FIN41360: PORTFOLIO & RISK MANAGEMENT

Group Project Assignment 2

Market Risk Modelling

Group 13

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I / We declare that all material included in this project is the result of my/our own work and that due acknowledgement has been given in the bibliography and references to all sources be they printed, electronic or personal.

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1 Executive Summary

This report analyses the risk exposure of three $1 million portfolios: equity, interest rate, and a 50-50 combined portfolio. Portfolios were constructed using leading S&P 500 sector stocks and major interest rate-sensitive ETFs, applying market capitalization and equal-weighted strategies. Risk was assessed using Value at Risk (VaR) and Expected Shortfall (ES) via parametric (Normal, Student-t, Cornish-Fisher, Monte Carlo) and non-parametric (Historical Simulation variants) methods. Backtesting using Kupiec's Unconditional Coverage Test and Lopez's QPS guided model selection. Historical Simulation, Student-t Distribution, and Bootstrap Historical Simulation were recommended for the respective portfolios. Sensitivity tests and recent market events were incorporated to confirm model robustness.

1.1 Literature Review

VaR and ES are fundamental risk measures, following frameworks like Basel (1996) and meeting coherence properties (Artzner et al., 1999). While parametric methods often assume normality, real-world returns exhibit skewness and fat tails (Dowd, 2005), motivating use of Student-t distributions and Cornish-Fisher expansions. Monte Carlo simulations (Glasserman, 2004) offer flexibility across assumptions, while non-parametric approaches like Historical Simulation (Hull, 2023) adapt directly from market data. Enhancements such as Volatility-Weighted and Age-Weighted Historical Simulations (Hull and White, 1998) improve responsiveness. Rigorous validation using Kupiec's UC Test (1995) and Lopez's QPS ensures model accuracy, with sensitivity analyses crucial for robustness under volatility shifts.

2. Portfolio Construction

2.1 Equity Portfolio

The equity portfolio included the largest stock from each of the eleven major sectors as of March 28, 2025, ensuring diversification. A value-weighted strategy (Sharpe, 1964) was used, with Apple Inc. (AAPL) holding the largest weight (~32.8%). Daily simple returns were computed using Yahoo Finance data.

Sector	Ticker	Company Name
Communication Services	GOOGL	Alphabet Inc.
Consumer Discretionary	AMZN	Amazon.com Inc.
Consumer Staples	PG	Procter & Gamble Co.
Energy	XOM	Exxon Mobil Corp.
Financials	JPM	JPMorgan Chase & Co.
Health Care	JNJ	Johnson & Johnson
Industrials	HON	Honeywell International
Information Technology	AAPL	Apple Inc.
Materials	LIN	Linde PLC
Real Estate	PLD	Prologis Inc.
Utilities	NEE	NextEra Energy Inc.

Table 1 : Equity Portfolio Selection

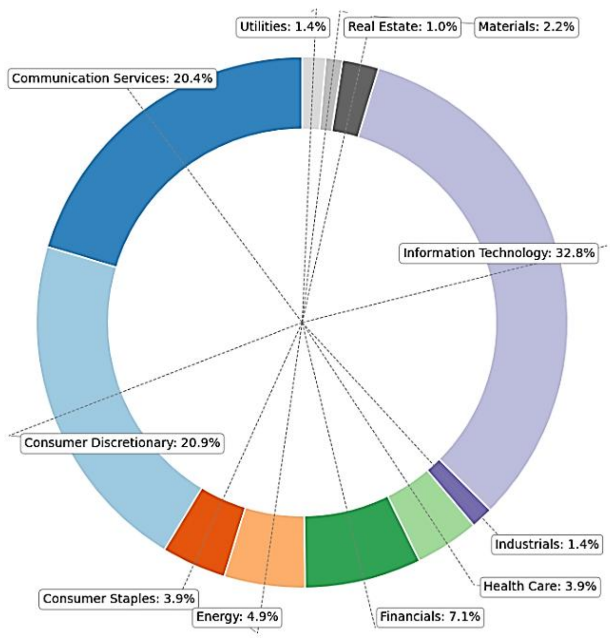


Figure 1: Equity Portfolio Sector Weights

2.2 Interest Rate Portfolio

This portfolio was constructed using four Treasury rates as proxies for duration-based fixed-income exposures. Each rate's return was modeled using a duration-convexity approximation based on daily changes in yields from FRED.

Instrument	Ticker	Focus
2-Year Treasury	DGS2	Short-duration rate proxy
5-Year Treasury	DGS5	Intermediate duration
10-Year Treasury	DGS10	Long-duration benchmark
30-Year Treasury	DGS30	Long-duration Exposure

Table 2 : Interest Rate Portfolio Selection

Returns were calculated using:

$$IR\ Return = -D.\Delta y + \frac{1}{2}.C.(\Delta y)^2$$

Where:

- D = Duration
- C = Convexity
- Δy = daily yield change in decimal format

All instruments were assigned equal weights (25%).

2.3 Combined Portfolio

The combined portfolio allocated 50% each to equity and interest rate assets, aiming to balance growth and stability. Daily returns were calculated as a weighted average, leveraging fixed-income steadiness to reduce equity-driven volatility (Sharpe, 1964).

Portfolio	Annualized Return (%)	Annualized Volatility (%)
Equity Portfolio	15.84%	20.93%
Interest Rate Portfolio	-5.29%	7.91%
Combined Portfolio	5.27%	11.44%

Table 3 : Portfolio Statistics

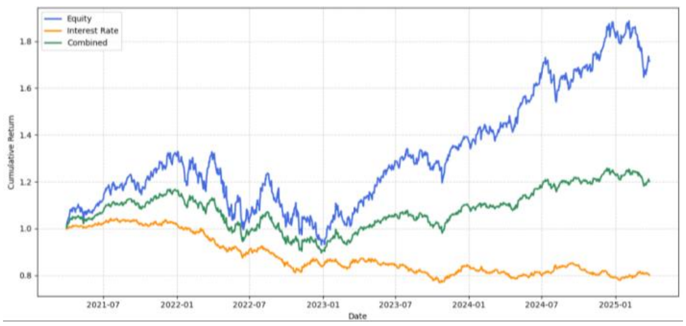


Figure 2: Cumulative Returns — Equity, Interest Rate, and Combined Portfolios

3. Risk Measurement: One-Day Horizon Estimates

This section presents the one-day Value at Risk (VaR) and Expected Shortfall (ES) estimates for three \$1 million portfolios: Equity, Interest Rate, and Combined, as of March 28, 2025.

3.1 Parametric Methods

3.1.1 Normal Distribution Approach

Under the assumption that returns are normally distributed, VaR and ES are computed using the sample mean and standard deviation (Jorion, 2007). Daily returns are modeled as:

$$R_{t+h} \sim N(\mu_h, \sigma_h^2)$$

where μ_h is the mean return and σ_h is the standard deviation over horizon h.

- VaR (Normal Distribution):

$$VaR_{h,\propto} = -\mu_h + \Phi^{-1}(1-\alpha)\sigma_h$$

- Expected Shortfall (Normal Distribution):

$$ES_{h,\propto} = -\mu_h + \frac{1}{\alpha}\phi(\Phi^{-1}(1-\alpha))\sigma_h$$

where Φ denotes the inverse standard normal CDF and ϕ denotes the standard normal PDF and we approximate $\mu_h = 0$

Metric	Equity	Interest Rate	Combined
Mean	626.35	-210.03	208.16
Volatility	13,206.32	4,981.92	7,215.14
VaR (95%)	-22,348.81	-7,984.49	-12,076.02
ES (95%)	26,614.48	10,486.29	14,674.61
VaR (99%)	-31,348.84	-11,379.64	-16,993.09
ES (99%)	34,571.31	13,487.91	19,021.74

Table 4 : Normal Distribution VaR and ES Results

The Equity portfolio exhibited the highest risk under the normality assumption, with a 99% Expected Shortfall (ES) of \$34,514, followed by the Combined portfolio (\$18,997) and the Interest Rate portfolio (\$13,488). This reflects the typical volatility-risk profile across asset classes. However, deviations from the normal distribution suggest that relying solely on parametric models may underestimate tail risk (Jorion, 2006).

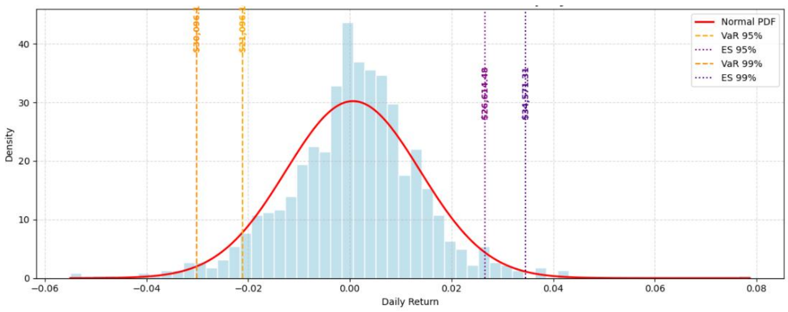


Figure 3 : Normal Distribution - Equity Return Fit

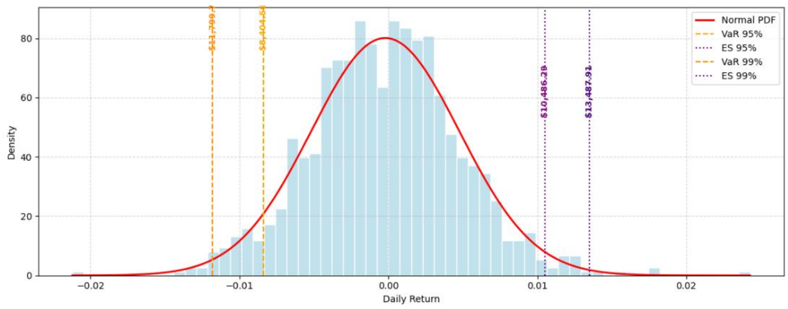


Figure 4 : Normal Distribution - Interest Rate Return Fit

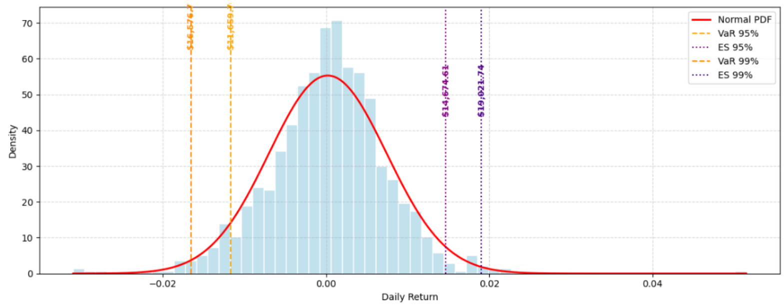


Figure 5 : Normal Distribution - Combined Return Fit

3.1.2 Student-t Distribution Approach

The Student-t distribution offers a better fit for financial return data by capturing heavy tails and extreme values, which are common in markets (McNeil, Frey, & Embrechts, 2005). Compared to the normal distribution, it typically yields higher Value at Risk (VaR) and Expected Shortfall (ES) estimates—especially at higher confidence levels—reflecting the greater likelihood of large losses.

- Student-t VaR Formula:

$$VaR_{\alpha} = -\mu + \sigma \times t_{v}^{-1}(\alpha)$$

where $t_v^{-1}(\alpha)$ is the inverse cumulative distribution function with v degrees of freedom.

- Student-t ES Formula:

$$ES_\alpha = -\mu + \sigma \times \frac{t_v(\alpha)}{1-\alpha} \times \frac{v + (t_v^{-1}(\alpha))^2}{v-1}$$

where $t_v(\alpha)$ is the probability density function.

Metric	Equity	Interest Rate	Combined
Degree of Freedom	5.17	13.1	6.04
VaR (95%)	$19,882.71	$8,340.05	$11,035.81
Standardized t-quantile (95%)	-2.00	-1.77	-1.94
Standardized t-density (95%)	0.06	0.08	0.06
Standardized t-ES (95%)	2.85	2.31	2.70
ES (95%)	-$30,787.54	-$10,366.39	-$16,236.21

Table 5 : Student-t Distribution VaR and ES at 95% Confidence

Metric	Equity	Interest Rate	Combined
Degree of Freedom	5.17	13.1	6.04
VaR (99%)	$33,649.84	$12,357.59	$18,050.01
Standardized t-quantile (99%)	-3.32	-2.65	-3.14
Standardized t-density (99%)	0.01	0.02	0.01
Standardized t-ES (99%)	4.37	3.16	4.02
ES (99%)	-$46,566.16	-$1,273.60	-$23,953.70

Table 6 : Student-t Distribution VaR and ES at 99% Confidence

The results demonstrate that the Combined portfolio, due to the integration of equity and interest rate exposures, records a notably higher Expected Shortfall (ES) at the 99% confidence level than either portfolio individually. This underscores the need to incorporate tail-risk measures when assessing portfolio risk. Furthermore, the Q-Q plots reveal that empirical return distributions closely match the theoretical quantiles of the Student-t distribution, validating its appropriateness for modeling heavy-tailed financial data (McNeil, Frey, & Embrechts, 2005).

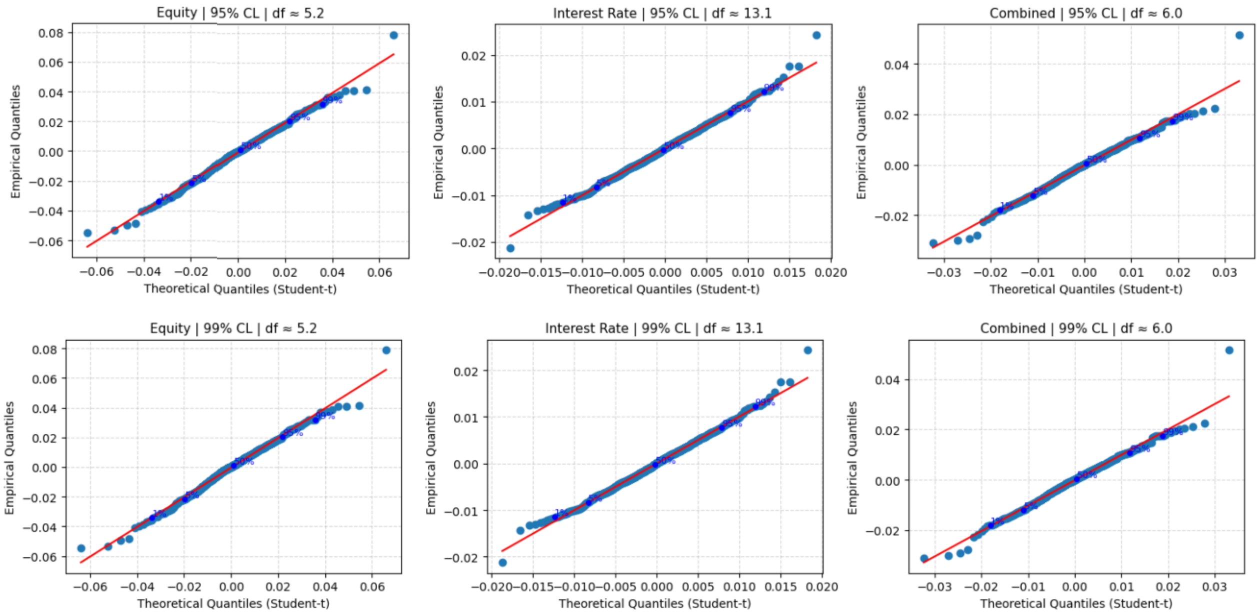


Figure 6 : Student-t Q-Q Plots for Equity, Interest Rate, and Combined portfolios

3.1.3 Cornish-Fisher Expansion Approach

The Cornish-Fisher expansion modifies the standard normal quantiles to account for skewness and excess kurtosis in financial return distributions (Dowd, 2005). This adjustment enables a more accurate estimation of Value at Risk (VaR) and Expected Shortfall (ES) when returns deviate from normality. The adjusted quantile z_{CF} using the Cornish-Fisher expansion is calculated as:

$$z_{CF} = z + \frac{(z^2 - 1)}{6} \times Skewness + \frac{(z^3 - 3z)}{24} \times Excess\ Kurtosis - \frac{(2z^3 - 5z)}{36} \times (Skewness^2)$$

where:

- z = standard normal quantile
- Skewness = third central moment normalized
- Excess Kurtosis = kurtosis minus 3

Then, the Cornish-Fisher VaR is computed as:

$$VaR_{CF} = -\mu + \sigma \times z_{CF}$$

where μ is the mean return, and σ is the standard deviation.

Metric	Equity	Interest Rate	Combined
Mean	628.67	-210.03	209.32
Standard Deviation	13,185.55	4,981.92	7,206.25
Skewness	-0.13	0.15	0.01
Excess Kurtosis	2.26	0.87	3.46
Normal Quantile (z)	-1.64	-1.64	-1.64
Cornish-Fisher VaR	$20,958.78	$8,103.55	$11,112.69
Expected Shortfall	$26,569.33	$10,486.29	$14,655.11

Table 7 : Cornish-Fisher Expansion VaR and ES at 95% Confidence Level

Metric	Equity	Interest Rate	Combined
Mean	628.67	-210.03	209.32
Standard Deviation	13,185.55	4,981.92	7,206.25
Skewness	-0.13	0.15	0.01
Excess Kurtosis	2.26	0.87	3.46
Normal Quantile (z)	-2.33	-2.33	-2.33
Cornish-Fisher VaR	$38,236.85	$12,221.47	$22,304.52
Expected Shortfall	$34,513.65	$13,487.91	$18,996.89

Table 8 : Cornish-Fisher Expansion VaR and ES at 99% Confidence Level

The expansion demonstrates that when accounting for real-world asymmetries and fat tails in return distributions, both VaR and ES estimates tend to rise, better capturing the potential for extreme outcomes.

3.1.4 Extreme Value Theory (EVT) Approach

Extreme Value Theory (EVT) captures the behavior of extreme returns, focusing on the distribution's tail (Embrechts, Klüppelberg, & Mikosch, 1997). By fitting a Generalized Pareto Distribution to the worst 10% of losses (90% threshold), EVT produces significantly higher risk estimates than traditional methods—particularly for the equity portfolio—highlighting its utility in stress testing and capital planning.

Metric	Equity	Interest Rate	Combined
Mean	628.67	-210.03	209.32
Volatility	13,178.93	4,979.41	7,202.63
Selection Period	90.0	90.0	90.0
Alpha	0.5072	0.4694	0.4656
Beta	0.00556	0.00181	0.00305
Theta	9e-06	2.6e-05	1.6e-05
EVT VaR (return)	40,648.23	13,912.55	21,468.67
EVT VaR ($)	76,999.74	23,983.53	38,172.00
EVT Expected Shortfall ($)	628.67	-210.03	209.32

Table 9 : Extreme Value Theory (EVT) VaR & ES Results

The significantly higher ES values reflect the heavy-tailed nature of the return distributions, particularly for the equity portfolio, highlighting EVT's importance in capturing potential catastrophic losses.

3.1.5 Monte Carlo Simulation Approach

Monte Carlo simulations model portfolio returns by generating thousands of hypothetical scenarios. In this study, simulations were based on both Normal and Student-t distributions (Glasserman, 2004; McNeil, Frey, & Embrechts, 2005).

- Normal assumes symmetric, light-tailed returns.
- Student-t captures fat tails, reflecting higher chances of extreme losses.

The basic formula for calculating Monte Carlo VaR at a confidence level α is:

$$VaR_\alpha = -Quantile_\alpha(Simulated\ Portfolio\ Losses)$$

The Expected Shortfall (ES) under Monte Carlo simulation is computed as:

$$ES_\alpha = -E[Loss \mid Loss > VaR_\alpha]$$

where:

- Loss represents the simulated portfolio losses,
- $Quantile_\alpha$ is the α - th quantile from the simulated distribution,
- E[·] denotes the expectation operator.

Results showed consistently higher VaR and ES under the Student-t distribution, especially at the 99% level, supporting its use for more conservative risk estimation.

Metric	Equity	Interest Rate	Combined
VaR (95%)	$21,293.94	$8,228.74	$11,656.55
ES (95%)	$26,902.03	$10,297.47	$14,789.65
VaR (99%)	$30,794.23	$11,839.23	$16,955.77
ES (99%)	$35,494.30	$13,606.86	$19,534.06

Table 10 : Monte Carlo Simulation (Normal Distribution)

Metric	Equity	Interest Rate	Combined
VaR (95%)	$19,751.61	$8,530.20	$10,974.58
ES (95%)	$28,412.67	$11,008.03	$15,607.34
VaR (99%)	$34,886.53	$11,970.13	$17,856.58
ES (99%)	$45,444.79	$14,530.02	$23,492.38

Table 11 : Monte Carlo Simulation (Student-t Distribution)

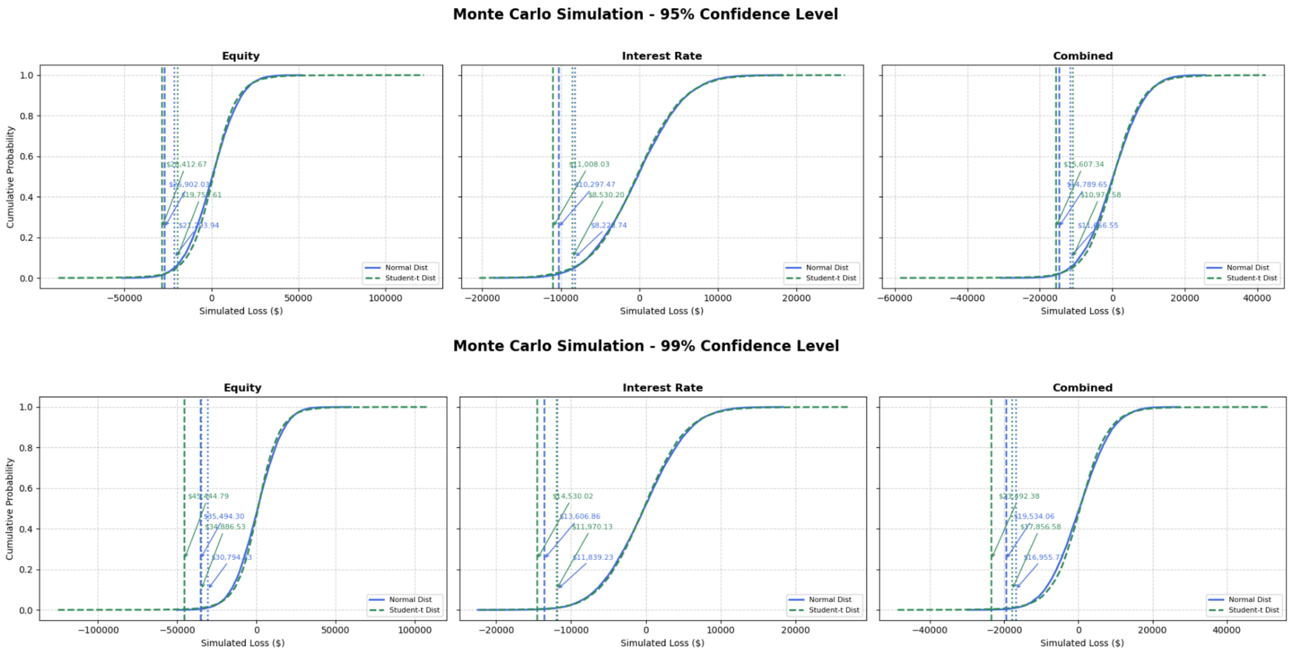


Figure 7 : Cumulative distribution functions (CDFs) of simulated portfolio losses under Normal and Student-t assumptions for Equity, Interest Rate, and Combined portfolios.

3.2 Non-Parametric Methods

3.2.1 Historical Simulation Approach

Historical Simulation (HS) estimates Value at Risk (VaR) and Expected Shortfall (ES) by directly using historical returns without assuming any distribution (Hull, 2023; Boudoukh et al., 1998).

- VaR is the empirical quantile at confidence level α.
- ES is the average of losses beyond the VaR threshold.

This method relies on the assumption that historical return patterns reflect future risks, as emphasized by Hull (2023).

Metric	Equity	Interest Rate	Combined
VaR (95%)	$21,478.00	$8,394.69	$12,029.45
ES (95%)	$30,078.19	$10,588.88	$16,191.83
VaR (99%)	$35,612.14	$11,571.89	$18,120.91
ES (99%)	$44,253.41	$13,669.17	$24,467.52

Table 12 : Historical Simulation VaR Results

3.2.2 Bootstrap Historical Simulation Approach

Bootstrap Historical Simulation, introduced by Hendricks (1996), improves basic historical simulation by randomly resampling returns with replacement. It better captures sampling variability and uncertainty in the tails. However, in this case, Bootstrap produced broadly similar or slightly lower VaR and ES estimates compared to simple historical simulation, indicating enhanced robustness without consistently higher risk measures.

Metric	Equity	Interest Rate	Combined
VaR (95%)	$21,089.58	$8,175.13	$11,696.70
ES (95%)	$29,231.17	$10,293.58	$15,793.39
VaR (99%)	$34,200.80	$11,237.00	$17,802.47
ES (99%)	$40,535.36	$12,816.87	$22,002.87

Table 13 :Bootstrap Simulation VaR Results

3.2.3 Age-Weighted Historical Simulation (AWHS) Approach

The Age-Weighted Historical Simulation (AWHS) method (Boudoukh et al., 1998) modifies historical simulation by giving exponentially higher weights to recent returns, thus reacting faster to changing market conditions. The weight assigned to each return at time t − i is given by:

$$w_i = (1 - \lambda)\lambda^{i-1}$$

where:

- wiw_iwi = weight of the i-th lag,
- λ∈(0,1) = decay factor (usually around 0.94),
- recent returns have higher weights.

The VaR is then calculated as the quantile from the weighted empirical distribution.

Metric	Equity	Interest Rate	Combined
VaR (95%)	$22,890.19	$6,632.32	$9,426.31
ES (95%)	-$126.82	$198.00	$137.45
VaR (99%)	$31,317.12	$7,007.29	$11,887.19
ES (99%)	$936.42	$268.74	$482.28

Table 13: Age-Weighted Historical Simulation (AWHS) VaR & ES Results

AWHS adjusted dynamically for recent volatility, but due to a calmer recent period, ES values were unexpectedly low. This underscores the importance of validating model sensitivity during changing market conditions.

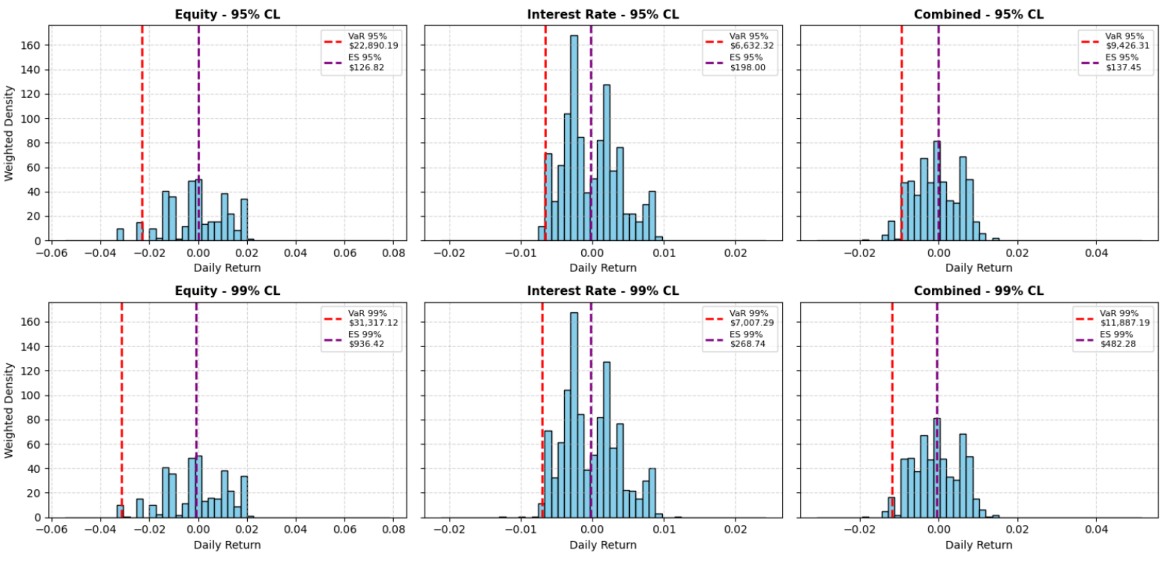


Figure 8 : AWHS Weighted Return Histograms

3.2.4 Volatility-Weighted Historical Simulation (VWHS) Approach

The Volatility-Weighted Historical Simulation (VWHS) (Pritsker, 2006) modifies past returns by scaling them according to the ratio of current to historical volatilities:

$$r_i^{adj} = r_i \times \frac{\sigma_t}{\sigma_i}$$

where:

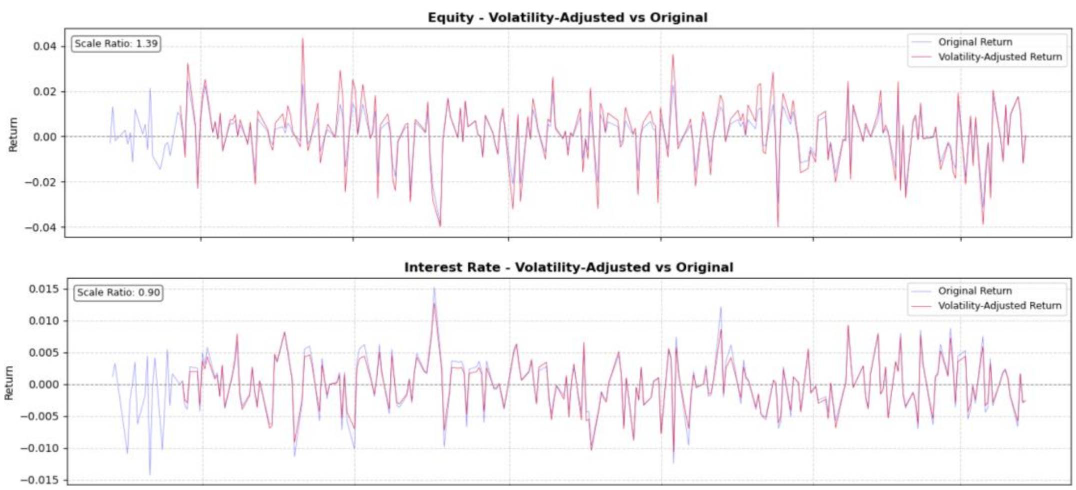
- r_i^{adj} = adjusted historical return,
- r_i = original return at time i,
- σ_t = current volatility estimate,
- σ_i = historical volatility at time i.

After adjusting returns, VaR and ES are calculated in the same way as in basic historical simulation.

Metric	Equity	Interest Rate	Combined
VaR (95%)	$23,941.53	$6,844.13	$11,024.18
ES (95%)	$29,484.04	$8,341.56	$13,502.92
VaR (99%)	$32,153.07	$9,372.65	$15,035.55
ES (99%)	$38,319.83	$10,689.79	$17,141.78

Table 14: Volatility-Weighted Historical Simulation (VWHS) VaR & ES Results

VWHS delivered stable and realistic risk estimates, properly adjusting historical shocks to the current market volatility. It is particularly effective under shifting volatility regimes.



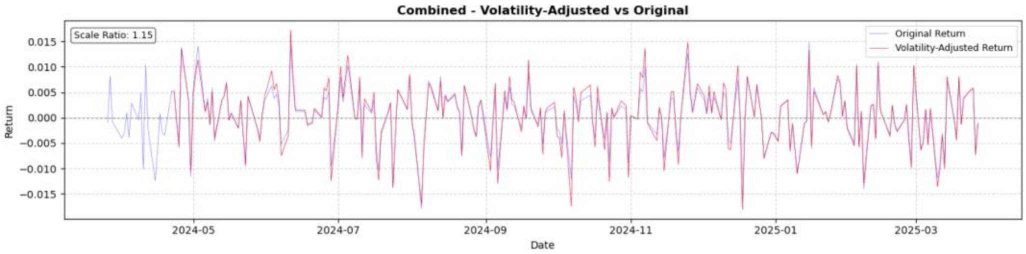


Figure 9 : VWHS Time Series Plots

3.3 Alternative Risk Measures: Spectral Risk Measure (SRM), Volatility, and Downside Deviation

To complement the traditional Value at Risk (VaR) and Expected Shortfall (ES) estimates, additional risk measures were calculated for the three portfolios — Equity, Interest Rate, and Combined — as of March 28, 2025. These measures ensure a broader and more robust view of portfolio risk beyond standard quantile-based approaches.

3.3.1 Spectral Risk Measure (SRM)

The Spectral Risk Measure (SRM) was computed following the method proposed by Acerbi (2002). SRM incorporates an investor's risk aversion by weighting outcomes more heavily for worse losses, using an exponential weighting function.

The SRM formula used is:

$$SRM = \int_0^1 q(p)\phi(p)dp$$

where:

- q(p) = quantile function of portfolio loss at probability level p,
- ϕ(p)= risk aversion weighting function, here set to an exponential form:

$$\phi(p) = \frac{ke^{-k(1-p)}}{1-e^{-k}}$$

with k = 400 representing strong risk aversion. Numerical integration was performed using 10,000 slices evenly spaced between p = 0.001 and p = 0.9999, avoiding extreme outliers but still capturing almost the full range of losses.

3.3.2 Volatility (Standard Deviation)

The standard deviation (σ) of returns was calculated as the traditional risk measure of overall variability:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i - \bar{r})^2}$$

where:

- r_i = portfolio returns,
- $\bar{r}$ = mean return.

It captures both upside and downside volatility equally.

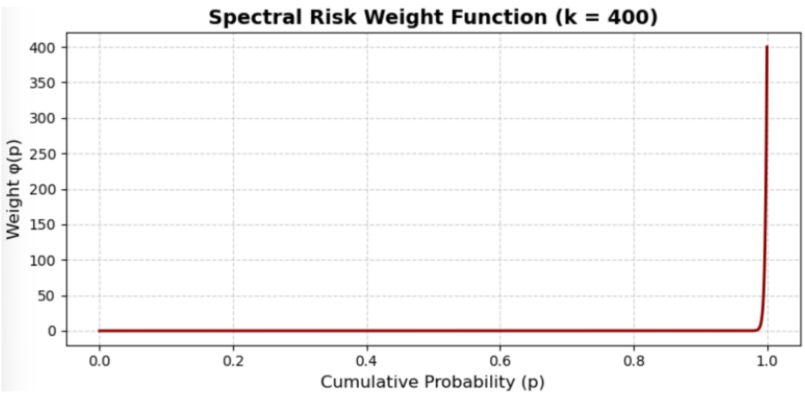
3.3.3 Downside Deviation

The Downside Deviation focuses only on negative returns, ignoring upside moves. It is computed as:

$$Downside\ Deviation = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\min\ (0, r_i)^2}$$

This measure better aligns with investor concerns by penalizing only losses.

Portfolio	Spectral Risk Measure	Standard Deviation	Downside Deviation
Equity	$48,520.48	$13,185.55	$13,710.34
Interest Rate	$14,561.12	$4,981.92	$5,026.26
Combined	$27,376.56	$7,206.25	$7,411.84



The Equity portfolio shows the highest risk, while the Interest Rate portfolio is the most stable. The Combined portfolio benefits from diversification. SRM values are higher across the board, especially for Equity, reflecting stronger sensitivity to tail risks due to investor risk aversion (McNeil et al., 2005).

4. Backtesting Using Expanding Windows

This section validates the one-day Value at Risk (VaR) estimates for the Equity, Interest Rate, and Combined portfolios over the period from March 30, 2023, to March 28, 2025. An expanding window approach was used, beginning with an estimation period from March 30, 2021, to March 29, 2023 (504 observations).

Backtesting was conducted using:

- Kupiec's Unconditional Coverage (UC) Test (Kupiec, 1995)
- Christoffersen's Independence and Conditional Coverage Tests (Christoffersen, 1998)
- Lopez's Quadratic Probability Score (QPS) (Lopez, 1999)

All parametric (Normal, Student-t, Cornish-Fisher, Monte Carlo) and non-parametric models (Historical Simulation, Bootstrap Historical Simulation, Age-Weighted, Volatility-Weighted) were evaluated at a 95% confidence level.

4.1 Methodology and Key Formulas

Backtesting evaluations were performed using the following statistical measures:

- Exceedance is recorded when the realized return r_t breaches the VaR threshold VaR_t

$$\text{Exceedance occurs if } r_t < VaR_t$$

- Kupiec's Unconditional Coverage Test (1995): This test evaluates if the proportion of exceedances matches the expected confidence level. The likelihood ratio statistic is:

$$LR_{uc} = -2[\log((1-\pi)^{n-x} \times \pi) - \log((1-\alpha)^{n-x} \times \alpha^{x})]$$

Where:

- n = total out-of-sample observations

- x = observed number of breaches
- $\pi = \frac{x}{n}$ = observed exceedance frequency
- α = VaR confidence level (e.g., 5% for 95% VaR)

LR_{uc} follows a χ2(1) distribution, and p-values are derived accordingly. A p-value > 5% indicates that the model's predictions are statistically acceptable.

- Lopez's Quadratic Probability Score (QPS) (1999): The QPS is calculated as:

$$QPS = \frac{2}{n}(b_t - \alpha)^2$$

Where $b_t = 1$ if a breach occurred at time t, and 0 otherwise. Lower QPS values indicate better predictive accuracy.

4.2 Backtesting Results for the Equity Portfolio

The Equity portfolio's backtest results are summarized in Table 15 below:

Model	Expected Breaches	Observed Breaches	Kupiec p-value	Kupiec Test	LR_uc	QPS Score
Normal Distribution	24.85	17	0.9407	Accept	2.9219	0.0666
Student-t Distribution	24.85	17	0.9407	Accept	2.9219	0.0666
Cornish-Fisher Expansion	24.85	17	0.9407	Accept	2.9219	0.0666
Monte Carlo Normal	24.85	17	0.9407	Accept	2.9219	0.0666
Monte Carlo Student-t	24.85	17	0.9407	Accept	2.9219	0.0666
Historical Simulation	24.85	15	0.9787	Accept	4.7599	0.0593
Bootstrap Historical Simulation	24.85	14	0.9884	Accept	5.8810	0.0557
Age-Weighted Historical Simulation	24.85	35	0.0181	Reject	3.8941	0.1318
Volatility-Weighted Historical Sim.	24.85	35	0.0181	Reject	3.8941	0.1318

Table 15: Backtesting Results for Equity Portfolio

Most parametric models and standard historical simulation methods passed the Kupiec test, indicating acceptable VaR performance at the 95% confidence level. In contrast, Age-Weighted and Volatility-

Weighted Historical Simulations showed a significantly higher number of breaches, resulting in model rejection. Among all, the Historical Simulation and Bootstrapped Historical Simulation had the lowest QPS (Quadratic Probability Score), suggesting superior predictive accuracy.

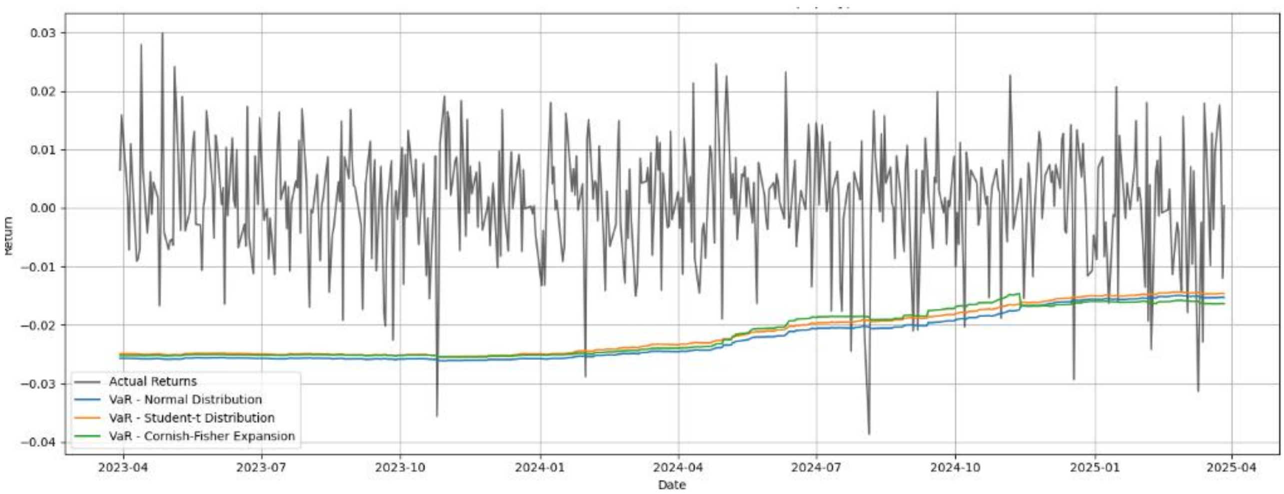


Figure 10: Parametric VaR Models vs Actual Returns (Equity Portfolio)

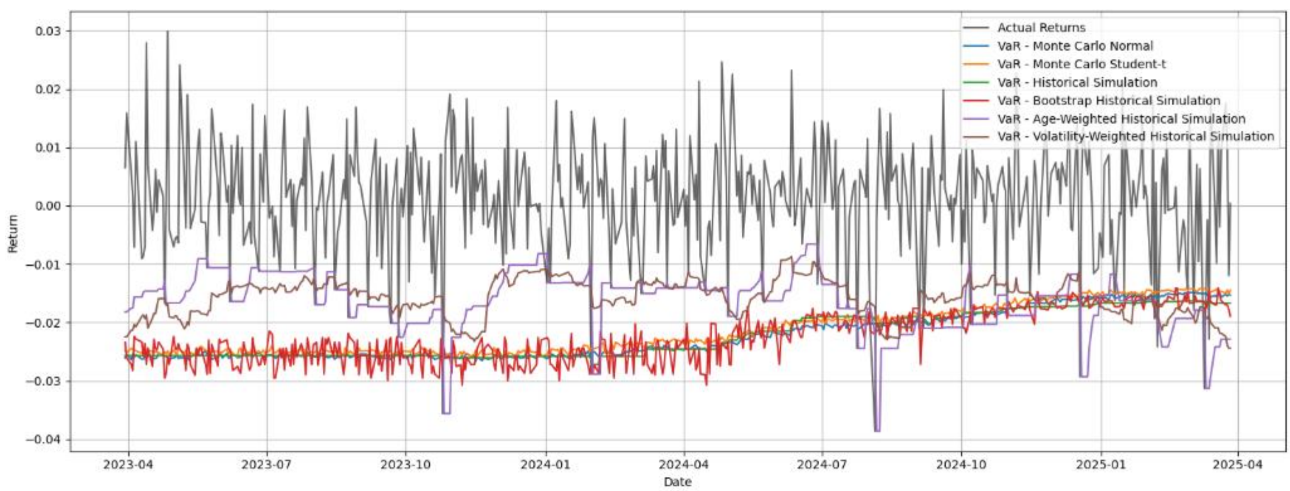


Figure 11: Non-Parametric VaR Models vs Actual Returns (Equity Portfolio)

4.3 Backtesting Results for the Interest Rate Portfolio

The Interest Rate portfolio's backtesting results are summarized in Table 16:

Model	Expected Breaches	Observed Breaches	Kupiec p-value	Kupiec Test	LR_uc	QPS Score
Normal Distribution	24.85	22	0.6771	Accept	0.3573	0.0847
Student-t Distribution	24.85	21	0.7496	Accept	0.6612	0.0811
Cornish-Fisher Expansion	24.85	23	0.5983	Accept	0.1485	0.0883
Monte Carlo Normal	24.85	21	0.7496	Accept	0.6612	0.0811
Monte Carlo Student-t	24.85	22	0.6771	Accept	0.3573	0.0847
Historical Simulation	24.85	23	0.5983	Accept	0.1485	0.0883
Bootstrap Historical Simulation	24.85	19	0.8665	Accept	1.5723	0.0738
Age-Weighted Historical Simulation	24.85	29	0.1683	Accept	0.6940	0.1100
Volatility-Weighted Historical Sim.	24.75	31	0.0857	Accept	1.5435	0.1177

Table 16: Backtesting Results for Interest Rate Portfolio

No major violations were found. The Student-t and Bootstrap models performed best, with few breaches and low QPS scores, highlighting their advantage in capturing fat tails.

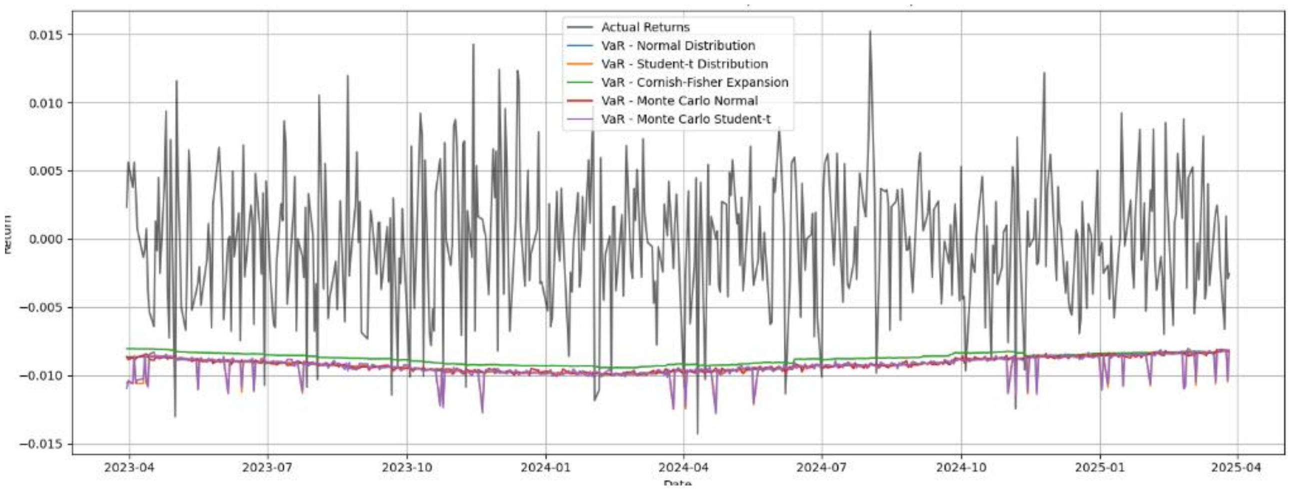


Figure 12: Parametric VaR Models vs Actual Returns (Interest Rate Portfoliio)

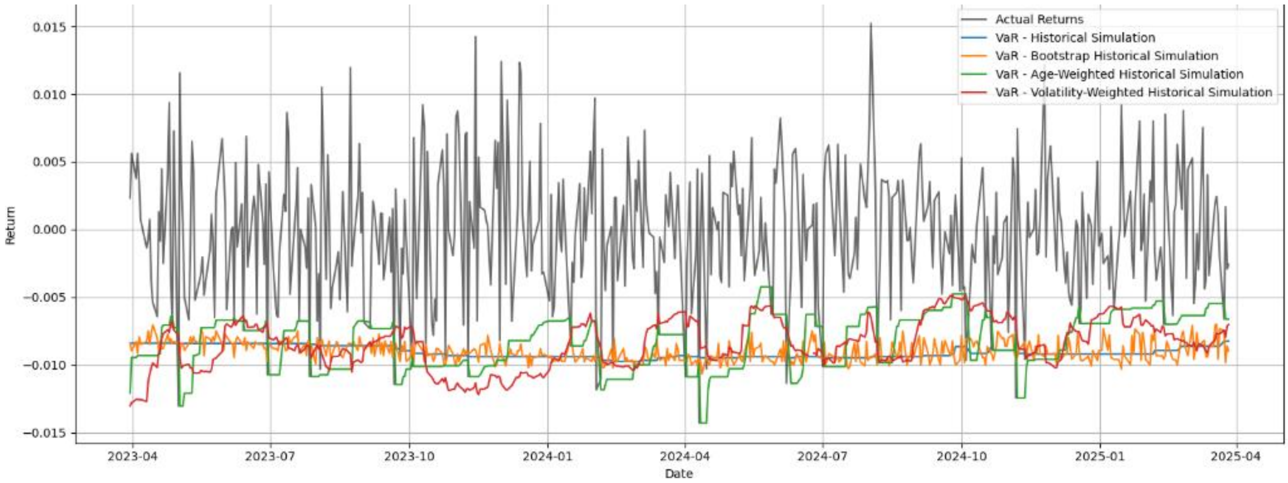


Figure 13: Non-Parametric VaR Models vs Actual Returns (Interest Rate Portfolio)

4.4 Backtesting Results for the Combined Portfolio

The Combined portfolio's backtesting results are presented in Table 17:

Model	Expected Breaches	Observed Breaches	Kupiec p-value	Kupiec Test	LR_uc	QPS Score
Normal Distribution	24.85	12	0.9972	Accept	8.5758	0.0485
Student-t Distribution	24.85	12	0.9972	Accept	8.5758	0.0485
Cornish-Fisher Expansion	24.85	15	0.9787	Accept	4.7599	0.0593
Monte Carlo Normal	24.85	12	0.9972	Accept	8.5758	0.0485
Monte Carlo Student-t	24.85	12	0.9972	Accept	8.5758	0.0485
Historical Simulation	24.85	9	0.9998	Accept	13.9448	0.0376
Bootstrap Historical Simulation	24.85	11	0.9987	Accept	10.1732	0.0448
Age-Weighted Historical Simulation	24.85	31	0.0891	Accept	1.4905	0.1173
Volatility-Weighted Historical Sim.	24.85	29	0.1683	Accept	0.6940	0.1100

Table 17: Backtesting Results for Combined Portfolio

Both parametric and non-parametric models demonstrated strong performance in the backtesting results. Among these, the **Bootstrap Historical Simulation** achieved the lowest QPS score, indicating the best overall performance, closely followed by the **Student-t models**. These models consistently exhibited accurate risk estimation, with low observed breaches and high Kupiec p-values, further validating their reliability in assessing the combined portfolio's risk

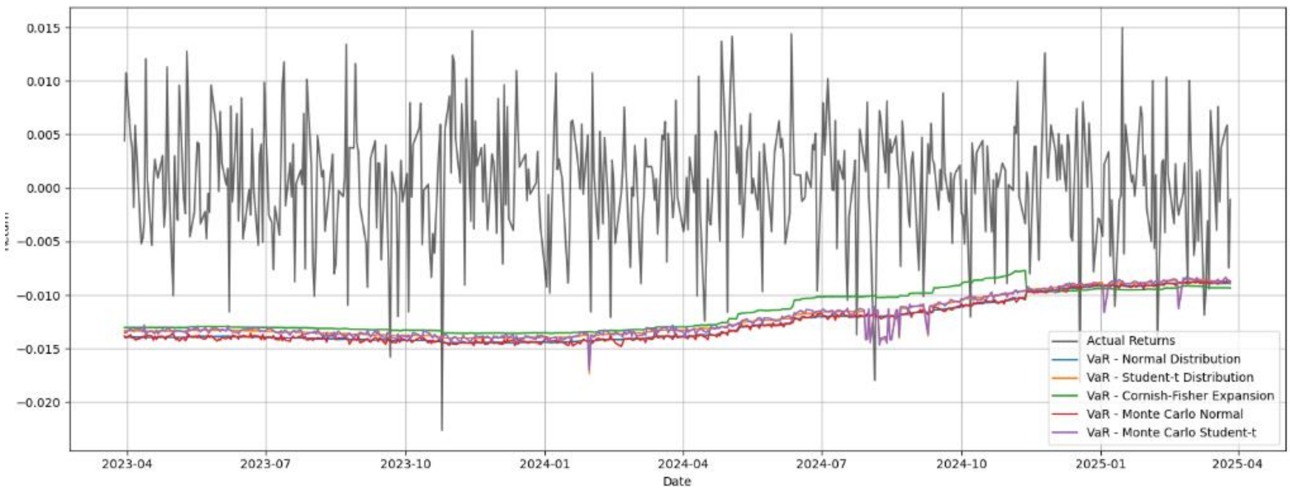


Figure 14: Parametric VaR Models vs Actual Returns (Combined Portfolio)

Figure 15: Non-Parametric VaR Models vs Actual Returns (Combined Portfolio)

5. Sensitivity Testing and Robustness

This section examines the sensitivity of Value at Risk (VaR) and Expected Shortfall (ES) estimates to two key parameters: (i) the confidence level and (ii) the decay factor λ in Volatility-Weighted Historical Simulation (VWHS). Sensitivity testing ensures that the selected risk models remain robust across different risk environments (Dowd, 2005).

5.1 Confidence Level Sensitivity (90% to 99%)

To explore how VaR and ES behave under varying risk tolerances, the confidence level was varied from 90% to 99% in 1% increments. Historical Simulation (HS) was applied to the Equity portfolio, while VWHS was used for the Interest Rate and Combined portfolios based on prior backtesting performance.

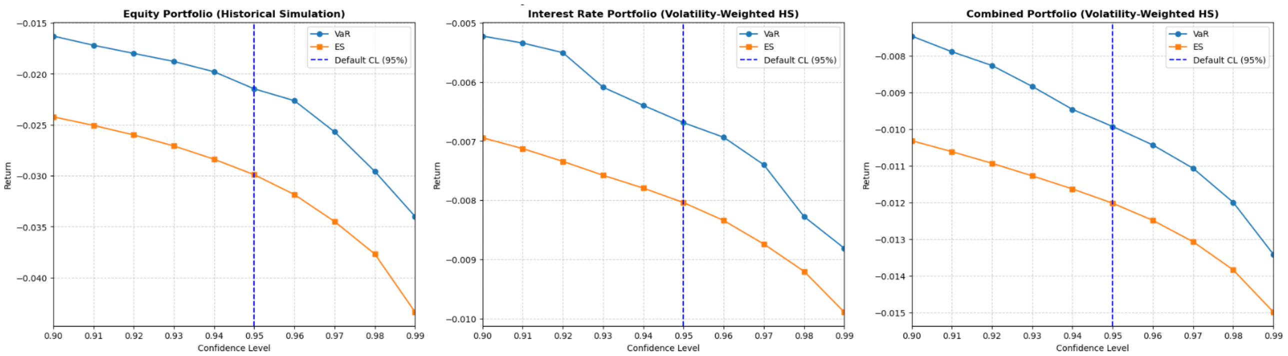


Figure 5.1: Confidence Level Sensitivity for VaR and ES

As expected, both VaR and ES increased with higher confidence levels, indicating greater potential losses under extreme market conditions. Notably, ES rose more sharply than VaR, highlighting deeper tail risk. The Equity portfolio showed the greatest sensitivity due to higher return volatility, while the Interest Rate portfolio had a flatter curve, reflecting lower market risk. The Combined portfolio displayed moderate sensitivity, benefiting from diversification. These results align with theoretical expectations from Jorion (2007).

5.2 Lambda Sensitivity (Decay Factor Sensitivity)

We further analysed how varying the exponential decay factor λ in VWHS affects risk estimates. Lambda was tested between 0.90 and 0.99 in increments of 0.01. A higher λ places more weight on older returns, while a lower λ emphasizes recent volatility.

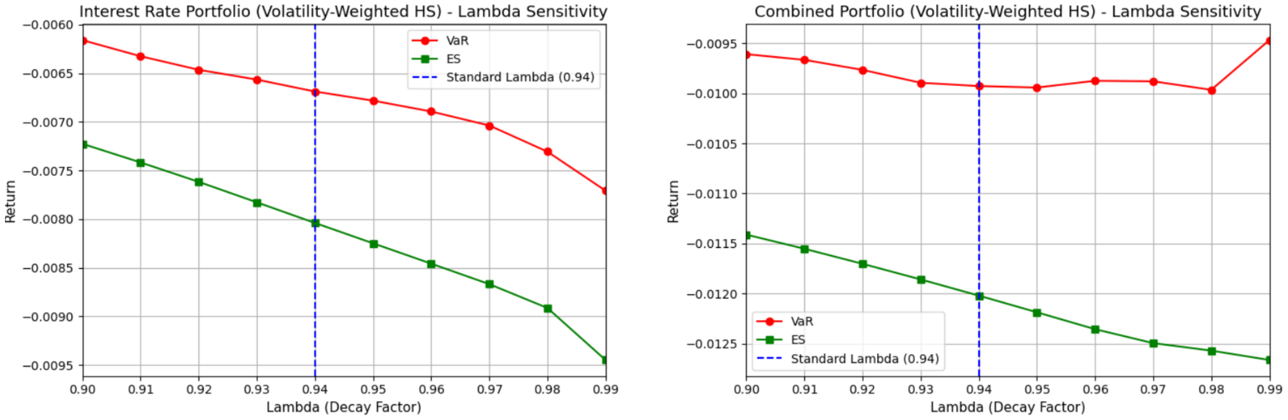


Figure 5.2: Lambda Sensitivity for VaR and ES

Findings show that reducing λ from 0.99 to 0.90 leads to larger (more negative) VaR and ES values, as the model becomes more responsive to recent volatility spikes. Conversely, a higher λ smoothens volatility but may underreact to new market shocks. A decay factor around λ = 0.94 provided a good balance between responsiveness and stability, aligning with recommendations from Pritsker (2006).

6 Conclusion

This report assessed the risk exposures of three $1 million portfolios — Equity, Interest Rate, and Combined — using various VaR and ES approaches, including Normal and Student-t distributions, Cornish-Fisher expansion, Monte Carlo simulations, and Historical Simulation. Backtesting results with Kupiec's Unconditional Coverage Test and Lopez's QPS recommended Historical Simulation for the Equity portfolio, Student-t for the Interest Rate portfolio, and Bootstrap Historical Simulation for the Combined portfolio. Sensitivity analyses confirmed model robustness under varying confidence levels and volatility assumptions. Given recent market trends, especially inflation and monetary tightening, the study emphasizes the need for flexible, distribution-aware models to support effective risk management for the Board Risk Committee.

7. References

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Appendix B: Rationale for Interest Rate Portfolio Selection

The Interest Rate portfolio was constructed to ensure balanced exposure to interest rate risk across different maturities of U.S. Treasury securities (2-year, 5-year, 10-year, and 30-year). The portfolio employs a DV01-balanced allocation, where each maturity contributes 25% to the total DV01 ($105.71 per maturity), ensuring equal sensitivity to a 1 basis point change in interest rates. This approach mitigates bias toward any single maturity and provides a diversified exposure to the yield curve, capturing dynamics such as steepening or flattening (Hull, 2023). The 2Y, 5Y, 10Y, and 30Y Treasuries were selected to represent short, medium, and long-term interest rate risks, aligning with standard market risk management practices (Jorion, 2007). Notional amounts were calibrated to achieve the target DV01 (e.g., $587297.15 for 2Y), reflecting the inverse relationship between bond duration and notional exposure. This structure supports robust backtesting and sensitivity analyses, as it standardizes risk contributions across maturities, facilitating accurate VaR and ES estimates for the Interest Rate and Combined portfolios.