The Short-End of the Curve

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Traditional Swap Curve[1] construction does not treat the very front of the curve consistently. The use of Libor deposits prevents the accurate pricing of FRAs and short-term interest rate swaps, due to a lack of consideration of market expectations of Central Bank Monetary Policy. Furthermore, risk management based upon deposit rates is not useful, due to the inability of most trading desks to trade them. This paper provides an introduction to an approach that uses Overnight Indexed Swaps and Money Market basis swaps to infer market policy expectations and construct the front of the curve with a one day resolution. Furthermore, it demonstrates the usefulness of risk managing with respect to changes in the policy rate.[2]

1. IDENTIFYING RISK FACTORS

Funding risk is the fundamental factor when considering position exposures in various currencies. It is possible to break this down into several primary elements:

- Central Bank Policy rates
- Day-to-day cash liquidity issues that cause variance around the central bank's policy rate
- Cross-currency funding demand (e.g. – the carry trade)
- Term liquidity premium

Clearly, the most significant exposure is to changes in the Central Bank Policy rate, and thus, a good a risk management and pricing system should have some understanding of this.

Traditionally, the very short-end of the curve has been constructed as a kludge of deposit rates and interest rate futures, but this introduces consistency problems over the rates used. For example, including a 1M deposit rate in a 3M Forward curve ignores the lower credit risk of lending for 1M vs 3M. Secondly, if cash deposits are used, the rates implied are often quite different to the Libor fixings due to commercial banks dominating the market, but if the Libor fixings are used, then the forward curve sub 3M is stationary. This may not seem like too much of a problem initially, but consider a surprise interest rate move[3] after the fixing has occurred; clearly the forwards in the curve will be very bizarre at the very front of the curve as the first future is ~25bp[4] different to the (overlapping) 3M Libor fix. All of these issues have the effect of mis-pricing very short-term interest rates, most obviously when comparing market traded FRAs[5] (especially the 1x4 and 2x5 FRAs). A more difficult question is where FRAs starting the day after a policy meeting should be priced and how to hedge them. The existing methodology does not provide any information about the 1D forwards priced into the curve in this respect.

Furthermore, risk management based upon deposit rates does not make sense as very few market participants may actually trade the deposits themselves (how many Swap traders have ever traded a deposit?). Moreover, how much "real" risk is involved in a 1M deposit position with \$100k DV01 when there is no Central Bank meeting in this period? The current VaR is around \$200k, but given that the only factors affecting this position

[1] *The Swap Curve, Lehman Brothers Fixed Income Research, Fei Zhou, 2002*
[2] *The author thanks Zhengyun Hu, Francis Butterworth, Alan Hookham and Jean-Baptiste Home for helpful comments and suggestions. Chris Pulman works on the FX Trading Desk.*
[3] *Or indeed any move in rates post-fixing*
[4] *In the event of a 25bp rate move*
[5] *Forward Rate Agreements*

are liquidity based funding issues, surely the risk is far lower? Conversely, what if there *is* a policy meeting in this period, surely the risk is much higher?

A further issue is how to represent the risk between different instruments that are very similar. For example, it is well-known that FX Forwards are deposit-like instruments, but how should they trade in relation to FRAs, or Overnight Indexed Swaps (OIS)? How should one account for the term spread of 1D funded assets vs 3M funded assets, or indeed Cross-Currency funded assets?

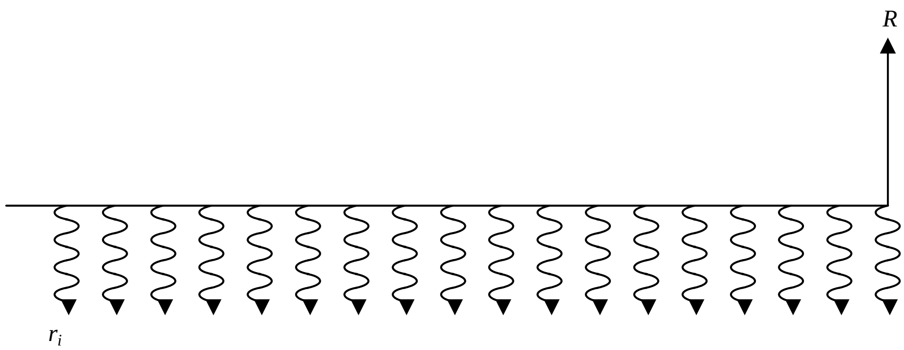
Clearly, existing methods of risk-management and pricing are insufficient in both their consistency and accuracy at the very front of the curve, and a new approach that fully accounts for the above effects needs to be considered.

2. OVERNIGHT INDEXED SWAPS (OIS)

OIS are a form of (generally) short-term interest rate swap in which the floating leg fixes to a daily interest rate, making them useful for gaining exposure to monetary policy changes. The OIS market grew out of the French T4M swap market in the 1990s, and has gradually expanded into many currencies over the past few years to be extremely liquid in the majors. These swaps are generally quoted in monthly tenors out to 1Y, or even beyond in some currencies, as well as weekly out to 1M. Recently, an inter-bank market has developed for swaps that are dated between Central Bank meetings, although these are generally only quoted via voice brokers.

An OIS is a swap which has a single payment at the end of the swap (usually on the day after the maturity of the swap), representing the difference in interest on the two legs of the swap. The fixed leg can be considered much like a synthetic deposit, and is quoted in the market as a yield that is applied over the tenure of the swap. The floating leg, on the other hand, has a daily fix, usually to the weighted-average of overnight cash deposits traded that day. The interest on the floating leg is then compounded up (apart from Fed Fund swaps, where it is averaged) and at the end of the swap, the net difference in interest is paid to the successful counterparty. As the interest is paid in a different period to that which it applies, there is a convexity issue; however, this can be shown to be negligible[6].

Figure 1. Overnight Indexed Swap Cash Flows



R is the swap rate, and r_i are the daily overnight fixes

The fact that these swaps have exposure to overnight interest rates makes them extremely useful in determining what the market is pricing in terms of the expectation of the path of central bank policy rates. Specifically, if we one is able to find some way of stripping out the short-term liquidity effects that are priced into these swap rates then one

[6] *Overnight Indexed Swaps and Floored Compounded Instrument in HJM One-Factor Model, Bank for International Settlements, Marc Henrard, 2004*

is left with an instrument that describes the market expectations of central bank policy. Indeed, these swaps provide a resolution that interest rate futures do not[7], being available at monthly tenors out to a year. Thus, noting that there is hardly ever more than one central bank policy meeting per month[8], it is possible to attribute each Swap tenor as representing a single meeting – e.g. the 1M OIS represents the 1st meeting, the 2M the 2nd, and so on.

How can this be done, quantitatively? As a first approximation, assume that there are no liquidity effects, so that they can be ignored. Then, note that the overnight rate should only change on the day of the policy meeting. Thus, knowing the current policy rate, r_0, compound up the interest for the number of days until the meeting, and then compound up at the new rate, r_1, for the remaining number of days, and solve for r_1. The below equation has the NPV of the fixed leg of the swap on the left-hand side, and the NPV of the floating leg on the RHS:

$$\left(1+\frac{R_1(T_1-t_0)}{D}\right)dF_T=\left(\prod_{l=0}^{T_1-t_0-1}\left(1+\frac{n_l q_l}{D}\right)\right)dF_T$$

Where R_1 is the 1M OIS rate, T_1 is the maturity date of the swap, t_0 is the start date of the swap, T is the payment date on both legs of the swap (usual one business day after T_1, but this varies by currency), q_l is the overnight rate on day l and n_l is the number of days for which that rate compounds (e.g.- usually equal to one unless day l is followed by a weekend, when it will be three, or a holiday), D is the day-count basis and dF_T is the discount factor to date T.

If, as discussed above, the q_l are allowed only to change after the meeting date, t_m, where t_m is associated with the m-th period OIS, then $q_l=r_0$ for $0 < l < t_m$ and $q_l=r_1$ for $t_m \le l < T_1$. Finally, cancelling the dF_T, the above equation becomes:

$$\left(1+\frac{R_1(T_1-t_0)}{D}\right)=\prod_{l=0}^{t_m-t_0-1}\left(1+\frac{n_l r_0}{D}\right)\prod_{l=0}^{T_1-t_m-1}\left(1+\frac{n_l r_1}{D}\right)$$

This can then be solved for r_1. For the purposes of maintaining a suitable abstraction in this article, the binomial approximation is used so that:

$$\left(1+\frac{n_l r_0}{D}\right)\approx\left(1+\frac{r_0}{D}\right)^{n_l}$$

Then, re-labelling t_m as t_1, for the meeting date in the first month, the equation reduces to[9]:

$$\left(1+\frac{R_1(T_1-t_0)}{D}\right)=\left(1+\frac{r_0}{D}\right)^{(t_1-t_0)}\left(1+\frac{r_1}{D}\right)^{(T_1-t_1)}$$

This is easily solved:

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[7] With the exception of Fed Funds Futures in the US and Cash Rate Futures in Australia and New Zealand
[8] If there is more than one meeting per month, then additional inputs, such as forward-starting swaps, that cover unique periods are needed, otherwise the problem is not generally soluble.
[9] *Note* $\prod_{l=0}^{t_m-t_0-1}(n_l)=t_m-t_0$ *and that* $\prod_{l=0}^{T_1-t_m-1}(n_l)=T_1-t_m$

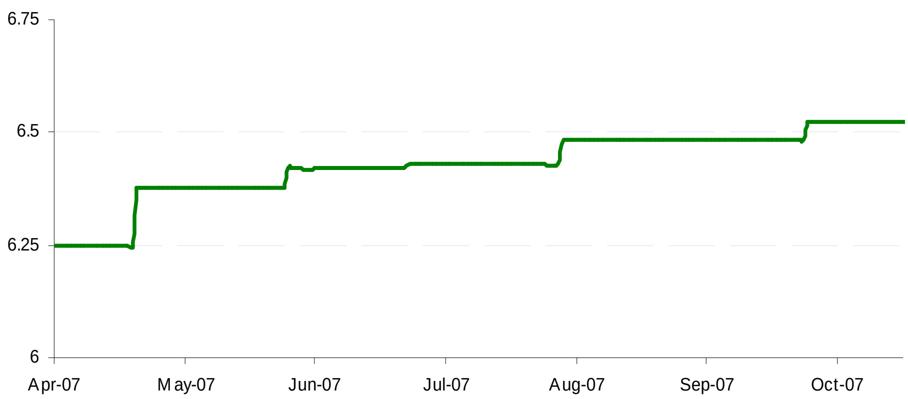
$$r_1 = D\left[\left(\frac{\left(1+\frac{R_1(T_1-t_0)}{D}\right)}{\left(1+\frac{r_0}{D}\right)^{(t_1-t_0)}}\right)^{\left(\frac{1}{T_1-t_1}\right)} - 1\right]$$

One can do this recursively, using the swap rates out to say 6M, to find the expected policy rates, r_i, for each meeting. Actually, this is somewhat an over-simplification, as a treatment of cash liquidity (see §5) must be considered when solving for these rates, which is modelled as a daily funding spread s_l, so the real equations to solve are:

$$\left(1+\frac{R_1(T_1-t_0)}{D}\right) = \prod_{l=0}^{t_m-t_0}\left(1+\frac{n_l\left(r_0+s_l\right)}{D}\right)\prod_{l=0}^{T_1-t_m}\left(1+\frac{n_l\left(r_1+s_l\right)}{D}\right) \quad ,,, \text{(A)}$$

However, the end result is almost the same: information that represents the 1D Central Bank Forward Curve, which is constrained to change level only on Monetary Policy meetings (fig 2). Once this has been determined, one can add back in the liquidity adjustments, s_i, to produce a 1D OIS Forward Curve that contains the required 1 day resolution in the forwards.

Figure 2. Reserve Bank of Australia (RBA) 1D Forward Curve



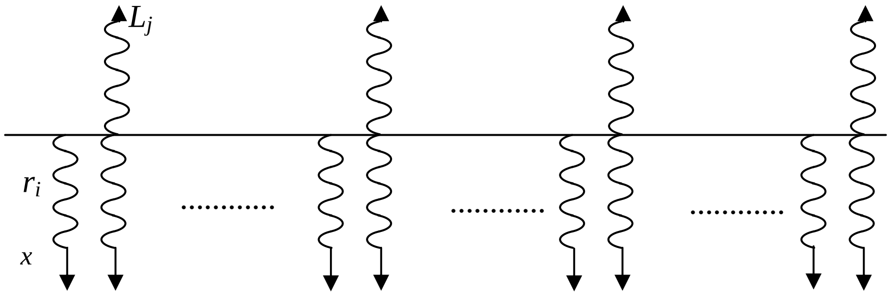
3. MONEY-MARKET & CROSS-CURRENCY BASIS SWAPS

It's all very well building a step-curve for pricing OIS, but how can this information be used to better construct the front of the Libor curve? Money-Market and Cross-Currency basis swaps have been discussed in detail elsewhere[10], so for brevity they shall just be briefly commented upon here. These are swaps where two floating rate streams are exchanged, for example 3M Libor vs. 6M Libor. There will be a basis-spread on one of the legs of the swap that generally represents investor preference of one index over the other and depends upon things such as the credit risk of lending for one term vs. another (e.g. lending for 6M is riskier than for 3M. So generally, one would expect to pay/receive 6M vs. 3M + spread. Similarly, one could enter a basis swap in which one paid the 1D OIS fix compounded up quarterly and received the 3M Libor fix + a spread, or almost equivalent, paid 1D OIS + spread vs. 3M Libor. One would expect the latter spread to be positive, as lending for 1 day is much less risky than for 3M, and also, one would expect interest-rate expectations to be priced into the next 3M which would also

[10] *Interest Rate Parity, Money Market Basis Swaps, and Cross-Currency Basis Swaps, Lehman Brothers Fixed Income Liquid Markets Research, Bruce Tuckman & Pedro Porfirio, 2003.*

affect that rate – in an upward sloping curve this would be more-positive, and in a downward sloping curve this would be less-positive (and possibly negative, depending on the amount and rate of easing priced in).

Figure 3. OIS – Libor Basis Swap



L_j is the 3m Libor payment on day j, r_i is the1d OIS fixing, and x is the OIS-Libor basis spread. The payments on the 1d leg of the swap are compounded up and paid quarterly.

Given that there are usually FRA prices for 1x4, 2x5 etc, and OIS out to 1Y with monthly resolution, one can bootstrap the FRAs and the OIS separately and price the 1D-3M basis swaps to maturities 1M – 1Y[11]. This can be done once a day as a calibration, or left free-floating. These basis spreads can then be applied to the step-shape OIS forward curve by recursively solving the following equation, to create a 1D OIS Index curve and a 3m Libor Index curve, which represents the NPV of the each leg of the OIS-Libor basis swap:

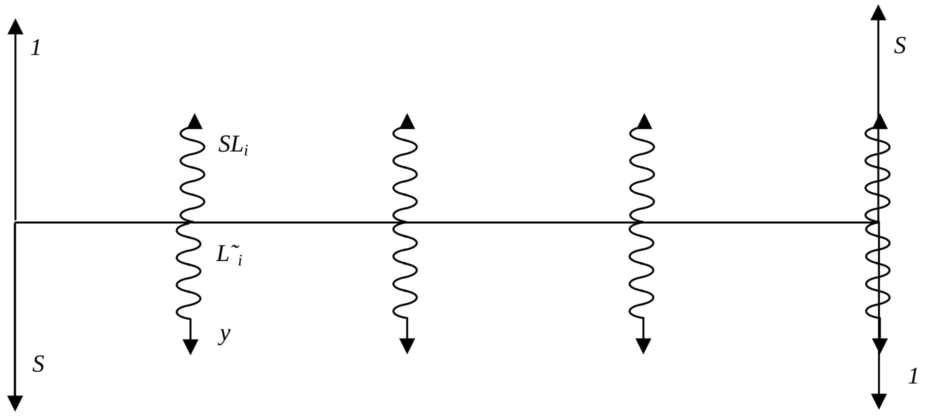
$$\sum_j^T \left(\prod_{i=j}^{T_{j+1}-T_j-1} \left(1 + \frac{n_i (r_i + x)}{D} \right) - 1 \right) dF_j = \sum_j^T L_j \Delta_j dF_j \qquad ,,, \text{(B)}$$

The product is over the 1d fixings within the *j*-th 3m period (i.e. - compounding over the 3m period), L_j is the *j*-th 3m Libor fixing, Δ_j is the day count fraction for the Libor coupons and each element of the sums is the 3-monthly payment on the basis swap. The result is a new curve that represents the 3M Libor forwards, the 3M Libor Index Curve, with a 1D resolution out to say 6M. The remaining portion of the curve can be constructed from the 2nd future out to the 8th or 12th future, and then with swap rates as per normal swap-curve construction.

For completeness, it is possible to extend this methodology to build the Funding Curve (often referred to as the 'Cross-Currency Basis Curve', as it is constructed from Cross-Currency Basis Swaps). The discount factors in the above equation can be found by including the Cross-Currency Basis spread, either directly (for 1Y+, these are market traded), or by calibration from FX Forwards. A Cross-Currency Basis Swap is a swap in which a 3m Libor stream in one currency is exchanged for one in a second currency plus a spread, *y*, with an exchange of principal at the beginning and the end, and is used in pricing FX Forwards[12].

[11] Note that the 1M, 2M & 3M basis swaps will just be single stub payments.
[12] See Tuckman & Porfirio.

Figure 4. Cross-Currency Basis Swap



Where 1 unit of notional in foreign currency is exchanged for S dollars, L_i is 3m USD Libor and $\tilde{L}_i$ is 3m foreign Libor

The discount factors can be found by recursively solving the following equation, which represents the NPV of the Cross-Currency basis swap legs:

$$S + \sum_{j}^{T} (\tilde{L}_j + y)\tilde{\Delta}_j d\tilde{F}_j + d\tilde{F}_T = 1 + S\sum_{j}^{T} L_j \Delta_j dF_j + SdF_T \qquad ,,, \text{(C)}$$

Where $d\tilde{F}_j$ are the foreign currency discount factors, $d\tilde{F}_T$ is the foreign currency discount factor to maturity, dF_T is the dollar discount factor to maturity, S is the number of US dollars per foreign unit of notional, and $\tilde{\Delta}_j$ is the day count fraction for the foreign Libor coupons. The USD discount factors and Libor forwards are found by constructing the Swap curve as discussed earlier, with the step-shape at the very front, and traditionally, from the 2nd future out, similarly for the foreign Libor forwards. It is purely the foreign discount factors that are different. Finally, for sub 1Y discount factors, either the 1Y Basis swap can be used (providing the short-term liquidity model is good enough) or the Basis swap spreads can be implied by noting that the FX Forwards can be described by the following equation:

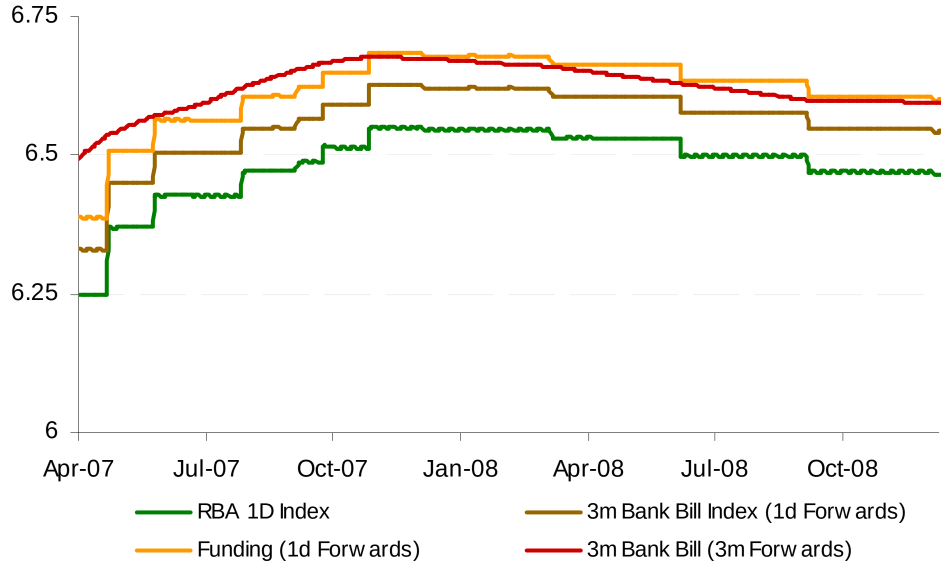
$$F_T = S\left(\frac{d\tilde{F}_T}{dF_T}\frac{dF_S}{d\tilde{F}_S}\right)$$

Where dF_S and $d\tilde{F}_S$ are the discount factors to spot in USD and in the foreign currency respectively. This equation may be substituted into equation (C).

To summarise, by including the solution of equations (A), (B) and (C) in a curve-building algorithm, one is able to construct a fully consistent set of Index and Funding curves that are capable of pricing Swaps, FRAs, OIS, Cross-Currency Basis Swaps, OIS-3m Libor Basis Swaps[13] and FX Forwards[14]. Significantly, they will also have proper knowledge of the 1D forwards at the very short-end of the curve (see fig 5).

[13] Further Index curves can be constructed by supplying basis spreads to the floating term – e.g. 3m-6m basis spreads.
[14] FX Forwards are priced by taking discount factors from the Funding Curve.

Figure 5. Australian Index & Funding Curves



4. CENTRAL BANK RISK REPRESENTATION

Now that it has been shown how to construct the curve in terms of policy adjustments, it is instructive to represent risk in terms of these adjustments. Continuing in the above frame-work, ignoring the liquidity effect, one has a set of equations that represent the curve. The usual method for estimating delta risk is to calculate the analytic Jacobian of the price of the assets used in the curve[15].For ease, below, the Jacobian of the swap rates with respect to the meeting rates has been calculated and then multiplied by the (diagonal) Jacobian of discount factors to the Swap tenors with respect to the Swap rates:

Using the earlier approximation and generalising to the *j*-month OIS and *j*-th overnight rate, the curve equations are:

$$\left(1+\frac{R_j(T_j-t_0)}{D}\right)=\prod_{i=0}^{j}\left(1+\frac{r_i}{D}\right)^{(t_i-t_{i-1})}$$

The products are over each of the overnight rates, with their interest factors raised to the power of the number of days for which they are valid (i.e. – to the previous meeting date, rather than to the previous OIS maturity date). Then, separate into two products, associated with the compounding up to a meeting date and that from the meeting date up to the OIS maturity date:

$$\left(1+\frac{R_j(T_j-t_0)}{D}\right)=\prod_{i=1}^{j}\left(1+\frac{r_{i-1}}{D}\right)^{(t_i-T_{i-1})}\prod_{i=1}^{j}\left(1+\frac{r_i}{D}\right)^{(T_i-t_i)}$$

Where the first product is set so that the compounding starts from the initial policy rate, r_0, and then also define $T_0 = t_0$ as the start date of the OIS, as noted earlier.

Taking the logarithm of both sides gives:

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[15] *A detailed description of this approach is beyond the scope of this paper, but "Fixed-Income Securities: Valuation, Risk Management and Portfolio Strategies" by Martellini, Priaulet & Priaulet contains an excellent exposition.*

$$\ln\left(1+\frac{R_j(T_j-t_0)}{D}\right)=\sum_{i=1}^{j}\left(t_i-T_{i-1}\right)\ln\left(1+\frac{r_{i-1}}{D}\right)+\sum_{i=1}^{j}\left(T_i-t_i\right)\ln\left(1+\frac{r_i}{D}\right)$$

Differentiate with respect to the k^{th} meeting rate, r_k:

$$\frac{\partial}{\partial r_k}\left(\ln\left(1+\frac{R_j(T_j-t_0)}{D}\right)\right)=\sum_{i=1}^{j}\left(\delta_{i-1,k}\frac{\left(t_i-T_{i-1}\right)}{\left(1+\frac{r_{i-1}}{D}\right)}\right)+\sum_{i=1}^{j}\left(\delta_{i,k}\frac{\left(T_i-t_i\right)}{\left(1+\frac{r_i}{D}\right)}\right)$$

Where $\delta_{i,k}$ is the Kronecker delta-function[16]. Re-arranging terms gives the Jacobian:

$$\frac{\partial R_j}{\partial r_k}=\left(\frac{D}{T_j-t_0}\right)\left(1+\frac{R_j(T_j-t_0)}{D}\right)\sum_{i=1}^{j}\left[\delta_{i-1,k}\frac{\left(t_i-T_{i-1}\right)}{\left(1+\frac{r_{i-1}}{D}\right)}+\delta_{i,k}\frac{\left(T_i-t_i\right)}{\left(1+\frac{r_i}{D}\right)}\right]$$

If the approximation[17] that the discount factors can be given by the below equations is made:

$$dF_i=\frac{1}{\left(1+\frac{R_i(T_i-t_0)}{D}\right)}$$

Then multiplying the diagonal matrix:

$$\frac{\partial(dF_i)}{\partial R_i}=-\frac{(T_i-t_0)}{D\left(1+\frac{R_i(T_i-t_0)}{D}\right)^2}$$

by the Jacobian above will give the Jacobian of discount factors with respect to the meeting dates:

$$\mathbf{J}=\frac{\partial(dF_j)}{\partial r_k}=-\frac{1}{\left(T_j-t_0\right)\left(1+\frac{R_j(T_j-t_0)}{D}\right)}\sum_{i=1}^{j}\left[\delta_{i-1,k}\frac{\left(t_i-T_{i-1}\right)}{\left(1+\frac{r_{i-1}}{D}\right)}+\delta_{i,k}\frac{\left(T_i-t_i\right)}{\left(1+\frac{r_i}{D}\right)}\right]$$

If the cash flows can be represented, by some interpolation method, as an equivalent set of cash flows on the swap maturity dates, then a reasonably good estimate of the CB01 – "PV change as a result of a 1bp move in the expectations of the Central Bank meeting outcome"[18] - can be provided.

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[16] *The Kronecker delta-function is defined by* $\delta_{i,k}=\begin{cases}1 & i=k\\ 0 & i\neq k\end{cases}$

[17] *As described in Section 3, the discount factors should actually be from the Funding Curve and the numerical examples are calculated using the Funding Curve.*

[18] *One could, of course, construct a set of curves, with consecutive meeting inputs perturbed, although this is computationally expensive.*

For completeness, note that given CB01 values for each meeting , inverting the Jacobian and multiplying by the vector, **c,** representing the CB01 for each meeting will return the notional, **n**, of each OIS that is required to hedge that given risk to Central Bank meetings:

$$\mathbf{n} = \mathbf{J}^{-1}\mathbf{c}$$

4.1 Worked Example

A demonstration of the usefulness of this approach is now appropriate. In Australia, the OIS fixes to the RBA target rate, so there is no liquidity effect to take into account. Figure 5 shows the RBA policy expectations as of April 10th 2007, and the corresponding expectations for rates (the current RBA policy rate is 6.25%) are:

Figure 6. RBA Policy Expectations

Date	Expected Policy Rate	Easing/Hiking[19]
02-May-07	6.36	+11
06-Jun-07	6.43	+7
04-Jul-07	6.43	0
08-Aug-07	6.47	+4
15-Sep-07	6.49	+2
03-Oct-07	6.51	+2

The corresponding Jacobian of discount factors with respect to the meeting dates is:

$$\begin{pmatrix} -2.44 & -9.49 & -9.49 & -9.49 & -9.49 & -9.49 \\ 0 & -1.61 & -7.51 & -7.51 & -7.51 & -7.51 \\ 0 & 0 & -1.86 & -9.29 & -9.29 & -9.29 \\ 0 & 0 & 0 & -1.31 & -9.97 & -9.97 \\ 0 & 0 & 0 & 0 & -1.04 & -5.19 \\ 0 & 0 & 0 & 0 & 0 & -1.54 \end{pmatrix}$$

The rows can be thought of as representing the monthly discount factors/swap rates, and the columns representing the meetings.

Suppose that one has the view that the RBA will hike rates on 2nd May 2007. He/she could enter a trade that would monetise the 14bp shown above if correct, and loose 11bp if incorrect. Consider risking \$100k – i.e. AUD -11,134 per bp of CB01, taking 0.8165 for AUD/USD spot. As this meeting falls within the 1M OIS, only the left-corner element of the Jacobian is needed (or invert it and multiply by the vector **c**, which has c_1=-11,134, and $c_i = 0$ for $i > 1$) which is -2.44 per 1m AUD. Thus, in order to generate the required level of risk, one should pay on AUD 4.563bn of the 1M OIS in order to get this CB01 (-11,134/-2.44 *AUD 1m). Compare this with the DV01 of AUD -38,323[20] (-\$31,291), which implies a risk value that is 244% too high.

4.2 Risk-Bucketing Comparison

A comparison of the bucketing of risk in both the new and traditional method would be useful. Figure 7 shows the market data used in constructing the curves. For the Central

[19] *Calculated as the basis point difference between in the expected policy rate vs. the previous expected policy rate.*
[20] *For a single cash flow, the DV01 is given by*

$$\frac{\partial(dF)}{\partial R}\times 0.0001\times Notional = -\frac{(T_i - t_0)}{D\left(1+\frac{R_i(T_i - t_0)}{D}\right)^2}\times 0.0001\times Notional$$

Bank approach, meetings have been used up to the 2nd, and then the usual bucketing structure thereafter.

Figure 7. Australian Market Data

Traditional		Step-based	
Bucket	**Price**	**Bucket**	**Step-based**
1M Deposit	6.3950	02-May-07 RBA	6.36
2M Deposit	6.4483	06-Jun-07 RBA	6.43
3M Deposit	6.4967	04-Jul-07 RBA	6.43
YBAM7 Future	9342.5	08-Aug-07 RBA	6.47
YBAU7 Future	9341.5	YBAU7 Future	9341.5
YBAZ7 Future	9334.5	YBAZ7 Future	9334.5
YBAH8 Future	9332.5	YBAH8 Future	9332.5
YBAM8 Future	9337.0	YBAM8 Future	9337.0
YBAU8 Future	9340.0	YBAU8 Future	9340.0
YBAZ8 Future	9340.5	YBAZ8 Future	9340.5
YBAH9 Future	9341.0	YBAH9 Future	9341.0
3Y Swap	6.6100	3Y Swap	6.6100
4Y Swap	6.6275	4Y Swap	6.6275
5Y Swap	6.5875	5Y Swap	6.5875
7Y Swap	6.5075	7Y Swap	6.5075
10Y Swap	6.4255	10Y Swap	6.4255

In the below examples, the PV01s and CB01s are defined as the change in value for 1bp increase in the interest rate. The cash flow dates have been chosen randomly to illustrate the risk-bucketing in the different regions of the curve.

Deposit Region

Figure 8. Comparison of risk-bucketing in the Deposit Region

Traditional		Step-based	
Bucket	**Risk**	**Bucket**	**Risk**
1M Deposit	394	02-May-07 RBA	-9,402
2M Deposit	-10,016	06-Jun-07 RBA	-5,379
3M Deposit	-10,491	04-Jul-07 RBA	0
YBAM7 Future	377	08-Aug-07 RBA	0
YBAU7 Future	0	YBAU7 Future	0
YBAZ7 Future	0	YBAZ7 Future	0
YBAH8 Future	0	YBAH8 Future	0
YBAM8 Future	0	YBAM8 Future	0
YBAU8 Future	0	YBAU8 Future	0
YBAZ8 Future	0	YBAZ8 Future	0
YBAH9 Future	0	YBAH9 Future	0
3Y Swap	0	3Y Swap	0
4Y Swap	0	4Y Swap	0
5Y Swap	0	5Y Swap	0
7Y Swap	0	7Y Swap	0
10Y Swap	0	10Y Swap	0
Total Risk	-19,736	**Total Risk**	-14,781

A single cash flow of AUD 1bn, receivable on 26th June 2007

Futures Region

Figure 9. Comparison of risk-bucketing in the Future Region

Traditional		Step-based	
Bucket	**Risk**	**Bucket**	**Risk**
1M Deposit	-54	02-May-07 RBA	-897
2M Deposit	-1,712	06-Jun-07 RBA	-719
3M Deposit	314	04-Jul-07 RBA	-897
YBAM7 Future	-2,525	08-Aug-07 RBA	-1,024
YBAU7 Future	-2,382	YBAU7 Future	-2,309
YBAZ7 Future	-2,131	YBAZ7 Future	-2,131
YBAH8 Future	41	YBAH8 Future	0
YBAM8 Future	0	YBAM8 Future	0
YBAU8 Future	0	YBAU8 Future	0
YBAZ8 Future	0	YBAZ8 Future	0
YBAH9 Future	0	YBAH9 Future	0
3Y Swap	0	3Y Swap	0
4Y Swap	0	4Y Swap	0
5Y Swap	0	5Y Swap	0
7Y Swap	0	7Y Swap	0
10Y Swap	0	10Y Swap	0
Total Risk	-8,449	**Total Risk**	-7,977

A single cash flow of AUD 100m, receivable on 10th March 2008

Swaps Region

Figure 10. Comparison of risk-bucketing in the Swaps

Traditional		Step-based	
Bucket	**Risk**	**Bucket**	**Risk**
1M Deposit	4	02-May-07 RBA	107
2M Deposit	131	06-Jun-07 RBA	86
3M Deposit	-28	04-Jul-07 RBA	107
YBAM7 Future	280	08-Aug-07 RBA	123
YBAU7 Future	217	YBAU7 Future	229
YBAZ7 Future	197	YBAZ7 Future	212
YBAH8 Future	156	YBAH8 Future	168
YBAM8 Future	139	YBAM8 Future	151
YBAU8 Future	98	YBAU8 Future	108
YBAZ8 Future	82	YBAZ8 Future	91
YBAH9 Future	92	YBAH9 Future	52
3Y Swap	1,580	3Y Swap	1,393
4Y Swap	7,538	4Y Swap	2,000
5Y Swap	-39,859	5Y Swap	-30,005
7Y Swap	-8,037	7Y Swap	-11,587
10Y Swap	741	10Y Swap	0
Total Risk	-37,410	**Total Risk**	-36,765

A single cash flow of AUD 100m receivable on 10th September 2012

It can be seen that the total risk for the Futures Region and the Swaps Region is very close in both methods. Indeed, the bucketing from around the forth future out is similar.

However, in the Deposit Region, the traditional method over-estimates the risk significantly.

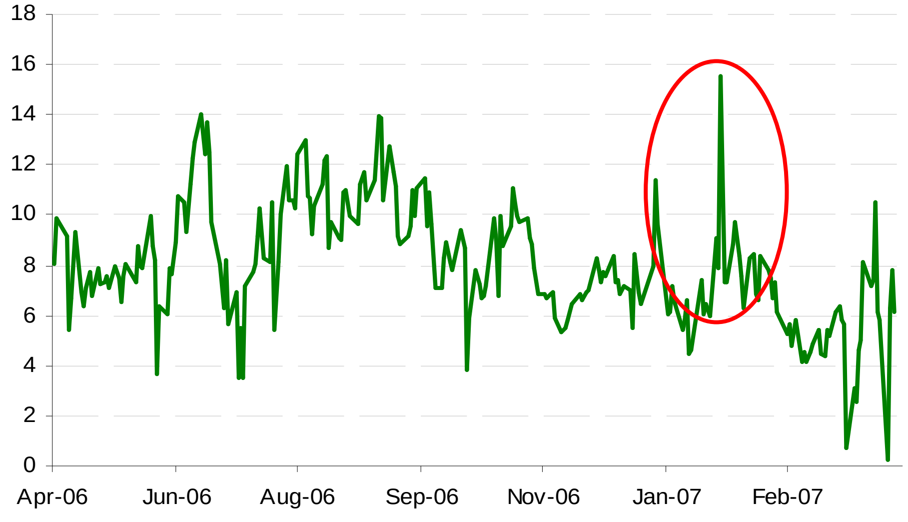
4.3 OIS – Libor Basis Risk

The next thing that needs to be monitored is the OIS-Libor basis risk. As there is not a generally traded basis swap at the short-end it is a risk to be monitored in terms of its volatility. Alternatively, one could trade OIS vs. FRAs or Swaps to synthetically create the Basis Swap.

The main driver of these basis spreads is the difference in credit quality in lending for 1d vs. 3m – there is a significantly higher risk of lending for 3m than for 1d. Across most G10 currencies, this is thought to be 10-15bp, as is assumed when one is trying to assess what is priced in to 3m Libor-based futures[21]. However, the spread between the Central Bank target rate and where 3m Libor is setting is also determined by what Monetary Policy is priced into the next 3m, which causes it to change depending whether there is cutting, easing or hiking priced into the curve. Therefore, it is not particularly useful to try and work out what is priced in from interest rate futures based upon this assumption, as one may deduce pricings that are somewhat different to the real expectations.

Furthermore, it is inappropriate to apply this spread to the constructed OIS curve. If a spread curve is to be properly constructed, then 3m Libor has to be compared with a 1d rate that is compounded up over 3m so that the expectations over the next 3m are properly included in the calculation of the spread. Conveniently, the 1d payments in an OIS-Libor basis swap are compounded up and paid quarterly vs. 3m Libor. Then, the 3m basis swap can almost be thought of as the difference between today's 3m Libor fix and the market quoted 3m OIS[22]. As noted in §1, if there is a surprise move in interest rates by the Central Bank, the market traded instrument will move to reflect this, and in this case, the 3m basis swap can move dramatically (see fig. 11). Indeed this spread will move throughout the day anyway, as market news can cause expectations of policy to change too. A stark example of this was in January 2007 when a large downward surprise on Australian CPI caused the market to completely price out the chance of an immediate RBA hike – see the spike on the chart.

Figure 11. Recent History of the 3m OIS-Libor Basis Swap Spread

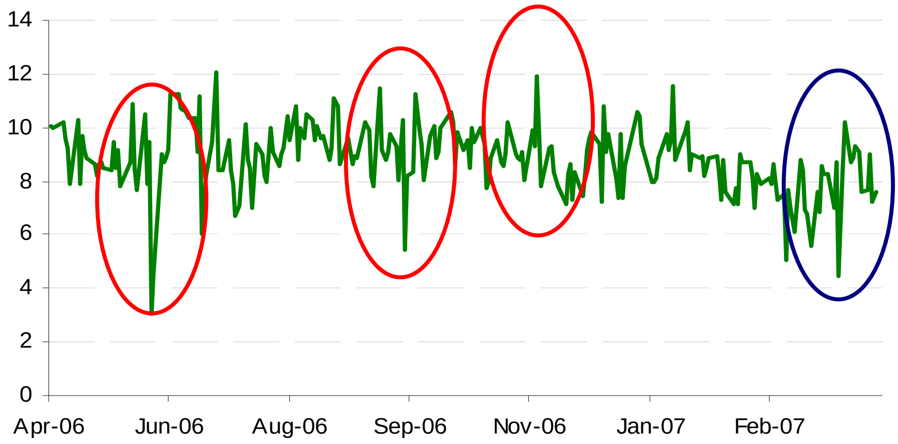


A large downward surprise in CPI caused the pricing out of an immediate RBA hike (red oval)

[21] *Introducing Reposcope: A tool for monitoring UK and euro-area monetary policy expectations, Liquid Markets Research Quarterly, Alexei Jiltsov and Adam Purzitsky, 2006*
[22] *Of course, the spread is also compounded so this is a small approximation.*

Generally, however, for longer-dated basis swaps, such as the 6M or the 12M, this effect is more muted, as it will only apply to the first payment on the swap. Indeed, the 12M Basis Swap spread (fig. 12) is pretty stable, with mean 8.82bp and standard deviation 1.46bp over the past year. It only moves significantly around policy meetings, but this is not too much of an issue if one has hedged their CB01, as this move in the spread is perfectly offset with the CB01 position.

Figure 12. Recent History of the 12m OIS-Libor Basis Swap vs. RBA Policy Meetings



The red ovals show when there was an RBA Monetary Policy meeting that wasn't fully priced for a hike going in, and the RBA subsequently hiked. The blue ovals show when it wasn't fully priced for a hike, and then the tightening wasn't delivered

The spread move highlights the inadequacy of the traditional method of constructing the short-end, as it uses instruments that are stationary, when the real curve is moving – the spread move captures that movement. To wit, consider the 1m forward-starting 3m OIS-Libor basis swap spread, which can be effectively thought of as the difference between the 1x4 Libor FRA and the 1x4 OIS[23]. Both of these instruments should have the market expectations of policy built into them, and because they have not had a fixing yet, they will not be distorted by the fixing effect.

5. FUNDING AND LIQUIDITY ISSUES

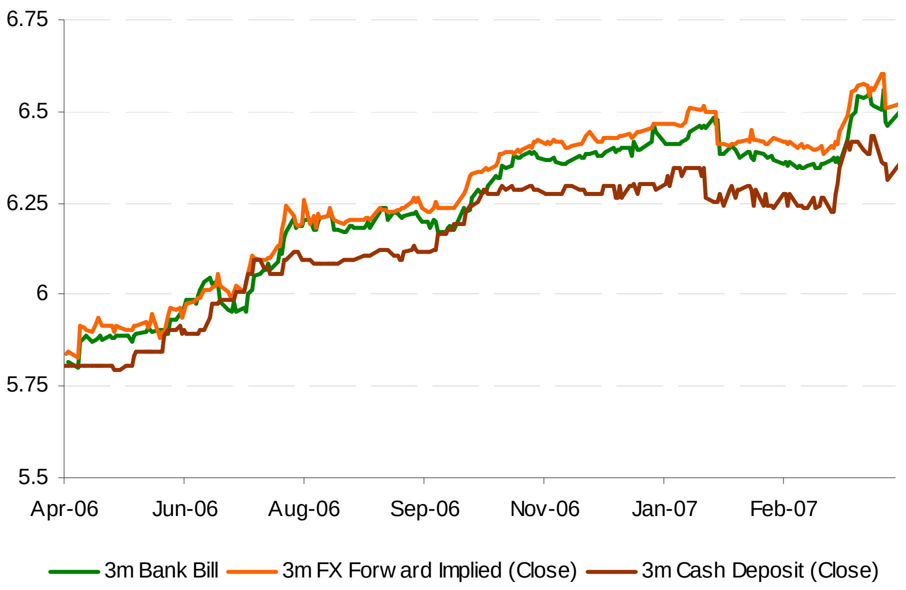
Once the risk with respect to Central Bank policy moves has been removed, positions still need to be funded. While this is less of an issue for Libor swap books, OIS, FX Forwards and Bonds still have either a cash position to be rolled or a fixing to funding instruments (in the case of OIS). There are, of course, subtle differences between the financing of these positions, in that the OIS fixes to a weighted average funding rate, while the FX Forwards and Bonds (via repo) are funded wherever the trader is able to during the day.

It is also worth further distinguishing between Libor financing of swaps and real financing of physical cash positions. Libor is a daily fixing that is determined by where several banks think that they "ought" to lend to AA-rated banking counterparties. Thus, it is not a real lending rate, and is often manipulated by constituent banks' swap fixing requirements. As it is used as the primary fixing rate for 1M, 3M & 6M Libor swaps, the other fixings may bear no resemblance to real market funding rates at all. Furthermore, it

[23] Again, remembering the spread is also compounded.

is an un-secured lending rate, which makes it further different to FX Forward implied rates[24] and cash deposit rates. (figure 13).

Figure 13. Comparison of 3m Deposits



After stripping out the central bank and basis spread risk, the remaining exposure is due to overnight funding, which is driven by many liquidity factors. For example, figure 14 shows the daily EONIA[25] - ECB[26] refinancing rate spread. The mean spread between the two rates is about 7bp, and this is stable, apart from around certain dates when it moves markedly. One can imagine that certain events such as corporate tax days, year-ends and month-ends would yield a higher demand for cash, and thus push the funding rate up. Of course, there are a great many drivers of this funding spread, and the ECB has produced a very sophisticated model of the EONIA - ECB rate spread[27], but below is a qualitative discussion of a few of these factors.

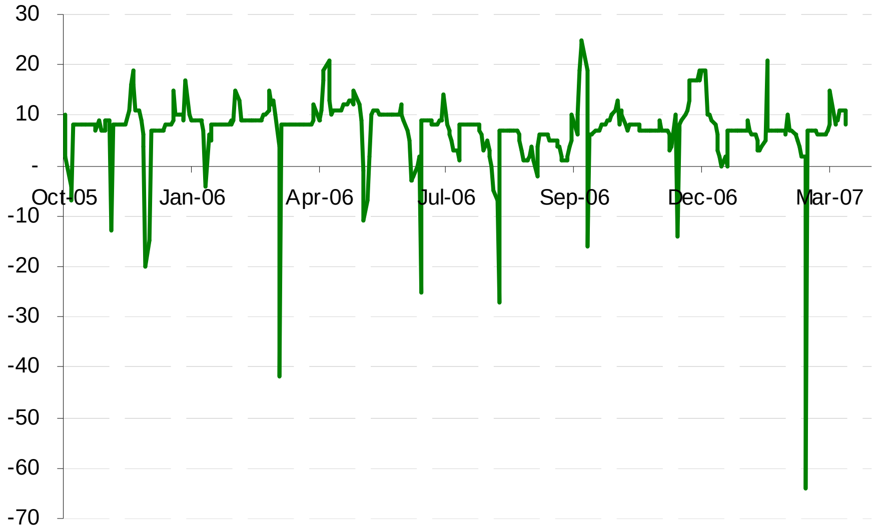
[24] *It is also worthwhile commenting on the non-existence of the so called "arbitrage" between deposits and FX Forwards. This arises from the risk of lending to one counterparty and borrowing from another – if the counterparty holding one's funds defaults, one loses the entire principal, while still having to pay back the currency that has been borrowed. On an FX Forward, however, the entire transaction is with a single counterparty, so if they default, one can withhold payment of "borrowed" currency, and hence is just left with an FX spot position arising from the initial exchange of principal. Thus the difference between FX Forward implied rates and cash deposits can be thought of as representing the difference between the risk of default on the notional and that of an FX position – i.e. this "arbitrage" is really selling the risk of default.*

[25] *Euro Overnight Index Average*

[26] *European Central Bank*

[27] *A Comprehensive Model of the Euro Overnight Rate, ECB Working Paper No. 207, Flemming Reinhardt, 2003*

Figure 14. EONIA – ECB Refinancing Rate Spread



Mean spread 7.09bp, standard deviation 7.06bp

5.1 Example Factors

Corporate Tax Day

All companies are required to pay their tax, and thus, free cash-flow held in bank accounts will be withdrawn in order to pay these bills. This creates a deficit in the banking system, as banks borrow to ensure that they have enough cash for this withdrawl. This pushes up borrowing costs.

Financial year-end

Various accounting regulations require firms not to lend out their balance sheet when year-end accounts are produced. Again, this drives up financing costs.

Month-end

As for above, but to less an extent. Often, the effect is even larger at quarter-ends.

Weekends

On a micro-economical level, the consumer generally withdraws money from his/her bank account on a Friday evening in order to have cash to spend over the weekend. As a weekend roll is generally for 3 days rather than 1 day, it is also slightly more risky and will also have this premium built in. Again, an upward move in funding costs.

Mondays

On a Monday, spare cash left over from the weekend is often deposited back into current accounts, as well as many companies paying in wages on a Monday. This exerts downward pressure on funding, as banks struggle to lend out their cash.

Central Bank reserve periods

Most Central Banks, in order to minimise systemic risk in the banking system, require banks to post collateral with them to a certain average value per month. This can create interesting effects around central bank policy meetings. As an example, the Federal Reserve requires banks to deposit an average amount of cash over a monthly period, dependent upon their liabilities. Over the course of the recent hiking cycle, these bank treasury desks have sort to deposit a large amount for just a few days prior to the Federal Open Market Committee (FOMC) meetings. This ensured that the reserve requirement was met, on average, for the next month, by borrowing capital at the lower funds rate and therefore avoiding the higher cost of financing post-FOMC hike. This put upward pressure on the overnight deposit rate in the days leading up to the meeting. Correspondingly, in the few days after the meeting, the very large amount of maturing repo and deposits caused a net cash surplus in the banking system, which caused rates to fall significantly (figure 15). The reverse of this would be expected in a cutting cycle.

Maturing Repo

Term repo executed with the Central Bank is part of the Central Bank's efforts to ensure that money market funds are trading close to its target rate. Clearly, a large amount of maturing repo will flood the market with cash on that maturity date, and will thereby put downward pressure on the funding rate. This is very evident if the maturity date of occurs shortly after a hike in the central bank rate. An example of this is the ECB bi-weekly open market operation which generally matures on the Tuesday following an ECB Governing Council Meeting, and the average cash rate on that day over the past couple of years has been ~30bp below the re-financing rate.

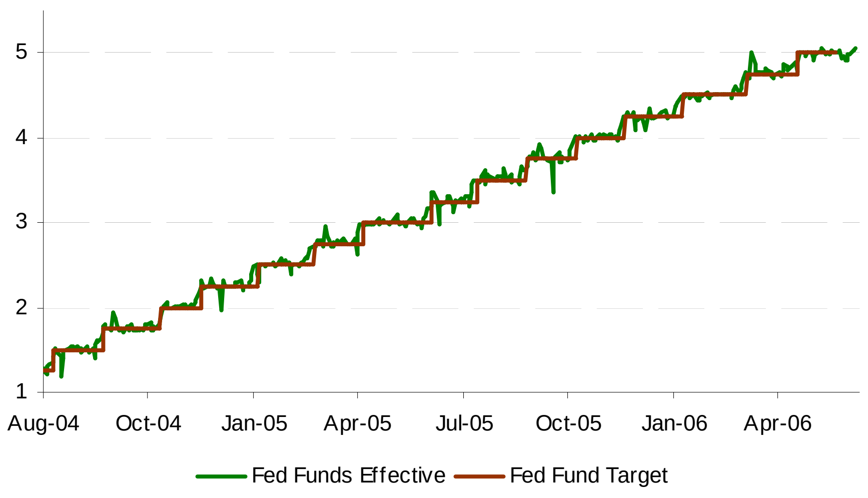
Yield-curve effects

There are also effects deriving from the slope of the curve – for example, if the Fed are hiking rates, and the expectation is that they will continue to do so, there may be a large portion of the market short Treasuries – and this will put upward pressure on repo[28].

Catastrophes/One-off events

The one thing that everybody wants in the event of a catastrophe is very quick access to cash. For example, there was a serious worry that the Millennium bug might cause computer systems globally to fail, and this pushed up short term borrowing costs over the year-end turn by 45bp. Nobody wanted to have to borrow cash back if they were short, as no-one would want to lend cash if their computer systems were down. In the face of possible systemic risk to the banking system, the Federal Reserve announced that it would supply all of the liquidity that would be needed. Subsequently, the year-turn premium collapsed to zero. However, it has become customary, ever since, for traders to price in a year-turn premium, even though this is rarely realised. In fact, the overnight deposit has regularly collapsed on the actual day of funding.

Figure 15. US Overnight Fed Funds Effective vs. Target



Note that the Fed Funds effective rate creeps up shortly before the policy meetings when the Fed is hiking

[28] *Measures of Asset Swap Spreads and their Corresponding Trades, Lehman Brothers Fixed Income Research, Bruce Tuckman, 2003*

5.2 Isolating the funding spread

It is instructive to construct a quantitative model of spread of the funding rate to policy rate will trade and risk manage with respect to this:

$$f_l = r_l + s_l + \varepsilon_l$$

Where f_l is the funding rate on day l, r_l is the r_i applicable on day l, and ε_l is the residual error term.

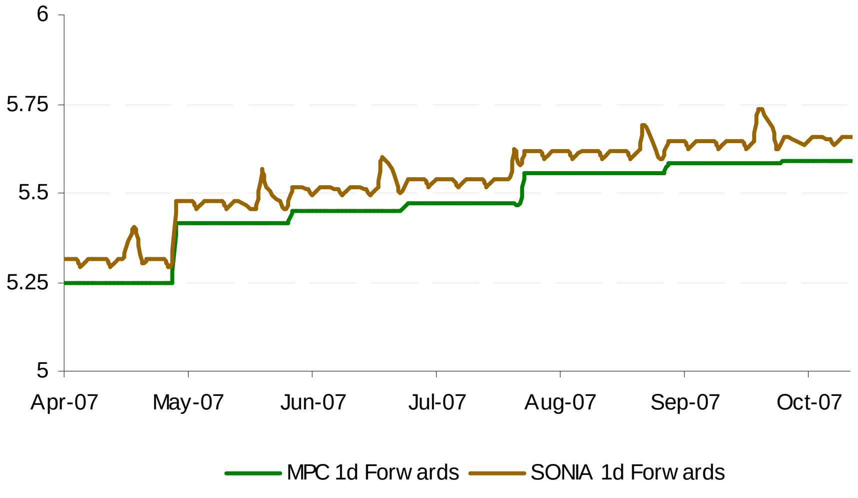
Ideally, ε_l would be small and mean-reverting over a period of 1M, and any model of the funding spread should have this goal in mind. The adjustments may be found either by averaging or some regression or weighting method. Figure 12 shows a trivial model of the SONIA[29]-MPC[30] Base Rate Spread that includes a month-end and premium and a daily average premium/discount based upon recent spread fixes. One can make the model as complex as required to reduce the residual error, or even keep it very simple by simply using the mean spread.

Once a spread model has been constructed, it can be used to extract the market policy expectations as described by the previously discussed equation:

$$\left(1+\frac{R_1(T_1-t_0)}{D}\right)=\prod_{l=0}^{t_m-t_0}\left(1+\frac{n_l\left(r_0+s_l\right)}{D}\right)\prod_{l=0}^{T_1-t_m}\left(1+\frac{n_l\left(r_1+s_l\right)}{D}\right)$$

Thus, provided the residual error term is mean reverting, or at least normally distributed, then, funding spread risk can be monitored using a standard VaR measure based upon the historical volatility of the residual error term ε_l.

Figure 16. Simple Model of the SONIA-MPC Base Rate Spread



A further point that is worth noting is that the only instrument with which one can hedge these funding differences is the Cross-Currency Basis Swap, which has an initial exchange of principal at the beginning and at the end. The swap is effectively the exchange of two floating rate notes (FRN) and so has no duration exposure, except after a fixing, when it effectively becomes a 3M FX Forward, plus a 3M forward-starting Cross-Currency Basis Swap. The near exchange of principal clearly has to be funded, and this is usually done by executing a 3M FX Forward. Alternatively, one might wish to

[29] *Sterling Overnight Index Average*
[30] *Bank of England Monetary Policy Committee*

lock in the OIS fixing each day, and this could be performed by executing an OIS-3M Libor basis swap in each currency – which effectively turns the Cross-Currency Basis Swap into two floating rate notes that pay the 1D OIS fixing. If the initial exchange is rolled daily, via an overnight FX Forward, the residual is the spread between when the trader is able to fund in the market, and where the market average was.

Clearly, there is little point in actually doing this, due to transaction costs, and generally, the funding exposure can be reasonably well hedged by executing 1W OIS (provided there is no meeting in the next week), and assuming that one is able to fund close to that. Alternatively, one could just execute Cross-Currency Basis Swaps to cover the funding risk, rolling quarterly via an FX Forward, and take on the risk of the Cross-Currency Spread moving (this is reasonably stationary, except for high-yield vs. Yen crosses, as a result of the carry trade via Uridashi Issuance[31]).

5.3 Funding and Index Curves

It is natural to differentiate between Funding and Index instruments. DV01 and CB01 are primarily defined and hedged by Index instruments:

- Libor Deposits
- FRAs
- Futures
- Libor Swaps
- OIS

Funding instruments, capture the (small) residuals

- Cross-Currency Basis Swaps
- Money-Market Basis Swaps
- FX Forwards
- Cash Deposits

The inclusion of OIS as an Index instrument, despite it containing information about funding, is that it still has an interest rate fix, like Libor swaps.

6. EXTRACTING MARKET PROBABILITIES

Finally, it is natural to try and extract the probabilities of certain outcomes occurring and compare them to personal views for trade decision-making. The market in options on Fed Funds futures has allowed market participants to transparently see what the market-based probability[32] of Fed actions is, and thus take positions based purely on these. However, for other markets, futures[33] and options upon the policy rate do not exist[34], and despite efforts to try and link options on Libor futures[35], the lack of resolution (3m futures generally contain three meetings, so there is an infinite number of solutions to the policy moves in that period[36]) and the problems in applying the policy-Libor spread (see §4.3).

[31] Uridashi issuance is the issuance of high yield debt, in New Zealand, for example, sold to Japanese investors. The financing of this cross-currency issuance causes banks to be paying JPY Libor and receiving NZD Bank Bills (the New Zealand equivalent of Libor) which causes them to hedge this basis risk by paying NZD Bills and receiving JPY Libor on the basis swap, causing the spread to widen.
[32] The Fediscope, Bruce Tuckman and Dana Calistru, Lehman Brothers Fixed Income Research, 2003
[33] Although extraction of expectations from OIS is equivalent.
[34] Actually, futures have recently been introduced for Australia and New Zealand, but they are not very liquid as of yet.
[35] See Jiltsov and Purzitsky
[36] Analogous to the three-body problem in Physics.

Given the rapid expansion of the OIS market, it seems natural that a swaption market will soon develop[37], allowing the similar direct trading of the probabilities of central bank action. However, until then, the approach, which is well known from the trading of Fed Funds futures, is to apply some assumptions to the outcomes of meetings and then use the expected policy rates to infer estimates of the probabilities. Essentially, these assumptions sell tail-risk for the purpose of maintaining simplicity – when analysing what is priced into a future, one assumes that only two outcomes can occur – e.g. if the future implies a higher rate than the current rate, then there is a chance of a 25bp hike and a chance of unchanged, but the assumption is that there is no probability of cuts. That inference is merely an estimate of the probability, however, as there is still a real probability of a rate cut, or a 50bp hike etc. The problem is that the future does not capture this probability – it merely provides the expectation. Options on Fed Funds futures are required to fully describe the probability distribution of market expectations of Fed Policy[38].

One approach, similar to that mentioned above, for estimating the probabilities is to assume that if the expected rate outcome is greater than the current policy rate then there is zero probability of a rate cut at that meeting. Then, say that the difference in rates, Δ, (column 3 in figure 6) divided by 25bp is the probability of a 25bp hike, as for the Fed Funds futures above. For example, at the first RBA meeting in figure 5, there is 11bp priced into the expectation, so there is an 11/25 = 44% chance of a 25bp hike, and a 100% - 44% = 56% chance of no move. Conversely, if the expected rate outcome is lower than the policy rate, then assume there is zero probability of a hike, and then apply the same logic. If the difference in rates is greater than 25bp, then there is a real probability of a 50bp move. In this case, attribute (Δ-25bp)/25bp*100% as the probability of a 50bp move and 100% minus this as the probability of a 25bp hike. This then assumes a zero probability of a rate cut or an unchanged rates outcome. Furthermore, it assumes that moves of greater than 50bp do not occur (50bp moves, themselves, are pretty rare these days). Each meeting is then treated in isolation. Obviously, this approach is far from perfect, but it does provide at least some transparency.

A more sophisticated method could include an application of the Reposcope[39] approach to calibrate the probabilities at the Libor future maturity dates and then some suitable form of interpolation (although, what form of interpolation, remains to be answered). However, the above approach, when applied to the RBA, gives the following probability distribution:

Figure 17. Estimated Probability Distribution for the RBA

Date	-50bp	-25bp	0bp	+25bp	+50bp	Expectation	bps
02-May-07	0%	0%	56%	44%	0%	6.36	+11
06-Jun-07	0%	0%	72%	28%	0%	6.43	+7
04-Jul-07	0%	0%	100%	0%	0%	6.43	0
08-Aug-07	0%	0%	84%	16%	0%	6.47	+4
15-Sep-07	0%	0%	92%	8%	0%	6.49	+2
03-Oct-07	0%	0%	92%	8%	0%	6.51	+2

[37] *Marc Henrard has discussed options on OIS in his article.*
[38] *See Fediscope, Tuckman*
[39] *See Jiltsov and Purzitsky*

7. CONCLUSION

This paper presents a new approach to the construction of the short-end of the Swap Curve which avoids the inconsistencies of the traditional approach. Through the use of Overnight Indexed Swaps and Money Market Basis Swaps, market expectations of Central Bank policy can be included that allow the accurate pricing of short-term interest rate instruments such as FRAs. This method is extended, through the inclusion of Cross-Currency Basis Swaps to allow the pricing of FX Forwards. Furthermore, this framework allows risk-management in terms of the outcomes of Monetary Policy meetings, whilst maintaining consistency across different instruments, through the use of basis swaps. Finally, integration of this approach with the usual instruments used in Swap Curve construction (Futures & Swaps) from around the 6m point on the curve out, produces a fully consistent framework that allows the accurate pricing and simple risk-management of the entire curve

8. REFERENCES

- *The Swap Curve, Lehman Brothers Fixed Income Research, Fei Zhou, 2002*
- *Overnight Indexed Swaps and Floored Compounded Instruments in HJM One-Factor Model, Bank for International Settlements, 2004*
- *Interest Rate Parity, Money Market Basis Swaps, and Cross-Currency Basis Swaps, Lehman Brothers Fixed Income Liquid Markets Research, Bruce Tuckman and Pedro Porfirio 2003*
- *Fixed-Income Securities: Valuation, Risk Management and Portfolio Strategies, Martellini, Priaulet and Priaulet, 2003*
- *Introducing Reposcope: A tool for monitoring UK and euro-area monetary policy expectations, Lehman Brothers Liquid Markets Research Quarterly, Alexei Jiltsov and Adam Purzitsky, 2006*
- *A Comprehensive Model of the Euro Overnight Rate, ECB Working Paper No. 247, Flemming Reinhardt, 2003*
- *Measures of Asset Swap Spreads and their Corresponding Trades, Lehman Brothers Fixed Income Research, Bruce Tuckman, 2003*
- *The Fediscope, Lehman Brothers Fixed Income Research, Bruce Tuckman and Dana Calistru, 2003*