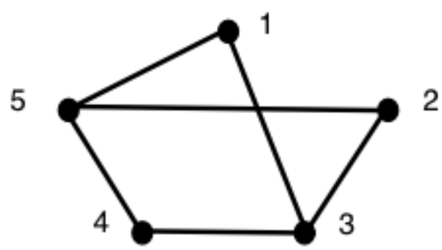
IMPORTANT FIRST STEPS:

1. **Close your laptops and put them away (if necessary, you may refer to your course notes).**
2. **Form a group of 2-3 students.**
3. **Clearly put your names and IDs on 1 copy of this worksheet.**
4. **Be sure to turn this exercise in at the end of class.**

Graph Theory

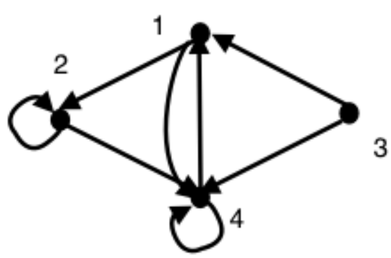
Draw the following graphs. When you're done, write out the adjacency list for each node (make sure to leave yourself enough room):

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$



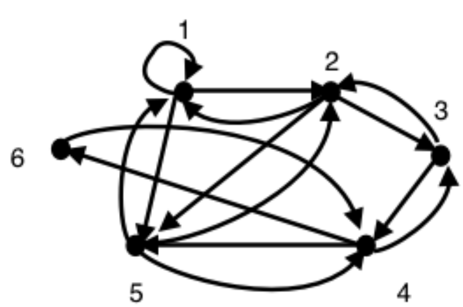
Adjacency list:
1: 5,3
2: 5,3
3: 4,2,1
4: 5,3
5: 1,2,4

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$



Adjacency list:
1: 2,4
2: 2,4
3: 1,4
4: 4,1

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

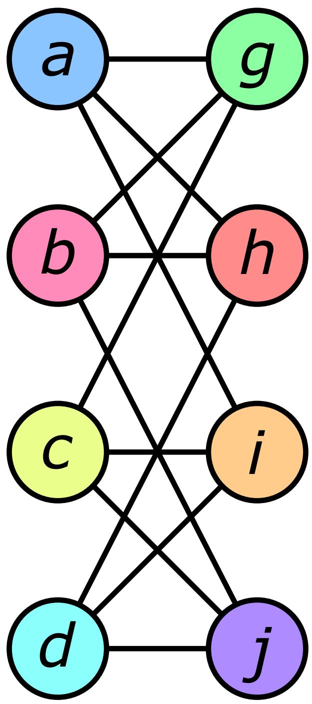


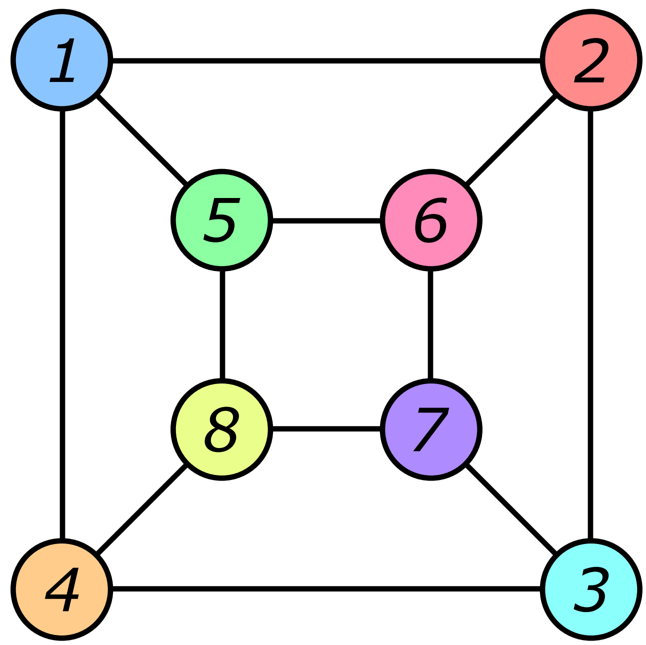
Adjacency list:
1: 1,2 5
2: 1,3,5
3: 2,4
4: 3,5,6
5: 1,2,4
6: 4

(note I would have accepted undirected here.

Two graphs, G and H, are **isomorphic** if there exists a 1-1 function/mapping f, such that: f: V(G) --> V(H)

(That is, any two vertices, u and v, are adjacent in a graph, G, if and only if, f(u) and f(v) are adjacent in H).





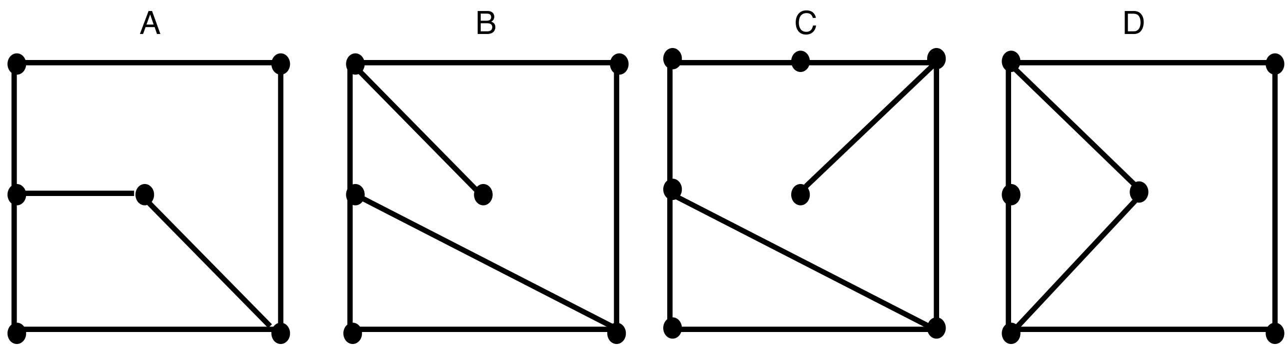
Images from Wikipedia

The above graphs are isomorphic. There exists a 1-1 mapping between the like-coloured nodes.

If you don’t have the colour version, here’s the mapping:

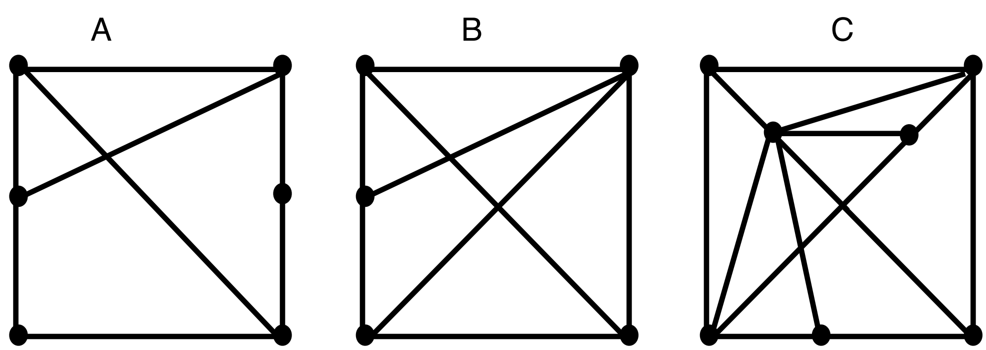
a : 1
b : 6
c : 8
d : 3
g : 5
h : 2
i : 4
j : 7

Given the graphs below, which, if any, are isomorphic? Label each vertex uniquely and provide the mapping between the vertex set of each graph, if so.



Hint: Try eliminating graphs first-- what criteria can you use to determine right away which graphs cannot have an isomorphic counterpart in the list of graphs above?

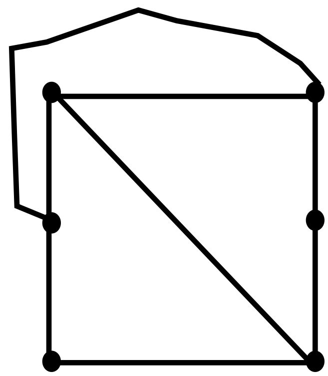
Solution: A & D are isomorphic.



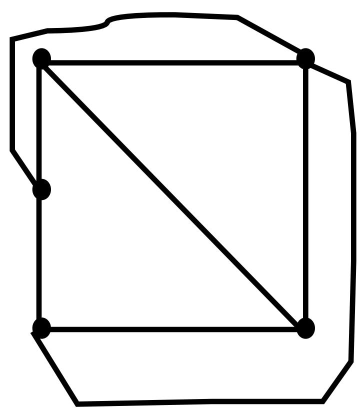
A graph is **planar** if it *can* be drawn on a surface such that no edges intersect (or cross one another). Such a drawing is called a *planar embedding*. Does a planar embedding exist for any or all of the graphs above? If so, draw the planar embedding on the next page. Technically the graphs you draw are *automorphic*, that is they are permutations of the *same* graph, or isomorphic with themselves (i.e. the vertices are identical, whereas above the vertices were simply mapped together, but were not the same vertices as they came from different graphs).

Draw your planar embeddings here, if they exist, otherwise draw as much as you can and show which edge you cannot connect without crossing another edge:

A

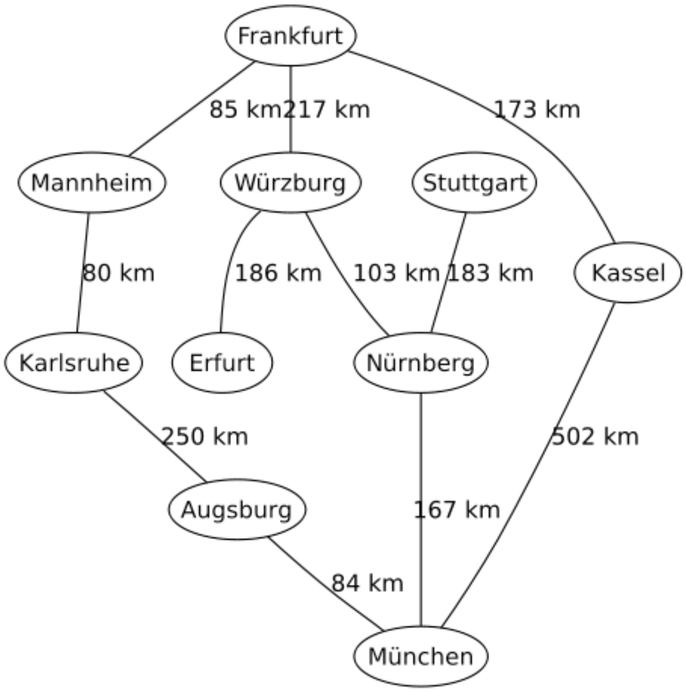


B

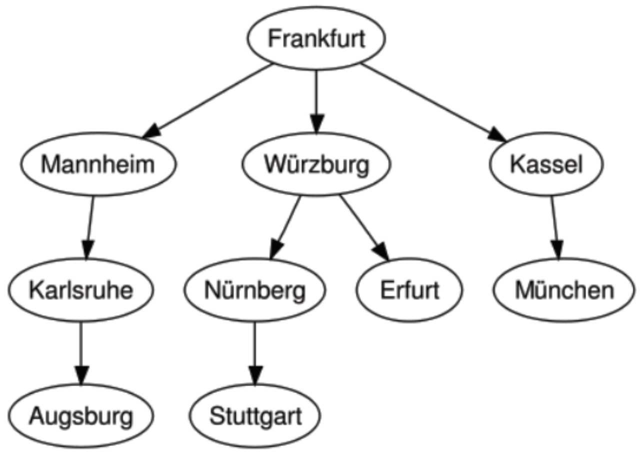


C not possible

Test your knowledge of traversals on the following graphs:



Graph A



Graph B

Write out the following for each graph above:

Starting from Frankfurt, write out any valid breadth-first traversal:

Graph A: **Frankfurt, Mannhein, Wurzberg, Kassel, Karlsruhe, Nurnberg, Erfurt, Munchen, Augsburg, Stuttgart (one possible BF traversal)**

Graph B: **Frankfurt, Mannheim, Wurzburg, Kassel, Karlsruhe, Erfurt, Nurnberg, Munchen, Augsburg, Stuttgart (one possible BF traversal)**

Starting from Frankfurt, write out any valid depth-first traversal:

Graph A: **Frankfurt, Manneheim, Karlsruhe, Augsburg, Munchen, Kassel, Nurnberg, Wurzburg, Erfurt, Stuttgart (one possible DF traversal)**

Graph B: **Frankfurt, Manneheim, Karlsruhe, Augsburg, Wurzburg, Nurnberg, Stuttgart, Erfurt, Kassel, Munchen (one possible DF traversal)**

What is the maximum number of edges of a complete graph with 10 vertices? Hint: this is a *counting* problem.

10 choose 2

Can you have an undirected graph with 40 edges and 9 vertices? Prove why or why not (do not draw the graph).

No loops are allowed.

There are a possible 9 choose 2 edges

9! / 7! * 2! = 9*8 / 2!
= 9 * 4
= 36

36 < 40, so it is impossible