COGS 200 Supplementary Note: Holographic

The description of “holographic” given in the COGS 200 notes is:

Any part of any given representation carries information about every part of the content represented by it

Let’s explore “holographic” further using sound as an example. A direct representation of audio (i.e., sound) is as a function, $f(t)$, of time. Indeed, it is $f(t)$, as an electrical signal, that one wants to send to headphones or speakers in order to generate sound and it is $f(t)$, as an acoustic wave, that one wants to have arrive at our ears in order to hear sound.

Consider the specific audio signal given by

$$f(t) = \sin(\omega t)$$

for a fixed ω. This $f(t)$ is a pure sinusoidal tone with frequency ω.

What is perhaps counter intuitive[1] is that any sound, $f(t)$, can be represented as the mixture (i.e., sum) of sinusoidal tones of differing frequencies and phases.[2] That is, one can represent sound as a function, $f(t)$, as we started with, or equivalently as a function, $F(\omega)$, of frequency.

Let’s summarize:

1. Any sound can be represented either as a function of time, $f(t)$, or as a function of frequency, $F(\omega)$.
2. Given $f(t)$, one can determine the equivalent $F(\omega)$. Similarly, given $F(\omega)$, one can determine the equivalent $f(t)$.
3. $f(t)$ represents sound explicitly as a function of time. But, $f(t)$ also represents sound implicitly as a function of frequency.
4. $F(\omega)$ represents sound explicitly as a function of frequency. But, $F(\omega)$ also represents sound implicitly as a function of time.

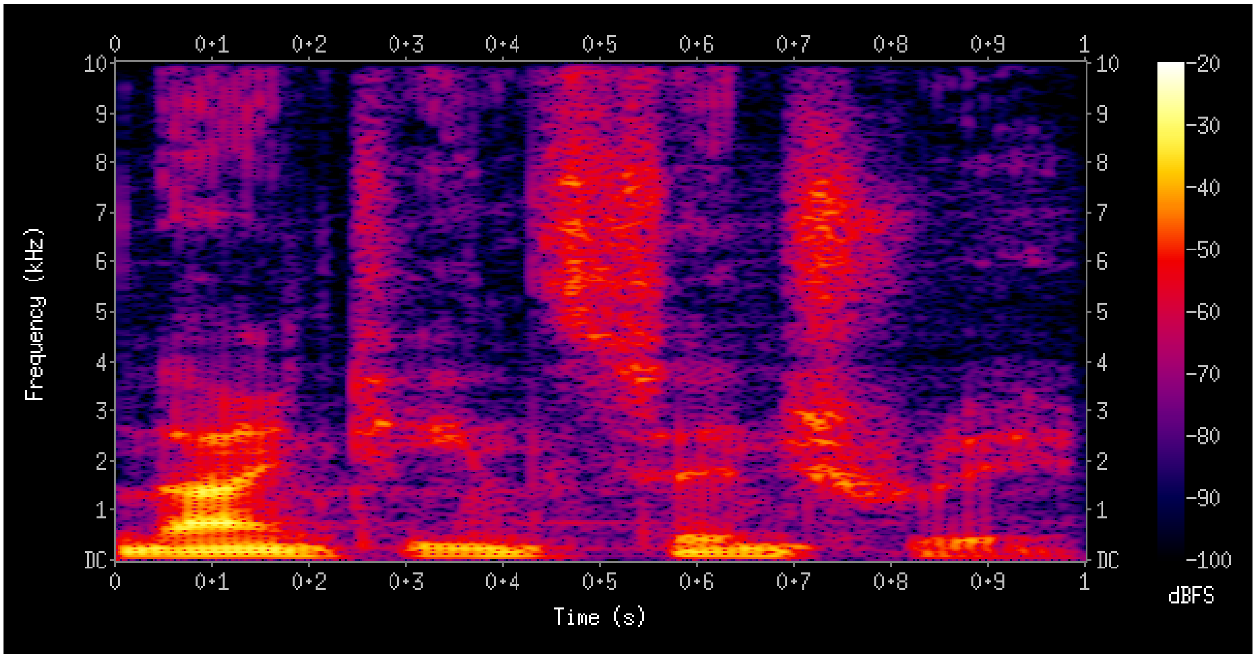
A spectrogram is a visualization of the frequency of sound as a function of time. (See the Wikipedia article, Spectrogram. The sample spectrogram that follows comes from the Wikipedia article. The article contains other example spectrograms).

Example 1 is a spectrogram of the spoken words “nineteenth century.” The vertical axis is frequency, ω, in the range 0–10,000 Hz. The horizontal axis is time (the utterance lasted about 1 second). $|F(\omega)|$, the magnitide of $F(\omega)$, is called the frequency spectrum of $f(t)$. A spectrogram is a colour-coded rendering of $|F(\omega)|$ as it changes with time. According to the legend given, yellow–white is large magnitude, blue–black is small magnitude.

[1] but intuitive to people familiar with music

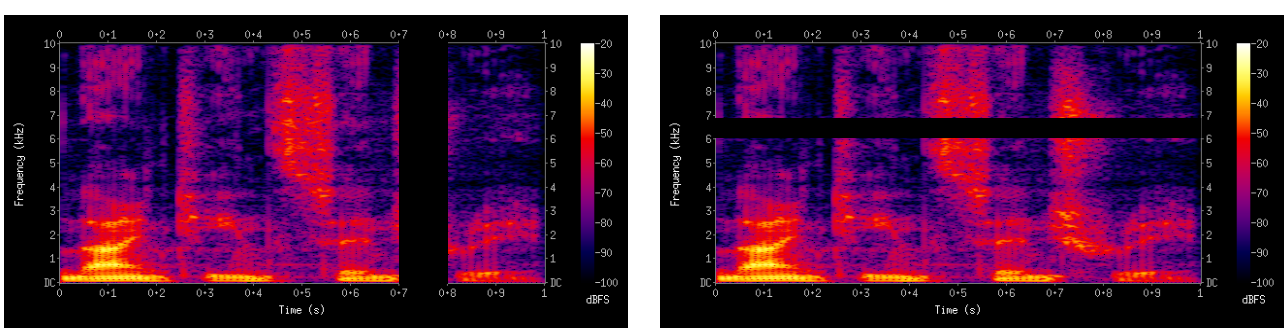
[2] To keep the discussion simple, I ignore the role that phase plays in what follows.

Example 1: Spectrogram of a male voice speaking the words “nineteenth century”



Let’s now address the sense in which a spectrogram is a holographic representation of sound. The spectrogram shown represents frequencies from 0–10,000 Hz for 1 second of sound. Suppose we were to lose 10% of the information represented in the temporal domain. The degraded spectogram on the left below eliminates all sound from the 0.7 to 0.8 second mark. What remains is an undegraded representation of the other 0.9 seconds of the utterance. Suppose instead we were to lose 10% of the information represented in the frequency domain. The degraded spectogram on the right below eliminates all frequencies from 6,000 Hz to 7,000 Hz. What remains is the entire 1 second of sound, but with loss of a specific band of frequencies. The entire utterance would still be understandable as speech. Indeed, there would be no loss at all if the utterance were broadcast on AM radio (since AM radio itself eliminates all frequencies above 5,000 Hz). Aside: Humans nominally can hear frequencies up to 20,000 Hz. AM radio was not intended as a medium for listening to high quality music.

Example 1 (cont’d): Degraded spectrograms



For a given sound, $f(t)$, pick a frequency, say 1,000 Hz. The specific value, $F(\omega)$, $\omega = 1,000$ Hz, represents information across time (since it is the "weight" that tells us how much of the specific sinusoid, $\sin(\omega t)$, $\omega = 1,000$ Hz, we need to represent $f(t)$ indefinitely across time. The same argument holds for any other specific value of ω (and therefore the same argument holds for all values of ω). Thus, $F(\omega)$, for any given ω, carries information about every part of the sound, $f(t)$, that it represents.

Aside: With images, as with sound, an equivalent frequency-based representation exists. In image processing, the (spatial) frequency-based representation of an image is called a hologram. Said another way, a hologram is to images as a frequency spectrum (and spectrogram) are to sound.[3]

[3] Aside: Holograms preserve phase information while spectrograms, as normally defined, don't.