

Package ‘semiIVreg’

August 2, 2024

Title Semi-Instrumental Variable (semi-IV) Regression

Version 1.0.0

Description semi-Instrumental Variable (semi-IV) estimation for general models, from Bruneel-Zupanc (2024). The main `semiivreg()` function is designed to work as easily as `lm()` or `ivreg()`. It automatically provides estimation of the marginal treatment effects (MTE) as well as eventual ATE (if the treatment effects were, in fact, homogenous).

Encoding UTF-8

Suggests knitr, rmarkdown, roxygen2, ivreg

Roxygen list(markdown = TRUE)

RoxygenNote 7.3.2

Imports data.table, ggplot2, gridExtra, MASS, stats

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Depends R (>= 2.10)

LazyData true

URL <https://www.cbruneel.com/>,
<https://cbruneelzupanc.github.io/semiIVreg/>

VignetteBuilder knitr

NeedsCompilation no

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construct_data	<i>Data construction functions</i>
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Description

These functions are used to construct the data from a given formula. Handles change from factor into several dummies for example. They also create reference individuals at which to evaluate the MTE and MTR (if no default is provided). For the numerical variables, take the average of the variable; for the factors, take the first level.

Usage

```
construct_data(formula, data)

create_ref_indiv(formula, data)

transform_factor(formula, data)
```

Arguments

formula	The formula of the model
data	The original dataset

Kappa_fun	<i>Control functions transformations for selection probabilities</i>
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Description

These functions provides pre-specified transformations to control flexibly for the selection probabilities in the regression.

These correspond to $\kappa_d(p)$ and corresponding $k_d(u)$ in Bruneel-Zupanc (2024).

Special functions are used for homogenous treatment effect specifications because the code is different. July 2024: for now, only polynomial transformations are encoded.

Usage

```
Kappa_fun(p, pol_degree = 5)

kdu_transform_fun(u, d, pol_degree = 5)

Kappa_homogenous_fun(p, pol_degree = 5)

ku_transform_homogenous_fun(u, pol_degree = 5)
```

Arguments

p	Vector of propensity scores to transform into a flexible function.
pol_degree	Degree of the polynomial transformation.
d	Which potential outcome to consider (only needed for $k_d(u)$ with heterogenous treatment effects).

Details

See Andresen (2018) or Bruneel-Zupanc (2024) for computation details linking $\kappa_d(p)$ and the corresponding corresponding $k_d(u)$.

$\kappa_1(p) = E(U_1|U_D \leq p)$ and $\kappa_0(p) = E(U_0|U_D > p)$ while $k_d(u) = E(U_d|U_D = u)$.

In the case of homogenous treatment effects: $k_0(u) = k_1(u)$. This provides some restriction on κ , hence the special functions.

Examples

```
v = seq(0.1, 0.9, by=0.1)
# Transformations for general Heterogenous TE functions:
Kappa_fun(p=v, pol_degree=6)
k1u = kdu_transform_fun(v, d=1, pol_degree=6)

# Transformations for Homogenous TE functions:
Kappa_homogenous_fun(p=v, pol_degree=6)
ku = ku_transform_homogenous_fun(v, pol_degree=6) # no d anymore, same for both d here;
```

mtr_fun	<i>MTE and MTR sub-estimation functions</i>
---------	---

Description

These functions allow to estimate the mte and mtr, and their confidence intervals, based on coefficients estimated from the main model in the main function. More details can be found in Bruneel-Zupanc (2024).

Different formulas must be applied depending on whether the treatment is homogenous or heterogeneous.

Usage

```
mtr_fun = function(d, data, ref_indiv, seq_u,
                  bwd = NULL, bw_y = NULL, bw_method = "plug-in",
                  pol_degree1, pol_degree2, var_outcome, var_treatment, var_w0, var_w1, var_covaria

lpolys_regress(x, y, bw=NULL, bw_method="plug-in", degree, drv)

mtr_fun_sieve(coeff, vcov, var_treatment, var_cov_2nd, ref_indiv, seq_u,
              homogenous=FALSE, pol_degree, conf_level, t_value, Xdat)
```

Arguments

bw_method	Method to compute the bandwidth (if bandwidth is NULL)
x	Vector of x values
y	Vector of y values
bw	Pre-specified bandwidth
degree	Degree of the polynomial: recommended to set to $drv + 1$
drv	Derivative order of the function to be estimated.

mtr_plot_fun	<i>Marginal Treatment Effect (MTE) and Responses (MTR) plots</i>
--------------	--

Description

Plot the MTR and MTE estimated curves with their confidence intervals.

Usage

```
mtr_plot_fun(dat_plot, common_supp)
```

```
mte_plot_fun(dat_plot, common_supp)
```

Arguments

dat_plot	Data frame with the estimated MTE and MTR values and their confidence intervals. Must contain specific variables: Phat, mtr0, mtr1, mtr0_lwr, mtr1_lwr, mtr0_upr, mtr1_upr, mte, mte_lwr, mte_upr.
common_supp	Vector of two values indicating the common support of the plot. Default is the full support 0,1 .
conf_band	Indicates whether to plot the confidence intervals. Default is "TRUE".
colMTE, colD0, colD1	Color of the MTE, MTR0 and MTR1 curves.

Details

Attention: by default in `semiivreg` the confidence intervals are computed analytically, and include an error because the first stage propensity score. This is corrected in `semiivreg_boot` by bootstrapping the entire estimation to obtain the confidence intervals.

roydata	<i>Generalized Roy Data with Heterogenous Treatment Effects</i>
---------	---

Description

A data frame of 100,000 observations drawn from a simulated Roy model with heterogenous treatment effects using [simul_data\(\)](#).

Usage

```
data(roydata)
```

Format

The data contains the following information which would be observed in a standard dataset:

y The observed outcome.

d The treatment.

w0, w1 The semi-IVs entering only D=0 and D=1.

Xbinary, Xcontinuous Two covariates, one binary and one continuous.

It also reports the typically unobserved potential outcomes and shocks:

y0, y1 The unobserved potential outcomes.

P The unobserved true treatment probability.

latent, V, Ud, U0, U1 The unobserved shocks V. Ud is the normalized V ranks. U0 and U1 are the outcome shocks. latent gives the latent utility term in the selection equation.

The data was generated using the following R code:

```
N=100000; set.seed(1234)
model_type = "heterogenous"
param_error = c(1, 1, 0.6, 0.5)
param_Z = c(0, 0, 0, 0, 1.5, 1.5, 0.9)
param_p = c(0, -0.7, 0.7, 0, 0, 0)
param_y0 = c(3.2, 0.8, 0, 0)
param_y1 = c(3.2+0.4, 0.5, 0, 0)
param_genX = c(0.4, 0, 2)

roydata = simul_data(N, model_type, param_y0, param_y1,
                     param_p, param_Z, param_genX, param_error)
```

roydata2

Generalized Roy Data with Homogenous Treatment Effects

Description

A data frame of 100,000 observations drawn from a simulated Roy model with homogenous treatment effects using [simul_data\(\)](#).

Usage

```
data(roydata2)
```

Format

The data contains the following information which would be observed in a standard dataset:

y The observed outcome.

d The treatment.

w0, w1 The semi-IVs entering only D=0 and D=1.

Xbinary, Xcontinuous Two covariates, one binary and one continuous.

It also reports the typically unobserved potential outcomes and shocks:

y0, y1 The unobserved potential outcomes.

P The unobserved true treatment probability.

latent, V, Ud, U0, U1 The unobserved shocks V. Ud is the normalized V ranks. U0 and U1 are the outcome shocks. latent gives the latent utility term in the selection equation.

The data was generated using the following R code:

```
N = 100000; set.seed(1234)
model_type = "homogenous"
param_error = c(1, 1.5, -0.6)
param_Z = c(0, 0, 0, 0, 1.5, 1.5, 0.9)
param_p = c(0, -0.5, 0.5, 0, 0, 0)
param_y0 = c(3.2, 0.8, 0, 0)
param_y1 = c(3.2+0.4, 0.5, 0, 0)
param_genX = c(0.4, 0, 2)

roydata2 = simul_data(N, model_type, param_y0, param_y1,
                      param_p, param_Z, param_genX, param_error)
```

semiivreg

Semi-IV Regression Function

Description

Semi-IV regression function from [Bruneel-Zupanc \(2024\)](#). Syntax inspired from ivreg. Returns MTE and MTR curves with confidence intervals. The estimation is almost *instantaneous* (a few seconds at most).

By default, return analytic standard errors not accounting for the fact that the propensity score is estimated in a first stage in semiivreg. Use semiivreg_boot to obtain 'correct' bootstrapped confidence intervals (takes a bit longer).

Usage

```
semiivreg(formula, data, propensity_formula=NULL,
          ref_indiv=NULL, firststage_model="probit",
          est_method="locpoly", # "locpoly", "sieve", or "homogenous".
          bw0=NULL, bw1=NULL, bw_y0=NULL, bw_y1=NULL, bw_method="plug-in",
          pol_degree_locpoly1=1, pol_degree_locpoly2=2,
          pol_degree_sieve=5, conf_level=0.05,
          common_supp_trim=c(0,1), trimming_value=NULL, automatic_trim=FALSE,
          plotting=TRUE)

semiivreg_boot(formula, Nboot=500, data, propensity_formula=NULL, ref_indiv=NULL,
               firststage_model="probit",
               pol_degree_transform=5, common_supp_trim=c(0,1), trimming_value=NULL,
               automatic_trim=FALSE, plotting=TRUE, conf_level=0.05, CI_method="delta")
```

Arguments

formula	<p>Formula of the regression, of the form <code>outcome ~ treatment semi-iv0 semi-iv1 commoncovariates</code>.</p> <p>The treatment variable should be binary (0, 1).</p> <p>covariates with an effect that differs on $D=1$ and $D=0$ should be included in each <code>semi-iv0</code> and <code>semi-iv1</code>.</p> <p>with <code>est_method = "locpoly"</code>: cannot restrict covariates to have common effects (not implemented), so <code>commoncovariates</code> will just be estimated as having generally a different effect on Y_0 and Y_1.</p>
data	Dataframe containing the data.
propensity_formula	<p>Formula for the 1st stage. If nothing specified, just runs a probit of <code>d ~ semi-iv0 + semi-iv1 + covariates</code> (removing the redundant variables).</p>
ref_indiv	<p>Specify the reference individual (in terms of covariates) at which we will evaluate the function.</p> <p>by default takes the average value for the numerical covariates, and the reference level for factors.</p>
firststage_model	<p>By default, the first stage is a probit model. Can specify another model (e.g., "logit").</p>
est_method	<p>Estimation method: default is "locpoly" for Robinson (1988) double residual regression for partially linear model. Other options include "sieve" to specify flexibly the control function as a polynomial with <code>pol_degree_sieve</code>, and "homogenous" which is a sieve where we also impose homogenous treatment effect.</p>
bw0, bw1	<p>Bandwidth of the first residual regressions of W_d and X on Phat. Need to be specified in the order of the covariates as specified in the model. Be very careful with factors. Default NULL and computed using the specified <code>bw_method</code>.</p>
bw_y0, bw_y1	<p>Bandwidth of the second regression of Y (net of the effects of the covariates) on Phat. Default NULL and computed using the specified <code>bw_method</code>.</p>
bw_method	<p>Method to compute the bandwidth of the local polynomial regressions. Default is simple "plug-in" method.</p>
pol_degree_locpoly1	<p>Degree of the local polynomial regression of the covariates on Phat. Default is 1 as recommended by Fan and Gijbels (1996) because we want to estimate the regular function.</p>
pol_degree_locpoly2	<p>Degree of the local polynomial regression of Y (net of the effects of the covariates) on Phat. Default is 2 as recommended by Fan and Gijbels (1996) because we want to estimate the derivative function.</p>
pol_degree_sieve	<p>Degree of the polynomial transformation for the control function.</p>
conf_level	<p>Confidence level for the confidence intervals.</p>
common_supp_trim	<p>Vector of two values indicating the set of propensity scores at which we will evaluate the function.</p> <p>Default is the full support $[0, 1]$. But can be trimmed manually.</p>

trimming_value	Can either be a vector <code>c(0.05, 0.95)</code> indicating the quantile of the propensity score above which and below which we keep the observations for both $D=0$ and $D=1$. Can also be a single value, in which case symmetric trimming up and down. Inserting a <code>trimming_value</code> generates <code>automatic_trim = TRUE</code> automatically.
automatic_trim	If TRUE, the estimation of the second stage is done on the common_support only.
plotting	TRUE if wants to plot at the end of the function, FALSE otherwise.
Nboot	Number of bootstrap samples.
CI_method	"delta" for delta method, "curve" for bootstrap the MTE curves directly. With <code>est_method = "locpoly"</code> , only "curve" method is possible.

Value

A list with the following elements:

`$data` Returns data of output estimation used to plot the MTE and MTR. In details:

`$RES` Dataframe with the estimated MTE and MTR values (and their confidence intervals if `est_method="sieve"` or "homogenous") for a sequence of unobservable resistance to treatment in the identifiable common support.

`$data` Original data used for the estimation where we added the propensity score estimated, named `Phat`, and where we made the transformation of the eventual factor variables as dummies.

`$ref_indiv` Reference individual(s) at which we evaluate the MTE and MTR.

`$Xdat` Set of covariates (this output is used for the bootstrap).

`$estimate` Returns the estimation of:

`$est`, or `$est0` and `$est1` If `est_method = "locpoly"`, `est0` and `est1` returns the second stage estimates of the effect of the covariates and semi-IVs on their respective potential outcomes. Coming out of the double residual regression à la Robinson (1988), running a no-intercept OLS of the residuals $Y-E(Y|P)$ on the residuals of every semi-IVs, $Wd-E(Wd|P)$, and covariates, $X-E(X|P)$.

If `est_method = "sieve"` or "homogenous", `$est` returns the second stage estimates of $E(Yd | D, Wd, X, P)$ where the propensity score is controlled for with a flexible control function (polynomial), $Kd(P)$.

`$propensity` First stage estimate of the propensity score.

`$avg_MTE` Average of the MTE over the identified common support. If full common support, it is an estimate of the $ATE(x, w0, w1)$. If `est_method="homogenous"`, the MTE is constant so it also gives the $ATE(x, w0, w1)$.

`$bw` Returns the bandwidth used (or estimated via `bw_method`) in the Robinson double residual regression.

`bw0` and `bw1` are the bandwidths of the first residual regressions of Yd , Wd and X on `Phat`.

`bw_y0` and `bw_y1` are the bandwidths of the second regression of Y (net of the effects of the covariates) on `Phat`. These are the one that matters for the smoothness of the MTE and MTR estimates.

`$plot` Returns separately the following plot objects: `supp` (support), `mtr`, `mte`.

`$supp` Returns the common support of the propensity score `Phat` between the two treatment group.

`$call` Returns the call of the function and the covariates and semi-IVs used.

The estimated model

semiivreg estimates the marginal treatment effect (MTE) and marginal treatment response (MTR) of a binary treatment variable using semi-IVs, W0 and W1. As with standard IVs (see Andresen, 2018), we estimate a semi-parametric partially linear model, as described in Bruneel-Zupanc (2024). For more details on the model and estimation procedure, see the vignette `vignette("semiIVreg", package = "semiIVreg")`, also available online [here](#). For more details on the use of the semiivreg function, see also the vignettes `vignette("semiIVreg_heterogenousTE", package = "semiIVreg")` and `vignette("semiIVreg_homogenousTE", package = "semiIVreg")`. For more details about causal inference with semi-IVs in general, see Bruneel-Zupanc (2024).

Caution about the Estimated Standard errors

By default, `est_method="locpoly"` returns no standard errors. If `est_method="sieve"` or `est_method="homogenous"`, it returns **analytic standard errors**: but these are wrong because they do not account for the fact that the propensity score is estimated. In any case, we recommend to use `semiivreg_boot` to obtain 'correct' bootstrapped confidence intervals. Implemented separately because the bootstrap takes more time, while the baseline semiivreg function is almost instantaneous.

Author(s)

Christophe Bruneel-Zupanc, cbruneel.com

References

Bruneel-Zupanc, C. (2024). Don't (fully) exclude me, it's not necessary! Identification with semi-IVs. arXiv preprint arXiv:2303.12667.

For empirical applications of the estimation of Marginal Treatment Effects with standard IVs, see for example:

Carneiro, P., Heckman, J. J., & Vytlacil, E. J. (2011). Estimating marginal returns to education. *American Economic Review*, 101(6), 2754-2781.

Brinch, C. N., Mogstad, M., & Wiswall, M. (2017). Beyond LATE with a discrete instrument. *Journal of Political Economy*, 125(4), 985-1039.

In particular, see Andresen, M. E. (2018). Exploring marginal treatment effects: Flexible estimation using Stata. *The Stata Journal*, 18(1), 118-158.

For double residual estimation of partially Linear models, see Robinson, P. M. (1988). Root-N-consistent semiparametric regression. *Econometrica: Journal of the Econometric Society*, 931-954.

For local polynomial regressions choice of degree: Fan, J., & Gijbels, I. (1996). Local polynomial modelling and its applications.

Examples

```
# Load data:
data(roydata)

# Run the semi-IV regression
semiiv = semiivreg(y~d|w0|w1, data=roydata)
semiiv = semiivreg(y~d|w0|w1|Xbinary + Xcontinuous, data=roydata) # with covariates
```

```

semiiv = semiivreg(y~d|w0+Xbinary|w1+Xbinary|Xcontinuous, data=roydata)
# Xbinary has different effect on Y0 and Y1, Xcontinuous has the same.
semiiv = semiivreg(y~d|w0|w1, data=roydata, propensity_formula = d~w0+w1+w0:w1)
# if want to specify another first stage

semiiv$plot$mtr # if want to plot mtr_plot

```

simul_data

Simulate data from the Generalized Roy Model with semi-IVs

Description

This function simulates data from the Generalized Roy Model with semi-IVs, following the simulation specified in Bruneel-Zupanc (2024).

For more details about the exact specification, see the vignettes [here](#) or by running `vignette("simul_data", package = "semiIVreg")`.

Usage

```

simul_data(N, model_type="heterogenous",
           param_y0, param_y1, param_p, param_Z, param_genX, param_error)

```

Arguments

N	Number of observations
model_type	Type of model: "heterogenous" or "homogenous"
param_y0	Parameters for Y0 = (delta0, beta0, beta0X1, beta0X2) i.e., intercept, effects on w0, X_1, X_2 on Y0.
param_y1	Parameters for Y1: (delta1, beta1, beta1X1, beta1X2). i.e., intercept, effects w1, X1, X2 on Y1.
param_p	Parameters for the selection: (alpha, alpha0, alpha1, alpha2, alphaX1, alphaX2) i.e., intercept and effects of w0, w1, w0w1, Xbinary, Xcontinuous on the latent utility.
param_Z	Parameters for the simulation of the semi-IVs: mean of W0 when X1=0, of W1 when X1=0, of W0 when X1=1, of W1 when X1=1; then variance of W0, W1, and covariance of W0 and W1.
param_genX	Parameters for the covariates: p_X1, mu_X2, sigma_X2.
param_error	Parameters for the error terms: depends on model_type: if heterogenous: variance of U0, U1, covariance of U0 and U1, variance of the cost (which has mean 0). if homogenous: variance of U, variance of V, covariance of U and V.

Details

This function simulates data from the Generalized Roy Model with semi-IVs, following the simulation specified in Bruneel-Zupanc (2024).

For more details about the exact specification, see the vignette [here](#) or by running `vignette("simul_data", package = "semiIVreg")`. One can use it to simulate general model with heterogenous treatment effects, but also restricted ones with homogenous treatment effects.

`simul_data` was used to simulate the dataset available with this package, `data(roydata)` to obtain the simulated model with heterogenous treatment effect, and `data(roydata2)` to obtain the simulated model with homogenous treatment effect.

Value

A data frame with the following columns:

y The observed outcome.

d The treatment.

w0, w1 The semi-IVs entering only D=0 and D=1.

Xbinary, Xcontinuous Two covariates, one binary and one continuous.

y0, y1 The unobserved potential outcomes.

P The unobserved true treatment probability.

latent, V, Ud, U0, U1 The unobserved shocks V. Ud is the normalized V ranks. U0 and U1 are the outcome shocks. latent gives the latent utility term in the selection equation.

References

Bruneel-Zupanc, C. (2023). Don't (fully) exclude me, it's not necessary! Identification with semi-IVs. arXiv preprint arXiv:2303.12667.

Andresen, M. E. (2018). Exploring marginal treatment effects: Flexible estimation using Stata. The Stata Journal, 18(1), 118-158.

Heckman, J. J., Urzua, S., & Vytlačil, E. (2006). Understanding instrumental variables in models with essential heterogeneity. The Review of Economics and Statistics, 88(3), 389-432.

Heckman, J. J., & Vytlačil, E. J. (2007). Econometric evaluation of social programs, part II: Using the marginal treatment effect to organize alternative econometric estimators to evaluate social programs, and to forecast their effects in new environments. Handbook of econometrics, 6, 4875-5143.

Examples

```
N = 10000; set.seed(12345)
```

```
# Example 1: Heterogenous Treatment Effects.
```

```
model_type = "heterogenous"
```

```
param_error = c(1, 1, 0.6, 0.5) # var_u0, var_u1, cov_u0u1, var_cost
```

```
param_Z = c(0, 0, 0, 0, 1.5, 1.5, 0.9)
```

```
# meanW0 Xbinary0, meanW1 Xbinary0, meanW0 Xbinary1, meanW1 Xbinary1, varW0, varW1, covW0W1
```

```
param_p = c(0, -0.7, 0.7, 0, 0, 0) # constant, W0, W1, W0xW1, Xbinary, Xcontinuous
```

```
param_y0 = c(3.2, 0.8, 0, 0) # intercept, W0, Xbinary, Xcontinuous;
```

```
param_y1 = c(3.2+0.4, 0.5, 0, 0) # the +0.4 = ATE; W1, Xbinary, Xcontinuous;
```

```
param_genX = c(0.4, 0, 2)
```

```
data = simul_data(N, model_type, param_y0, param_y1, param_p, param_Z, param_genX, param_error)
```

```
# Example 2: Homogenous Treatment Effects (constant MTE)
```

```
model_type = "homogenous"
```

```
param_error = c(1, 1.5, -0.6) # var_u, var_v, cov_uv
```

```
param_Z = c(0, 0, 0, 0, 1.5, 1.5, 0.9)
```

```
param_p = c(0, -0.5, 0.5, 0, 0, 0) # the constant <=> mean_V
```

```
param_y0 = c(3.2, 0.8, 0, 0)
```

```
param_y1 = c(3.2+0.4, 0.5, 0, 0)
```

```
param_genX = c(0.4, 0, 2)
```

```
data1 = simul_data(N, model_type, param_y0, param_y1, param_p, param_Z, param_genX, param_error)
```

```
# Set the effects of w1 or w0 on its outcome to zero if want a valid IV, e.g.,
# param_y1 = c(3.2+0.4, 0, 0, 0) # w1 is a valid IV
# or: param_y0 = c(3.2, 0, 0, 0) # w0 is a valid IV
```

supp_plot_fun	<i>Propensity score support plot</i>
---------------	--------------------------------------

Description

Returns the support plot by treatment status of the propensity score Phat included in a dataset.

Usage

```
supp_plot_fun(data, common_supp)
```

Arguments

data	Dataframe containing the treatment status, under a factor variable Treatment and the propensity score under the name Phat.
common_supp	Vector of two values indicating the common support of the plot. Default is the full support 0,1.
colMTE, colD0, colD1	Color of the MTE, MTR0 and MTR1 curves.

Examples

```
# Plot the true common support (with true - unobserved - propensity score)
# Using simulated data.
data(roydata); data=roydata;

# Syntax adjustment to use the function
data$Treatment = factor(data$d)
data$Phat = data$P # P is unobserved, we only know it because simulation here

#common_supp can be determined by looking at the plot - it's not necessary, just a graphical option
supp_P0 = c(min(data$Phat[which(data$d == 0)]), max(data$Phat[which(data$d== 0)]))
supp_P1 = c(min(data$Phat[which(data$d == 1)]), max(data$Phat[which(data$d == 1)]))
common_supp = c(max(supp_P0[1], supp_P1[1]), min(supp_P0[2], supp_P1[2]))

supp_plot = supp_plot_fun(data, common_supp); supp_plot
```

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