PHYS 371 - Assignment III

Celil Buğra Karacan
Deepartment of Physics, Bilkent University
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I. INTRODUCTION

A vertical spring, oscillating with mass `m` was given. It was asked to plot numerical solutions for position(`y(t)`) and velocity(`v(t)`) by given analytical solutions for them.

II. METHOD

`h` is the time step. $k = 1N/m, \quad m = 1.2kg,$ $y_0 = 10m, \quad v_0 = 0m/s.$

$$a_n = \ddot{y}_n = -\frac{k}{m} y_n$$

A. Euler Method

$$v_{n+1} = v_n + ha_n + O(h^2)$$

$$y_{n+1} = r_n + hv_n + O(h^2)$$

Plots are in sync with analytical solutions, however magnitude is exponentially increasing.

B. Euler-Cromer Method

$$v_{n+1} = v_n + ha_n + O(h^2)$$

$$y_{n+1} = r_n + hv_{n+1} + O(h^2)$$

The problem of exponentially increasing magnitude is solved by using $\mathrm{v}_{n+1} instead of \mathbf{v_n} for \mathtt{y}$(*see* : *FIG.4 and FIG.5*)

C. Leapfrog Method

$$v_{n+\frac{1}{2}} = v_n + \frac{h}{2} a_n$$

$$y_{n+1} = y_n + hv_{n+\frac{1}{2}}$$

$$v_{n+1} = v_{n+\frac{1}{2}} + \frac{h}{2} a_{n+1}$$

D. Verlet Method

$$y_n + 1 = 2y_n - hy_{n-1} + h^2 a_n + O(h^4)$$

$$v_n = \frac{y_{n+1} - y_{n-1}}{2h} + O(h^2)$$

III. VERIFICATION OF THE PROGRAM

As seen in the plots, with sufficient evaluations, methods above are well fit to analytical solutions provided, exception being Euler Method's problem.

IV. ANALYSIS

Since Euler Method calculates next step from present step, explicitly, trancation error accumulates over a few steps and thus exponential magnitude increase. Though as $\frac{\text{time}}{\text{steps}}$ ratio decreses, that error is less highlighted for longer. Euler-Cromer Method is also called Semi-Implicit Euler Method, since next position is not explicitly derived from present position and velocity. Leapfrog is superior to Euler-Cromer buy improving over the idea and using midpoints for latter calculations. Verlet, however, acquires even higher accuracy through increasing initial conditions, yet keeps position calculation not velocity dependent. That way is superior (given that initial conditions' abundance) yet keeps calculations on a short leash (relatively).

V. INTERPRETATION

It seems the more number of steps, initial conditions, and the less step length; the better. Best approach seems using Verlet out of those four, using simply Euler-Cromer for lacking necessary initial conditions.

VI. DATA

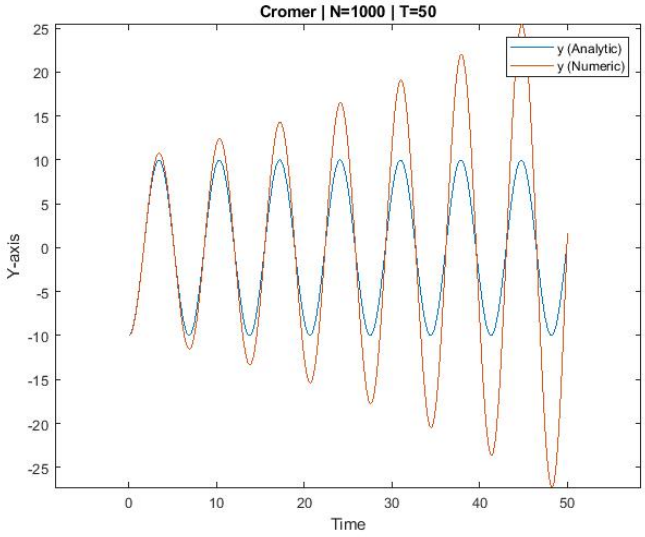

FIG. 1. Euler Method - y(t)

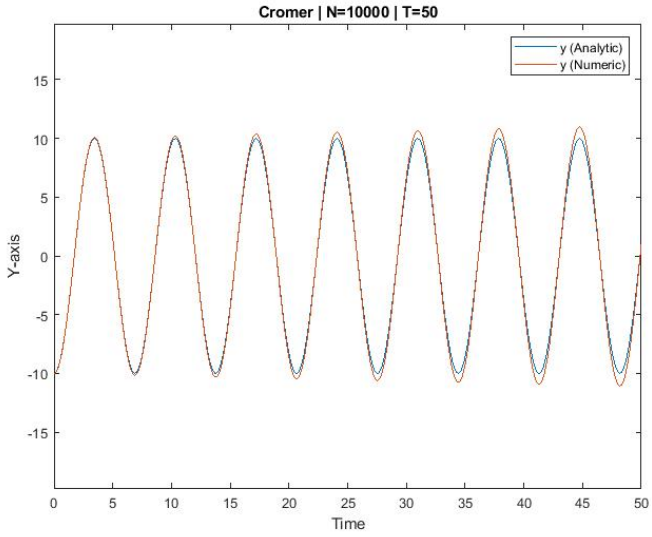

FIG. 3. Euler Method - y(t) (second)

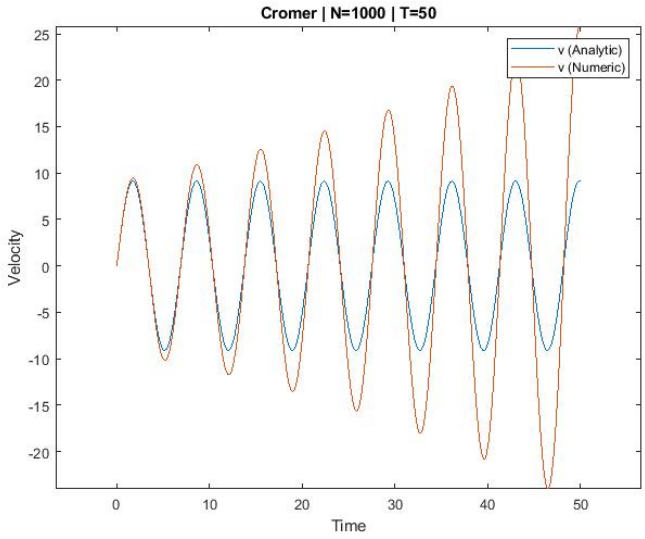

FIG. 2. Euler Method - v(t)

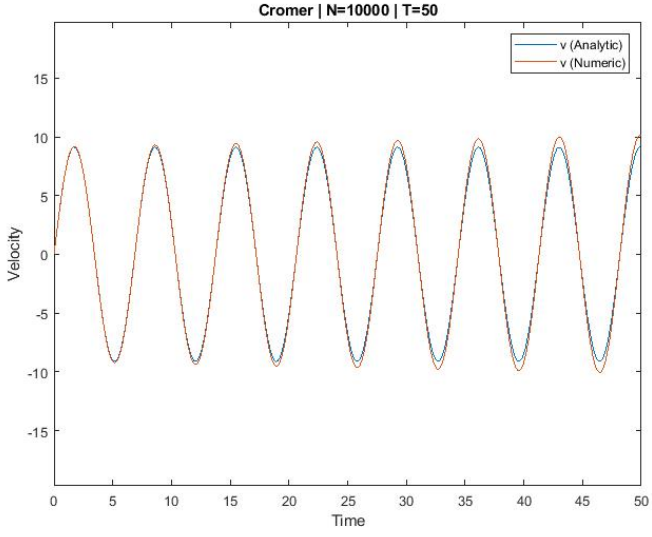

FIG. 4. Euler Method - v(t) (second)

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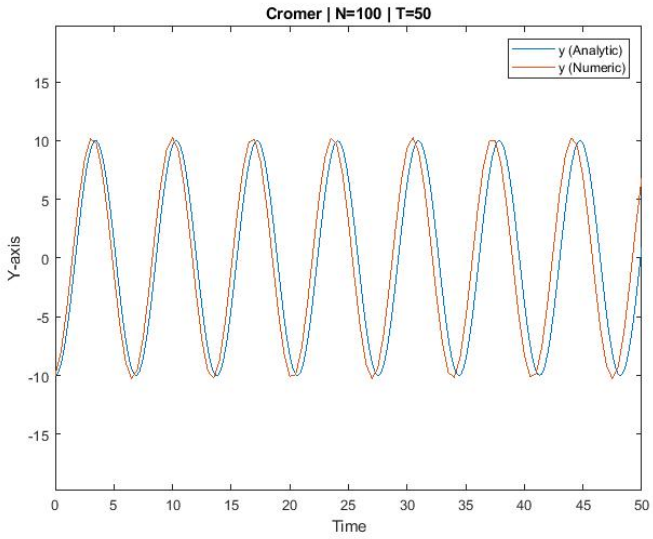

FIG. 5. Euler-Cromer Method - y(t)

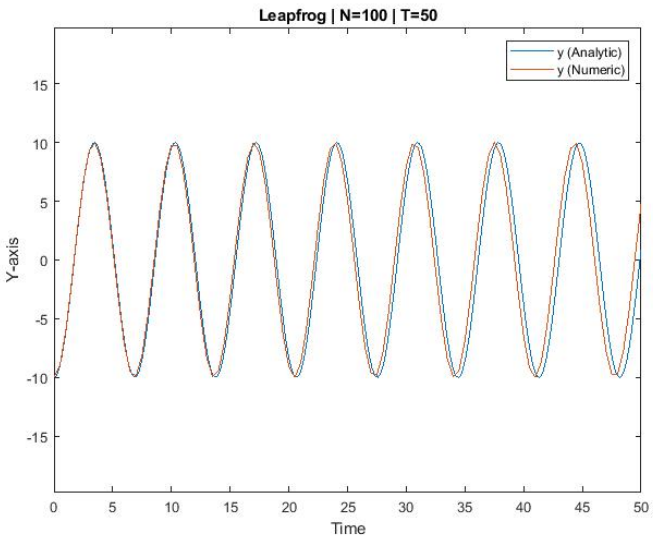

FIG. 7. Leapfrog Method - y(t)

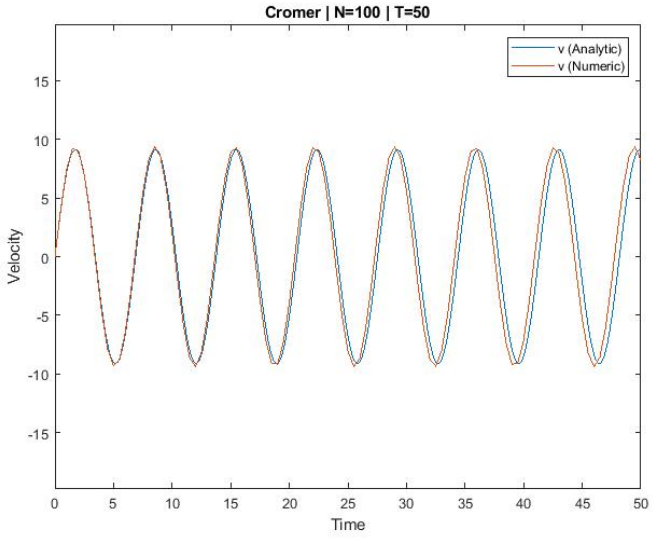

FIG. 6. Euler-Cromer Method - v(t)

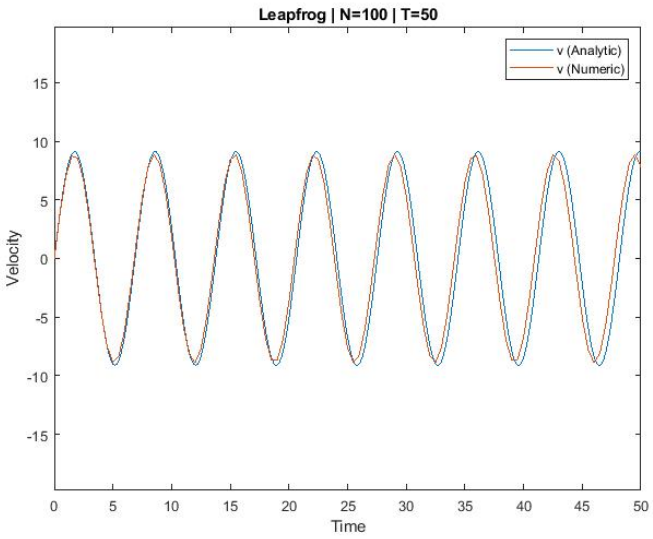

FIG. 8. Leapfrog Method - v(t)

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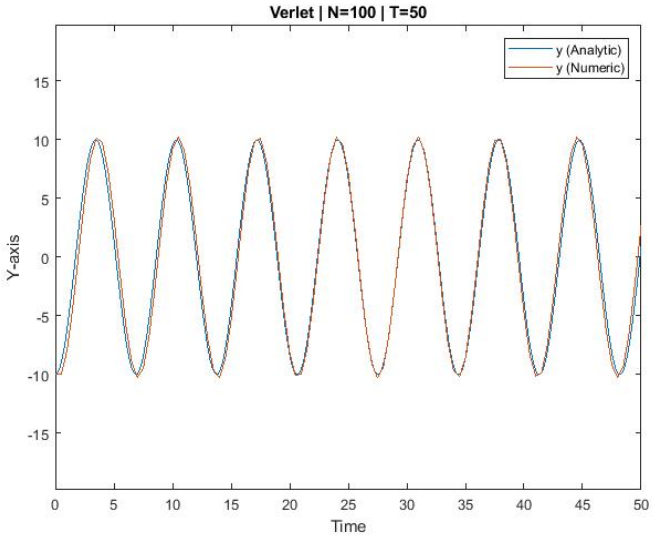

FIG. 9. Verlet Method - y(t)

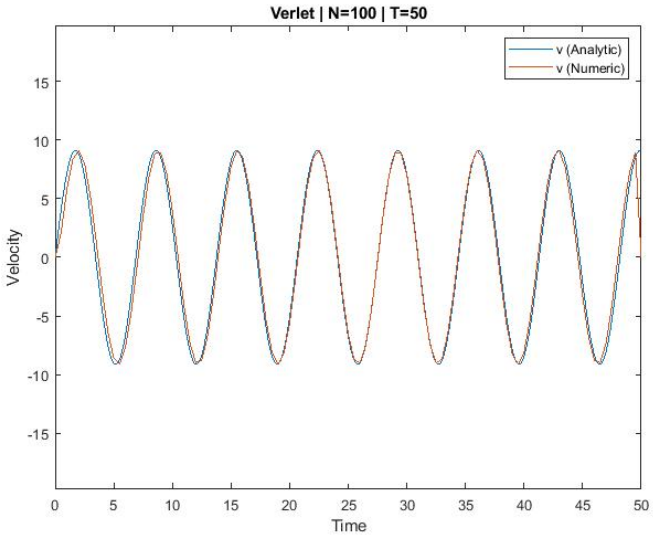

FIG. 10. Verlet Method - v(t)