PHYS 371 - Assignment III

Celil Buğra Karacan
Deepartment of Physics, Bilkent University
(Dated: November 26, 2018)

I. INTRODUCTION

Signal $s(t) = \frac{1}{1-0.9sin(t)}$ with expansion: $s(t) = 1 + 0.9sin(t) + (0.9sin(t))^2 + (0.9sint(t))^3 + ...$ was provided. Computing signals DFT (Discrete Foruier Transform) $S(\omega)$ and its power spectrum, autocorrelation function of $s(t)$ -$A(\tau)$- and its DFT -$A(\omega)$- with its power spectrum were assigned. Power spectrum was square of absolute of a function. Moreover, a noise such that $y(t) = s(t) + \alpha(2r_i - 1)$ was to be taken into consideration, where $0 \leq r_i \leq 1$.

II. METHOD

MATLAB® was put to use since its library had necessary functions. ACF (AutoCorrelation Function) was tried to be written from scratch, yet sketch written did not perform well.

The period of $s(t)$ is 2π, since it's a function of $sin(t)$. Moreover, period of $A(\tau)$ is the same since integral formula: $A(\tau) = \int_{-\infty}^{+\infty} s^*(t)s(t+\tau)d\tau$, notice $\tau + 2k\pi$ gives same result as τ, k being integer. An array of length `n` = 250 spanning $[0, 2\pi]$ was initiated as `t`. Then, array `s` was initiated accordingly, later on within a `for loop` the noise was added forming `y`. `fft()` function was used to take Fast Fourier Transfrom, and `autocorr()` for calculating ACF. However, first approach was to use integral formula provided, that was *possible*, yet it took so long to run the code, thus was commented out.

III. VERIFICATION OF THE PROGRAM

There seems no evident/frank way of verifying the program for the time being. The signal is known, and physical analogies to relate/verify further calculations on the signal are absent to the author.

IV. ANALYSIS

It is apparent that first calculating ACF and proceeding with its DFT reveals a consistent solution. Thus, it helps greatly to overcome the problem of noise. On contrary, calculating DFT right-away exeggerates the noice introduced.

V. INTERPRETATION

It will be proposed here, most probably not for the first time, that any further calculations to be done on the original signal should be carried out in ω space on DFT of ACF of the signal. That way noise's influence *might* be neglegible to say the least.

VI. DATA

Plots for α values $0, 10, 40$ are provided below.

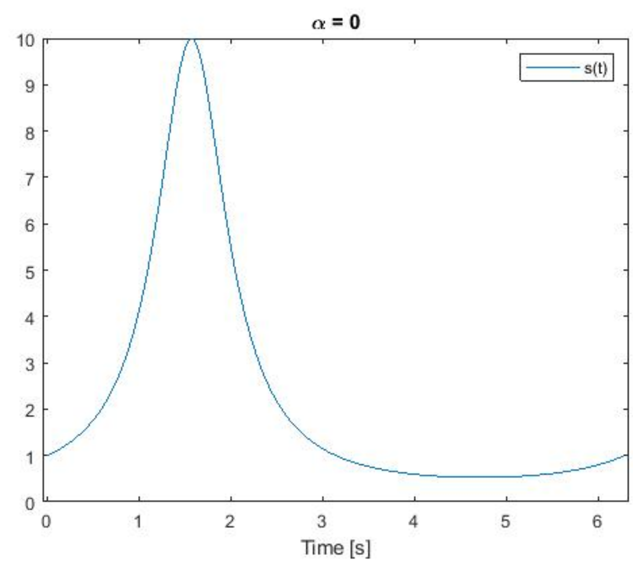

FIG. 1. $s(t)$ vs t, $\alpha = 0$.

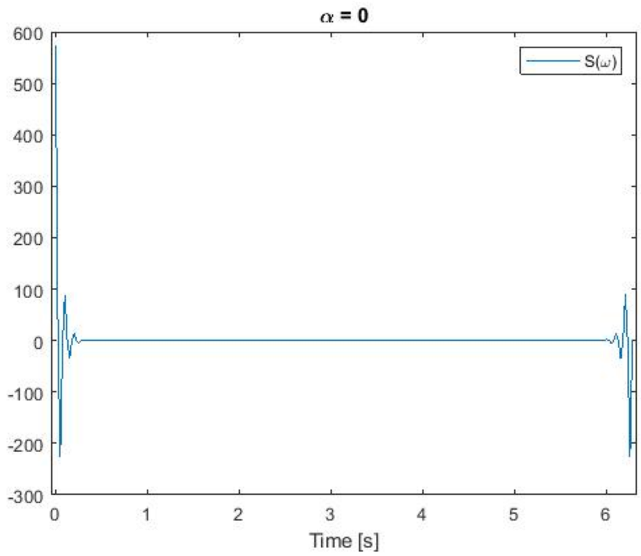

FIG. 2. $S(\omega)$ vs ω, $\alpha = 0$.

.

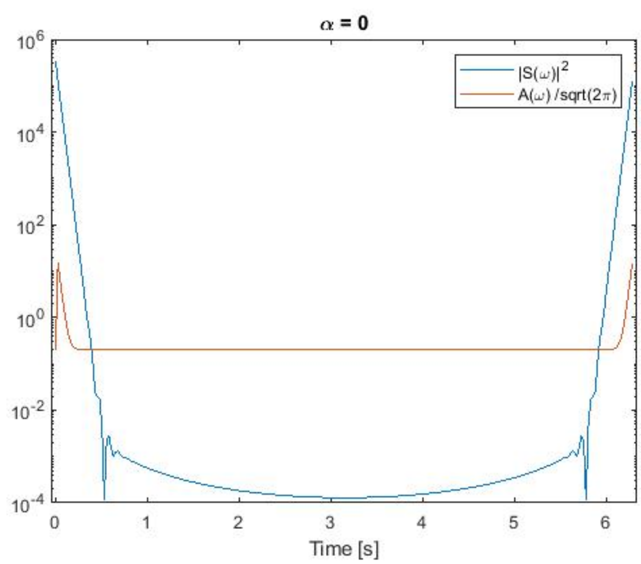

FIG. 3. $\frac{A(\omega)}{\sqrt{2\pi}}$ & $|S(\omega)|^2$ vs ω, $\alpha = 0$.

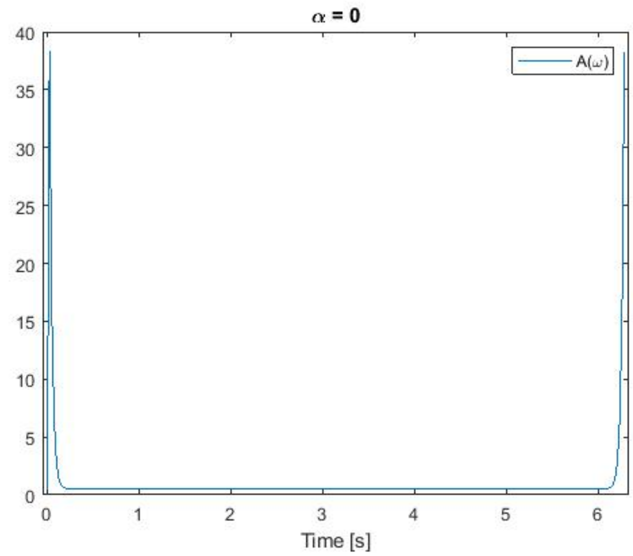

FIG. 4. $A(\omega)$ vs ω, $\alpha = 0$.

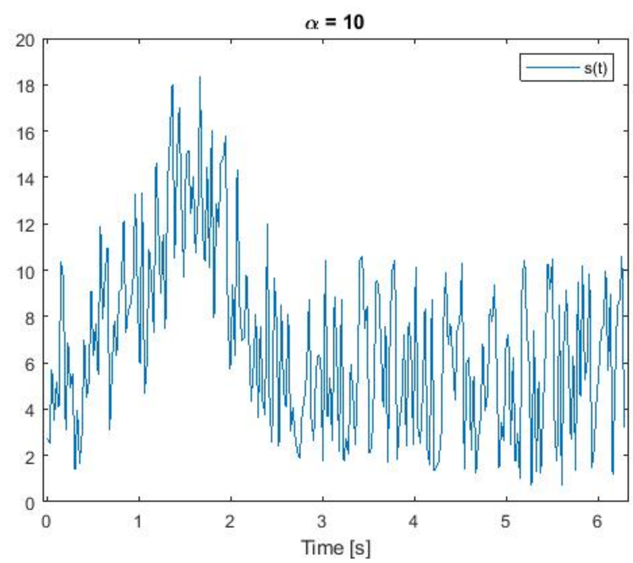

FIG. 5. $s(t)$ vs t, $\alpha = 10$.

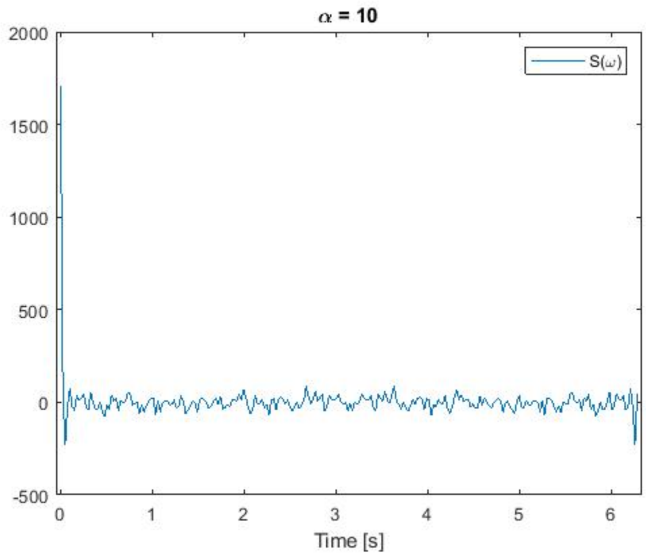

FIG. 6. $S(\omega)$ vs ω, $\alpha = 10$.

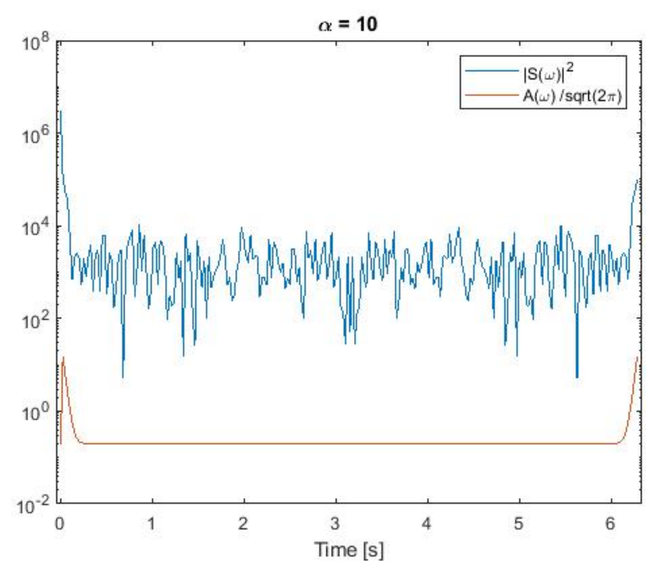

FIG. 7. $\frac{A(\omega)}{\sqrt{2\pi}}$ & $|S(\omega)|^2$ vs ω, $\alpha = 10$.

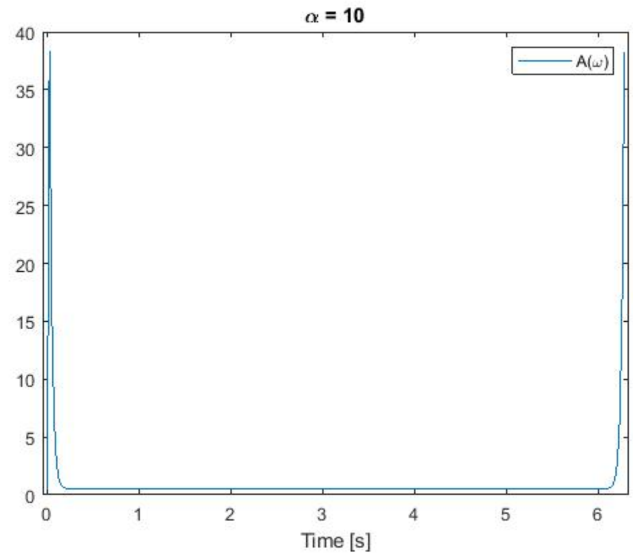

FIG. 8. $A(\omega)$ vs ω, $\alpha = 10$.

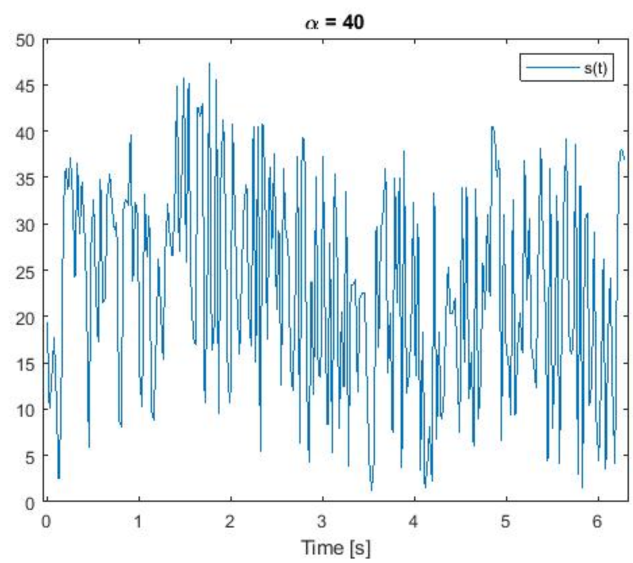

FIG. 9. $s(t)$ vs t, $\alpha = 40$.

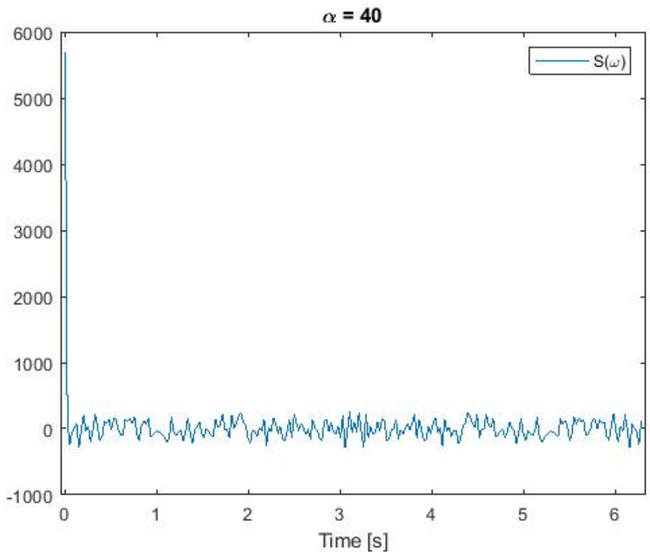

FIG. 10. $S(\omega)$ vs ω, $\alpha = 40$.

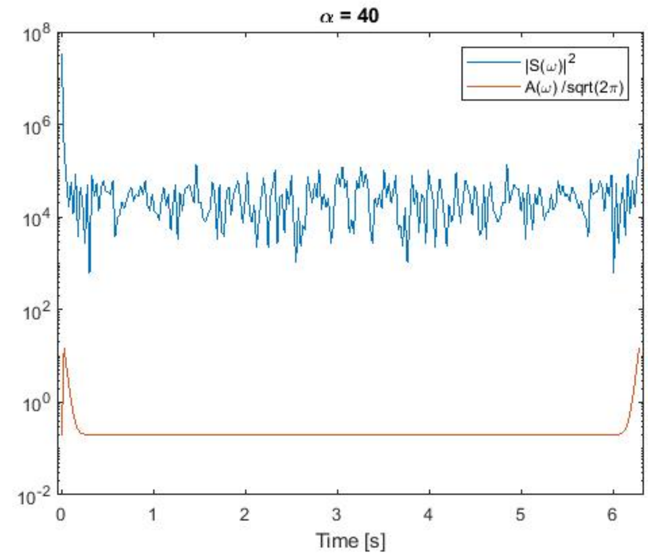

FIG. 11. $\frac{A(\omega)}{\sqrt{2\pi}}$ & $|S(\omega)|^2$ vs ω, $\alpha = 40$.

,

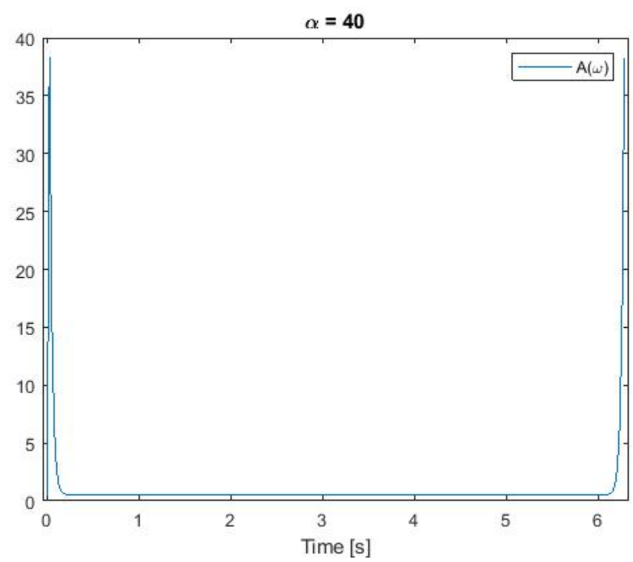

FIG. 12. $A(\omega)$ vs ω, $\alpha = 40$.

.