Discussion 0A Recap

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1 Sets

Here's some basic notation you should be familiar with. For this section, let $A = \{1, 2\}$ and $B = \{1, 2, 3, 4\}$.

- Cardinality, i.e. size of a set. $|A| = 2$.
- Subsets, i.e. when all members of one set belong to another. $A \subseteq B$.
- Intersection, i.e. everything in common. $A \cap B = \{1, 2\}$.
- Union, i.e. everything that appears. $A \cup B = \{1, 2, 3, 4\}$.
- Set difference, i.e. everything in one but not the other. $B \setminus A = \{3, 4\}$.
- Cartesian product, i.e. all possible pairs (or tuples) of elements:
  $A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), \dots\}$.
- Power set, i.e. the set of all subsets. $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$. Note that $|\mathcal{P}(A)| = 2^{|A|}$.

Some important sets that you'll come across this semester:

- The naturals (which includes 0): $\mathbb{N} = \{0, 1, 2, \dots\}$
- The integers: $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
- The rationals: $\mathbb{Q} = \{\frac{a}{b} | a, b \in \mathbb{Z}, b \neq 0\}$
- The reals: $\mathbb{R}$
- The complexes: $\mathbb{C}$

As we saw in discussion, if you want to show that two sets are equal, i.e. $A = B$, you need to show $A \subseteq B$ and $B \subseteq A$. This is the same as showing $x = y$ by proving $x \leq y$ and $y \leq x$.

2 Propositional Logic

We work with logical statements, or propositions. Operators operate on propositions. The common ones are

- and, $P \vee Q$: True only if both propositions are True.
- or, $P \wedge Q$: False only if both propositions are False (unlike the English or, which is commonly either or).
- not, $\neg P$: Negates the truth value, i.e. True becomes False and vice versa.
- implies, $P \implies Q \equiv \neg P \wedge Q$: False only if P is True and Q is False.

Finally, we have two quantifiers: *for all*, which is $(\forall x \in \mathbb{R})$, and *there exists*, which is $(\exists x \in \mathbb{Z})$.

3 Tips

- Use DeMorgan's Laws to move negations past $\wedge, \vee$. Negate the clauses and swap between the two.

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

- To simplify expressions, distribute operators by following the "distributive law," e.x.

$$(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R).$$

  The operator linking P and Q remains as the operator linking the two resulting clauses.

- Operators cannot be arbitrarily switched, i.e. $\forall x \exists y P(x, y) \not\equiv \exists y \forall x P(x, y)$. Take $P(x, y) = x < y$ as a counterexample.