Target: Y-M: $\int_F DA\, e^{iS_{YM}[A]} / \int D\lambda$

$\downarrow$ gauge fixing

$\int_{\cdot} DA\, e^{iS_{YM}[A] + S_{fix}[A] + S_{gh}[A,b,c]} Db\, Dc.$

$\| $

$\int_{F_0 \times F_{bc}} DA\, e^{iS + S_{gh}} Db Dc.$ — different in field easy in string

String:

String: $\int_F Dg\, DX\, e^{iS[g,X]}$

$\|$

$\int_{F_0 \times F_{bc}} Dg\, DX\, e^{iS[g,X] + S_{gh}[g,X](b,c)} Db\, Dc.$

$\Downarrow$ fixed $g \Rightarrow$ sum of "canonical"

$\sum_g Z_g = \int_{F_{d_X} \times F_{bc}} DX\, e^{iS_0[g,X] + S_{gh}(g,X,b,c)}$

$\downarrow$

we need it inv when g varies

$\hbar \to 0$; but we have Weyl anomaly $= 0$ (β func $= 0$)

$\Downarrow$

GR: $R_{ab} = 0$.

Weyl Anomaly — Computation

▷ How String Couple with Space-time?

$$S[X, g] = \frac{1}{4\pi\alpha'} \int g^{1/2} d^2\sigma\, g^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)$$

$\uparrow$ 度规 $\quad g = |g^{ab}|$ $\quad \downarrow$ 弯曲时空 $\eta_{\mu\nu} + \Sigma_{\mu\nu}$

$$\exp(S) = \exp(S_0) \cdot \Big(1 + \underbrace{\int d^2\sigma\, g^{1/2} g^{ab} \partial_a X^\mu \partial_b X^\nu \Sigma_{\mu\nu}(X)}_{\text{vertex operator of "graviton"}} + O(\Sigma^2)\Big)$$

— [illegible]

"Vierbein": Diff-Geom. Story.

M: Riemannian mfd dim n.

$\mathbb{R}^n \to TM \to M$. "bundle"

取 local orthonormal frame. (先 frame 再 Schmidt 正交化).

$SO(n) \to PM \to M$.

此时: $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$. 自变量由 $g_{\mu\nu}$ 变为 A^i_μ

"Connection": $D_\mu e^a = (W_\mu)_b{}^a e^b \leftarrow$ basis.

$D_\mu \Sigma_a = -(W_\mu)_a{}^b \Sigma_b \leftarrow$ coordinate

generator of $so(1,3)$

$\downarrow$

$(W_\mu)_b{}^a = A^i_\mu (T_i)_b{}^a$

$(T_i)_b{}^a$: $i = 1,2,3,4,5,6$; $a,b = 1,2,3,4$.

此时, couple to $S[\Psi, A^i_\mu] = \int_M \bar\Psi \not{D} \Psi$. ; $\not{D} = D_\mu \gamma^\mu = (\partial_\mu + \tilde{W}_\mu)\gamma^\mu$

$= (\partial_\mu + A^i_\mu (\tilde{T}_i)_b{}^a)\gamma^\mu.$

T_i: $so(1,3)$ — canonical Matrix Rep.

$\tilde{T}_i$: 6个 $\frac{1}{4}[\gamma^\mu, \gamma^\nu]$ — Spinor Rep.

▷ Ghost's Action — Start! Gauge-fixing.

Group: Diff × Weyl — 生成共形变换!

$$S = \int d^2\sigma\, |g|^{1/2} \Big(\frac{1}{2} g^{ij} \frac{\partial X^\mu}{\partial \sigma^i} \frac{\partial X^\nu}{\partial \sigma^j} \eta_{\mu\nu}\Big) + \underbrace{b_{ab} (\hat{P}_1 c)^{ab}}_{\text{ghost of diff}} + \underbrace{b_{ab} g^{ab} c_w}_{\delta(b^a_a) \text{ weyl-ghost}}.$$

Computation: (Polchinski P.91 & 92)

$$\langle O \rangle_g = \int [dX\, db\, dc]\, e^{-S[X,b,c,g]}\, O$$

$\downarrow \delta g$ — 能动张量, under Diff.

$$\delta \langle O \rangle_g = \underbrace{-\frac{1}{4\pi} \int d^2\sigma\, g(\sigma)^{1/2}\, \delta g_{ab}(\sigma) \langle T^{ab}\, O \rangle_g}_{\text{diff}} + \underbrace{\Big(-\frac{1}{2\pi}\Big) \int d^2\sigma\, g(\sigma)^{1/2}\, \delta w(\sigma) \langle T^a{}_a(\sigma) \ldots \rangle_g}_{\text{Weyl}}$$

classical action is Weyl-inv.

quantum: $T^a{}_a = a_1 R$.

then $\langle T^a{}_a O \rangle$ possibly non-zero

} Weyl anomaly.

$\downarrow$ 用OPE计算.

CFT & OPE:

$:O: = O - \langle O \rangle \cdot \mathrm{id}_H$.

Recall: classical: $T^{ab}(\sigma) \overset{\text{classical}}{=\!=} \frac{4\pi}{g(\sigma)^{1/2}} \frac{\delta}{\delta g_{ab}(\sigma)} S$.

$G_{\mu\nu} = \eta_{\mu\nu} + \chi_{\mu\nu}$

quantum:

$$\frac{\delta}{\delta g} \langle O \rangle_g = \frac{\delta}{\delta g} \int D\phi\, e^{S(\phi, g)} = \int D\phi\, e^{S(\phi, g)}\, T \cdots = \langle T O_g \rangle$$

Today: What's anomaly?

Recall:

Slavnov - Taylor Equivalency

$Z[J] = \int D\phi\, e^{S(\phi)+J\phi}$ $\quad \phi + \varepsilon\delta_\xi\phi \quad \xi \in \mathfrak{X}(F)$ $\downarrow \exp$

$\downarrow \phi \Rightarrow \phi'$: $\phi' = \phi'(\phi)$ 为某种变换. , $\mathrm{Jac}(\phi'(\phi)) = (A_\xi(\phi)\cdot\varepsilon + 1)$

$\int D\phi'\, e^{S(\phi')+J\phi'}$ $\qquad$ Assume $A_\xi(\phi) = \text{const}$ (与ϕ无关)

$= \int D\phi\,(1+\varepsilon A_\xi + \varepsilon J\delta_\xi\phi)\, e^{S(\phi)+J\phi}$

$= \varepsilon\langle A_\xi + J\delta_\xi\phi\rangle_J + Z[J]$

$\overset{\shortparallel}{0}$!

注: "广义 Slavnov - Taylor."

$\xi \in \mathfrak{X}(F)$ 非 Sym. 是否有.

$\langle \delta_\xi S\rangle = \langle A_\xi\rangle$

$\downarrow$ vanish on-shell, 故 VEV 相等 0.

计算: $\langle O\rangle_J = \dfrac{\int D\phi\, O\, e^{-S[J,\phi]}}{\int D\phi\, e^{-S[J,\phi]}} = \dfrac{\langle O + \varepsilon A_\xi O + \varepsilon J\delta_\xi\phi O + \frac{\delta O}{\delta\varphi}\varepsilon\delta_\xi\varphi\rangle_J}{\langle 1 + \varepsilon A_\xi + \varepsilon\int J\delta_\xi\varphi\rangle_J}$

(此处 O 是换了记号)

若 $\frac{\delta O}{\delta\varphi}\varepsilon\delta_\xi\varphi = 0$; $\varepsilon A_\xi + \varepsilon\int J\delta_\xi\varphi = ST$;

Ep: $\langle O\rangle_J\langle ST\rangle_J = \langle O\cdot ST\rangle_J$. $\rightsquigarrow$?!

Chiral - Symmetry,

Setting: $S_A(\psi,\bar\psi) = \bar\psi \not{D} \psi$. 无质量.

对称性 (Classical):

$\delta\psi = \gamma_5\psi$.

$[D\psi] \to [D\psi](1+a)$; where $a \propto F\wedge F$ (诸's work: $\int F\wedge F = 0 \rightsquigarrow$ 广义 S-T 成立)

Recall: $Z[J] = e^{iW[J]}$. $\Phi_J = \frac{\delta W[J]}{\delta J}$: "$\Phi(J)$"

then $J = J(\Phi)$; $\Gamma(\Phi) = W(J(\Phi)) - \Phi\cdot J(\Phi)$

"Effective Action"

[$A_\xi(J\phi) = \delta_\xi\Gamma(\phi)$ & $A_\xi(\omega) = 0$?] — !!?

J线数学的计算:

$S_A(\psi,\bar\psi) = -\bar\psi(\not\partial - i\not{A})\psi$

$D_A\psi D_A\bar\psi \rightsquigarrow D_A\psi D_A\bar\psi\,(1 - i\varepsilon\,\mathrm{Tr}(\delta(x-y)\gamma_5)^{x_i}_{y_i}$

$\infty \times 0$ — ?!! — $\boxed{\text{参 "Atiyah - Singer"}}$!

物理方法: (Fujikawa's method)

$\bar\psi$ & ψ 可 Fourier 展开; 视为 Fourier 系数.

记 $i\not{D}_A\phi_n = \lambda_n\phi_n$. — λ_n & ϕ_n 与 A 有关. $\quad \psi = \sum a_n\phi_n$

$D\bar\psi D\psi = \prod da_n^* \prod da_n$. where $\int \bar\phi_n\phi_n = 1$. $\qquad$ Grassmann $\quad$ Base

$\delta\phi = i\varepsilon\gamma_5\phi \sim \sum\delta a_n\phi_n = \sum i\varepsilon\gamma_5\phi_m\cdot a_m$

$X = i\varepsilon\int\bar\phi_n\gamma_5\phi_m$; $\quad \delta a_n = i\varepsilon\int d^4x\,\bar\phi_n\gamma_5\phi_m$.

即 $a_n' = a_n + \delta a_n = \sum(\delta + X)_{nm}a_m$

计算 $\int D\bar\psi D\psi\, e^{iS[\psi]} = \int\prod da_n^*\prod da_n\, e^{\sum\lambda_n a_n^* a_n}$ (a_n 变来变去)

由 Grassmann $\searrow$ $\qquad \downarrow a_n \rightsquigarrow a_n'$

$\int \det^{-1}(\delta + X)\prod da_n^*\prod da_n\, e^{\sum\lambda_n a_n^* a_n}$

> $\det^{-1}(\delta + X)$ 之计算:

对于 ε 展开至一阶, 做正规化

$\sim \exp(-i\varepsilon\sum_n\int\bar\psi_n\gamma_5 e^{-(i\not{D}/\Lambda)^2}\psi_n)$ 并 $\Lambda \to \infty$ (相当于截断 Λ)

recall: $\int\bar\psi_n O\psi_n\, d^4x$ (用狄拉克 记法表示)

$\sim \sum_n\int\langle\phi_n|x\rangle\langle x|O_{ij}|\psi_{n_j}\rangle$

$\sim \sum_n\int\langle x|O_{ij}|\phi_{n_j}\rangle\langle\phi_n|x\rangle$

$\sim \sum_n\int\langle x|\mathrm{tr}\,O|x\rangle$: then: 原式 $= \int\frac{d^4k}{(2\pi)^4}\mathrm{tr}(\gamma_5 e^{-ikx} e^{\not{D}^2/\Lambda}(e^{ikx}))$

$= \int\frac{d^4k}{(2\pi)^4}\mathrm{tr}(\gamma_5 e^{ikx} e^{(D+ik)^2/\Lambda^2 - \frac{1}{2}e\not{F}/\Lambda^2})$

BCH: up to 1阶: $e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]} e^{\cdots}$

$\Downarrow$

$e^{(D+ik)^2/\Lambda}\, e^{-\frac{1}{2}\not{F}/\Lambda^2} = e^{\cdots + \cdots}$

$\downarrow$

$\sim I \quad \Longrightarrow \quad$ 且 higher term vanish!

$e^{-\frac{1}{2}\not{F}/\Lambda^2} = 1 - \frac{1}{2}\not{F}/\Lambda^2 + \underbrace{\cdots}_{F F \& \text{ higher term}}$ $\Rightarrow$ 仅有 $\not{F}$ 项存留.

我认为的 Weyl Anomaly:

Weyl anomaly in all textbooks're wrong!

$S(g,\phi) = \cdots$, $\quad e^{W(j)} = Z[g] = \int[D_g\phi]\,e^{S(g,\phi)} \rightsquigarrow$ part. function.

$g^{\mu\nu}\frac{1}{\sqrt{g}}\frac{\delta Z[g]}{\delta g^{\mu\nu}} = \int[D\phi]\,g^{\mu\nu}\frac{1}{\sqrt{g}}\frac{\delta S}{\delta g^{\mu\nu}}e^{S(g,\phi)} + \int g^{\mu\nu}\frac{1}{\sqrt{g}}\frac{\delta D\phi}{\delta g^{\mu\nu}}e^{S(g,\phi)}$

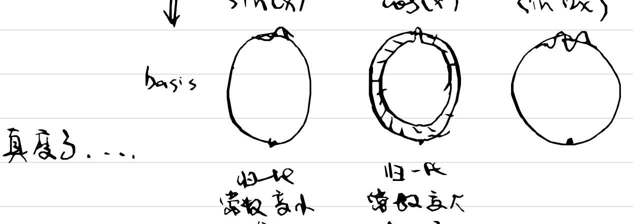
$\overset{\shortparallel}{T^\mu_\mu} \sim$ off-shell 0 $\qquad \downarrow$?

(这应该是0吧? 不然太离谱了..)

故: 对线性表示给出测度区别疑问.

$D_g\phi$ 与 $D_{g'}\phi$ 应仅差常数... 变了什么?

我猜如下形状是一个例子:



还真变了....

Finite Equivariant Cohomology. $\mathfrak{g} \to TM$

$G \curvearrowright M$: C^∞ mfd (not necessarily cpt). $\mathfrak{g}$. $T_a \mapsto V_a = V_a^\mu \partial_\mu$, $[V_a, V_b] = f_{ab}^c V_c$

Weyl model (A) (Cartan model (C) special) BRST model (B).

$A \simeq B$, $A_{basic} \simeq C$

(A). Ring structure $\simeq W(\mathfrak{g}) \otimes \Omega^*(M)$

$\Lambda^\cdot(\mathfrak{g}^*) \otimes S^\cdot(\mathfrak{g}^*)$, w^a (+1) ϕ^a (+2)

d: $dw^a = \phi^a - \frac{1}{2} f^a_{bc} w^b w^c$

$d\phi^a = -f^a_{bc} w^b \phi^c$ d on Ω is natural.

d graded Leibniz: (grade: $W(\mathfrak{g})$的grading + $\Omega^*(M)$ grading)

$\mathcal{L}_a$: $\mathcal{L}_a w^b = -f^b_{ac} w^c$

$\mathcal{L}_a \phi^b = -f^b_{ac} \phi^c$ ($\mathcal{L}_a$ on $\Omega^*(M)$ 由右不变向量场)

ι_a: $\iota_a w^b = \delta_a^b$, $\iota_a \phi^b = 0$

$\iota_a dx^\mu = V_a^\mu$, $\iota_a x^\mu = 0$

(B) Ring structure $\simeq W(\mathfrak{g}) \otimes \Omega^*(M)$

$\Lambda^\cdot(\mathfrak{g}^*) \otimes S^\cdot(\mathfrak{g}^*)$ (但基 W basis 记为 w'^a 和 ϕ'^a)

$\delta = d - w^a \mathcal{L}_a + \phi^a \iota_a \big|_\Omega$ (ι_b 穿过 $W(\mathfrak{g})$ 时会变号)

$\mathcal{L}_a$, ι_a 只作用在 Ω 之上, 是我们知道的作用

$\mathbb{L}_a$: $\mathbb{L}_a w^b = -f^b_{ac} w^c$, $\mathbb{L}_a \phi^b = -f^b_{ac} \phi^c$, $\mathbb{L}_a x^\mu = V_a^\mu$. d与$\mathbb{L}$交换.

I_a: $I_a w^b = \delta_a^b$, $I_a \phi^b = 0$, $I_a dx^\mu = 0$, $I_a x^\mu = 0$.

Thm (同构)

A & B 之间有同构, 且 $(d, \mathcal{L}_a, \iota_a) \longleftrightarrow (\delta, \mathbb{L}_a, I_a)$ 作用对应

$A_{basic} = \{\alpha \in A \mid \iota_a \alpha = 0, \mathcal{L}_a \alpha = 0\}$

$$\begin{array}{ccc} B & \xrightarrow[\simeq]{\psi} & A \\ \cup & & \cup \\ C & \xrightarrow[\simeq]{\psi} & A_{basic} \end{array}$$

$\psi_B = \exp(-w'^a \iota_a)$ 作用于 B $\psi_B: B \to B$

$\varepsilon: B \to A$ 改写算子. $w'^a \mapsto w^a$, $\phi'^a \mapsto \phi^a$

则 $\psi = \varepsilon \circ \psi_B$

ψ的逆: $\psi^{-1} = \psi_B^{-1} \circ \varepsilon^{-1}$; $\psi_B^{-1} = \exp(w^a \iota_a)$

Cartan Model:

C: $\{\beta \in B \mid I_a \beta = 0;\ \mathbb{L}_a \beta = 0\}$

$\Downarrow$ β没有w部分. $\Downarrow$ 群作用下不变.

C ring structure: $(S(\mathfrak{g}^*) \otimes \Omega(M))^{(G, \mathbb{L}_a)}$

如何实现 w^a 与 ϕ^a 为联络及曲率?

(分类空间之上)

希望把 BRST 的 Thom class 变为真正的 class.

希望: $A \longrightarrow \Omega^*(M)$ 由 $w^a \mapsto$ 1-form, $\phi^a \mapsto$ 2-form 构造.

$$\begin{array}{ccc} A & \longrightarrow & \Omega^*(M) \\ \uparrow & \nearrow & \\ \Omega^*(M) & & \end{array}$$

确定一个 G-bundle with connection 之后, 是否可以

让
$$\begin{array}{ccc} A & \longrightarrow & \Omega(M) \\ \downarrow{\scriptstyle \iota_a/d/\mathcal{L}_a} & & \downarrow{\scriptstyle \iota_a/d/\mathcal{L}_a} \\ A & \longrightarrow & \Omega(M) \end{array}$$

答案: 在 Free 作用下可以.

△ Thom class. $\mathbb{R}^d \to E \to M$. $E_0 = E - 0$-section. 定义 $H^d(E, E_0)$ s.t. 逐点 $H^d(\mathbb{R}^d, \mathbb{R}^{d*})$

而 $H^d(\mathbb{R}^d, \mathbb{R}^{d*}) \simeq$ homology of d-forms vanish outside ball over 0.

$\simeq H^d_{dR}(\mathbb{R}^d, \mathbb{R}^d - B_0)$.

拓扑上: $G \curvearrowright \mathbb{R}^d \longrightarrow V \longrightarrow M$

z^i locally.

$dz^i = \psi^i$, $d\psi^i = 0$.

$d\bar\psi_i = B_i$, $dB_i = 0$.

BRST model.

$\delta z^i = \psi^i + w^a M_{a\,j}^{\ i} z^j$

$\delta \psi^i = -\phi^a M_{a\,j}^{\ i} z^j + w^a M_{a\,j}^{\ i} \psi^j$

$\delta \bar\psi_i = B_i - w^a M_{a\,i}^{\ j} \bar\psi_j$

$\delta B_i = \phi^a M_{a\,i}^{\ j} \bar\psi_j - w^a M_{a\,i}^{\ j} B_j$

Thom class 用 $\int \exp \delta(i z^k \bar\psi_k - t B_i \delta^{ik} \psi_k) dB\, d\bar\psi$ 计算 (防止引入 d-form)

($d = 0$. $\int_{\text{fibre}} \omega = 1$.)

而 $\delta(i z^k \bar\psi_k - t B_i \bar\psi_k \delta^{ik}) = -t(B_i - \frac{i}{2t} \delta_{ij} z^j)^2 - \frac{1}{4t} z^k z^l \delta_{kl} + i\psi^k \bar\psi_k - t\phi^a M_{a\,i}^{\ k} + \delta^{ij} \bar\psi_k \bar\psi_j$

以上 BRST

以下 Weyl

$\downarrow$ 积 dB, 再用 Weyl model

$\int \exp(-\frac{1}{4t} z^k z^l \delta_{kl} + i(\psi^k - w^a M_{a\,j}^{\ k} z^j)\bar\psi_k - t\phi^a M_{a\,i}^{\ l} \bar\psi_l \bar\psi_m \delta^{im}) d\bar\psi$.

而 $\phi^a M_{a\,i}^{\ j} = R_{\mu\nu i}^{\ \ \ j} dx^\mu dx^\nu \cdot \frac{1}{2}$

$w^a M_{a\,i}^{\ j} = -\Gamma_{\mu i}^{\ \ j} dx^\mu$.

现在, 考虑 $\mathbb{R}^d_{s^i} \to V \to M_{x^\mu}$, 有表 (G没有了, w&φ用 Γ&R代入)

$\delta s^i = \Psi^i - \Gamma_{\alpha j}^{\ \ i} dx^\alpha s^j$

$\delta x^\alpha = dx^\alpha$ $\delta \Psi^i = \frac{1}{2} R_{\alpha\beta\ \gamma}^{\ \ \ i} dx^\alpha dx^\beta s^\gamma - \Gamma_{\alpha j}^{\ \ i} dx^\alpha \Psi^j$

$\delta \bar\Psi_i = B_i + \Gamma_{\alpha i}^{\ \ j} dx^\alpha \bar\Psi_j$

$\delta B_i = -\frac{1}{2} R_{\alpha\beta i}^{\ \ \ \ j} dx^\alpha dx^\beta \bar\Psi_\nu + \Gamma_{\mu i}^{\ \ \nu} B_\nu dx^\mu$.

将相应 Equivariant form 变化为真的 form.

从而计算 $\int \exp(\delta(s^i \bar\Psi_i - t B_i \bar\Psi_j g^{ij}))$

为何用 δ?

$dg^{ij} = \partial_\mu g^{ij} dx^\mu = 2\Gamma_\mu^{\ ik} g^{kj} dx^\mu \ldots$

可被 δ 消掉. $\delta g^{ij} = \partial_\mu g^{ij} \delta x^\mu = \partial_\mu g^{ij} dx^\mu$.

$\Rightarrow = i(\Psi^i - \Gamma_{\alpha j}^{\ \ i} dx^\alpha s^j)\bar\Psi_j + i s^i (B_i + \Gamma_{\alpha i}^{\ \ j} dx^\alpha \bar\Psi_j) - t(-\frac{1}{2} R_{\alpha\beta i}^{\ \ \ \ j} dx^\alpha dx^\beta \bar\Psi_j + \Gamma_{\alpha i}^{\ \ k} B_k dx^\alpha) \times \bar\Psi_j g^{ij}$
$- tB_i(B_j + \underbrace{\Gamma_{\alpha j}^{\ \ k} dx^\alpha \bar\Psi_k}) g^{ij} + \underbrace{tB_i \bar\Psi_j \partial_\alpha g^{ij} dx^\alpha}$

~ 三项消掉, 得到

$= -tB_i B_j g^{ij} + i s^i B_i + i \Psi^i \bar\Psi_i + \frac{1}{2} t R_{\alpha\beta i}^{\ \ \ \ j} dx^\alpha dx^\beta \bar\Psi_j \bar\Psi_k g^{ik}$

$= -t(B_i - \frac{i}{2t} s^j g_{ij})^2 - \frac{1}{4t} s^i s^j g_{ij} + i\Psi^i \bar\Psi_i + \frac{1}{2} t R_{\alpha\beta i}^{\ \ \ \ j} dx^\alpha dx^\beta \bar\Psi_j \bar\Psi_k g^{ik}$

$\downarrow$ 应用 $\exp(-w_a \iota_a) = \exp(\Gamma_{\alpha i}^{\ \ j} dx^\alpha s^i \iota_{\frac{\partial}{\partial s^j}})$

再积掉B

RMK: $\Psi^i = ds^i$

$\sim \int \exp(-\frac{1}{4t} s^i s^j g_{ij} + i(\Psi^i + \Gamma_{\alpha j}^{\ \ i} dx^\alpha s^j)\bar\Psi_i + \frac{1}{2} t R_{\alpha\beta i}^{\ \ \ \ j} dx^\alpha dx^\beta \bar\Psi_j \bar\Psi_k g^{ik}) d\bar\Psi$

$t \to \infty$ $\downarrow$ 拉回;

代入的拉回. $\therefore s = 0$.

$\int \exp(R_{\alpha\beta i}^{\ \ \ \ j} dx^\alpha dx^\beta \bar\Psi_j \bar\Psi_k g^{ik}) d\bar\Psi$

$t \to 0$ $\downarrow$

$\int \exp(-\frac{1}{4t} s^i s^j g_{ij}) \left[\bigwedge_i^d (\Psi^k + \Gamma_{\alpha_k j_k}^{\ \ \ k} dx^{\alpha_k} s^{j_k}) + O(t)\right]$

而 $s^i = V_\alpha^i x^\alpha + O(x^2)$, $\Psi^i = V_\alpha^i dx^\alpha$ } 拉回到零点附近, 由于 $\exp(-\frac{1}{4t} s^i s^j g_{ij})$ 的存在.

从而 · 所值为: $\underbrace{\exp(-\frac{1}{4t} V_\alpha^i V_\beta^j x^\alpha x^\beta g_{ij})} \bigwedge_{k=1}^d \left[(V_{\alpha_k}^k + \Gamma_{\alpha_k j_k}^{\ \ \ k} x^{\beta_k} V_{\beta_k}^{j_k}) dx^{\alpha_k}\right]$

相当于 $\mathrm{Sign}(V)$.

Covariant Phase space [以标量场为例]

物理公理层次:
- Lagrangian ☆ (参考 Ntab: phase space; <V22>)
- ↓ Lagrangian 力学 / Hamiltonian 力学 (参考 QoGS) H-BRST
- 作用量 (Mrieb) — FP / L-BRST / BV
- Functional (Mrieb)

双复形: $(M\times \mathcal{F})$ variational bicomplex
d δ

$$\mathcal{L}\in\Omega^{n,0}(M\times\mathcal{F}),\quad P_\phi=(-1)^\mu\frac{\delta L/d\phi}{\delta\partial_\mu\phi}dx^1\cdots\widehat{dx^\mu}\cdots dx^n\in\Omega^{n-1,0}(M\times\mathcal{F})$$

$\Theta=-P_\phi\wedge\delta\phi\in\Omega^{n-1,1}(M\times\mathcal{F})$ 辛势密度.

$\omega=\delta\Theta$ 辛密度 $\in\Omega^{n-1,2}$.

$\boxed{\text{注: 若场是 } d \text{ 形式场, } P \text{ 应为 } n-1-d \text{ 形式. (似乎从右求导}}$

$\therefore\ \alpha_\Sigma=\int_\Sigma\Theta$ 辛势　$\omega_\Sigma=\int_\Sigma\delta\Theta$ 辛结构. 给出解流形 Σ^{n-1} 上的 1-形式 / 2-形式

$$\mathcal{L}\xrightarrow{\int dM}S.\qquad \delta\mathcal{L}\to\delta S=\int_M\left[\left(\frac{\partial\mathcal{L}}{\partial\phi}-\partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\right)\delta\phi+\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\delta\phi\right)\right]$$

$\therefore\ \delta\mathcal{L}=(E\mathcal{L}\wedge\delta\phi + d\Theta)\quad\in\Omega^{n,1}$.

D 对称性: $\xi\in\mathfrak{X}(\mathcal{F})$ s.t. $\mathcal{L}_\xi(\mathcal{L})=dK_\xi$. $K_\xi\in\Omega^{n-1,0}$.

- Nöther 流

$$J_\xi\in i_\xi\Theta+K_\xi\in\Omega^{n-1,0}\ \longrightarrow\ dJ_\xi\overset{\text{on-shell}}{=\!=}0\in\Omega^{n,0}.$$

RMK: $J_\mu=\frac{\delta L}{\delta\partial_\mu\phi}\delta_\xi\phi+K_\mu\in\Omega^{1,0}$. $K_\mu=\star K_\xi$.

$\partial^\mu J_\mu=0$ on shell

$\left(=-\frac{\delta L}{\delta\phi}\delta_\xi\phi+\partial_\mu K^\mu=-\delta_\xi L+\delta_\xi L=0.\right)$

Pf. $dJ_\xi=d i_\xi\Theta+dK_\xi=i_\xi(d\Theta)+\mathcal{L}_\xi(\mathcal{L})=i_\xi(d\Theta)+i_\xi\delta\mathcal{L}+\delta(\cancel{i_\xi\mathcal{L}})$

$=0$ Onshell.

- Nöther 荷. $Q_\xi^\Sigma:=\int_\Sigma J_\xi\in\Omega^0(\mathcal{F})$.

Prop (1): $\omega_\Sigma(\xi,\cdot)=-\delta Q_\xi^\Sigma$ (δQ_ξ^Σ 为 J_ξ 的 Hamiltonian)

(2). $\omega_\Sigma|_{ELM}$ 可逆

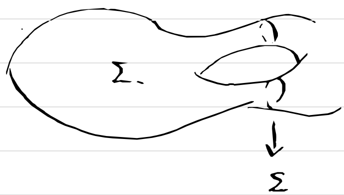
则 $\{Q_\xi^\Sigma,f\}=\xi(f)$, $\forall f\in C^\infty(ELM)$.

Pf: LHS $=i_\xi\omega_\Sigma=\int_\Sigma i_\xi\delta\Theta$

RHS $=\int_\Sigma\delta J_\xi=\int_\Sigma\delta i_\xi\Theta+\delta K_\xi=\int_\Sigma-\mathcal{L}_\xi\Theta+i_\xi\delta\Theta+\delta K_\xi$.

仅需 $\int_\Sigma-\mathcal{L}_\xi\Theta+\delta K_\xi=0$.　$\int_\Sigma-d\mathcal{L}_\xi\Theta+d\delta K_\xi=0$

即 $\mathcal{L}_\xi\left(\int_\Sigma-d\Theta+\delta\mathcal{L}\right)=0$.



$\mathcal{L}_\xi\int_\Sigma(\delta\mathcal{L}-d\Theta)=0$ 即 $\sim\langle E\mathcal{L}\rangle$.

(用到 $\mathcal{L}_\xi$ 与 δ 交换)

Example (Minkowski Maxwell) - Noether 荷悖论

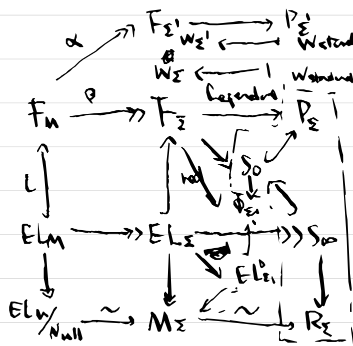
$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ 仅在 λ local 时消失.

$P_{A_\mu} = \frac{\delta L}{\delta \partial_0 A_\mu} = -F^{0\mu}$ $(\mu \neq 0)$.

$\delta_\lambda A_\mu = \partial_\mu \lambda$. $Q_\lambda = -\int_\Sigma F^{0\mu}\partial_\mu\lambda = -\int_\Sigma \partial_\mu(\lambda F^{0\mu}) + \int_\Sigma \lambda \partial_\mu F^{0\mu}$

λ local. $= \int_\Sigma \lambda \partial_\mu F^{\mu 0} \overset{\text{on-shell}}{=} 0$.

可 $\{Q_\lambda, A_\mu\}$ 不应为 $\partial_\mu\lambda$ 吗? 因为 $w_\Sigma|_{EL_M}$ 退化. $\rightsquigarrow \{,\}$ 无定义.



$\alpha^*(w_{\Sigma'}) \neq \beta^*(w_\Sigma)$

但 EL_M 上是一样的!

$\iota^*\alpha^*(w_{\Sigma'}) = \iota^*\beta^*(w_\Sigma)$

$EL^0_{\Sigma_1}$ 为 Φ_{Σ_1} 的拉回.

Example (Chern-Simons) 此处无 Hamiltonian

$L = AdA + \frac{2}{3}A\wedge A\wedge A$.

$P_A = A$ [此处 A 为 1-形式场, 动量因而为 1-form]

$W_\Sigma = \int_\Sigma \delta A \wedge \delta A$ (为拉回至 F_M 的形式)

$F_\Sigma = \{A|_\Sigma, \dot{A}|_\Sigma\}$

$\downarrow$ reduction, 由于 $\ker w$ 为 Frobenius 分布.

$\Phi_\Sigma = \{A|_\Sigma\} = \mathrm{Conn}(\Sigma)$ 为商去 $\ker w$ 作为分布的相空间.

$$\left(\begin{array}{l} dw = 0 \Rightarrow \overbrace{w([X,Y],Z) + w(X,[Z,X]) + \cdots}^{0} \\ \qquad \underbrace{\cdots + Xw(Y,Z) + \cdots}_{0} = 0 \\ \therefore [X,Y] \in \ker w. \end{array} \right)$$

$EL^0_{\Sigma_1} \subset \Phi_\Sigma$. 为分布流形等价类中与 EL_Σ 交者, 即

Flat conn Σ.

而 $M_\Sigma = {}^{\text{Flat conn}\,\Sigma}/_{\text{Gauge}}$ 为其相空间.

RMK: 此处体系无用 Hamiltonian 描述 Poisson bracket.

只用 Symplectic structure 给 Poisson bracket

▷ 证明: $\iota^*(\omega_\Sigma)$ 与 Σ 无关.

$$\begin{tikzcd} & & F_{\Sigma_2} \\ EL_M \arrow[r, "L"] & F_M \arrow[ur, "\omega_{\Sigma_1}"] \arrow[dr, "\omega_{\Sigma_1}"'] & \\ & & F_{\Sigma_1} \end{tikzcd}$$

Pf. $\iota^*(\omega_{\Sigma_1}-\omega_{\Sigma_2})$
$= \iota^*\int_{\Sigma_1-\Sigma_2} \delta p\wedge\delta\phi$
$= \iota^*\int_{\Sigma_1-\Sigma_2} (\delta\Theta) = \iota^*\int_N d(\delta\Theta) + \iota^*\int_N \delta(\delta L - EL\wedge\delta\phi)$

$\downarrow$ $= 0$ $= 0$ □

即证。

Fact 2: $EL_M \to F_{\Sigma_1}\oplus F_{\Sigma_2}$ 迷向.
Pf: $(-\omega_{\Sigma_1}+\omega_{\Sigma_2})\overset{(-\omega_{\Sigma_1},\,\omega_{\Sigma_2})}{}(\delta_\phi,\delta_\phi) = 0$.

▷ Suppose $EL_M \to \Phi_{\Sigma_1}\oplus\Phi_{\Sigma_2}$ Image Lagrangian. (类比: ΣL 的边界条件这样).

Functorial Classical Field Theory.

$(n-1)$ $\Sigma_1 \longrightarrow (\Phi_{\Sigma_1}, \omega_{\Sigma_1})$
$M \uparrow \rightsquigarrow L_M \subseteq \Phi_{\Sigma_1}\oplus\Phi_{\Sigma_2}\ ((-\omega_1)\oplus(\omega_2))$ Lagrangian
$\Sigma_2 \longrightarrow (\Phi_{\Sigma_2}, \omega_{\Sigma_2})$

s.t. $\Sigma\times[0,\varepsilon] \longrightarrow \{(x,y) \mid (x,y) \mid x \overset{\text{Character distribution}}{\underbrace{\sim\sim\sim}_{\text{of } C_\Sigma}} y\} \subseteq \bar{\Phi}_\Sigma\oplus\Phi_\Sigma$.

for $C_\Sigma \subseteq \Phi_\Sigma$ coisotropic. 即 x & y 为差一个规范变换的两 EL 解.
注: Characteristic distribution: 即演化仅在 Gauge 上变化.
$\ker(\omega_\Sigma|_{C_\Sigma}) \subseteq \mathfrak{X}(C_\Sigma)$

What's Gauge Transformation? [非紧支的 U(1)-Gauge 非 Gauge!].

$\xi\in\mathfrak{X}(F_M)$ s.t. $\begin{cases} \delta_\xi S = 0 \\ \xi|_{EL_M} \in \ker(\omega|_{EL_M}) \end{cases}$. (not $\xi\in\omega|_\Sigma$!)

例子 (Maxwell):
$A_\mu \to A_\mu + \partial_\mu\lambda$ $\quad \omega(\varepsilon\partial_\mu\lambda, B) = ?$
$\Theta = \delta P_\phi\wedge\delta\phi$. $\delta P_\phi = \delta F^{0\mu}(A)$ ∴ $\omega = \int\delta F^{0\mu}(A)\wedge\delta A_\mu$
$= \star dA$ $\qquad = \int\delta\star dA\wedge\delta A_\mu$.

$\iota_\xi\omega = \int\delta_\xi\star dA\wedge\delta A \mp \int\delta\star dA\wedge\delta_\xi A$
$= \int\underbrace{\star d\delta_\xi A}_{=0}\wedge\delta A \mp -\int\delta\star dA\wedge d\lambda \overset{?}{=} -\int(\star d\delta A)\wedge d\lambda = -\int d(\star d\delta A\wedge\lambda)$
(by $d\star dA = 0$.)

∴ $\boxed{\lambda \text{ local}} \overset{才}{\Rightarrow} \iota_\xi\omega = 0$! 因此可相信大 Gauge 非 Gauge。 $\boxed{\Sigma L}$ ☆

RMK: ① 必须 on-shell! ② 必须 λ local (cpt supp!).

▷ Reduction: Covariant → BV/BFV.
$EL_M \to F_M \searrow F_\Sigma \to P_\Sigma$.

一个 Gauge: $\xi\in\mathfrak{X}(F_M)$ $\begin{cases} \xi(S) = 0 \\ \xi|_{EL_M} \in \ker(\omega|_{EL_M}) \end{cases}$

一族 Gauge: $\mathfrak{g} \to \mathfrak{X}(F_M)$ s.t. 1° Lie bracket kept
2° $\mathrm{Im}(\mathfrak{g}) = \ker(\omega|_{EL_M})$

此时 $EL_M/\ker(\omega|_{EL_M}) = EL_M/\mathfrak{g}$。

$\boxed{\text{但是!}}$ $\mathrm{Im}(\mathfrak{g}\to\mathfrak{X}(F_M)) \neq \Gamma(\ker(\omega|_{EL_M}))$.
例: $\mathfrak{g} = \Omega^0(M,\mathfrak{g})$ in Maxwell.
$\Gamma(\ker(\omega|_{EL_M})) = \partial_\mu\lambda(x,A)$. 不必用"最大的"规范群。

BV & renormalization: BV基础(未完)

Recall: FP: F_{cl} 场空间, $S: F_{cl}\to\mathbb{R}\in C^\infty(F_{cl})^{\mathfrak g}$

$\mathfrak g \to \mathfrak X(F_{cl})$

↓ 构造

F_{BV}, ω_{BV}: odd symplectic form on F_{BV}

Q_{BV}: deg +1 $C^\infty(F_{BV})\to C^\infty(F_{BV})$, $Q_{BV}=\{S_{BV},-\}$.

$\in \mathfrak X(F_{BV})$

Construction. $F_{BV}=T^*[-1](F_{cl}\oplus\mathfrak g[1]) = \underbrace{\mathfrak g[1]}_{\text{ghost}}\oplus \underbrace{F_{cl}\oplus F_{cl}^\vee[-1]\oplus \mathfrak g^\vee[-2]}_{F_{BRST}}$

ω_{BV} = canonical for T^*.

$S_{BV}=S_{cl}(\phi)+\phi^*\delta_c\phi + c^*_a f^a_{bc}c^bc^c = S_{BV}(\phi,\phi^*,c,c^*)$

限制在 $L=\begin{cases} c\ \text{任意} \quad GF(\phi)=0 \\ \phi^*=(dGF)^\dagger b \ \text{for some } b \\ c^*=0 \end{cases}$ 上 (Lagrangian subspace)

∴ FP表示为 $\int_{GF} D\phi\, e^{S(\phi)} e^{b\delta_c GF}$ = F-P 积分.

Eg: C-S: $F_{cl}=\Omega^1(M,\mathfrak g)$, $\mathfrak g=\Omega^0(M,\mathfrak g)$ 对应于 Ω^2,Ω^3.

$F_{BV}=\Omega^\bullet(M,\mathfrak g)[1]$;

$S_{BV}=\int \frac12 AdA+\frac13 A\wedge A\wedge A + \langle A^*, \underbrace{dc+[c,A]}_{\delta_c A}\rangle + \langle c^*,\underbrace{[c,c]}_{\delta_c c}\rangle$

$Q_{BV}^2=0 \iff \{S_{BV},S_{BV}\}=0$

证明见上次. (?)

QME: $\delta_S=\{S,-\}-i\hbar\Delta$; $\delta_S^2=0 \iff \frac12\{S_{BV},S_{BV}\}+\hbar\Delta S_{BV}=0$

BV 构造: (Z-graded space)

Σ: 场空间, $\omega\in\Lambda^2\Sigma$ 辛形式

$Q:\Sigma\to\Sigma$, $Q^2=0$. 自由=二次型 $I_{BV}\in C^\infty(\Sigma)$ 相互作用.

RMK: Q与Q_{BV}完全不一样!

Q: L_∞代数结构 可构造 $Q^0_{BV}: C^\infty(\Sigma)\to C^\infty(\Sigma)$

($Q^0_{BV}=\{\langle e,Qe\rangle,-\}$

as function, 而为 $(Q,0,0,0,\dots)$ 给出的L^∞结构.

Eg (KG) 0 deg +1 deg

$\Sigma=C^\infty(M)\oplus C^\infty(M)$

$\Rightarrow \int \phi\partial^2\phi=\langle e,Qe\rangle$

$Q:(\phi_1,\phi_2)\mapsto(0,\partial^2\phi_1)$

CME $Q^0_{BV}I_{BV}+\frac12\{I_{BV},I_{BV}\}=0$ (注: $\{\langle e,Qe\rangle,\langle e,Qe\rangle\}=0$

因为 $Q^2_{BV}=0$, 且无 fermion 常数。

$\mathcal H\to\mathbb C[\Sigma,\Sigma^*]$

φ: $\Sigma\in\boxed{\mathbb{CP}^1}$

$\mathbb{CP}^1\to \mathrm{Alg}(\mathcal H,\mathbb C)$

$g=\mathbb{CP}^1-\{\infty\}/\mathbb{CP}^1-\{0\}$

g_+ g_-

↓

$\varphi_+: \mathbb{CP}^1-\{\infty\}\to \mathrm{Alg}(\mathcal H,\mathbb C)$

$\ni 0$

L_∞ & BV: (?)

$$\begin{tikzcd}
\mathrm{Sym}(L[1]) \arrow[r, "D"] \arrow[d] & \mathrm{Sym}(L[1]) \arrow[d] \\
\mathrm{Sym}(L^\vee[1]) \arrow[d, equal] & \mathrm{Sym}(L^\vee[1]) \arrow[l, "D^\vee"'] \arrow[d, equal] \\
C^\infty(F_{BV}) & C^\infty(F_{BV}) \arrow[l, "Q"']
\end{tikzcd}$$

" $F_{BV} = L[1]$ " — 线性空间.

BV 结构 = L_∞ algebra + inner product.

HMC action. $\{S_{HMC}, -\} = D^\vee$

$$\sum \frac{1}{(i+1)!} \langle a, \mu_i(a, \dots a) \rangle \hookrightarrow \text{symplectic form}$$

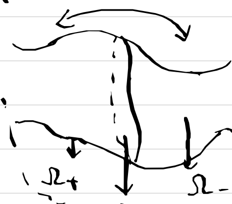
$\mu_i : L^i \longrightarrow L$. 为 Q 的按次数分解.

$$\sum \Phi_1(z_1) \cdots \Phi_n(z_n)\, e^{iS(\Phi)/\hbar} = \sum_{\rho \in I(\Phi_1 \cdots \Phi_n)} h_\rho(z_1, \dots z_n)\, \overline{h_\rho}(\bar z_1, \dots \bar z_n).$$

New One:

(用于构造 B_Σ: Σ上 的构型空间. F_Σ: Σ上 场上的空间+边界

▷ Reflection Positivity:



$M = \Omega_- \sqcup \Sigma \sqcup \Omega_+$,

$\mathcal{E}_M = L^2(F_M)$ $\mathcal{E}_\pm = L^2(F_{\Omega_\pm})$

$\Theta: M\to M \to \Theta: L^2(F_M) \to L^2(F_M)$ 拉回 ↑Σ

$\Theta\begin{cases}\Theta(\Sigma)=\Sigma\\ \Theta(\Omega_\pm)=\Omega_\mp\\ \Theta^2=\mathrm{id}\end{cases}$

$\mathcal{E}_M$: Observable 空间 (类似 GNS 构造)

▷ Reflection Positivity (物理原因 必要).

Σ上内积: $\langle A,B\rangle = \int_{F_M}[D\phi]e^{S(\phi)}\bar A(\phi)B(\phi)$.

$\langle\Theta A, A\rangle \ge 0 \quad \forall A\in\mathcal{E}_+$.

Pf(切割复合): $\int[D\phi]\bar A(\phi_-)A(\phi_+) = \int D\phi_\Sigma(\ )^2 \ge 0$. $(\bar A(\phi_-)=A(\phi_+))$

RP ~ 闵氏 Unitarity.

$\langle\Theta A, A\rangle = \langle 0|\Phi(t_E,\vec x)\Phi(t_E,\vec x)|0\rangle$

∴ 闵氏 Unitarity ⇒ H Hermitian ⇒ $\Phi(t)^\dagger = \Phi(-t)$ if $\Phi(0)$ Hermitian

欧氏: 反射 ⟺ 闵氏: Unitarity.

▷ 用 RP 构造态空间 — 构造非负定内积.

$\langle\Theta A, A\rangle$ 为非负定.

$N = \ker\langle\Theta A, A\rangle$

$\mathcal{H} = \mathcal{E}_+/N$. $\langle\Theta -, -\rangle$ 为内积, 内积是路径积分.

RMK: $\mathcal{E}_+ : L^2(F_{\Omega_+})$;

固定 $A\in\mathcal{E}_+$, 取 $B\in\mathcal{E}_+$.

$$\langle\Theta B, A\rangle_\Sigma = \int D\phi_\Sigma \int_{\phi_-|_\Sigma=\phi_\Sigma} D\phi_-\, e^{S(\phi_-)}\bar B(\phi_-)\int_{\phi_+|_\Sigma=\phi_\Sigma}D\phi_+\, e^{S(\phi_+)}A(\phi_+).$$

仅与 $\tilde A(\phi_\Sigma) = \int_{\phi_+|_\Sigma=\phi_\Sigma} D\phi_+\, e^{S(\phi_+)}A(\phi_+)$ 有关.

有规范层次下 FQFT: 不好搞!

$\langle \delta[\phi_2], U\delta[\phi_1]\rangle \stackrel{?}{=} \int_{[\phi|_{\Sigma_1}]=[\phi_1],\ [\phi|_{\Sigma_2}]=[\phi_2]} [D\phi]\, e^{S(\phi)}$ 当如PI空间有Gauge.

$\hookrightarrow$ 带边界条件 Path Int.

$\downarrow$

阅领

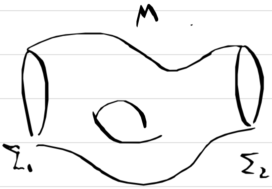
BV-BFV.

$\Sigma_1 \xrightarrow{M} \Sigma_2$

$\downarrow$

F_M

$F^\partial_{\Sigma_1} \leftarrow F_M \rightarrow F^\partial_{\Sigma_2}$



Construction $F_M = \{\phi, c, \phi^+, c^+\} \longrightarrow$ By classical BV.

$\omega_M = \int_M (\delta\phi\wedge\delta\phi^+ + \delta c\wedge\delta c^+)$

$S_M = S_{cl}(\phi) + \phi^+\delta_c\phi + \frac{1}{2}\langle c^+, [c,c]\rangle$

$Q_M = \left(\frac{\delta S_{cl}}{\delta\phi} + \phi^+\frac{\delta(\delta_c\phi)}{\delta\phi}\right)\frac{\delta}{\delta\phi^+} + \delta_c\phi\frac{\delta}{\delta\phi} + c^+c\frac{\delta}{\delta c^+} + \frac{1}{2}[c,c]\frac{\delta}{\delta c}$.

无边界时, $\delta S_M = \iota_{Q_M}\omega_M$

有边界时: $\delta S_M = \iota_{Q_M}\omega_M + \pi^*(\alpha_\Sigma) \longrightarrow$ 边界项

$F_M|_\Sigma = \{\phi|_\Sigma, c|_\Sigma, \phi^+|_\Sigma, c^+|_\Sigma, \dot\phi|_\Sigma, \dot c|_\Sigma, \dot\phi^+|_\Sigma, \dot c^+|_\Sigma\}$.

与拉氏密度一样, 可证:

$\alpha = \int_\Sigma\left(\frac{\partial L^{BV}}{\partial\dot\phi}\delta\phi + \frac{\delta L^{BV}}{\delta\dot c}\delta c\right) = \int_\Sigma \left(P_\phi + \phi^+\frac{\delta(\delta_c\phi)}{\delta\dot\phi}\right)\delta\phi + \left(\phi^+\frac{\delta(\delta_c\phi)}{\delta\dot c}\right)\delta c$; 边界项.

$\downarrow$

比如: $\delta_c A_0 = \partial_0 c$ 的 $\dot c$ 时有关.

$F^\partial_\Sigma = (F_M|_\Sigma)/\ker\delta\alpha$.

$\downarrow$

构造出辛空间.

文献: BV-BFV. Classical BV BFV: Cattaneo; Mnev. Reshetikhin.

2012.

Quantum 2015.

BFV: $(B^\partial_\Sigma, \omega^\partial_\Sigma = \delta\alpha^\partial_\Sigma, S^\partial_\Sigma)$; Q^∂_Σ 是 Q 的推出 (用 $F_{M(\epsilon)} \to F_\Sigma$ 关联).

S^∂_Σ 与 Q^∂_Σ 亦为 ω 之关系. [ω^∂_Σ 与 ω_M 可能无关联]

Quantum BV-BFV.

$\boxed{\text{rmk: } \Sigma \text{ 实际为 } \bar{\Sigma}_1 \sqcup \Sigma_2}$

Classical: $(F_\Sigma^\partial, \omega_\Sigma^\partial = \delta\alpha_\Sigma^\partial, Q_\Sigma^\partial)$ & $(F_M, \omega_M, S_M, Q_M)$

$\Downarrow$ quantum.

P: polarization. 给 F_Σ^P 极化，使之变为 $T^*B_\Sigma^P$，且需使 ω_Σ 的选择在 $T^*B_\Sigma^P|_x$ 上不变.

把 S_Σ^∂ 量化为 Ω_Σ^P

核心. H_Σ^P 定义: $L^2(B_\Sigma^P)$. $V_M^P = EL_M^{S_0}/Q_{M_0}$ ↔ S_0 的 Gage vector field ??

$\uparrow$ B_Σ 作用量的二次部分.

做分解 $F_M = B_\Sigma^P \oplus V_M^P \oplus \mathcal{O}$. 选 $\mathcal{O} = L \oplus \Delta$

(residual) 慢场 快场 (fluctuation)

取 $\Phi \in B_\Sigma^P$,

$\Psi_M^P(\Phi) \stackrel{\Delta}{=} \int_{\Phi \times L} e^{S_M} \Rightarrow \Psi_M^P(\Phi) \in \mathrm{func}(B_{\bar{\Sigma}_1 \sqcup \Sigma_2}^P \otimes V_M^P)$

$= \mathrm{func}(B_{\bar{\Sigma}_1}^P) \otimes \mathrm{func}(B_{\Sigma_2}^P) \otimes \mathrm{func}(V_M^P)$

$\Psi_M^P(\Phi)$ 为时间演化算符.

$\boxed{\alpha_\Sigma \text{ 与 } S_M \text{ 的问题}}$

Homological FQFT: 如何处理掉 (V_M^P)

$$Z_M^P(\Phi_\Sigma) = \int_{V_M^P \text{的 lagrangian}} [D\Phi_V] \int_{\Phi_L \in L} [D\Phi_L] e^{S_M^P(\Phi_\Sigma, \Phi_V, \Phi_L)}$$

$$= \int_{F_M \ominus B_\Sigma^P \text{ 的 lagrangian}} e^{S_M^P}$$