PHYS 371 - Assignment III

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I. INTRODUCTION

Two phenomenon was assigned to be numerically simulated, which are an isolated and a non-isolated wires length L with one dimensional heat loss due to contact with (supposedly infinete amount of) ice at the ends, and for the latter radiation causes it to cool too. Positions $x = 0$ and $x = L$ are at $T = 0K$ through out the simultion, and initially wire inbetween is uniformly at $100K$.

II. METHOD

For `part a`,

$$\frac{d^2f}{dx^2}\Big|_i = \frac{f_{i+2} - 2f_{i+1} + f_i}{\Delta x^2} \tag{1}$$

(Forward Difference Aprrox.) is used. Note that, subscript i denotes time step. Thus:

$$\frac{\partial T(x,t)}{\partial t} = \frac{K}{C\rho}\frac{\partial^2 T(x,t)}{\partial x^2} \tag{2}$$

, becomes

$$\frac{\partial T_i^j}{\partial t} = \frac{K}{C\rho}\frac{T_{i+2}^j - 2T_{i+1}^j + T_i^j}{\Delta x^2} \tag{3}$$

, where superscript j denotes position step.

From there on, Euler's Method and formula above (3) will be used to calculate T_i^j for n many steps for position and m many for the time period. Therefore eventually will form a matrix of $n \times m$. Then:

$$T_i^j = T_{i-1}^j + \Delta t \frac{\partial T_{i-1}^j}{\partial t} \tag{4}$$

is the algorithm to be implemented. Where
$\Delta t = time/m$ and $\Delta x = L/n$. Now notice, that there is not a dependence on Δt as time passed can be taken as any value, that is a bit confusing to be honest.

For the `part b`,

`meshc()` function of MATLAB ®was used to plot the matrix of temperature values, function also provides a contour below the graph for the good.

For `part c`,

all that was asked was to plot for various plots for "several times". That was interpreted as to use different time steps (m), resulting in varying details due to scale.

For `part d`,

`contour()` function was all neded, giving out isothermal lines right away, since only provided data was temperature after all.

For e,

Crank-Nicolson Method (or Central Difference Approx.) was asked to replace (1).

$$\frac{d^2f}{dx^2}\Big|_i = \frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta x^2} \tag{5}$$

Thus:

$$\frac{\partial T_i^j}{\partial t} = \frac{K}{C\rho}\frac{T_{i+1}^j - 2T_i^j + T_{i-1}^j}{\Delta x^2} \tag{6}$$

Proceeding there on with (3), yields a better solution.

For `part f`.

Analytic solution for isolated wire problem was provided and has been solved within a triple `for loop`, two for the temperature matrix and one for the finite sum with negligible truncation error.

$$T(x,t) = \sum_{n=1,3,\ldots}^{\infty} \frac{4T_0}{n\pi} sin(k_n x) e^{-{k_n}^2 Kt/(C\rho)} \tag{7}$$

, where $k_n = n\pi/L$.

However, it should be noted that, middle of the wire was getting cooler for some reason. That was not expected and it is suspected that there lies a mistake which is to be found out.

For `part g`,

equation below were provided:

$$\frac{\partial T}{\partial t} = -h(T - T_e) \tag{8}$$

, T_e being surrounding temperature.

and first partial derivative of T being sum of equations (2) and (8). so `for loops` constructed earlier were adhered together to solve for non-isolated wire problem.

III. ANALYSIS

In part g, fast drop of temerature is due to there being less influenced by ices located both terminals. Thus, it radiates faster for the beginning and drops earlier therefore. Also there is exponential drop for Crank-Nicolson Method.

IV. DATA

Plots for several times could not be uploaded due to scarsity of time unfortunately.

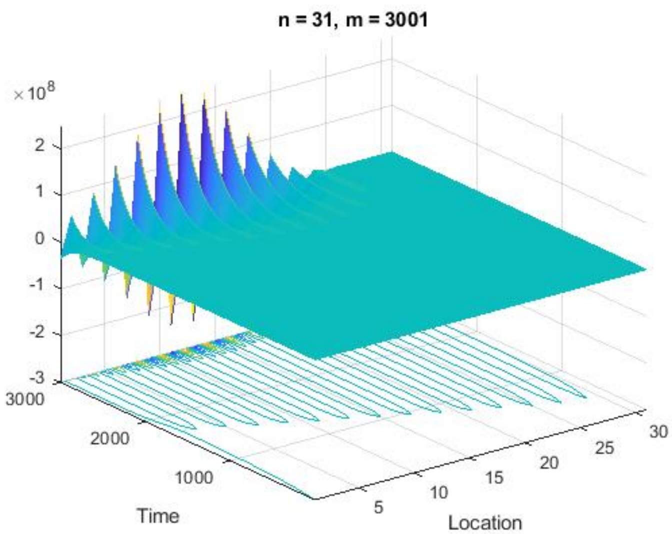


FIG. 1.

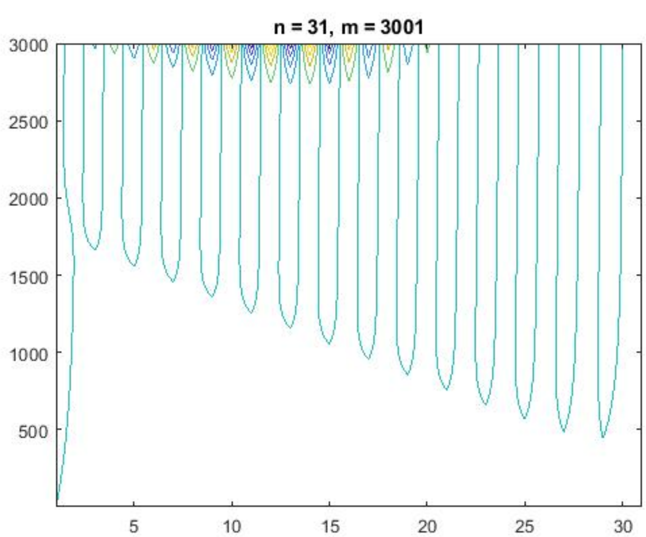


FIG. 2.

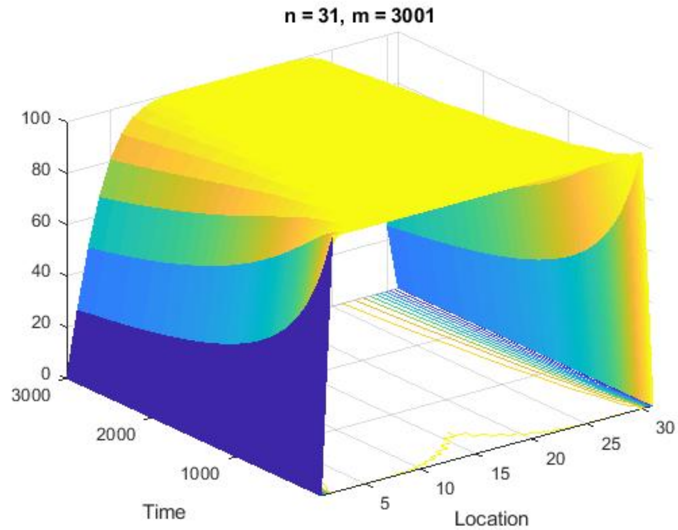


FIG. 3.

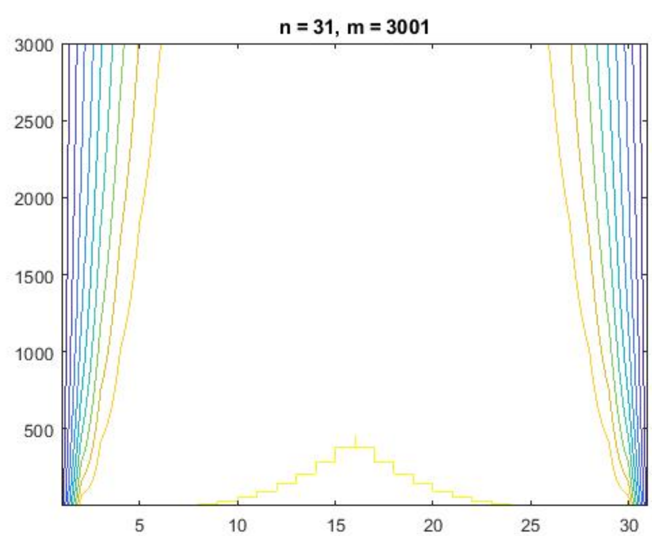


FIG. 4.

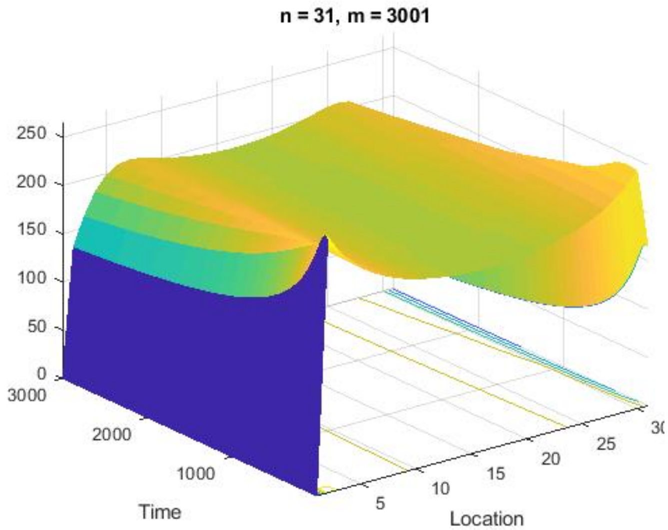

n = 31, m = 3001
250
200
150
100
50
0
3000
2000
1000
Time
5
10
15
20
25
Location

FIG. 5.

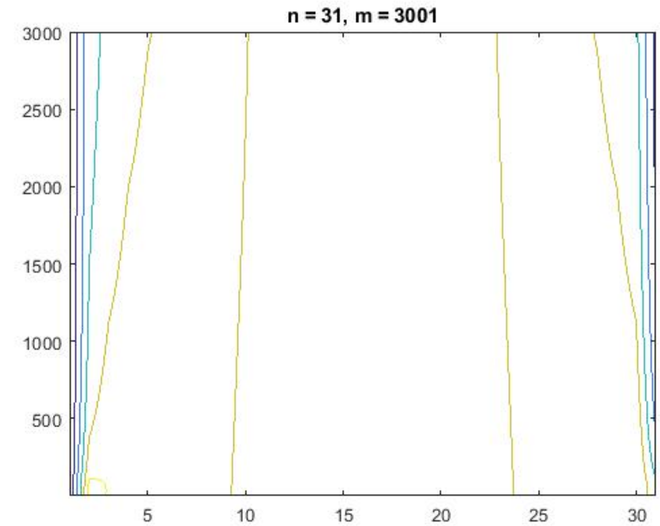
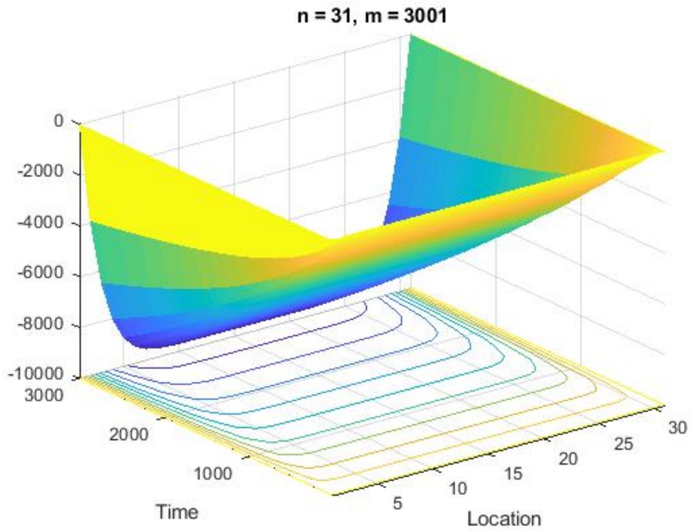
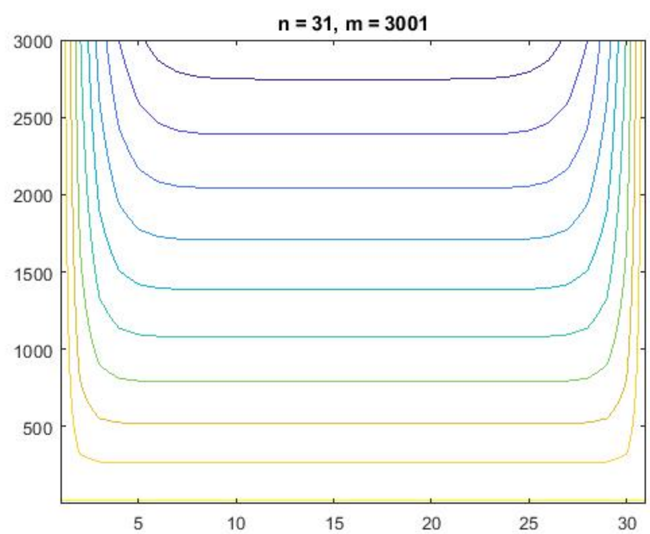

n = 31, m = 3001
3000
2500
2000
1500
1000
500
5
10
15
20
25
30

FIG. 6.



n = 31, m = 3001
0
-2000
-4000
-6000
-8000
-10000
3000
2000
1000
Time
5
10
15
20
25
30
Location

FIG. 7.



n = 31, m = 3001
3000
2500
2000
1500
1000
500
5
10
15
20
25
30

FIG. 8.