SAMPLE SOLUTION

IMPORTANT FIRST STEPS:

1. **Close your laptops and put them away (if necessary, you may refer to your course notes).**
2. **Form a group of 2-3 students.**
3. **Clearly put your names and IDs on 1 copy of this worksheet.**
4. **Be sure to turn this exercise in at the end of class.**

Counting

How many ways can you arrange 8 books, such that a particular book is in the second place?

Given 8 different books, we must select a particular book to go in the second place (8 choices), and then we must arrange the remaining 7 books (7! choices)... so 8 * 7!, or 8! choices.

If we have 35 green marbles and 12 purple marbles, how many ways can we select a collection of marbles so that we have 8 green ones and 3 purples ones?

If the marbles are not distinct, then there really is only one way to elect 8 green and one way to select 3 purple. In that case since we want the order that we select them then G, G, P, G... would be different than P, P, P, G, etc. So 1 x 1 x 11!.

If the marbles are distinct (e.g. green marble 1, green marble 2 can choose 8 green marbles, with no repetition (we have exactly 35 marbles of which we are selecting 8), and the order in our final collection does not matter (same for the purple): C(35,8) * C(12,3)

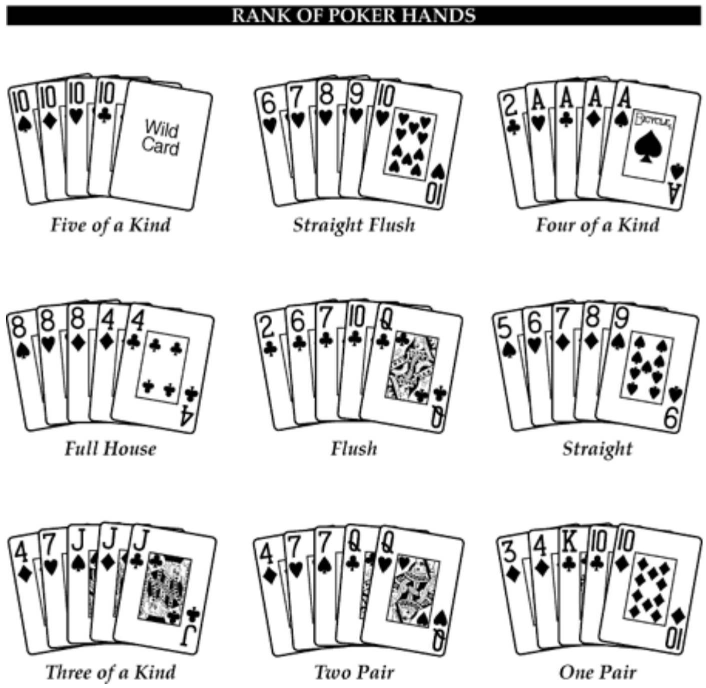
On a exam I will be explicit, but for here I want you to think about both.

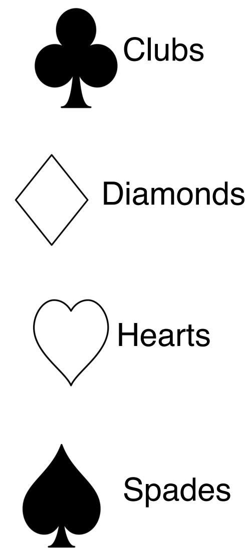
How many ways can you arrange m identical stones into k piles so that each pile has exactly one stone in it?

If the piles are indistinguishable, then just one. :)

SAMPLE SOLUTION

For the next few questions, consider the following:





Rank (13 total): Ace, King, Queen, Jack, 10, 9, 8, 7, 6,5,4,3,2

How many 3-of-a-kind (but not full house) hands are possible?
The task is to choose a rank, then the three suits for the three cards, then choose the remaining cards (there are two ways to do that-- the same as the first, or just based on what's left in the deck)

C(13,1)*C(4,3)*C(12,2)*C(4,1)*C(4,1) OR C(13,1)*C(4,3)*C(48,1)*C(44,1)/2 (I divide by two because I don't care about the order of the other two cards)

How many straights, but not straight flushes are possible?
(C(9,1)*C(4,1)^5) - C(9,1)*C(4,1)
Explanation: There are 9 possible "starts" to a straight: A,2,3,4,5,6,7,8,9 (after 9 there are not enough cards to get 5 in a row) and we need to choose one (reps no, order no). We need to choose a suit for each one (reps no, order no). This, however, gives us a result with the straight flushes included, so we need to remove those. That is 9 * 4 (choose the start, then choose the suit for all of the cards).

How many different hands of 5 cards are possible?
C(52,5): We are simply choosing a combination of cards from 52, no reps, order doesn't matter.

How many different hands of 5 cards with no diamonds are possible?
C(52-13,5)

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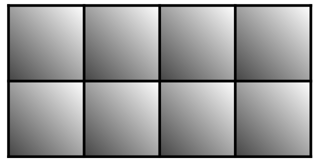

Suppose you are to place square tiles in a 2x4 rectangular pattern on a bathroom wall. How many different patterns can you make if you have 15 distinct tiles to use?

If you have exactly 15, distinct tiles, then 15^(8). (15 choices for the first * 14 for the second * ... * 8 choices for the eighth). If you wanted to tile a larger area than 2x4, then you would have to allow duplicates, otherwise you would run out of tiles! So it would become 8^n, where n is the number of tiles spots that you need to fill.

How many ways are there to place 10 distinct marbles in 3 distinct baskets?
For each marble you have 3 choices, or 3 * 3 * 3...: 3^10

What if the marbles are not distinct, but the baskets are?
This is the same problem as x1 + x2 + x3 = 10, so (10+3-1) choose (3-1). 66 total.

What if the baskets are not distinct, but the marbles are?
This next two are quite challenging! I have not yet had a student give me a solution to this question. A question this challenging will NOT be on the exam . :)

We have 10! marble arrangements and then (10+3-1) choose 10 possible ways to split them among the indistinct baskets.

E.g.

x1x2x3x4x5x6x7x8x9x10 (one possible arrangement of marbles; 10! total)

...we then need to split them into the baskets:

I x1x2 I x3x4x5x6x7x8x9x10
I x1x2x3 I x4x5x6x7x8x9x10
I x1x2x3x4 I x5x6x7x8x9x10
...
x1 I x2 I x3x4x5x6x7x8x9x10
x1 I x2x3 I x4x5x6x7x8x9x10
x1 I x2x3x4 I x5x6x7x8x9x10
...
x1x2 I x3 I x4x5x6x7x8x9x10
x1x2 I x3x4 I x5x6x7x8x9x10

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...
etc...

This will over count, though...note the following examples, which are the same (since the baskets are indistinct):

x1 l x2 l x3x4x5x6x7x8x9x10
x2 l x1 l x3x4x5x6x7x8x9x10

But these are not the same (since the marbles are not distinct):

x1x2 l x3x4 l x4x5x6x7x8x9x10
x1x4 l x3x2 l x4x5x6x7x8x9x10

Here are the cases

0 0 10 (1 way to get this, i.e. 10 C 10)
0 1 9 (10 ways to choose the one, the rest go in the final spot, i.e. 10 C 1)
1 1 8 (10 choices for first, 9 for second, divide by 2, so 45, i.e (10 C 2))
0 2 8 (10 C 2)
0 3 7 (10 C 3)
1 2 7 (10 C 1) * (7 C 2)
2 2 6 (10 C 2) * (10 C 2)
1 3 6 (10 C 1) * (9 C 3)
0 4 6 (10 C 4)
0 5 5 (10 C 5)
3 4 3 (10 C 3) (7 C 4)

So add up the row results.

What if both the marbles and the baskets are not distinct?
For each marble you have 3 choices, or 3 * 3 * 3...: 3^10, but you have to discount the permutations of the baskets. E.g. b1=(1 2 4), b2=(3 5 6), b3=(7 8 9 10) is the same as b1=(7 8 9 10), b2=(1 2 4), b3=(3 5 6). We also have to take into account the case where all of the marbles go into the same basket: b1=(1-10) with b2 and b3 empty is the same as b2=(1-10) with b1 and b3 empty, for example. But this has fewer permutations (3, to be exact).

Here are the cases:

0 0 10
0 1 9
1 1 8
0 2 8
0 3 7
1 2 7

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2 2 6
1 3 6
0 4 6
0 5 5
3 4 3

Here's one attempt:

There are (10+3-1) choose 10 possible locations among the 3 baskets where we could put a marble (if the marbles were indistinguishable, but the baskets were distinguishable, we'd be done).

But this doesn't take into account that the marbles are distinguishable, so for each of those (10+3-1) choose 10 distributions of marbles, we need to pick which marble is which, so we have (10+3-1) choose 10 * 10! (E.g. if I have x1 + x2 + x3 = 10, we have (10+3-1) choose 10 distributions of the "things" across the three "baskets". But if I come along and label each thing so they are no longer indistinguishable, then I also have (10+3-1) choose 10 * 10! ways in which I could distribute those things). This is waaaay too big, though.

However we've counted the baskets as distinguishable here, which they're not, so alternatively we could divide out the number of permutations of baskets (3!).

So, as a rough guess: (10+3-1) choose 10 / 3! = 11, which is correct... but it doesn't generalize! :)

E.g. if we have 3 baskets and 8 marbles that would predict:

(8+3-1) choose 8 / 3! =

10 C 8
10! / 8!2!
10*9/2
45 / 2!
22.5... WRONG

0 0 8
0 1 7
0 2 6
0 3 5
0 4 4
1 1 6
2 2 4
3 3 2
4 4 0

SAMPLE SOLUTION

Feel free to throw your hat in the ring!

Here's a dynamic programming solution (thanks to Steve W.)-- not a formula, like what we're looking for, but an easy way to check your answers.

```
def jarways2(table, n, k):
  if n < 0 or k < 0 or table[n][k] is None:
      ways = 0
      if n == 0:
          ways = 1
      elif k == 0:l
          ways = 0
      elif n < 0 or k < 0:
          return 0
      else:
          ways = jarways2(table, n, k-1) + jarways2(table, n-k, k)
      table[n][k] = ways
  return table[n][k]
```