PHYS-371 FINAL REPORT

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INTRODUCTION

Synchronized random walk in one dimension, is a stochastic process involving (in this case) two randomized subjects. Both moving in same direction with each iteration for the exception of one of them bouncing back from a boundary. As their next steps are not dependent on the position or past steps except for the boundaries, they might be considered as Markov chains with symmetric probabilities. As gambler's ruin can be utilized for modelling increase or decrease in a stock, synchronized random walk shall have its appliances.

METHOD

Matlab® code below have been written to model the problem on computer. For pseudo-random numbers, internal `rand()` function of Matlab® was put into use. Two variables, namely length of finite line (`m`) and number of trials per a given m (`n`) had been defined. Then spawn points `w1` and `w2` for the walkers were assigned (with a `while` statement), according to the condition that they could not be the same. Another while statement made sure, as many iterations as necessary where performed until two walkers had met. For their steps at each iteration, a random number ($b = 1, 2$), settled the orientation (left or right). To be able to plot the movements, a matrix called `Journey` records each iteration's position values for the walkers. At the mean time, an array called `Distance` records the distance thorough out iterations. Those two are plotted side by side in a figure to demonstrate that distance is decreasing whenever a walker bounces back from a boundary. Moreover, for σ calculation an array `T` (length `n`) records number of iterations found for each trial. Lengths of M = [25, 50, 75, 100, 125, 150, 175, 200, 225, 250] have been used for the calculations.

VERIFICATION

Since only way the distance between the walkers is decreased is one of them being at a boundary, it is expected that they would meet at a boundary and nowhere else. That is the case for all the graphs drawn so far. Also, the distance between walkers shall not increase due to there being no permitted way for such an event. Once more, there have been no graphs defying this rule. The author is confident the logic was set correctly on these manners.

ANALYSIS

As the length of finite line increases, mean value for iterations necessary increases as well (FIG. 1,3). The increase is exponential, and that is more evident for higher number of `n` (FIG. 5). At the mean time, the σ (standard deviation) increases exponential too (FIG. 2,4,6). The reason being, there is no correlation with the randomness and the finite line length. Note that, as `m` increases, the probability of distance between walkers getting higher increases too. Since, the distance sets number of bounce-backs from the boundaries, as the distance increases iterations necessary to end the game is expected to increase. However, that does not have to occur all the time, thatnks to increase in σ, even though the mean value for high `m`s are increased, small iteration numbers are still possible as can be seen in the (FIG. 5).

INTERPRETATION

Initially, the code was being written for a case, in which the walkers could take independent steps. Meaning, it was possible for them to move in different directions, therefore for distance to be able to increase. Even though that mistake had been noticed, the code structure was kept in such a way, that evaluating for the wrong scenario was still possible. All that was necessary is to swap code lines ending with `|*|`. These cases shall be called 'drunk' to separate from the original question. The drunk cases as can be seen in the (FIG. 7,9) have much higher iterations to finish the meeting-up game. However, their σ graphs (FIG. 8,10) seem quite the similar with synchronized steps'. They all indicate an exponential increase in σ after all, as `m` increases.

DATA

Plots obtained have been added at the end.

For mean value plots, errorbars are included, with σs there being plotted below them.

For the σ plots, take the number of x-axis as a reference to `m` within `M`, array of lengths used.

Also, last four plots are samples of `Journeys` for provided `m` values with a possible outcome of `t`.

FURTHER READING

For the mathematical aspect of the problem, Steve Lalley's "Statistics 312" course page have been come by. Yet, a direct match was far from being found, and the similar examples could not be comprehended fully. Nevertheless, his resources (PDF files) are suggested for further reading on the subject.

Lalley, Steve. The University of Chicago, Department of Statistics, Announcements and News, 2016, galton.uchicago.edu/ lalley/Courses/312/.

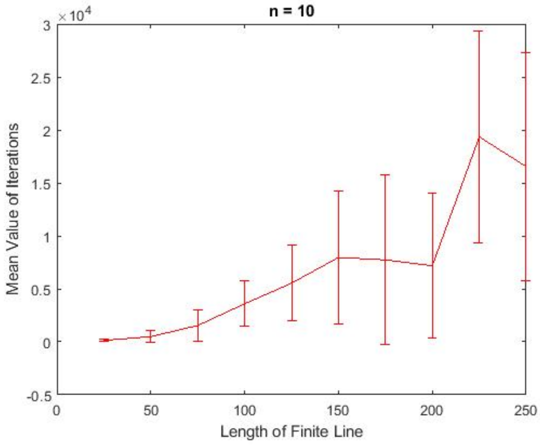

FIG. 1: synchronized steps

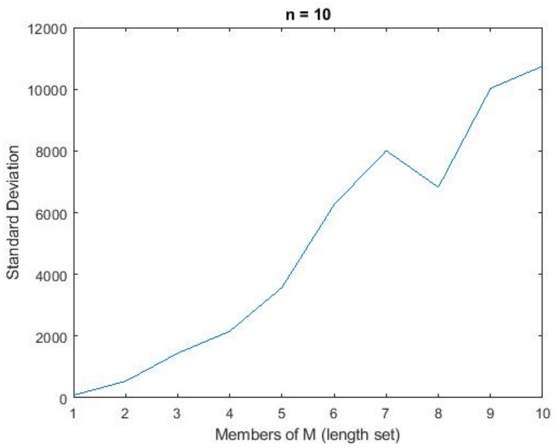

FIG. 2: synchronized steps

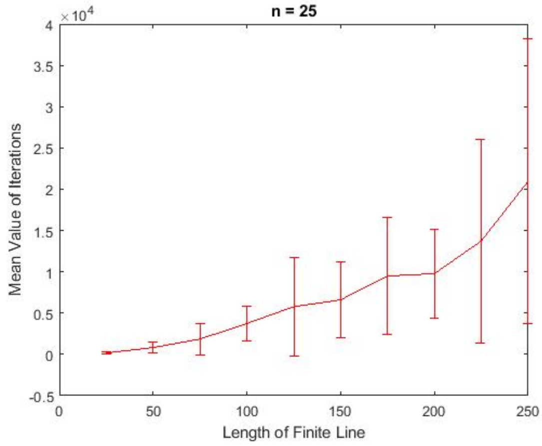

FIG. 3: synchronized steps

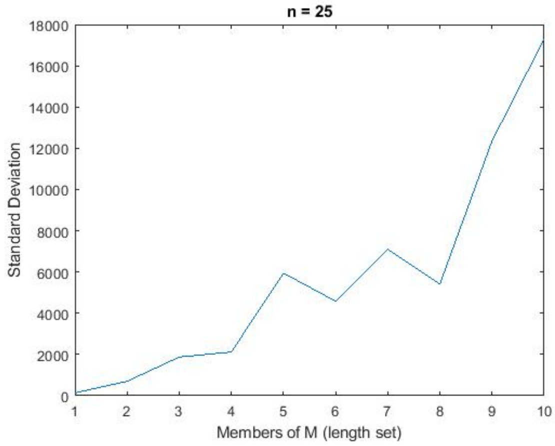

FIG. 4: synchronized steps

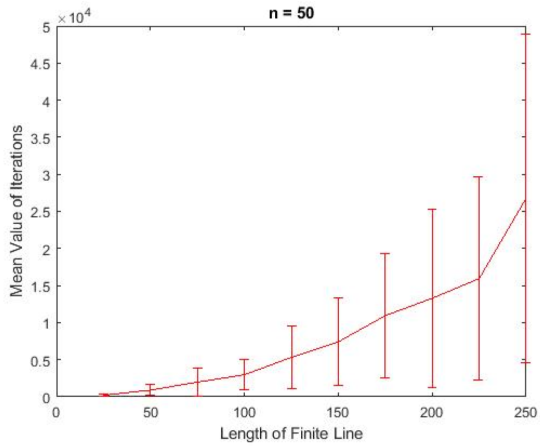

FIG. 5: synchronized steps

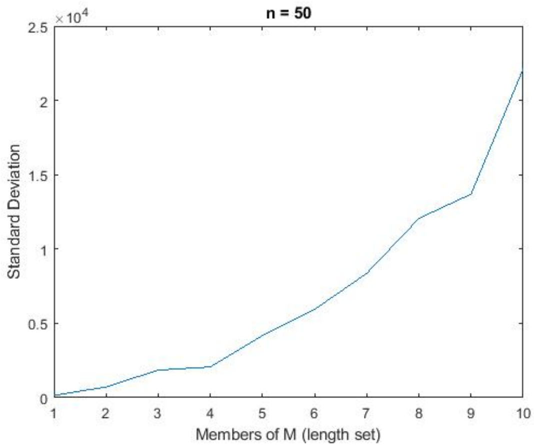

FIG. 6: synchronized steps

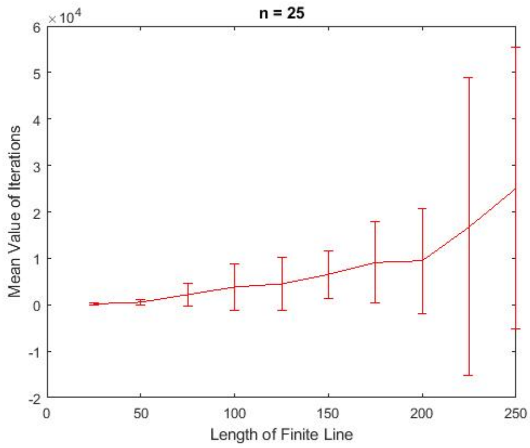

FIG. 9: independent steps (drunk)

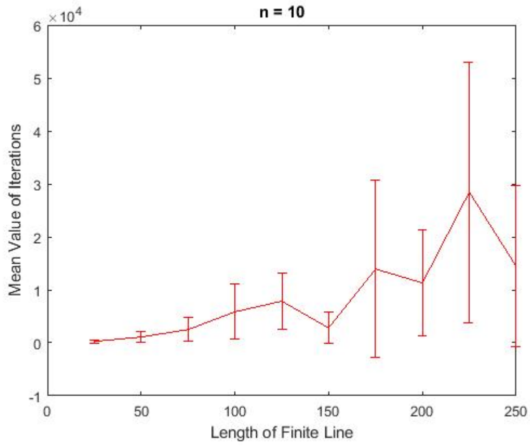

FIG. 7: independent steps (drunk)

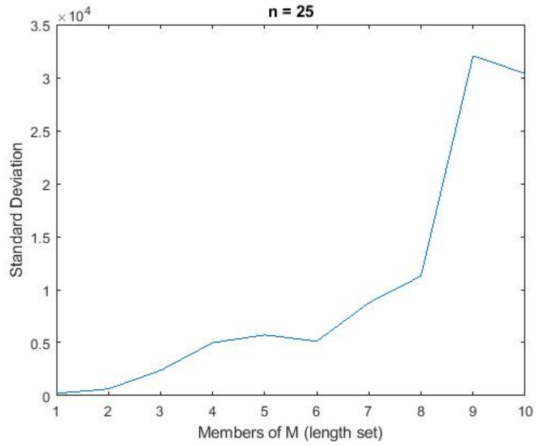

FIG. 10: independent steps (drunk)

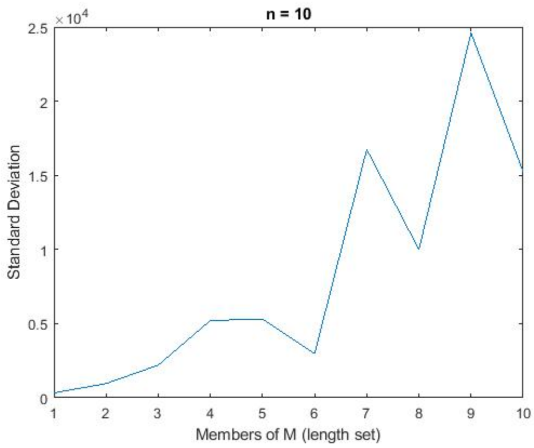

FIG. 8: independent steps (drunk)

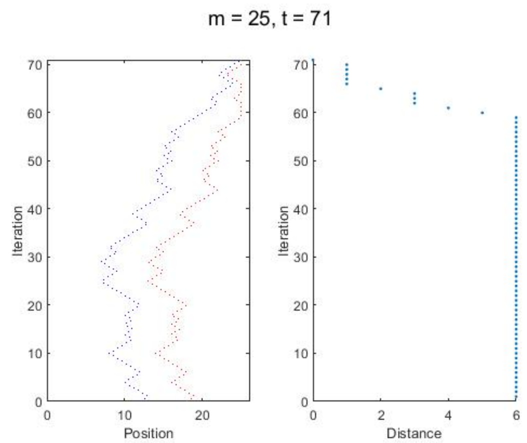

FIG. 11: synchronized steps

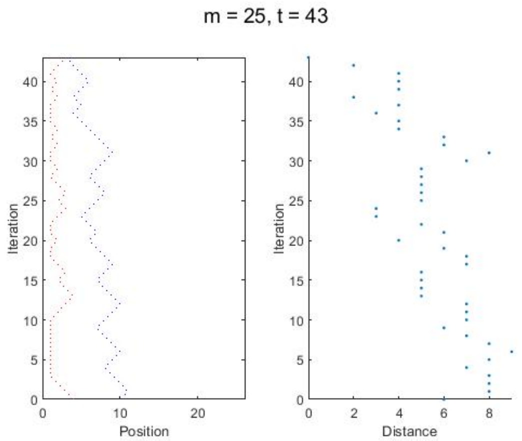

FIG. 12: independent steps (drunk)

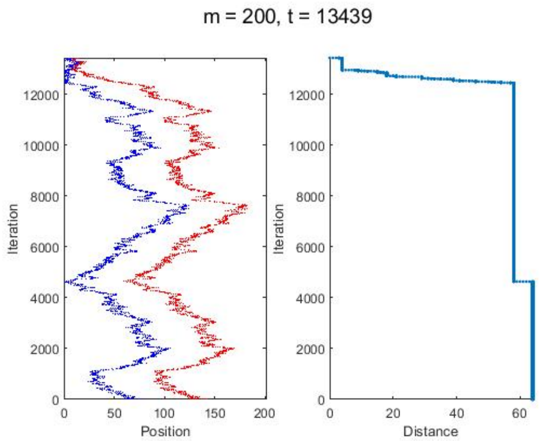

FIG. 13: synchronized steps

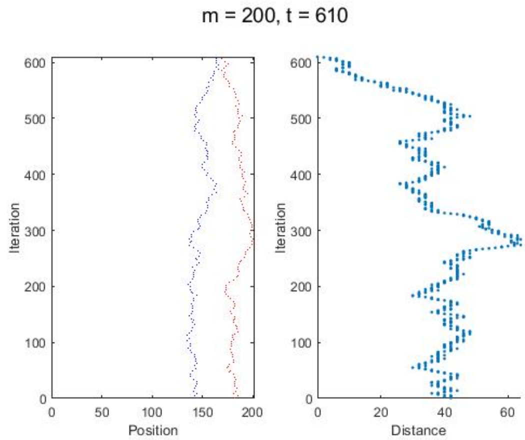

FIG. 14: independent steps (drunk)