PHYS 371 - Assignment III

Celil Buğra Karacan
Deepartment of Physics, Bilkent University
(Dated: November 12, 2018)

I. INTRODUCTION

Free fall (one dimensional) of a ball from 440m of height was to be calculated using Euler Method. Air resistance was included. To be able to use provided analytic solutions for velocity and position, positive y-axis was towards center of the Earth, and y_0 was set 0 (ground level is at $\mathtt{y} = 440$). ($c_d = 0.5$, $\rho = 1.2$, $A = \pi r^2$)

$$f_{air} = \frac{1}{2} c_d \rho A v^2$$

II. METHOD

`h` is the time step size. $\mathtt{y}_0 = 0m, \quad v_0 = 0m/s.$ ($h = 0.01$ unless specified).

$$a_n = \ddot{\mathtt{y}}_n = g - \frac{1}{m} f_{air}(\dot{\mathtt{y}}_n)$$

$$f_{air} = \frac{1}{2} \rho c_d A \dot{\mathtt{y}}_n{}^2$$

Euler Method

$$v_{n+1} = v_n + h a_n$$

$$\mathtt{y}_{n+1} = r_n + h v_n$$

For 3 desired balls (basketball, baseball, bowling ball) generic values of mass and radius were collected and are present in the MATLAB® code.

Two separate cases were implemented in the code,
Firstly, each balls trajectory and corresponding velocity while y <= 440 have been calculated and plotted separately (see: data section).
Secondly, every trajectory and velocity have been plotted in their respective figures cumulatively for comparison. However, that was only possible if extrapolation beyond $y = 440$ (ground level) was performed.
As for the code, functions: `setBall()`, `AnalyticalFun()`, `NumericalFun()` have been defined to be able to set ball specifications, and calculalate velocity and position in each step rather neatly. Those functions were results of a visial choice in code.

III. VERIFICATION OF THE PROGRAM

For $c_d = 0$, in other words without air resistance, v increases linearly (see: FIG.1, 2). That is also independent of mass or radius of the ball used as seen in the figures. However, for the cases $c_d = 0.5$ as seen in figures (FIG.5-10); although position fits well, velocities have huge errors. Problem here is that neither Euler-Cromer Method nor decreasing h tremendously did the job. It is suspected that a cause which can not be identified by the author (myself) is present and responsible for the situation.

IV. ANALYSIS

It seems from figures FIG.6, 8, 10 that, by the time baseball hits ground, it reaches its terminal velocity but not other balls. However, author is willing to remember he is not confident in the calculation of velocities, and in reality that might not be the case. Even though graphs of h other than 0.01 are not included in data section, there have been trials and position accuracy increased as suspected with decreasing h.

V. INTERPRETATION

Geerally, it would be wise to decrease step size as much as possible, in the limits of computational cost. Even then, as seen in the previous assignment (4), Euler-Cromer or if possible Verlet Methods might have better accuracy (with higher costs). In this particular case however, velocity calculation seems to not obey these reasonable suggestions.

VI. DATA

`data#` are respectively values for basketball, baseball, and bowling ball; odd ones specifying analytic and even ones specifying numeric solutions. Setting legend as desired in a `for()` loop has not been successfully implemented yet.

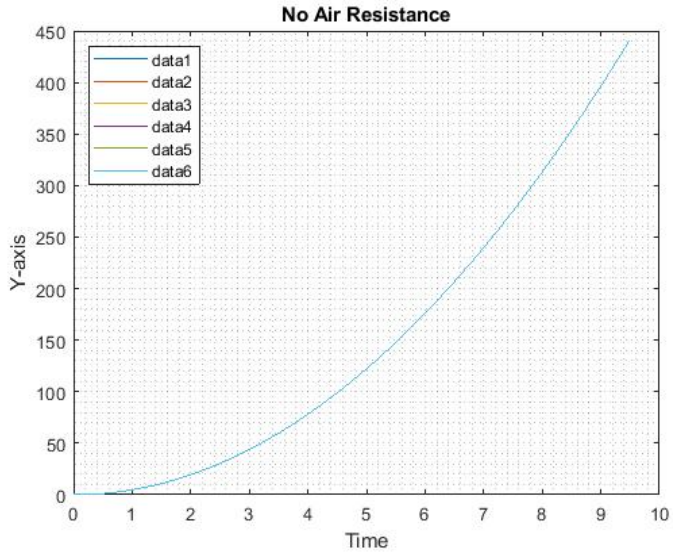

FIG. 1. $c_d = 0$

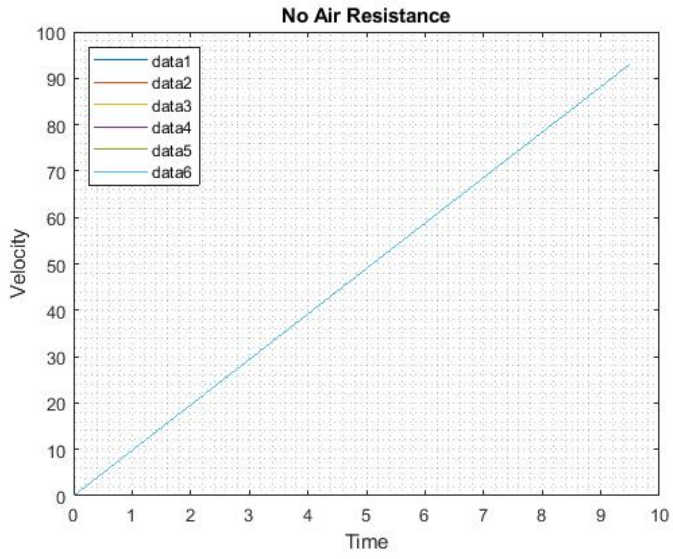

FIG. 2. $c_d = 0$

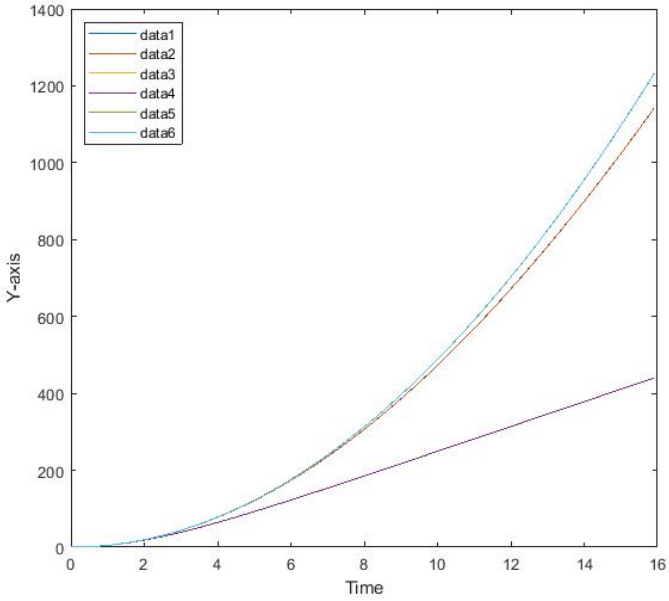

FIG. 3. Positions of balls compared

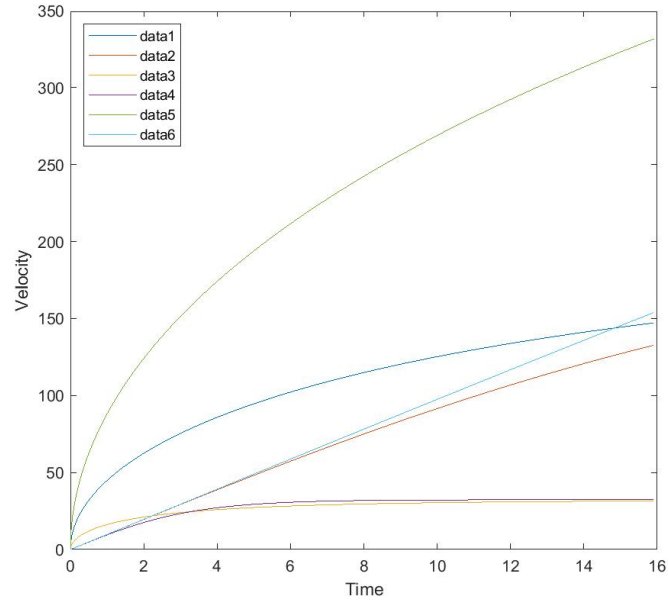

FIG. 4. Velocities of balls compared

.

.

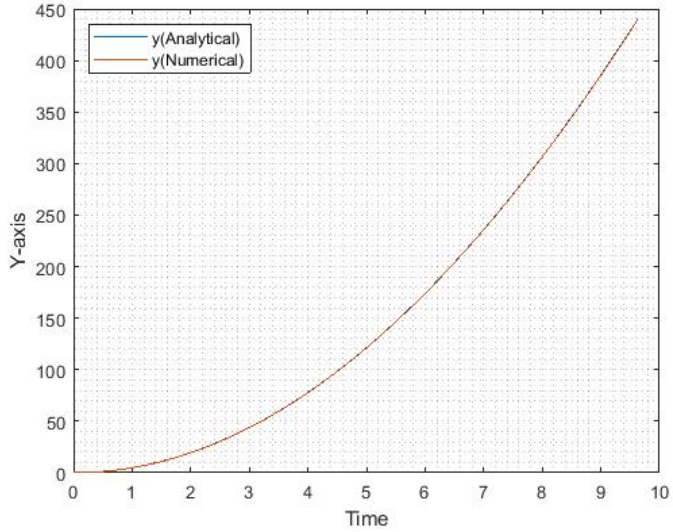

FIG. 5. Basketball - Position

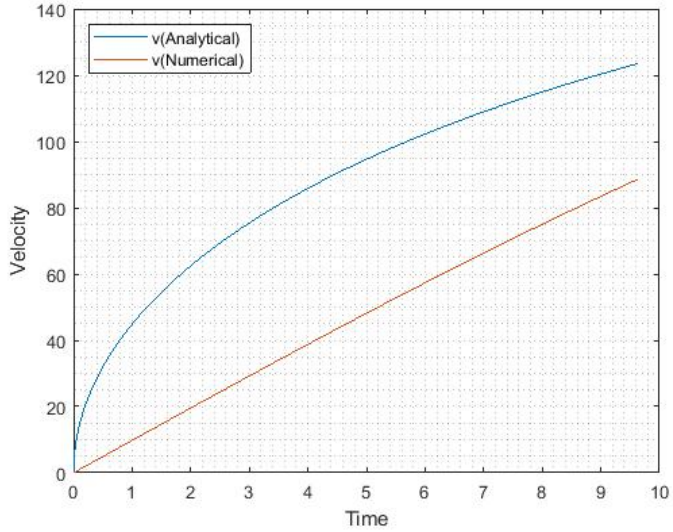

FIG. 6. Basketball - Velocity

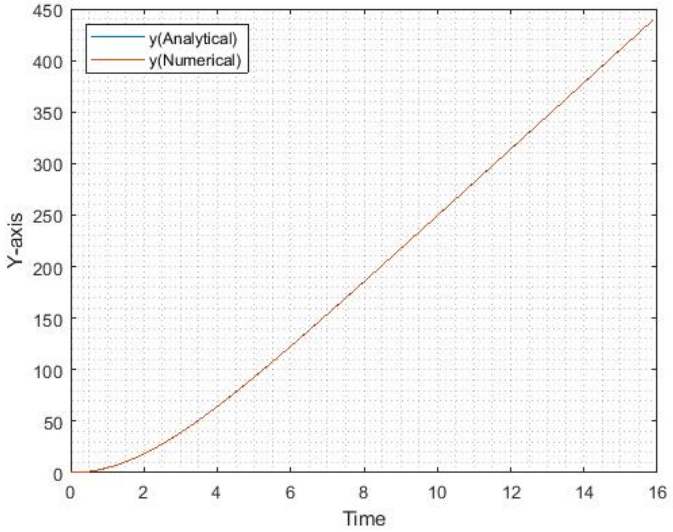

FIG. 7. Baseball - Position

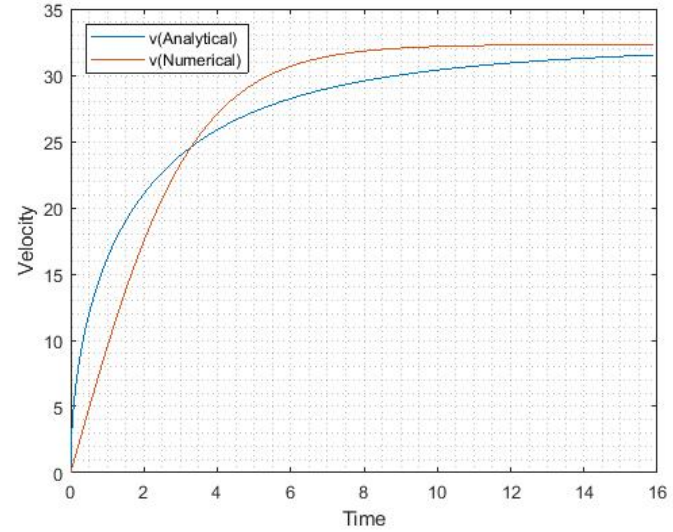

FIG. 8. Baseball - Velocity

.

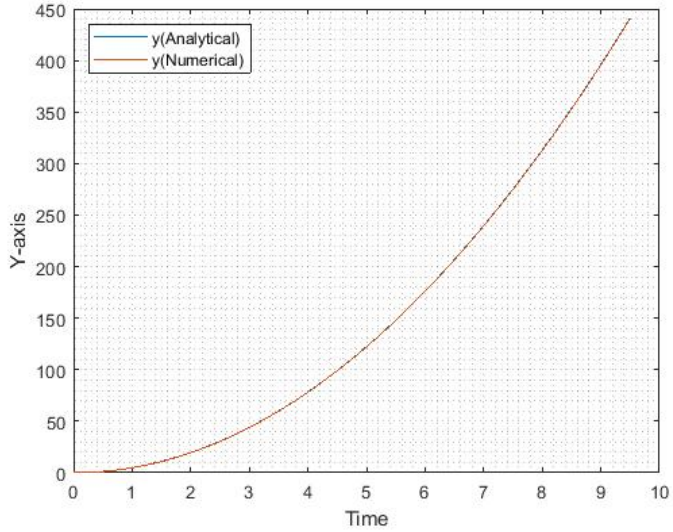

FIG. 9. Bowling Ball - Position

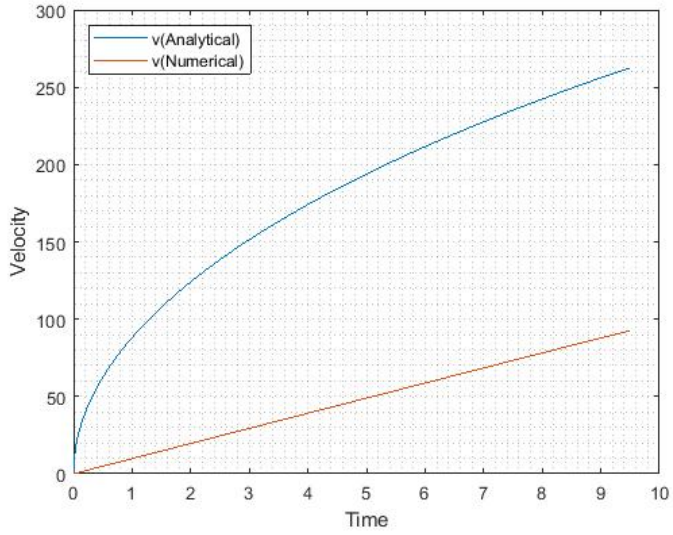

FIG. 10. Bowling Ball - Velocity