71.2
## 808
## 809
         69.5
               71.2
## 810
         69.5
               71.2
         69.5
               71.2
## 811
## 812
         69.5
               71.2
         69.5 71.2
## 813
## 814
         69.5 71.2
## 815
         69.5
               71.2
## 816
         69.5
               71.2
## 817
         69.5
               71.2
## 818
         69.5
               71.2
         69.5
               71.2
## 819
## 820
         69.5
               71.2
## 821
         69.5
               71.2
## 822
         69.5
               71.2
         69.5
               71.2
## 823
         69.5
## 824
               71.2
## 825
         69.5
               71.2
## 826
         68.5
               71.2
## 827
         68.5
               71.2
## 828
         68.5 71.2
## 829
         68.5
               71.2
               71.2
## 830
         68.5
## 831
         68.5
               71.2
## 832
         68.5
               71.2
## 833
         68.5
               71.2
               71.2
         68.5
## 834
## 835
         68.5
               71.2
## 836
         68.5
               71.2
               71.2
## 837
         68.5
## 838
         68.5
               71.2
         68.5
## 839
               71.2
## 840
         68.5
               71.2
         68.5 71.2
## 841
## 842
         68.5 71.2
```

```
## 843
         68.5 71.2
         67.5
## 844
               71.2
## 845
         67.5
               71.2
## 846
         67.5 71.2
## 847
         67.5
               71.2
## 848
         67.5
               71.2
## 849
         67.5
               71.2
               71.2
         67.5
## 850
## 851
         67.5
               71.2
## 852
         67.5
               71.2
## 853
         67.5
               71.2
         67.5
               71.2
## 854
## 855
         65.5
               71.2
         65.5
## 856
               71.2
## 857
         73.0
               72.2
               72.2
## 858
         72.5
         72.5 72.2
## 859
         72.5
               72.2
## 860
               72.2
## 861
         72.5
         72.5
               72.2
## 862
         72.5
## 863
               72.2
## 864
         72.5
               72.2
         71.5
               72.2
## 865
## 866
         71.5
               72.2
         71.5 72.2
## 867
## 868
         71.5 72.2
## 869
         71.5
               72.2
## 870
         71.5
               72.2
         71.5
## 871
               72.2
## 872
         71.5
               72.2
         71.5
               72.2
## 873
## 874
         70.5
               72.2
## 875
         70.5
               72.2
         70.5
               72.2
## 876
         70.5
               72.2
## 877
         69.5
## 878
               72.2
## 879
         69.5
               72.2
## 880
         69.5
               72.2
         69.5
               72.2
## 881
## 882
         69.5 72.2
## 883
         69.5 72.2
         69.5 72.2
## 884
## 885
         69.5
               72.2
## 886
         69.5
               72.2
## 887
         69.5
               72.2
               72.2
         69.5
## 888
## 889
         68.5
               72.2
## 890
         68.5 72.2
         68.5
               72.2
## 891
## 892
         68.5
               72.2
         67.5
## 893
               72.2
## 894
         67.5
               72.2
         67.5 72.2
## 895
## 896
         67.5 72.2
```

```
65.5 72.2
## 897
## 898
         73.0 73.2
## 899
         73.0 73.2
## 900
         73.0 73.2
## 901
         72.5
              73.2
## 902
         72.5 73.2
## 903
         71.5 73.2
## 904
         71.5 73.2
## 905
         70.5 73.2
## 906
         70.5 73.2
## 907
         70.5 73.2
         69.5 73.2
## 908
         69.5 73.2
## 909
## 910
         69.5 73.2
## 911
         69.5 73.2
## 912
         68.5 73.2
## 913
         68.5 73.2
## 914
         68.5 73.2
## 915
         72.5 73.7
## 916
         72.5 73.7
         72.5 73.7
## 917
## 918
         72.5 73.7
## 919
         71.5 73.7
## 920
         71.5 73.7
## 921
         70.5 73.7
## 922
         70.5 73.7
## 923
         70.5 73.7
## 924
         69.5 73.7
         69.5 73.7
## 925
## 926
         69.5 73.7
## 927
         69.5
              73.7
## 928
         69.5 73.7
y = Galton$child
x = Galton parent
galt = lm(y~x)
galt #b_0 and b_1
##
## Call:
## lm(formula = y \sim x)
##
## Coefficients:
## (Intercept)
                          Х
       23.9415
                     0.6463
##
summary(galt)$r.squared #the R^2
## [1] 0.2104629
summary(galt)$sigma #the RMSE
## [1] 2.238547
\#b_0 is 23.9415 inches, b_1 is 0.6463 inches, RMSE is 2.238547 inches, and R^2 is 0.2104629
```

Interpret all four quantities: b_0 , b_1 , RMSE and R^2 .

b_0 (23.9415 inches) isn't very significant because the intercept is far from any of the data points, b_1 indicates that our model predicts a 0.6463 inch height increase in the child for every 1.0 inch increase in parent height, RMSE (2.238547 inches) indicates that, on average, our model predicts a 2.238547 inch error from actual child height, and the R^2 (0.2104629) indicates that our linear model is not much better than the null model.

How good is this model? How well does it predict? Discuss.

This model is not very good. The RMSE of 2.238547 is significant, and the R^2 value indicates the model is not much better than the null model.

It is reasonable to assume that parents and their children have the same height? Explain why this is reasonable using basic biology and common sense.

It is reasonable to assume parents and their children will be similar in height, because height is mainly determined by genetics. However, the gender of the child is also a significant variable, because males are taller than females on average, and that is not considered in this model.

If they were to have the same height and any differences were just random noise with expectation 0, what would the values of β_0 and β_1 be?

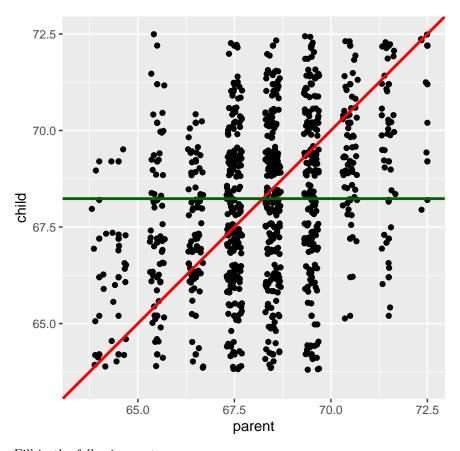
beta_1 would be 1, and beta_0 would be 0.

Let's plot (a) the data in \mathbb{D} as black dots, (b) your least squares line defined by b_0 and b_1 in blue, (c) the theoretical line β_0 and β_1 if the parent-child height equality held in red and (d) the mean height in green.

```
pacman::p_load(ggplot2)
ggplot(Galton, aes(x = parent, y = child)) +
    geom_point() +
    geom_jitter() +
    geom_abline(intercept = b_0, slope = b_1, color = "blue", size = 1) +
    geom_abline(intercept = 0, slope = 1, color = "red", size = 1) +
    geom_abline(intercept = avg_height, slope = 0, color = "darkgreen", size = 1) +
    xlim(63.5, 72.5) +
    ylim(63.5, 72.5) +
    coord_equal(ratio = 1)
```

```
## Warning: Removed 76 rows containing missing values (geom_point).
```

Warning: Removed 88 rows containing missing values (geom_point).



Fill in the following sentence:

Children of short parents became shorter on average and children of tall parents became taller on average.

Why did Galton call it "Regression towards mediocrity in hereditary stature" which was later shortened to "regression to the mean"?

"...he found that the heights of the children tended to be more moderate than the heights of their parents. For example, if parents were very tall the children tended to be tall but shorter than their parents. If parents were very short the children tended to be short but taller than their parents were. This discovery he called "regression to the mean," with the word "regression" meaning to come back to." source: http://www.biostat.jhsph.edu/courses/bio653/misc/JMPer%20Cable%20Summer%2098%20Why%20is%20it%20called%20Regression.htm

Why should this effect be real?

There are multiple other variables that contribute to a dult height that are unaccounted for. You now have unlocked the mystery. Why is it that when modeling with y continuous, everyone calls it "regression"? Write a better, more descriptive and appropriate name for building predictive models with y continuous.

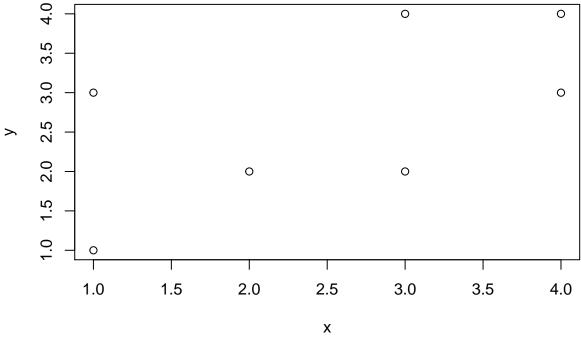
"regression" is a term that is historically derived from Galton's research on parental-child height, that has stuck even today. A better name for predictive models with y continuous is "least predictive error equation line".

Create a dataset \mathbb{D} which we call Xy such that the linear model as \mathbb{R}^2 about 50% and RMSE approximately 1.

```
x = c(1,1,2,3,3,4,4)

y = c(1,3,2,2,4,3,4)

plot(x,y)
```



```
Xy = lm(y~x)
summary(Xy)$r.squared #the R~2
```

[1] 0.3665158

summary(Xy)\$sigma #the RMSE

[1] 0.9701425

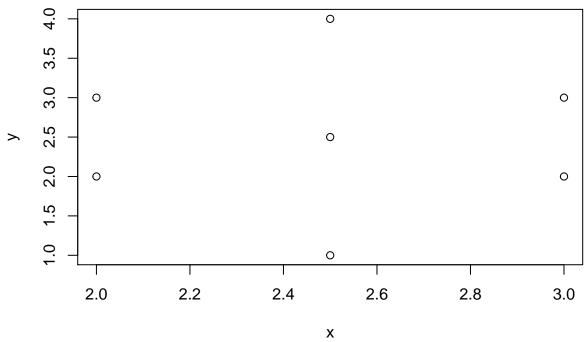
```
Xy = data.frame(x = x, y = y)
```

Create a dataset \mathbb{D} which we call Xy such that the linear model as \mathbb{R}^2 about 0% but x, y are clearly associated.

```
x = c(2,2,2.5,2.5,2.5,3,3)

y = c(2,3,1,2.5,4,2,3)

plot(x,y)
```



```
Xy = lm(y~x)
summary(Xy)$r.squared #the R~2
```

```
## [1] 2.151439e-31
Xy = data.frame(x = x, y = y)
```

Load up the famous iris dataset and drop the data for Species "virginica".

iris

##		Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
##	1	5.1	3.5	1.4	0.2	setosa
##	2	4.9	3.0	1.4	0.2	setosa
##	3	4.7	3.2	1.3	0.2	setosa
##	4	4.6	3.1	1.5	0.2	setosa
##	5	5.0	3.6	1.4	0.2	setosa
##	6	5.4	3.9	1.7	0.4	setosa
##	7	4.6	3.4	1.4	0.3	setosa
##	8	5.0	3.4	1.5	0.2	setosa
##	9	4.4	2.9	1.4	0.2	setosa
##	10	4.9	3.1	1.5	0.1	setosa
##	11	5.4	3.7	1.5	0.2	setosa
##	12	4.8	3.4	1.6	0.2	setosa
##	13	4.8	3.0	1.4	0.1	setosa
##	14	4.3	3.0	1.1	0.1	setosa
##	15	5.8	4.0	1.2	0.2	setosa
##	16	5.7	4.4	1.5	0.4	setosa
##	17	5.4	3.9	1.3	0.4	setosa
##	18	5.1	3.5	1.4	0.3	setosa
##	19	5.7	3.8	1.7	0.3	setosa
##	20	5.1	3.8	1.5	0.3	setosa
##	21	5.4	3.4	1.7	0.2	setosa

## 2		3.7	1.5	0.4	setosa
## 2		3.6	1.0	0.2	setosa
## 2	4 5.1	3.3	1.7	0.5	setosa
## 2	5 4.8	3.4	1.9	0.2	setosa
## 2	5.0	3.0	1.6	0.2	setosa
## 2	5.0	3.4	1.6	0.4	setosa
## 2	8 5.2	3.5	1.5	0.2	setosa
## 2	9 5.2	3.4	1.4	0.2	setosa
## 3		3.2	1.6	0.2	setosa
## 3		3.1	1.6	0.2	setosa
## 3		3.4	1.5	0.4	setosa
## 3		4.1	1.5	0.1	setosa
## 3		4.2	1.4	0.2	setosa
## 3		3.1	1.5	0.2	setosa
## 3		3.2	1.2	0.2	setosa
## 3		3.5	1.3	0.2	setosa
## 3		3.6	1.4	0.2	setosa
## 3 ## 4		3.0	1.3	0.2	setosa
		3.4	1.5	0.2	setosa
## 4		3.5	1.3	0.3	setosa
## 4		2.3	1.3	0.3	setosa
## 4		3.2	1.3	0.2	setosa
## 4		3.5	1.6	0.6	setosa
## 4		3.8	1.9	0.4	setosa
## 4		3.0	1.4	0.3	setosa
## 4		3.8	1.6	0.2	setosa
## 4		3.2	1.4	0.2	setosa
## 4		3.7	1.5	0.2	setosa
## 5		3.3	1.4	0.2	setosa
## 5	7.0	3.2	4.7		versicolor
## 5	6.4	3.2	4.5	1.5	versicolor
## 5	6.9	3.1	4.9	1.5	versicolor
## 5	5.5	2.3	4.0	1.3	versicolor
## 5	6.5	2.8	4.6	1.5	versicolor
## 5	5.7	2.8	4.5	1.3	versicolor
## 5	6.3	3.3	4.7	1.6	versicolor
## 5	8 4.9	2.4	3.3	1.0	versicolor
## 5	9 6.6	2.9	4.6	1.3	versicolor
## 6	5.2	2.7	3.9	1.4	versicolor
## 6	5.0	2.0	3.5	1.0	versicolor
## 6		3.0	4.2	1.5	versicolor
## 6		2.2	4.0	1.0	versicolor
## 6		2.9	4.7	1.4	versicolor
## 6		2.9	3.6		versicolor
## 6		3.1	4.4		versicolor
## 6		3.0	4.5		versicolor
## 6		2.7	4.1		versicolor
## 6		2.2	4.5		versicolor
## 7		2.5	3.9		versicolor
## 7		3.2	4.8		versicolor
## 7		2.8	4.0		versicolor
		2.8			versicolor
			4.9		
## 7		2.8 2.9	4.7 4.3		versicolor versicolor
## 7	5 6.4				

## 76	6.6	3.0	4.4	1.4 versicolor
## 77	6.8	2.8	4.8	1.4 versicolor
## 78	6.7	3.0	5.0	1.7 versicolor
## 79	6.0	2.9	4.5	1.5 versicolor
## 80	5.7	2.6	3.5	1.0 versicolor
## 81	5.5	2.4	3.8	1.1 versicolor
## 82	5.5	2.4	3.7	1.0 versicolor
## 83	5.8	2.7	3.9	1.2 versicolor
## 84	6.0	2.7	5.1	1.6 versicolor
## 85	5.4	3.0	4.5	1.5 versicolor
## 86	6.0	3.4	4.5	1.6 versicolor
## 87	6.7	3.1	4.7	1.5 versicolor
## 88	6.3	2.3	4.4	1.3 versicolor
## 89	5.6	3.0	4.1	1.3 versicolor
## 90	5.5	2.5	4.0	1.3 versicolor
## 91	5.5	2.6	4.4	1.2 versicolor
## 91	6.1	3.0	4.6	1.4 versicolor
		2.6		
## 93	5.8	2.3	4.0	1.2 versicolor
## 94	5.0		3.3	1.0 versicolor
## 95	5.6	2.7	4.2	1.3 versicolor
## 96	5.7	3.0	4.2	1.2 versicolor
## 97	5.7	2.9	4.2	1.3 versicolor
## 98	6.2	2.9	4.3	1.3 versicolor
## 99	5.1	2.5	3.0	1.1 versicolor
## 100	5.7	2.8	4.1	1.3 versicolor
## 101	6.3	3.3	6.0	2.5 virginica
## 102	5.8	2.7	5.1	1.9 virginica
## 103	7.1	3.0	5.9	2.1 virginica
## 104	6.3	2.9	5.6	1.8 virginica
## 105	6.5	3.0	5.8	2.2 virginica
## 106	7.6	3.0	6.6	2.1 virginica
## 107	4.9	2.5	4.5	1.7 virginica
## 108	7.3	2.9	6.3	1.8 virginica
## 109	6.7	2.5	5.8	1.8 virginica
## 110	7.2	3.6	6.1	2.5 virginica
## 111	6.5	3.2	5.1	2.0 virginica
## 112	6.4	2.7	5.3	1.9 virginica
## 113	6.8	3.0	5.5	2.1 virginica
## 114	5.7	2.5	5.0	2.0 virginica
## 115	5.8	2.8	5.1	2.4 virginica
## 116	6.4	3.2	5.3	2.3 virginica
## 117	6.5	3.0	5.5	1.8 virginica
## 118	7.7	3.8	6.7	2.2 virginica
## 119	7.7	2.6	6.9	2.3 virginica
## 120	6.0	2.2	5.0	1.5 virginica
## 121	6.9	3.2	5.7	2.3 virginica
## 122	5.6	2.8	4.9	2.0 virginica
## 123	7.7	2.8	6.7	2.0 virginica
## 124	6.3	2.7	4.9	1.8 virginica
## 125	6.7	3.3	5.7	2.1 virginica
## 126	7.2	3.2	6.0	1.8 virginica
## 127	6.2	2.8	4.8	1.8 virginica
## 128	6.1	3.0	4.9	1.8 virginica
## 129	6.4	2.8	5.6	2.1 virginica
	J. 1			

##	130	7.2	3.0	5.8	1.6	virginica
##	131	7.4	2.8	6.1	1.9	virginica
##	132	7.9	3.8	6.4	2.0	virginica
##	133	6.4	2.8	5.6	2.2	virginica
##	134	6.3	2.8	5.1	1.5	virginica
##	135	6.1	2.6	5.6	1.4	virginica
##	136	7.7	3.0	6.1	2.3	virginica
##	137	6.3	3.4	5.6	2.4	virginica
##	138	6.4	3.1	5.5	1.8	virginica
##	139	6.0	3.0	4.8	1.8	virginica
##	140	6.9	3.1	5.4	2.1	virginica
##	141	6.7	3.1	5.6	2.4	virginica
##	142	6.9	3.1	5.1	2.3	virginica
##	143	5.8	2.7	5.1	1.9	virginica
##	144	6.8	3.2	5.9	2.3	virginica
##	145	6.7	3.3	5.7	2.5	virginica
##	146	6.7	3.0	5.2	2.3	virginica
##	147	6.3	2.5	5.0	1.9	virginica
##	148	6.5	3.0	5.2	2.0	virginica
##	149	6.2	3.4	5.4	2.3	virginica
##	150	5.9	3.0	5.1	1.8	virginica

iris2 = iris[iris\$Species != "virginica",]
iris2

##		Sepal.Length	${\tt Sepal.Width}$	${\tt Petal.Length}$	Petal.Width	Species
##	1	5.1	3.5	1.4	0.2	setosa
##	2	4.9	3.0	1.4	0.2	setosa
##	3	4.7	3.2	1.3	0.2	setosa
##	4	4.6	3.1	1.5	0.2	setosa
##	5	5.0	3.6	1.4	0.2	setosa
##	6	5.4	3.9	1.7	0.4	setosa
##	7	4.6	3.4	1.4	0.3	setosa
##	8	5.0	3.4	1.5	0.2	setosa
##	9	4.4	2.9	1.4	0.2	setosa
##	10	4.9	3.1	1.5	0.1	setosa
##	11	5.4	3.7	1.5	0.2	setosa
##	12	4.8	3.4	1.6	0.2	setosa
##	13	4.8	3.0	1.4	0.1	setosa
##	14	4.3	3.0	1.1	0.1	setosa
##	15	5.8	4.0	1.2	0.2	setosa
##	16	5.7	4.4	1.5	0.4	setosa
##	17	5.4	3.9	1.3	0.4	setosa
##	18	5.1	3.5	1.4	0.3	setosa
##	19	5.7	3.8	1.7	0.3	setosa
##	20	5.1	3.8	1.5	0.3	setosa
##	21	5.4	3.4	1.7	0.2	setosa
##	22	5.1	3.7	1.5	0.4	setosa
##	23	4.6	3.6	1.0	0.2	setosa
##	24	5.1	3.3	1.7	0.5	setosa
##	25	4.8	3.4	1.9	0.2	setosa
##	26	5.0	3.0	1.6	0.2	setosa
##	27	5.0	3.4	1.6	0.4	setosa
##	28	5.2	3.5	1.5	0.2	setosa
##	29	5.2	3.4	1.4	0.2	setosa

	30	4.7	3.2	1.6	0.2	setosa
##	31	4.8	3.1	1.6	0.2	setosa
##	32	5.4	3.4	1.5	0.4	setosa
##	33	5.2	4.1	1.5	0.1	setosa
##	34	5.5	4.2	1.4	0.2	setosa
##	35	4.9	3.1	1.5	0.2	setosa
##	36	5.0	3.2	1.2	0.2	setosa
	37	5.5	3.5	1.3	0.2	setosa
	38	4.9	3.6	1.4	0.1	setosa
	39	4.4	3.0	1.3	0.2	setosa
	40	5.1	3.4	1.5	0.2	setosa
	41	5.0	3.5	1.3	0.3	setosa
	42	4.5	2.3	1.3	0.3	setosa
	43	4.4	3.2	1.3	0.2	setosa
	44	5.0	3.5	1.6	0.6	setosa
	45	5.1	3.8	1.9	0.4	setosa
	46	4.8	3.0	1.4	0.3	setosa
##	47	5.1	3.8	1.6	0.2	setosa
##	48	4.6	3.2	1.4	0.2	setosa
##	49	5.3	3.7	1.5	0.2	setosa
##	50	5.0	3.3	1.4	0.2	setosa
##	51	7.0	3.2	4.7	1.4 v	rersicolor
##	52	6.4	3.2	4.5	1.5 v	rersicolor
##	53	6.9	3.1	4.9	1.5 v	ersicolor
	54	5.5	2.3	4.0		ersicolor
	55	6.5	2.8	4.6		rersicolor
	56	5.7	2.8	4.5		rersicolor
	57	6.3	3.3	4.7		ersicolor
	58	4.9	2.4	3.3		ersicolor
	59		2.9	4.6		ersicolor
		6.6				
	60	5.2	2.7	3.9		rersicolor
	61	5.0	2.0	3.5		rersicolor
	62	5.9	3.0	4.2		rersicolor
	63	6.0	2.2	4.0		ersicolor
	64	6.1	2.9	4.7	1.4 v	rersicolor
##	65	5.6	2.9	3.6	1.3 v	rersicolor
##	66	6.7	3.1	4.4	1.4 v	rersicolor
##	67	5.6	3.0	4.5	1.5 v	rersicolor
##	68	5.8	2.7	4.1	1.0 v	rersicolor
##	69	6.2	2.2	4.5	1.5 v	rersicolor
##	70	5.6	2.5	3.9	1.1 v	rersicolor
##	71	5.9	3.2	4.8	1.8 v	ersicolor
	72	6.1	2.8	4.0	1.3 v	ersicolor
	73	6.3	2.5	4.9		ersicolor
	74	6.1	2.8	4.7		rersicolor
	75	6.4	2.9	4.3		ersicolor
	76	6.6	3.0	4.4		ersicolor
						ersicolor
	77 70	6.8	2.8	4.8		
	78	6.7	3.0	5.0		rersicolor
	79	6.0	2.9	4.5		rersicolor
	80	5.7	2.6	3.5		rersicolor
##		5.5	2.4	3.8		rersicolor
##		5.5	2.4	3.7		rersicolor
##	83	5.8	2.7	3.9	1.2 v	rersicolor

```
## 84
                 6.0
                              2.7
                                            5.1
                                                         1.6 versicolor
## 85
                 5.4
                                            4.5
                                                         1.5 versicolor
                              3.0
## 86
                                                         1.6 versicolor
                 6.0
                              3.4
                                            4.5
## 87
                 6.7
                              3.1
                                            4.7
                                                         1.5 versicolor
## 88
                 6.3
                              2.3
                                            4.4
                                                         1.3 versicolor
## 89
                                                         1.3 versicolor
                 5.6
                              3.0
                                            4.1
## 90
                                            4.0
                                                         1.3 versicolor
                 5.5
                              2.5
## 91
                 5.5
                              2.6
                                            4.4
                                                         1.2 versicolor
## 92
                 6.1
                              3.0
                                            4.6
                                                         1.4 versicolor
## 93
                 5.8
                              2.6
                                            4.0
                                                         1.2 versicolor
## 94
                 5.0
                              2.3
                                            3.3
                                                         1.0 versicolor
                              2.7
                                            4.2
                                                         1.3 versicolor
## 95
                 5.6
## 96
                 5.7
                              3.0
                                            4.2
                                                         1.2 versicolor
## 97
                 5.7
                              2.9
                                            4.2
                                                         1.3 versicolor
## 98
                 6.2
                              2.9
                                            4.3
                                                         1.3 versicolor
## 99
                 5.1
                              2.5
                                            3.0
                                                         1.1 versicolor
## 100
                 5.7
                                            4.1
                              2.8
                                                         1.3 versicolor
```

If the only input x is Species and you are trying to predict y which is Petal.Length, what would a reasonable, naive prediction be under both Species? Hint: it's what we did in class.

```
#Find the mean of the petal lengths of both species
setosa = iris[iris$Species == "setosa", ]
mean(setosa$Petal.Length)

## [1] 1.462
versicolor = iris[iris$Species == "versicolor", ]
mean(versicolor$Petal.Length)
```

[1] 4.26

Prove that this is the OLS model by fitting an appropriate 1m and then using the predict function to verify you get the same answers as you wrote previously.

```
iris_lm = lm(Petal.Length ~ Species, data = iris2)
iris_lm
##
## Call:
## lm(formula = Petal.Length ~ Species, data = iris2)
## Coefficients:
##
         (Intercept)
                      Speciesversicolor
##
               1.462
                                   2.798
predict(iris_lm, data.frame(Species = "setosa"))
##
       1
## 1.462
predict(iris_lm, data.frame(Species = "versicolor"))
##
## 4.26
```