Short-Rate Pricing after the Liquidity and Credit Shocks: Including the Basis *

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Abstract

The basis between swaps referencing funded fixings and swaps referencing overnight-collateralized fixings (e.g. 6 month Euribor vs 6 month Eonia) has increased in importance with the 2007-9 liquidity and credit crises. This basis means that new pricing models for fixed income staples like caps, floors and swaptions are required. Recently new formulae have been proposed using market models. Here we present equivalent pricing in a short-rate framework which is important for applications involving credit, like CVA, where this is often useful because default can occur at any time. Furthermore, in this new multiple-curve world, short-rate models are fundamentally altered and we describe these changes.

Keywords: credit crisis, liquidity crisis, forward curve, discount curve, basis swaps, bootstrapping, swaps, swaptions, counterparty risk, CVA, multi-curve term structure modeling, closed form formulas.

1 Introduction

The 2007-9 liquidity and credit crises included large basis spreads opening up between overnight-collateralized instruments (e.g. 6M Eonia) and non-

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collateralized fixings[1] (e.g. 6M Euribor). Deposits of the same maturity as fixings follow the same pattern. Yield curves built from instruments referencing these different instrument types demonstrate a significant basis (e.g. building from Eonia swaps vs building from Euribor swaps). This means that new formulae are required for pricing previously-standard instruments, e.g. swaps, caps, and swaptions. Recent work offers different approaches [Bia10, CS09, FST09, KTW09, Hen09, Mer09, Mer10, Mor09, PT10, AB10] using short-rate, HJM and BGM settings with or without some fx analogy. However, within the short-rate setup, analytic swaption pricing formulae are lacking. Thus calibration to volatility data is awkward. Whilst [KTW09] use a short rate setting for multiple bond qualities they did not provide direct analytic swaption pricing (they price via a Gram-Charlier expansion on the bond price distribution). Short-rate settings can be, but are not necessarily, more appropriate when credit risk needs to be included for example in CVA [BM06a, BPP09, BM06b], because default can occur at any time.

We provide direct pseudo-analytic swaption pricing formulae within a short-rate setup including the basis between overnight-collateralized instruments (e.g. 6M Eonia) and non-collateralized fixings (e.g. 6M Euribor). We use a discounting curve to represent riskless investment and a separate curve for market expectations of non-collateralized fixings. We call this second curve the fixing curve. Our approach differs from [KTW09] in that they use discounting and basis curves whereas we use discounting and fixing curves. We follow [Bia10] in using an explicit fx argument to deal with potential arbitrage considerations. Our approach can also be generalized as more data becomes available, especially with respect to Eonia-type swaptions (currently missing from the market). Thus we propose a short-rate solution for the observed basis enabling modeling of standard instruments and calibration to swaptions.

The fx analogy for Eonia/Euribor modeling, as in [KTW09, Bia10], is motivated by the fact that fixed-for-floating swaps have characteristics of quantos: they observe Euribor fixings, i.e. based on deposits of a specific tenor, so observe risky+funded, and pay exactly that number in a collateralized+unfunded[2] setting (i.e. riskless). Furthermore risky and riskless assets are not interchangeable in the market. The same (uncollateralized) asset will have a different price from entities with different risk levels, thus there

[1]In Fixed Income ”fixing” is a noun, it means the value at which a rate is fixed once a day for contract settlement, e.g. ”Euribor® is the rate at which Euro interbank term deposits are offered by one prime bank to another prime bank within the EMU zone, and is published at 11:00 a.m. (CET) for spot value (T+2).”.

[2]Of course when swaps go off-market they require funding in as much as they are out of the money, and that they are not part of an asset-swap package, etc.

is some analogy to an exchange rate between risk levels (or qualities in the language of [KTW09]). The general approach of quoting a spread to price bonds from different issuers serves the same purpose. [KTW09] use a pricing kernel approach to change units (from risky to riskless) whereas [Bia10] use an fx rate to change units. We regard risky+funded as analogous to one currency and collateralized+unfunded as analogous to another currency. Note that whilst individual risky entities do default, Euribor fixings are not strongly dependent on the default of any single entity. Market practice supports this view that different investment qualities exist in the separation of funded/collateralized (or risky/riskless) worlds. We do not attempt to model the drivers of this separation, we start from the observable Eonia and Euribor instruments and derivatives.

Each different Euribor tenor fixing, e.g. 1 month, 3 months, 6 months, represents a different level of risk (including liquidity risk). Thus we have a separate fixing curve for each tenor Δ which we label f_Δ to make the source-tenor explicit. (In forward-rate models each different tenor forward is a different product). We have a single discounting curve (Eonia). Unlike pre-crisis short-rate modeling different levels of risk are now priced significantly differently and are not interchangeable. Thus each fixing curve is specific to the tenor that it was constructed from. Whilst we could calculate a 1 month rate from a 6 month tenor fixing curve will not be the same as the 1 month fixing, because it represents a different level of risk. Thus a short-rate approach in the post-crisis world provides tenor-fixings at different future times, but not the fixings from different tenors (at least not with the level of risk of the different tenor). This is inherent in the post-crisis market: fixing curves multiply whether in short-rate or forward-rate models. Thus the new post-crisis world causes fundamental changes to short-rate modeling — previously short-rate models could provide future floating rates for any tenor, now a different fixing curve is required for each tenor.

A short-rate setup, for the observed basis between discounting and fixing, is potentially useful for a variety of problems, e.g. CVA (credit valuation adjustment), it offers a complementary approach to forward-rate modeling [BPP09, BM06a]. In addition, the credit world in general has a strong connection to short-rate modeling because default can happen at any time, not just at tenor-multiple-maturities. [BM06b] expands on this point and concludes that the forward rate approach will be less dominant in the credit space (see Chapter 23). As they state, a short rate approach combines naturally with intensity-based hazard rate modeling. Thus this paper complements existing forward-rate multi-curve setups (HJM and BGM) [Bia10, Hen09, Mer09, Mer10, AB10, PT10] and extends [KTW09].

2 Model

We put ourselves in the European context by using Euribor and Eonia. The equations are general and this naming is only for convenience.

Discount Curve We use a riskless curve as our discount curve. For a concrete example we take Eonia as the riskless discount curve. We aim to price standard fixed income instruments (e.g. swaps, swaptions) subject to collateral agreements, thus the yield curve for similarly collateralized instruments is appropriate for discounting *whatever fixings they reference.*

Fixing Curve We calibrate a second curve, or "fixing curve" to reproduce fixings and market expected fixings (when used together with the discount curve). Note that each different fixing tenor Δ leads to a different fixing curve f_Δ. For a concrete example consider the 6 month Euribor fixing and market-quoted standard EUR swaps. This follows the concept in [AB10, KTW09] of having a discount curve and a separate curve that, together with the discount curve, reproduces market expectations of fixings and instruments based on them e.g. forwards and swaps.

N.B. a FRA (Forward Rate Agreement) payment is not the same as a swap payment for the same fixing because a FRA is paid in advance (discounted with the fixing by definition), whereas a standard swap fixing is paid in arrears.

We use a short-rate model for the discounting curve D, and for the fixing curve f_Δ, with short rates $r_D(t)$ and $r_{f_\Delta}(t)$ respectively[3]. Note that the level of risk that the fixing curve represents is explicitly referenced in Δ. Although the short rates are both denominated in the same currency, say EUR, they represent theoretically different investment qualities (risks), as in [KTW09], thus there is an exchange rate between them. This setup also mirrors that of [Bia10] to preserve no-arbitrage between alternative investment opportunities.

Similarly to [KTW09], the second (fixing) curve f_Δ represents an investment opportunity in as much as market-Δ-fixing-tenor-risk-curve-bonds are available. Whilst it may be difficult in practice to find a provider with the appropriate level of risk for a given tenor this does not affect the consistency of the derivation. All modeling abstracts from reality — for example no riskless (or overnight-risk-level) bonds exist because even previously-safe sovereign government bonds (e.g. US, UK, Germany) have recently exhibited significant yield volatility.

[3] We need d later for an index, hence the use of D for the discounting curve.

2.1 Standard Instruments

The setup for standard instruments (swaps and swaptions) is very similar to [KTW09, Bia10, AB10, Mer09], our innovations are in the next section where we show how to price them within a specific short rate setup.

2.1.1 Definitions

We define each of our base items here: short rates r_D, r_{f_Δ}; bank accounts $B_l(t)$ ($l = D, f_\Delta$); and discount factors $P_l(t,T)$. $\mathcal{F}_t$ is the usual filtration at t.

- r_D the instantaneous rate of return of a riskless investment.
- r_{f_Δ} the instantaneous rate of return of a risky investment with risk level corresponding to LIBOR with tenor Δ.
- $B_l(t) = \exp(\int_0^t r_l(s)ds)$, bank account with level of risk $l = D, f_\Delta$.
- $P_l(t,T) = \mathbb{E}_l[B_l(t)/B_l(T)|\,\mathcal{F}_t]$ zero coupon bond with level of risk $l = D, f_\Delta$.

2.1.2 Swaps

Swaps in this dual curve set up are similar to differential floating for fixed swaps (aka floating for fixed quanto swaps) in [BM06b] (page 623). To price swaps we need to define the simply compounded (fixing curve) interest rate that the swaps fix on, $L_k(t)$, at T_{k-1} for the interest rate from T_{k-1} to T_k, i.e. tenor τ^{fix}. Note that f_Δ refers to tenor Δ or τ^{fix} (either can be clearer in context). We define this rate in terms of the fixing curve with tenor Δ:

$$L_k(t) := \frac{1}{\tau^{\text{fix}}}\left(\frac{P_{f_\Delta}(t,T_{k-1})}{P_{f_\Delta}(t,T_k)} - 1\right).$$

Using the terminology of [Mer09] and the equations above, we have for the the floating leg of a swap:

$$\mathbf{FL}(t:T_a,\ldots,T_b) = \sum_{k=a+1}^{b} \mathbf{FL}(t:T_{k-1},T_k) = \sum_{k=a+1}^{b} \tau_k P_D(t,T_k) L_k(t)$$

N.B. Since separate curves are used for discounting and fixing there is no reason for a floating leg together with repayment of par at the end to price at par. A riskless floating rate bond should price to par, but that bond

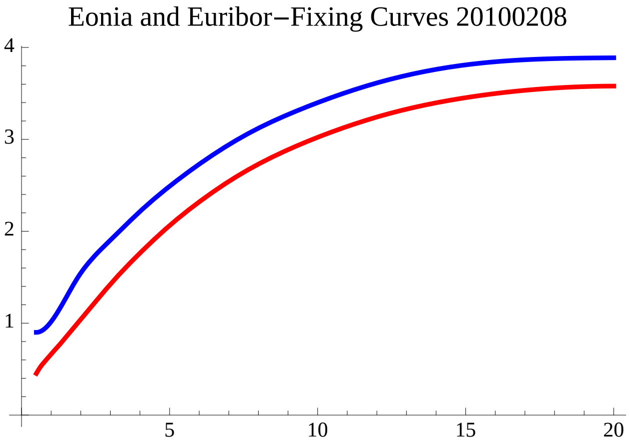

Figure 1: Discounting (lower) and fixing (upper) curves. Note that a conventional Euribor curve (not shown) would be very close to the fixing curve.

would not get Euribor coupons but Eonia coupons. This has previously been pointed out by [Mer09, KTW09].

From these definitions we can bootstrap both discounting and fixing curves from, in euroland, an Eonia discount curve (built conventionally) and Euribor deposits and swaps. Note that we will need the dynamics of $r_*(t)$, where * is either D or f_Δ, under the $T-$forward measure, i.e. using the zero coupon D-quality (riskless) bond with maturity T, for swaption pricing.

Figure 1 shows the discount curve (Eonia) and the fixing curve bootstrapped from Euribor swaps and 6M deposit. Note that a conventional Euribor curve (not shown) would be very close to the fixing curve. By conventional we mean one bootstrapped from swaps alone, i.e. done without reference to a riskless discount curve.

2.1.3 Swaptions

For a European swaption strike K and maturity T, directly following on from the formulae for a swap above, we have :

$$\begin{aligned}&\mathbf{ES}[\omega, T, K, a, b, c, d]\\ &= P_D(0,T)\,\mathbb{E}_D^T\left\{\left[\omega\left(\sum_{i=a+1}^{b}\tau_i^{\text{fix}}P_D(T,T_i)L_i(T) - \sum_{i=c+1}^{d}\tau_i K P_D(T,T_i)\right)\right]^+\right\}\end{aligned}$$

where $\omega = 1$ for a payer swaption (and -1 for a receiver swaption), and the $a, b, c, d, \tau_i^{\text{fix}}, \tau_i$ take care of the different tenors and payment frequencies of the two sides.

Note that because the floating leg does not price to par we must include the specification of the floating coupons. These floating coupons are the market expectation of the relevant fixings $L_i(T)$.

2.2 Short-Rate Pricing

We present semi-analytic pricing (i.e. simulation and formulae) for swaptions under general affine short-rate models and pseudo-analytic (i.e. integration and formulae) pricing in the gaussian case.

For full analytic tractability we use a 1-factor Hull-White type model for the discount curve (e.g. Eonia), expressed as a G1++ type model (analogous to the terminology in [BM06b]) for the discounting short rate $r_D(t)$ under the riskless bank-account numeraire for D:

$$\begin{aligned} r_D(t) &= x(t) + \varphi_D(t), \quad r_D(0) = r_{D,0} \\ dx(t) &= -a_D x(t)dt + \sigma_D dW_1(t) \end{aligned} \tag{1}$$

We also use a one factor model for the fixing curve, i.e. to fit the market-expected fixings, and we label this short rate $r_{f_\Delta}(t)$ under the Δ-tenor-level-risk bank-account numeraire for f_Δ.

$$\begin{aligned} r_{f_\Delta}(t) &= y(t) + \varphi_{f_\Delta}(t), \quad r_{f_\Delta}(0) = r_{f_\Delta,0} \\ dy(t) &= -a_{f_\Delta} y(t)dt + \sigma_{f_\Delta} dW_2(t) \end{aligned} \tag{2}$$

where $(W_1(t), W_2(t))$ is a two-dimensional Brownian motion with instantaneous correlation ρ. We also have a quality-exchange-rate (aka risk exchange rate) X with process under $\mathbb{Q}_{f_\Delta}$ for the quantity of f_Δ-quality (or risk level) investment required to obtain one unit of D-quality investment:

$$dX(t) = (r_{f_\Delta}(t) - r_D(t))X(t)dt + \nu X(t)dW_X(t)$$

where we assume W_X has zero correlation with $W_1(t)$ and with $W_2(t)$. This assumption implies that Equation 2 is unchanged under the measure change from $\mathbb{Q}_{f_\Delta}$ to $\mathbb{Q}_D$.

This is a very parsimonious representation relative to two G2++ models. It is, however, sufficient since we only wish to calibrate to Eonia discount rates and Euribor swaptions. N.B. neither Eonia caps/floors nor swaptions are liquid at present (early 2010). It is tempting to use 3m swaptions and 6m swaptions to get implied euribor swaption volatility, however, there is a significant difference in their liquidity so this is problematic.

2.2.1 Swaption Pricing

For a European swaption strike K and maturity T we have for any one-factor affine short rate model (i.e. bond prices available as $A()e^{B()}$):

$$
\begin{aligned}
&\mathbf{ES}[\omega, T, K, a, b, c, d] \\
&= P_D(0,T)\,\mathbb{E}^T\left\{\left[\omega\left(\sum_{i=a+1}^{b}\tau_i^{\text{fix}}P_D(T,T_i)L_i(T) - \sum_{i=c+1}^{d}\tau_i K P_D(T,T_i)\right)\right]^+\right\} \\
&= P_D(0,T)\,\mathbb{E}^T\left\{\left[\omega\left(\sum_{i=a+1}^{b}P_D(T,T_i)\left(\frac{P_{f_\Delta}(T,T_{i-1})}{P_{f_\Delta}(T,T_i)} - 1\right) - \sum_{i=c+1}^{d}\tau_i K P_D(T,T_i)\right)\right]^+\right\} \\
&= P_D(0,T)\int_{\mathbb{R}^2}\left\{\left[\omega\left(\sum_{i=a+1}^{b}A_D(T,T_i)e^{-B_D(T,T_i)x}\right.\right.\right. \\
&\qquad\qquad \times\left(\frac{A_{f_\Delta}(T,T_{i-1})}{A_{f_\Delta}(T,T_i)}e^{(-B_{f_\Delta}(T,T_{i-1})+B_{f_\Delta}(T,T_i))y} - 1\right) \\
&\qquad\qquad \left.\left.\left. - \sum_{i=c+1}^{d}\tau_i K A_D(T,T_i)e^{-B_D(T,T_i)x}\right)\right]^+\right\} f(x,y)dxdy \qquad (3)
\end{aligned}
$$

Where $A_*, B_*, * = D, f_\Delta$ are the affine factors for the riskless and f_Δ quality bond prices in their respective units of account. Note that up to this point the equations apply to any affine short rate model. However, to go further analytically we require the joint distribution of x and y under the $T-$forward measure, i.e. $f(x,y)$ above. This is available for Gaussian models but not for CIR-type specification. This formulae could be applied in a CIR-type specification by combining simulation up to T with the affine bond formulae used above.

We require Equations 2 and 1 in the T_D-forward measure (i.e. zero coupon bond from the discounting curve D as the numeraire). Using standard change-of-numeraire machinery we obtain:

$$
\begin{aligned}
dx(t) &= \left[-a_D x(t) - \frac{\sigma_D^2}{a_D}(1 - e^{-a_D(T-t)})\right]dt + \sigma_D dW_1^T \\
dy(t) &= \left[-a_{f_\Delta} y(t) - \rho\frac{\sigma_{f_\Delta}\sigma_D}{a_D}(1 - e^{-a_D(T-t)})\right]dt + \sigma_f dW_2^T
\end{aligned}
$$

This is because the Radon-Nikodym derivative dQ_D^T/dQ_D uses the zero coupon discounting bond, hence only x is involved in the measure change (not y as well).

These have explicit solutions:

$$
\begin{aligned}
x(t) &= x(s)e^{-a_D(t-s)} - M_x^T(s,t) + \sigma_D \int_s^t e^{-a_D(t-u)} dW_1^T(u) \\
y(t) &= y(s)e^{-a_{f_\Delta}(t-s)} - M_y^T(s,t) + \sigma_{f_\Delta} \int_s^t e^{-a_{f_\Delta}(t-u)} dW_2^T(u)
\end{aligned}
$$

where

$$
\begin{aligned}
M_x^T(s,t) &= \int_s^t \left[\frac{\sigma_D^2}{a_D}(1 - e^{-a_D(T-t)}) \right] e^{-a_D(t-u)} du \\
M_y^T(s,t) &= \int_s^t \left[\rho \frac{\sigma_{f_\Delta}\sigma_D}{a_D}(1 - e^{-a_D(T-t)}) \right] e^{-a_{f_\Delta}(t-u)} du
\end{aligned}
$$

Now we can express a European Swaption price as:

Theorem 1. *The arbitrage-free price at time $t = 0$ for the above European unit-notional swaption is given by numerically computing the following one-dimensional integral:*

$$
\begin{aligned}
\mathbf{ES}(\omega, T, K, a, b, c, d) &= -\omega P_D(0,T) \int_{-\infty}^{\infty} \delta(x) \frac{e^{-\frac{1}{2}(\frac{x-\mu_x}{\sigma_x})^2}}{\sigma_x \sqrt{2\pi}} \\
&\quad \left[\Phi(-\omega h_1(x)) - \sum_{i=a+1}^{b} \lambda_i e^{\kappa_i(x)} \Phi(-\omega h_2(x)) \right] dx
\end{aligned}
$$

where $\omega = 1$ for a payer swaption (and -1 for a receiver swaption),

$$
\begin{aligned}
\delta(x) &:= \sum_{i=a+1}^{b} A_D(T,T_i)e^{-B_D(T,T_i)x} + \sum_{i=c+1}^{d} \tau_i K A_D(T,T_i) e^{-B_D(T,T_i)x} \\
h_1(x) &:= \frac{\bar{y}(x) - \mu_y}{\sigma_y \sqrt{1-\rho_{xy}^2}} - \frac{\rho_{xy}(x-\mu_x)}{\sigma_x \sqrt{1-\rho_{xy}^2}} \\
h_2(x) &:= h_1(x) + \alpha_i \; \sigma_y \sqrt{1-\rho_{xy}^2} \\
\alpha_i &:= -(-B_{f_\Delta}(T,T_{i-1}) + B_{f_\Delta}(T,T_i)) \\
\lambda_i(x) &:= \frac{1}{\delta(x)} A_D(T,T_i) e^{-B_D(T,T_i)x} \frac{A_{f_\Delta}(T,T_{i-1})}{A_{f_\Delta}(T,T_i)} \\
\kappa_i(x) &:= -\alpha_i \left[\mu_y - \frac{1}{2}(1-\rho_{xy}^2)\sigma_y^2 \alpha_i + \rho_{xy}\sigma_y \frac{x-\mu_x}{\sigma_x} \right]
\end{aligned}
$$

Where $\bar{y} = \bar{y}(x)$ is the unique solution of:

$$\sum_{i=a+1}^{b} A_D(T,T_i)e^{-B_D(T,T_i)x}\left(\frac{A_{f_\Delta}(T,T_{i-1})}{A_{f_\Delta}(T,T_i)}e^{(-B_{f_\Delta}(T,T_{i-1})+B_{f_\Delta}(T,T_i))\bar{y}} - 1\right)$$
$$= \sum_{i=c+1}^{d} \tau_i K A_D(T,T_i)e^{-B_D(T,T_i)x}$$

and

$$\begin{aligned}
\mu_x &:= -M_x^T(0,T)\\
\mu_y &:= -M_y^T(0,T)\\
\sigma_x &:= \sigma_D\sqrt{\frac{1-e^{-2a_D T}}{2a_D}}\\
\sigma_y &:= \sigma_{f_\Delta}\sqrt{\frac{1-e^{-2a_f T}}{2a_{f_\Delta}}}\\
\rho_{xy} &:= \frac{\rho\sigma_D\sigma_{f_\Delta}}{(a_D+a_f)\sigma_x\sigma_y}\left[1-e^{-(a_D+a_{f_\Delta})T}\right]
\end{aligned}$$

and

$$\begin{aligned}
A_*(T,T_i) &= \frac{P_*(0,T_i)}{P_*(0,T)}\exp[\frac{1}{2}(V(T,T_i,a_*,\sigma_*) - V(0,T_i,a_*,\sigma_*) + V(0,T,a_*,\sigma_*))]\\
B_*(T,T_i) &= \frac{1-e^{a_*(T_i-T)}}{a_*}\\
V(T,T_i,a_*,\sigma_*) &= \frac{\sigma_*^2}{a_*^2}\left((T_i-T) + \frac{2}{a_*}e^{-a_*(T_i-T)}\frac{1}{a_*}e^{-2a_*(T_i-T)} - \frac{3}{2a_*}\right)
\end{aligned}$$

where $*$ is either f_Δ or D.

Proof. We manipulate Equation 3 of **ES** above into the same form as for Theorem 4.2.3 [BM06b] and provide an appropriate new definition of $\bar{y}$ to finish the proof.

The $[]^+$ part of Equation 3 of **ES** is of the form:

$$\sum_{i=1}^{n} U_i e^{-V_i x}(F_i e^{-G_i y} - 1) - \sum_{j=1}^{m} M_j e^{N_j x}$$

Expanding:

$$\sum_{i=1}^{n} U_i e^{-V_i x} F_i e^{-G_i y} - \sum_{i=1}^{n} U_i e^{-V_i x} - \sum_{i=j}^{m} M_j e^{N_j x}$$

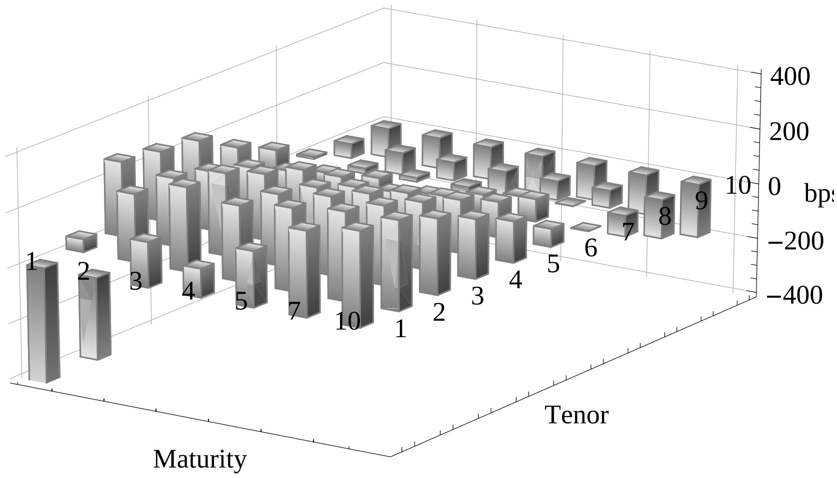

Figure 2: Swaption calibration for G1++/G1++ model, errors in bps.

Now if we multiply and divide by the two negative terms we obtain:

$$-\left(\sum_{k=1}^{n} U_k e^{-V_k x} + \sum_{j=1}^{m} M_j e^{N_j x}\right)\left(1 - \sum_{i=1}^{n} \frac{U_i e^{-V_i x}}{\sum_{k=1}^{n} U_k e^{-V_k x} + \sum_{j=1}^{m} M_j e^{N_j x}} F_i e^{-G_i y}\right)$$

Now define $\bar{y} = \bar{y}(x)$ as the unique solution of:

$$\sum_i U_i e^{-V_i x}(F_i e^{-G_i \bar{y}} - 1) = \sum_j M_j e^{N_j x}$$

So if we freeze x we have the same form as Theorem 4.2.3 of [BM06b], and the rest of the proof is immediate. □

As an example we calibrated to Eonia, Euribor swaps and Euribor swaptions using data asof 8/Feb/10. We find calibration parameters:

a_D	σ_D	a_{f_Δ}	σ_{f_Δ}	ρ
1.0	0.4	0.1	0.0105	-0.95

Swaption calibration with the G1++/G1++ model is similar to that displayed in [BPP09] using a G2++ model, which also uses post-crisis data, see Figure 2.

3 Conclusion

We provide pseudo-analytic pricing for swaptions in a discounting/fixing multi-curve gaussian short-rate setup. Each different tenor fixing Δ gives rise to a different fixing curve f_Δ. Semi-analytic swaption pricing for general affine short-rate models is possible by combining simulation up to maturity with analytic bond formulae at swaption maturity. Pricing of other standard instruments, e.g. caps, are simplifications of the swaption formulae provided. Structurally the pseudo-analytic calculations are very similar to G2++ pseudo-analytic swaption pricing in [BM06b]. This means that the current calculations could be directly extended to a G1++/G2++ setup or G2++/G2++ setup once Eonia swaptions, that are currently not available, become liquid. This work extends [KTW09] who also worked in the short-rate setting but did not provide pseudo-analytic swaption pricing.

The post-crisis world with significant basis spreads between tenors produces a fundamental change in short rate models apart from having separate discounting and fixing f_Δ curves. Each fixing curve can only provide fixings (floating coupons) for the tenor Δ from which it was constructed. Creating a coupon for a different tenor Δ_{other} will not reproduce market-expected fixings for the other tenor. This is because the other tenor represents a different level of risk. This is inherent in the post-crisis market: fixing curves multiply whether in short-rate or forward-rate models. Thus the new post-crisis world causes fundamental changes to short-rate modeling: fixings curves can produce floating coupons for any date but only for the tenor Δ associated with the fixing curve f_Δ.

There is a significant lack of data in the market at present, in early 2010. Eonia/OIS swaptions are missing for example. We calibrate our model jointly to give Eonia and Euribor volatility, however it is not possible to explicitly test our identification with current market data. Also 6M volatility data does not give full information about 3M volatility. Although futures options are liquid and can help, the long end of the volatility curve is simply missing for non-standard tenors. Without this data pricing options on non-standard tenor fixings (e.g. 3M Euribor, 12M USD Libor) is problematic.

The Gaussian framework is convenient analytically but, as is well known, permits negative rates. In this context it also permits negative basis spreads depend on the strength of the correlation between the driving processes, their relative volatilities and the strength of their respective mean reversions. It is possible to obtain analytic results with factor-correlated CIR processes where the correlation is done at the process level (not at the level of the driving Brownian motions which remain independent). However, this does not give rise to constant instantaneous correlations and is outside the scope of the present paper.

A short-rate setup for multiple yield curves, i.e. discounting and fixing, is potentially useful for a variety of problems, e.g. CVA (credit valuation adjustment) where forward-rate setups are less natural, as in [BPP09]. Thus this paper complements existing market model approaches [Hen09, Mer09, AB10].

Appendix

Here we provide the data used in the model (asof 8th February 2010). Note that the 6M money market instrument is typically a few basis points away from the 6M Euribor fixing, and the money market instrument changes throughout the day. Thus the swap rates given are one particular snapshot. (We ignore fixing lags in this paper, conventionally 2 business days for EUR).

Euribor & Swap Rates	
Tenor	Rate (%)
6M	0.97
2Y	1.586
3Y	1.952
4Y	2.264
5Y	2.532
6Y	2.761
7Y	2.959
8Y	3.122
9Y	3.254
10Y	3.366
12Y	3.546
15Y	3.708
20Y	3.798

Eonia Yields	
Tenor (Years in Act360)	Rate (%)
0.505556	0.45573
1.01389	0.66794
1.52222	0.85542
2.03333	1.05402
3.05278	1.44287
4.06389	1.78273
5.07778	2.08370
6.09167	2.34107
7.10833	2.56461
8.12778	2.75217
9.13889	2.90716
10.15000	3.04098
11.16670	3.15866
12.18060	3.25962
15.2250	3.46469
20.3000	3.57937

Swaption Implied Volatilities (maturity x tenor)										
	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y
1Y	51.1	39.2	33.7	30.2	27.6	26.2	25.1	24.4	23.8	23.3
2Y	37.3	30.4	27.5	25.7	24.2	23.3	22.7	22.2	21.8	21.4
3Y	28.8	24.8	23.1	22.	21.1	20.7	20.4	20.2	20.	19.8
4Y	24.1	21.4	20.3	19.6	19.1	19.	18.9	18.8	18.8	18.8
5Y	21.	19.1	18.4	18.	17.7	17.7	17.6	17.6	17.7	17.7
7Y	19.	17.8	17.3	17.1	16.9	16.9	16.8	17.	17.1	17.2
10Y	17.1	16.5	16.3	16.1	16.	16.1	16.1	16.3	16.4	16.6

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