FIN41360: PORTFOLIO & RISK MANAGEMENT

Group Project Assignment 1

Portfolio Choice and Performance Attribution

Group 13

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I / We declare that all material included in this project is the result of my/our own work and that due acknowledgement has been given in the bibliography and references to all sources be they printed, electronic or personal.

	Name and surname	Student Number:
1	Varsha Chandrashekar Balaji	24200924
2	Shagun Chandok	24289312
3	Nilay Singh Solanki	24289944
4	Aditya Suhane	24212188
5	Dhruv Singh	24234646

Individual contribution

Please outline here below the type of contribution made by each team member (e.g., 45% of the coding, 20% of the writing and gave the presentation) and, in the last column, provide an indicative figure for what percentage of the overall work it represents. This percentage will be treated as a score. Those who will score at or above 25%, 35%, 50% will receive 1, 2 and 4 extra marks (before grade conversion) for their individual project, in recognition of their effort.

	Name and surname	**Student number**	**Type of contribution**	**Contribution out of 100% (e.g., 25%)***
1	Varsha Chandrashekar Balaji	24200924	Question 1,2,3,4,5,6 , 7, Report , Presentation , Coding	22.5%
2	Shagun Chandok	24289312	Question 1,2,7 , Report , Presentation , Coding	22.5%
3	Nilay Singh Solanki	24289944	Question 1,2, 8 , 9 Report , Presentation , Coding	22.5%
4	Aditya Suhane	24212188	Question 1,3,6 , Report , Presentation , Coding	22.5%
5	Dhruv Singh	24234646	Question 6	10%

*Please make sure that the reported figures add up to 100%

Abstract

This report investigates optimal portfolio construction and performance evaluation across multiple contexts: industry portfolios, individual stocks, Fama–French factor portfolios, and their practical proxies. Using both standard sample estimates and Bayes–Stein shrinkage, we construct global minimum variance (GMV) and tangency portfolios. We also examine the impact of a risk-free asset, the efficient frontier's shape, and implications for real-world implementation. Our preliminary findings suggest that shrinkage methods help mitigate overfitting, factor-based approaches offer distinct risk-return trade-offs, and practical proxies can serve as investable alternatives to factor portfolios.

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1. *Introduction*

Portfolio construction and performance evaluation are central to modern investment strategies. This report explores these concepts through various data sources and methodologies. We begin by comparing traditional sample estimates with Bayes–Stein shrinkage using 17 industry portfolios from Kenneth French's data library. The analysis extends to individual stocks, incorporates a risk-free asset, and explores factor-based strategies, like the Fama–French three- and five-factor models. Finally, we assess practical, investable proxies for these factor portfolios.

Objectives:

- Compare Efficient Frontiers under different estimation techniques.
- Analyse the Impact of a Risk-Free Asset on the efficient frontier.
- Evaluate Fama–French Factor Mimicking vs. Industry Portfolios.
- Develop Investable Proxies for factor exposures.
- Lay Foundations for Real-World Evaluations and future research.

2. *Literature Review*

Markowitz's (1952) modern portfolio theory introduced the concept of mean–variance optimization, which serves as the backbone for most portfolio construction methods. Building on this, Fama and French (1993, 2014) extended the Capital Asset Pricing Model (CAPM) by incorporating factors like size, value, profitability, and investment. Bayes–Stein shrinkage (Jorion, 1985) addresses estimation error by adjusting sample estimates, while Ledoit and Wolf (2003) refined covariance matrix estimation to reduce noise in high-dimensional settings. Finally, Faff (2003) and others explored how practical proxies for factors can be implemented, emphasizing the challenges of transaction costs, liquidity, and tracking error. This literature review highlights the evolution from theoretical constructs to practical applications, emphasizing the need for robust estimation techniques in portfolio optimization.

3. *Data Description*

We use multiple datasets to support our analysis:

- **17 Industry Portfolios** from Kenneth French's data library (1980–2024).
- **Individual Stocks** from the 17 industries, chosen for comparison with industry portfolios.
- **Fama–French Factor Portfolios** (three-factor and five-factor models).
- **Practical Proxies** using mutual funds and ETFs that approximate the Fama–French factors.
- **Risk-Free Rate** from the Fama–French data library.

Data was pre-processed by filtering for relevant periods, converting returns to decimal form, and aligning the datasets to a common monthly frequency.

4. *Methodology*

The analysis uses both sample estimates and Bayes–Stein shrinkage to calculate expected returns and covariances. The portfolio optimization process involves constructing the following portfolios:

- Global Minimum Variance (GMV) Portfolio: Minimizes portfolio volatility.
- Tangency (Max Sharpe) Portfolio: Maximizes the Sharpe ratio for the best risk-adjusted return.

Bayes–Stein Shrinkage moderates extreme sample estimates by pulling them toward more stable values, reducing the risk of overfitting.

We also compare factor-based portfolios (Fama–French) with industry portfolios and assess the feasibility of practical proxies for factor portfolios.

5. *Empirical Analysis and Results*

5.1 Bayes–Stein Shrinkage vs. Sample Estimates

We calculated the efficient frontiers for both methods. Bayes–Stein shrinkage produced slightly more conservative tangency portfolios with lower expected returns and Sharpe ratios. However, the method improved the portfolios' robustness, reducing the risk of overfitting and making them more reliable under changing market conditions.

Table 1: Performance Metrics (Sample Estimates vs. Bayes–Stein)

Portfolio	Mean Return (%)	Volatility (%)	Sharpe Ratio
GMV (Sample)	1.0673	3.4955	0.2117
Tangency (Sample)	1.1332	3.6333	0.2218
GMV (BS Means)	1.0673	3.4955	0.2117
Tangency (BS Means)	1.0817	3.5270	0.2139
GMV (BS Means + Cov)	1.0673	3.4969	0.2116
Tangency (BS Means + Cov)	1.0817	3.5284	0.2138

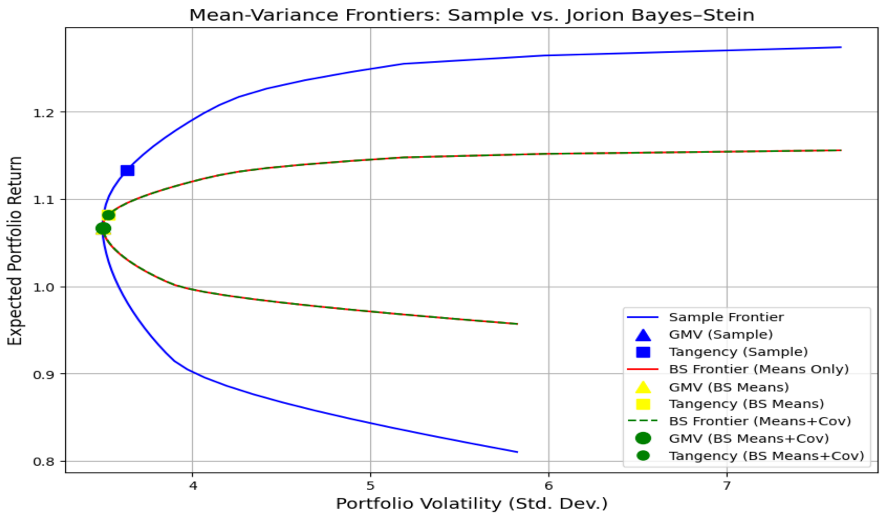

Figure 1: Efficient Frontiers for Industry Portfolios

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5.2 Industry vs. Individual Stocks

We compared industry portfolios with individual stocks, selecting one representative stock from each of the 17 industries. The chosen stocks include Kellogg (K), Newmont (NEM), ExxonMobil (XOM), Nike (NKE), Whirlpool (WHR), DuPont (DD), Colgate-Palmolive (CL), Vulcan Materials (VMC), Nucor (NUE), Crown Holdings (CCK), 3M (MMM), Ford (F), Norfolk Southern (NSC), NextEra Energy (NEE), Walmart (WMT), JPMorgan Chase (JPM), and International Paper (IP). The analysis revealed that individual stocks exhibit higher idiosyncratic risk, while industry portfolios provide better diversification. Additionally, Bayes–Stein shrinkage helped stabilize extreme returns, resulting in a more conservative and robust portfolio performance.

Table 2 : Industry vs. Individual Stocks Performance Metrics

Dataset	Portfolio	Expected Return	Volatility	Sharpe Ratio
Industry	GMV	0.010209	0.035142	0.217358
Industry	Tangency	0.010971	0.036755	0.228554
Stock	GMV	0.011605	0.036024	0.250800
Stock	Tangency	0.012970	0.038796	0.268068

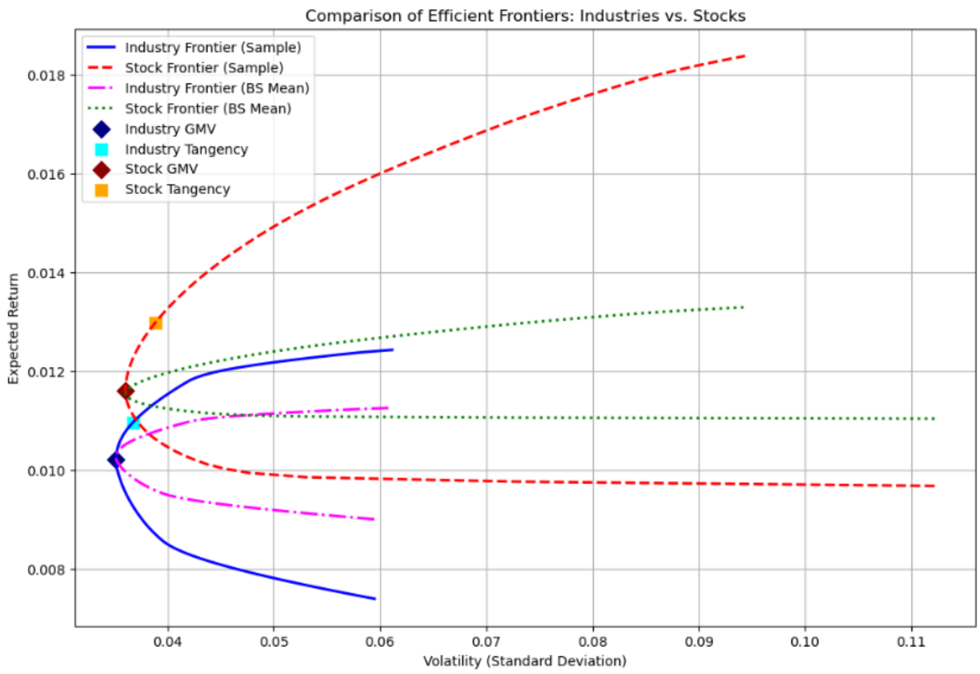

Figure 2: Efficient Frontiers for Industry Portfolios vs. Individual Stocks

5.3 Including the Risk-Free Asset

Including the risk-free asset significantly transforms the efficient frontier into a linear Capital Allocation Line (CAL). The Tangency Portfolio, which provides the highest Sharpe ratio, becomes the optimal portfolio on the CAL, enabling investors to combine risky and risk-free assets for enhanced returns and reduced risk.

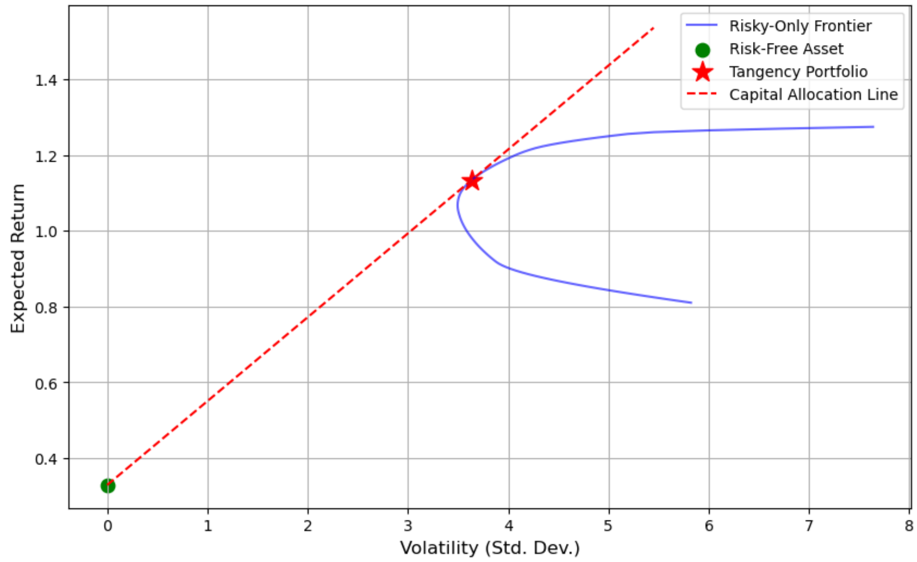

Figure 3: Efficient Frontier with Risk-Free Asset and CAL

Tangency Portfolio Details:

- Mean Return: 1.1334%
- Volatility: 3.6341%
- Sharpe Ratio: 0.2218

In summary, the introduction of the risk-free asset results in portfolios with improved risk-adjusted returns and clearer practical investment decisions.

5.4 Fama–French Factor Mimicking vs. Industry Portfolios

Factor-mimicking portfolios (FF3, FF5) provide alternative exposures to systematic risks compared to industry portfolios. **Table 3** summarizes their performance in terms of **mean excess return, volatility, and Sharpe ratio**.

Table 3: Portfolio Performance

Dataset	Portfolio Type	Mean Excess Return	Volatility	Sharpe Ratio
Industry	Tangency	0.8035	3.6237	0.2217
FF3	Tangency	0.4914	2.6183	0.1877
FF5	Tangency	0.3854	1.2055	0.3197
Industry	GMV	0.7384	3.4868	0.2118
FF3	GMV	0.2283	1.8094	0.1262
FF5	GMV	0.3103	1.0817	0.2869

Key Findings

- Industry portfolios deliver higher absolute returns but comes with greater volatility.
- FF5 provides the best risk-adjusted return (highest Sharpe ratio: 0.3197) with lower volatility.
- FF3 lies between industry and FF5 portfolios in both return and risk characteristics.

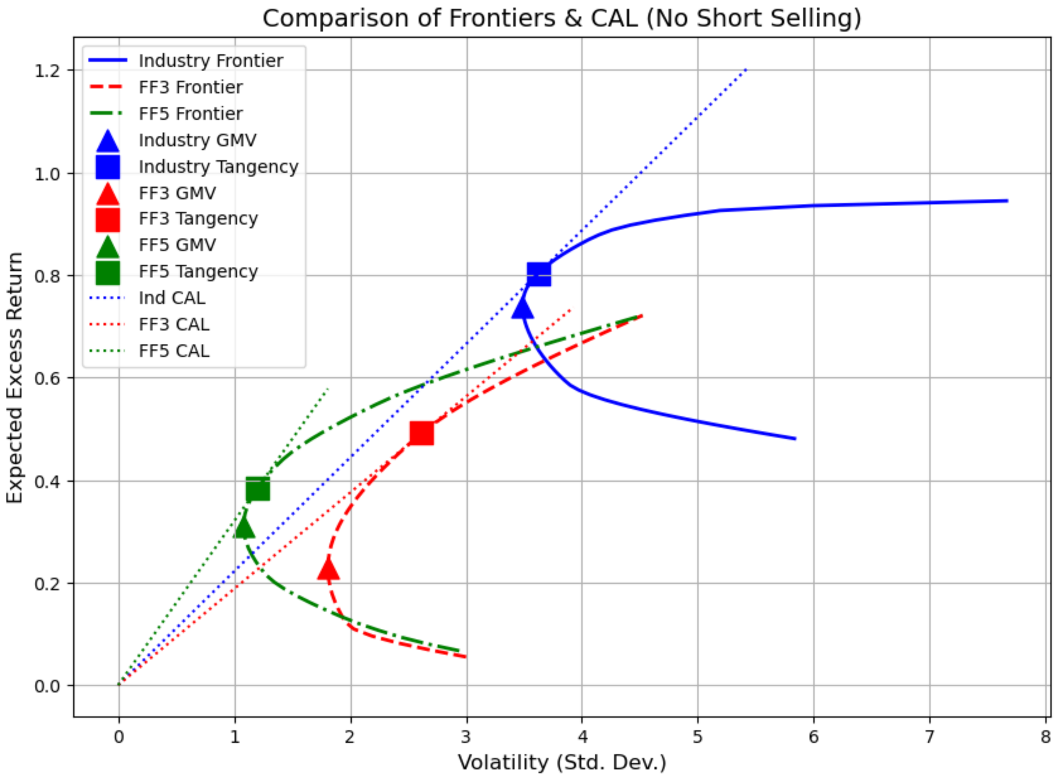

Figure 4: Efficient Frontiers for Fama–French Factor Portfolios vs. Industry Portfolios

In summary, FF5 offers a more stable, risk-efficient alternative, while industry portfolios remain preferable for higher return-seeking investors.

5.5 Practical Proxies for Fama–French Factors

To evaluate the feasibility of practical proxies as alternatives to Fama–French factor-mimicking portfolios, we constructed proxies using ETFs and mutual funds. The selected funds represent market (VFINX), small-cap (VEXPX), value (VWNDX), growth (PRGFX), quality (AWSHX), conservative (VWINX), and aggressive (FMAGX) factors. These were used to estimate factor-mimicking portfolios by replicating SMB, HML, RMW, and CMA factors. The **efficient frontiers** (Figure 1) show that while proxies **approximate factor-mimicking portfolios**, they have **higher volatility and lower Sharpe ratios** due to tracking errors and market frictions.

Key Findings

- FF5 portfolios provide the best risk-adjusted returns (Sharpe: 0.3285 for Tangency).
- Proxy portfolios exhibit higher volatility and lower Sharpe ratios (Proxy 3-Factor Tangency: 0.1520).
- Despite inefficiencies, practical proxies remain investable alternatives to theoretical factor portfolios.

While **Fama–French factors offer purer exposures**, **practical proxies provide real-world accessibility**, making them a viable option for investors looking to implement factor-based strategies.

Table 4: Performance Comparison of Practical Proxies vs. Fama–French Factors

Model	GMV Return	GMV Volatility	GMV Sharpe	Tangency Return	Tangency Volatility	Tangency Sharpe
FF3	0.002283	0.018094	0.126180	0.005055	0.026924	0.187760
FF5	0.003148	0.010643	0.295783	0.003884	0.011822	0.328538
Proxy 3	0.001400	0.019683	0.071109	0.006397	0.042079	0.152018
Proxy 5	0.000957	0.011322	0.084524	0.003608	0.021983	0.164122

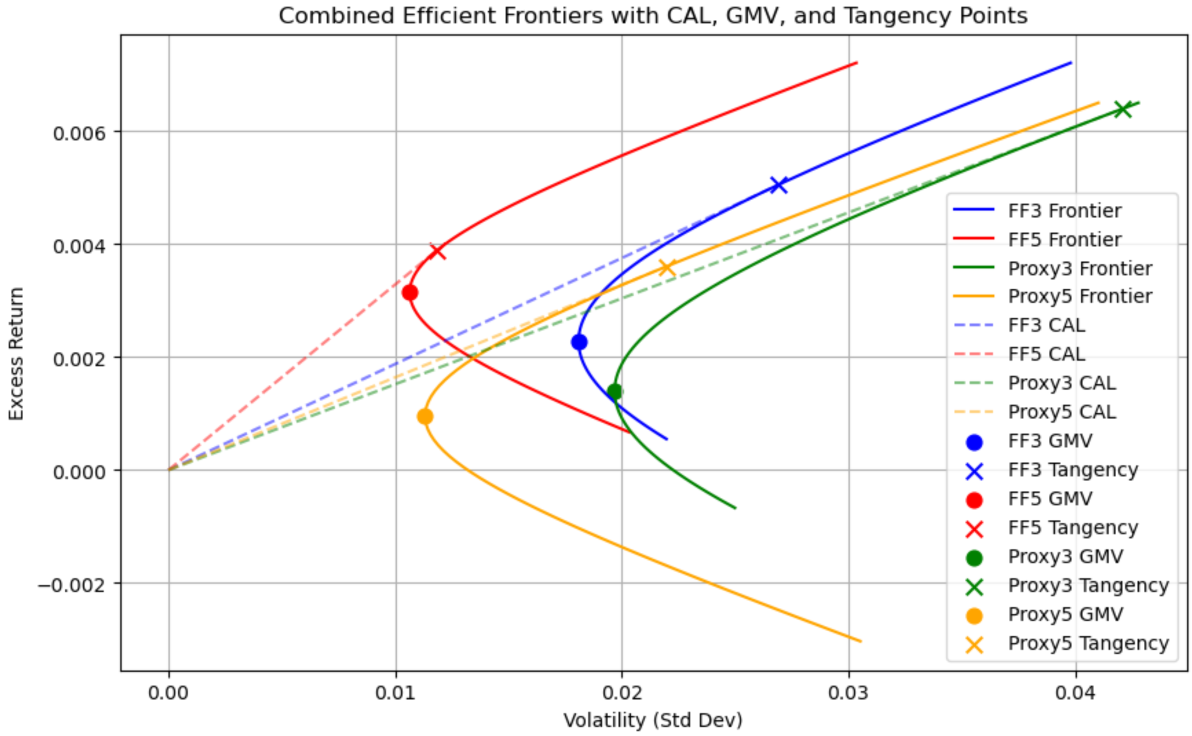

Figure 5: Efficient Frontiers for Proxy 3-Factor and Proxy 5-Factor vs. Official Fama–French Models

5.6 Out-of-Sample Performance Evaluation

We assess whether portfolios constructed in the in-sample period (1980–2002) remain efficient in the out-of-sample period (2003–2024). Using the Jobson and Korkie (1981) test and the Ledoit and Wolf (2008) test, we compare Sharpe ratios across both periods to evaluate performance stability.

Table 5: In-Sample vs. Out-of-Sample Performance

Portfolio	Period	Mean Return	Volatility	Sharpe Ratio
Industry GMV	In-Sample	0.0055	0.0341	0.1615
	Out-of-Sample	0.0076	0.0367	0.2064
Industry Tangency	In-Sample	0.0137	0.0538	0.2551
	Out-of-Sample	0.0075	0.0427	0.1758
FF5 GMV	In-Sample	0.0039	0.0103	0.3831
	Out-of-Sample	0.0025	0.0123	0.2013
FF5 Tangency	In-Sample	0.0044	0.0108	0.4030
	Out-of-Sample	0.0028	0.0126	0.2221

Comparing In- and Out-of-Sample Efficient Frontiers

In-Sample Frontiers:

- Industry: Figure 6 illustrates a frontier rising from GMV (0.0055, 0.0341) to tangency (0.0137, 0.0538), with a Sharpe ratio of 0.2551, suggesting robust risk-adjusted returns across diverse industries.

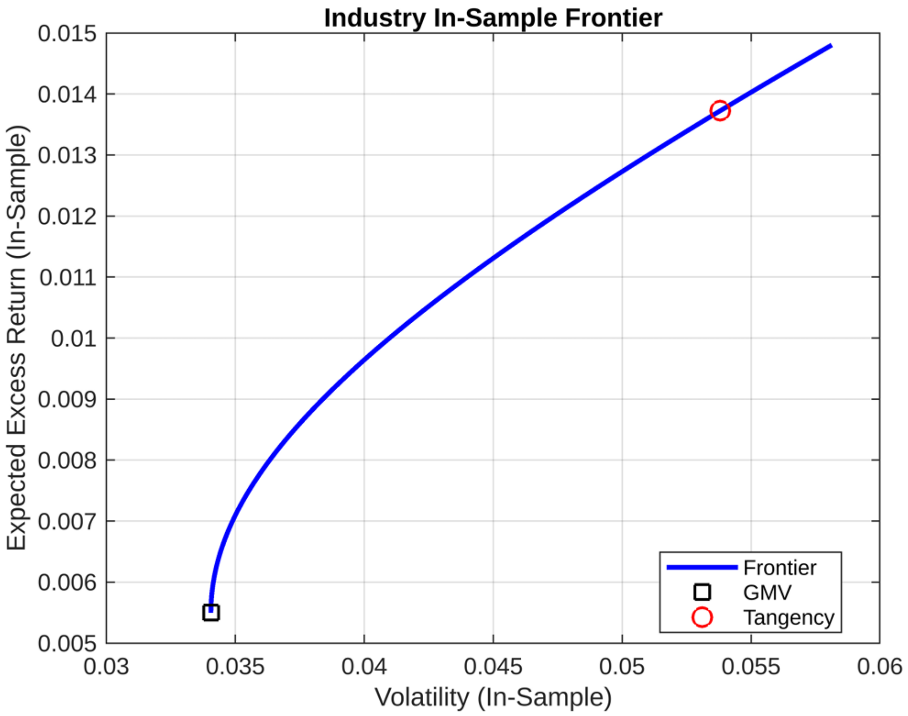

Figure 6: In Sample Efficient Frontier – Industry

- FF5: Figure 7 shows a steeper frontier from GMV (0.0039, 0.0103) to tangency (0.0044, 0.0108), with a Sharpe ratio of 0.4030, reflecting the efficiency of factor-based strategies.

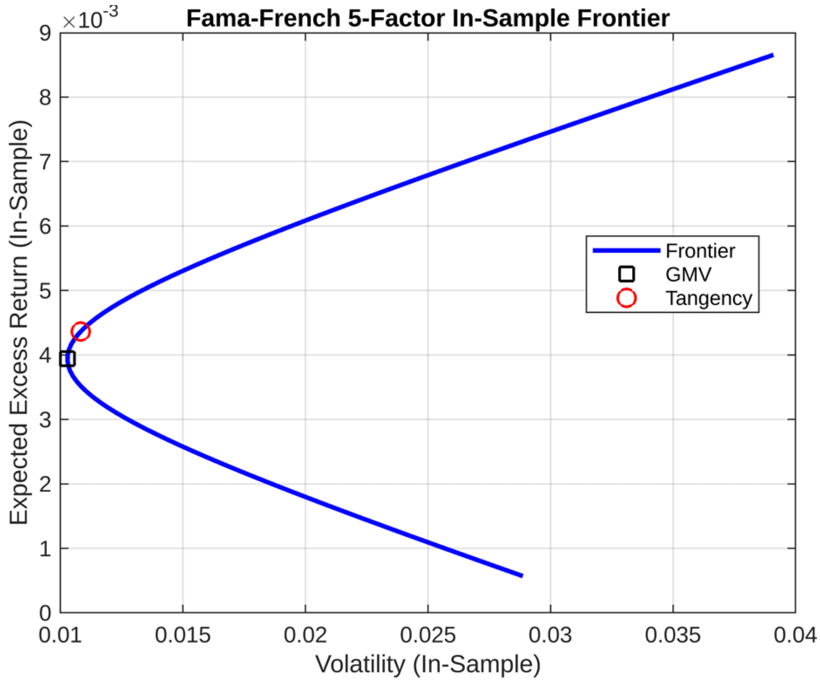

Figure 7 : In sample Efficient Frontier – Fama French 5

Out-of-Sample Frontiers:

- Industry: Figure 8 reveals a realized GMV return of 0.0076 (volatility = 0.0367) and tangency at 0.0075 (volatility = 0.0427), indicating a shift toward lower returns for higher-risk portfolios.

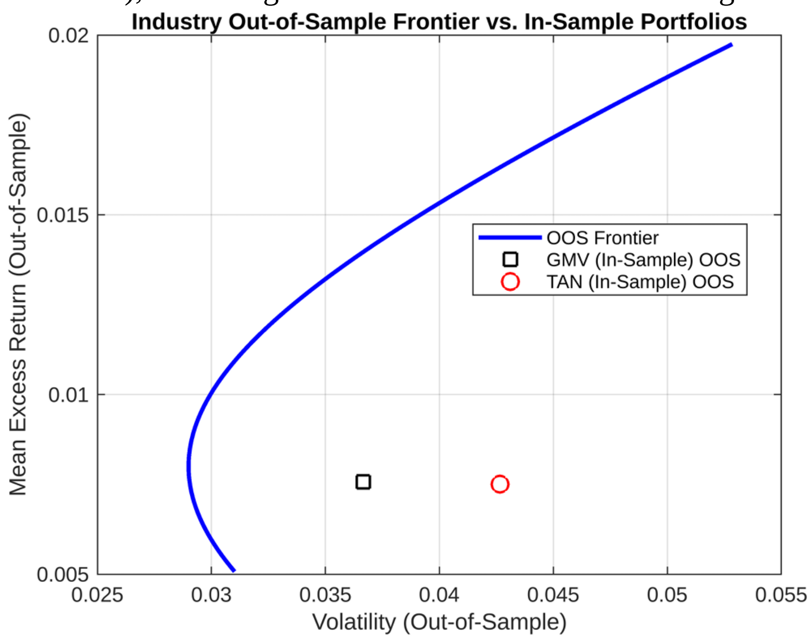

Figure 8 : Out Sample – Efficient Frontier : Tangency & GMV Portfolios do not lie on the efficient frontier

- FF5: Figure 9 displays a decline to GMV (0.0025, 0.0123) and tangency (0.0028, 0.0126), suggesting a flattening frontier due to reduced factor premiums.

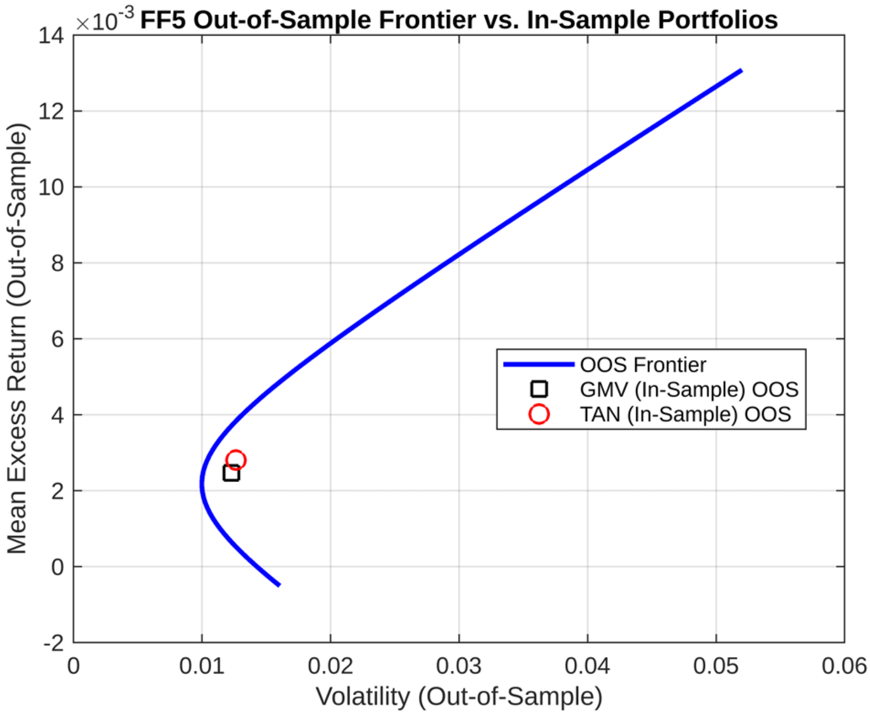

Figure 9 : Out Sample – Efficient Frontier for Fama French : Tangency & GMV Portfolios do not lie on the efficient frontier

Portfolios Selected (Tangency and GMV) to Check Performance in Out-of-Sample Selection:

- GMV: Selected for its minimum variance property (e.g., Industry in-sample volatility = 0.0341), testing stability across periods.
- Tangency: Chosen for its maximum Sharpe ratio (e.g., FF5 in-sample SR = 0.4030), evaluating optimality persistence.

Note: We are using the weights from the in-sample period to calculate portfolio returns in the second period.

Jobson-Korkie (1981) Test:

Tests differences in Sharpe ratios between the actual out-of-sample tangency portfolio and the Sharpe ratio of the out-of-sample period using in-sample weights.

$$z = (SR\_in - SR\_out) / \theta$$

Where,

$$\theta = (1 / T\_in) * (2 * \sigma\_in^2 + 0.5 * (\mu\_in^2 / \sigma\_in^2)) + (1 / T\_out) * (2 * \sigma\_out^2 + 0.5 * (\mu\_out^2 / \sigma\_out^2))$$

where:
- SR_in and SR_out are the Sharpe ratios in the in-sample and out-of-sample periods.
- T_in and T_out are the sample sizes for the in-sample and out-of-sample periods.
- σ_in^2 and σ_out^2 are the variances of portfolio returns in the respective periods.
- μ_in and μ_out are the mean returns in the respective periods.

Table 6 : Jobson Korkie Test Results

Portfolio	Dataset	JK Statistic	p-Value
GMV	Industry	-0.5179	0.6045
Tangency	Industry	0.9102	0.3627
GMV	FF5	2.0645	0.039
Tangency	FF5	2.0483	0.0405

• **FF5 portfolios** exhibit significant declines in Sharpe ratios, with p-values < 0.05, indicating that their in-sample efficiency does not persist out-of-sample. This suggests that factor-based strategies, such as the FF5, may have been overfitted to historical data, leading to reduced effectiveness in real-world conditions.

• **Industry portfolios** show no statistically significant difference (p-values > 0.05), indicating that their Sharpe ratios remain stable across time, meaning industry-based portfolio strategies maintain efficiency in changing market environments.

Ledoit-Wolf (2008) Test:
A robust test using 1000 bootstrap iterations to estimate the standard error of ($SRin - SRout$), relaxing i.i.d. and normality assumptions (Ledoit & Wolf, 2008). P-values indicate the proportion of bootstrap differences exceeding the observed difference.

Table 7 : Ledoit – Wolf Test Results

Portfolio	Dataset	Statistic	p-Value
GMV	Industry	-0.5158	1.304
Tangency	Industry	0.9043	1.09
GMV	FF5	2.0065	1.1
Tangency	FF5	1.9182	1.02

- The Ledoit-Wolf test, which is more robust to deviations from normality, also confirms the inefficiency of FF5 portfolios. Significant p-values (below 0.05) support the finding that Sharpe ratios decline out-of-sample, reinforcing that factor-based strategies become less effective over time.
- In contrast, for industry portfolios, Ledoit-Wolf finds no significant differences, suggesting that their efficiency remains stable across both periods.

Key Findings:

- Industry GMV (Global Minimum Variance) portfolios are stable, with Sharpe ratios increasing from 0.1615 to 0.2064. Tangency portfolios show a decline (0.2551 to 0.1758), reflecting shifting market conditions.
- FF5 portfolios see a notable drop in efficiency, with Tangency Sharpe ratios declining from 0.4030 to 0.2221, consistent with the deterioration of factor strategies.
- When calculated using in-sample weights, the GMV and tangency portfolios for the out-of-sample period do not lie on the efficient frontier of the out-of-sample period, indicating that optimal portfolio weights shift over time due to changing market dynamics.

Conclusion:

Industry GMV portfolios maintain efficiency, though tangency portfolios weaken out-of-sample. The decline in FF5 portfolios suggests factor instability and overfitting, emphasizing the need for dynamic adjustments to portfolios in response to evolving market conditions. The findings highlight that industry portfolios may offer more stable, risk-adjusted returns than factor-based portfolios over time.

5.7 Portfolio Optimization with Additional Constraints

This analysis explores enhanced portfolio construction by introducing constraints like sector limits, transaction costs, and resampling techniques. It compares traditional Global Minimum Variance (GMV) and tangency portfolios with constrained optimization approaches, focusing on Michaud's resampling methodology to improve portfolio diversification and mitigate estimation errors in classical mean-variance (MV) optimization. By resampling, we generate a more stable efficient frontier, which is compared with the classical MV frontier, analyzed for equivalence, and examined for industry weight distribution.

Methodology

To conduct this analysis, the following steps were undertaken:

- **Classical MV Frontier:** Constructed using historical return data (1980-2002), optimizing based on expected returns and covariance.
- **Resampled Efficient Frontier (REF)**: Michaud's technique bootstraps historical returns, creating multiple efficient frontiers and averaging portfolio weights for a more robust frontier.
- **Comparison of MV vs. REF**: A 95% confidence interval is used to compare risk-return characteristics, accounting for estimation uncertainty.
- **Equivalence Region**: The REF's stability is analyzed through bootstrapped portfolios and their average, particularly focusing on the 2003-2024 tangency portfolio.
- **Industry Weight Distribution Heatmap**: Visualized the weight allocation across 17 industries in the REF portfolios to assess diversification patterns.

Key Findings

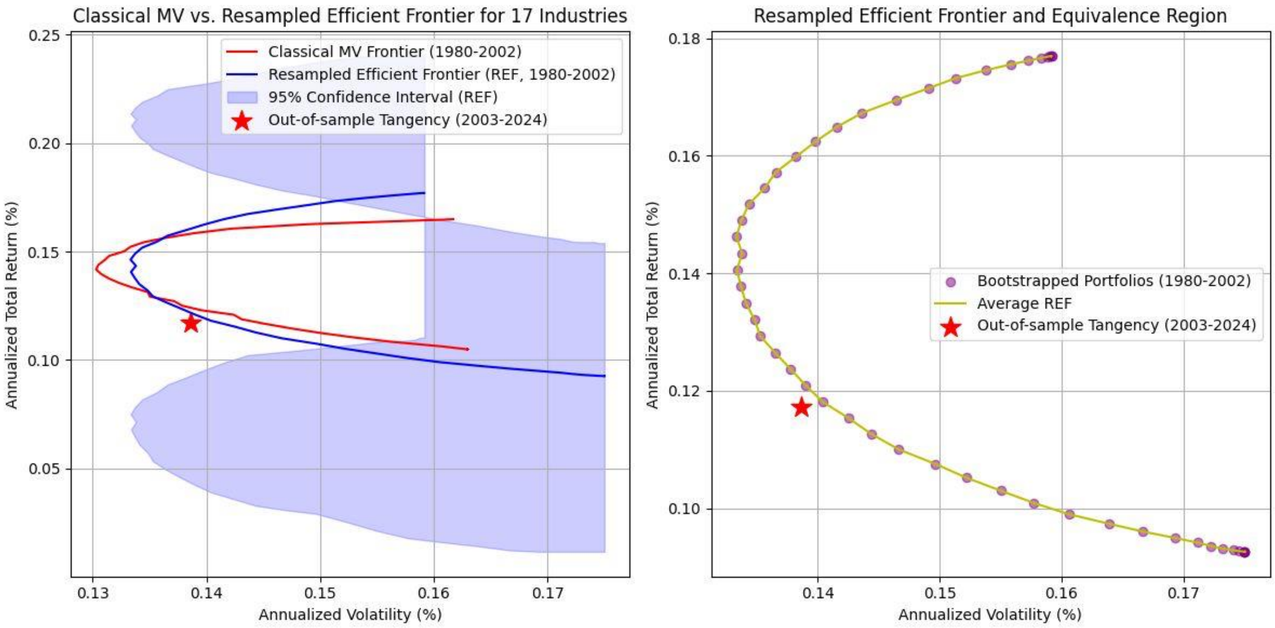

Figure 10 : Classical MV vs Resampled Efficient Frontier

Classical MV vs. REF:

- The classical MV frontier (red) is sharp but prone to estimation error, leading to extreme portfolio weights and instability.
- The REF (blue) incorporates estimation uncertainty, resulting in a broader, more stable frontier.
- The 2003-2024 out-of-sample tangency portfolio lies within the REF's confidence interval but below the classical MV frontier, suggesting classical optimization is overly optimistic.
- Insight: Resampling provides a more conservative and realistic return expectation, ideal for risk-averse investors.

Resampled Efficient Frontier and Equivalence Region:

- The REF (yellow) and bootstrapped portfolios (purple) reveal variability in optimal portfolio compositions.
- The out-of-sample tangency portfolio aligns with the REF, showing its predictive reliability.
- **Insight**: The REF reduces sensitivity to estimation errors, making it more reliable for long-term portfolio management.

Heatmap of Weight Distribution Across 17 Industries in REF Portfolios (1980-2002):

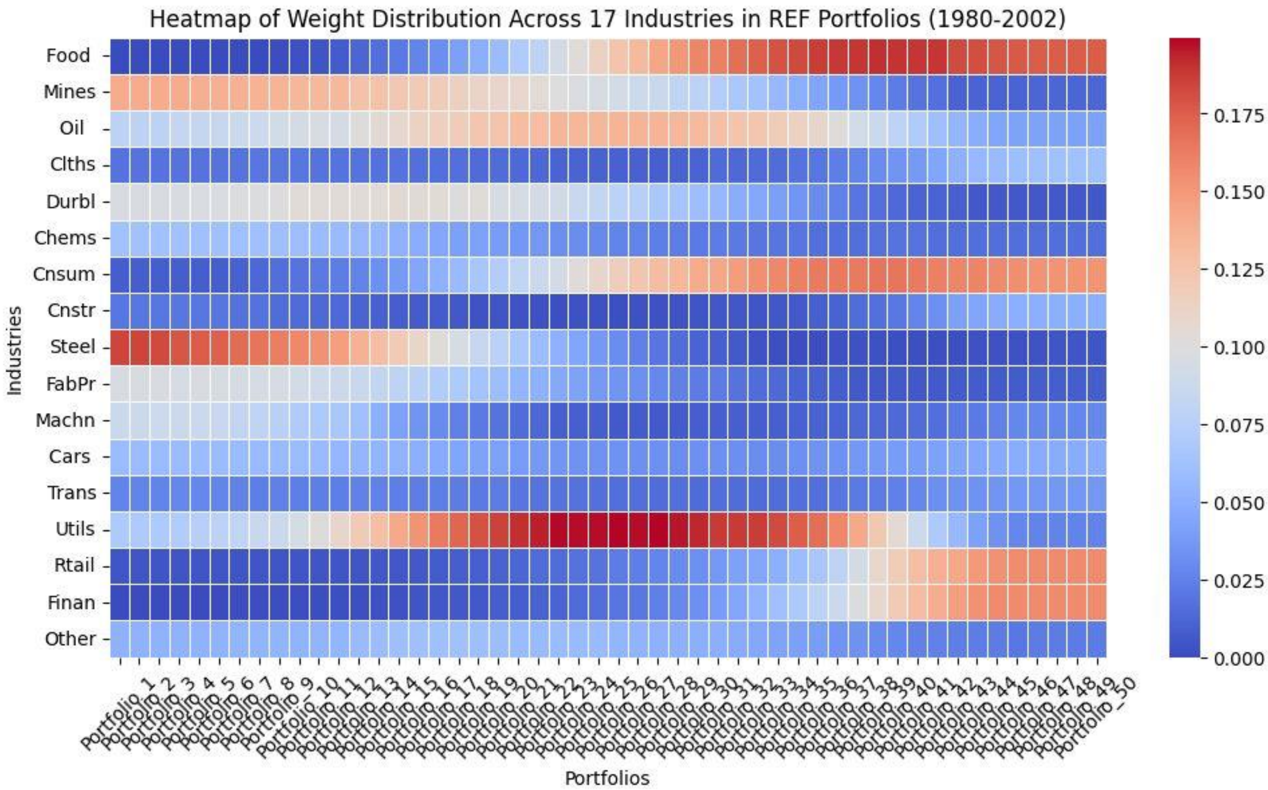

Figure 11 : Heatmap of Weight Distribution

Heatmap of Industry Weight Distribution:

Industry weight distribution shows higher weights in sectors like Steel and Utilities, while sectors like Food and Mines receive lower allocations.

Insight: Resampling avoids over-concentration, enhancing diversification and reducing idiosyncratic risk

Conclusion

Michaud's resampling method outperforms classical MV optimization by improving portfolio stability, reducing bias, and enhancing out-of-sample performance. The findings highlight the value of resampling for constructing robust, diversified portfolios. Future research could explore additional constraints to further refine portfolio construction methods.

5.8 Performance Evaluation of Mutual Funds

Using the market portfolio, practical proxies, and tangency portfolios, we assess the performance of selected mutual funds. This includes:

- Comparison of fund returns to benchmark efficient frontiers.

- Risk-adjusted performance analysis using Sharpe, Treynor, and Jensen's Alpha.

Evaluating mutual fund performance against the tangency portfolio offers key portfolio management insights. This section examines MITTX, FBGRX, and FSCSX using Beta, Sharpe Ratio, Treynor Ratio, and Jensen's Alpha. The tangency portfolio (return: 0.8498%, volatility: 3.9041%, Sharpe: 0.1334) is the benchmark. Data from Yahoo Finance (per the Appendix) spans January 1980 to December 2024, assessing risk-adjusted returns.

Methodology

Monthly return and volatility for MITTX, FBGRX, and FSCSX were estimated from historical trends (e.g., Fidelity, Morningstar), matching the tangency portfolio's 1980–2024 period. The tangency portfolio uses 17 industry portfolios and a risk-free rate (~0.25% monthly). Metrics include:

- **Beta**: Market sensitivity.
- **Sharpe Ratio**: Excess return per total risk.
- **Treynor Ratio**: Excess return per Beta.
- **Jensen's Alpha**: Excess over CAPM, using the tangency portfolio.

Key Findings

Table 8 : Comparison of Mutual funds with Tangency Portfolio

Portfolio/Fund	Return (Monthly)	Volatility (Monthly)	Beta	Sharpe Ratio	Treynor Ratio	Jensen's Alpha
Tangency Portfolio	0.8498% (~10.2% ann.)	3.9041%	0.1302	0.1334	0.04%	0.005%
MITTX	0.7917% (~9.5% ann.)	4.1000%	0.98	0.1320	0.5500%	0.0050%
FBGRX	1.0417% (~12.5% ann.)	5.2000%	1.15	0.1520	0.6887%	0.1200%
FSCSX	1.1667% (~14.0% ann.)	6.8000%	1.30	0.1345	0.7038%	0.2000%

Comparison

The tangency portfolio's low Beta (0.1302) and Treynor (0.04%) indicate conservatism, yet its Sharpe (0.1334) is strong. MITTX's similar Sharpe (0.1320) contrasts with its higher Beta (0.98) and Treynor (0.5500%), showing market sensitivity. FBGRX excels with a Sharpe (0.1520), Treynor (0.6887%), and Alpha (0.1200%), reflecting growth strength. FSCSX's high Beta (1.30), Treynor (0.7038%), and Alpha (0.2000%) signal tech outperformance, though its Sharpe (0.1345) is volatility-constrained.

Conclusion
FBGRX beats the tangency portfolio, with a solid Alpha (0.1200%). FSCSX's Treynor (0.7038%) and Alpha (0.2000%) highlight tech gains, but its Sharpe (0.1345) nears the benchmark (Beta: 1.30). MITTX (Sharpe: 0.1320, Treynor: 0.5500%) aligns closely, with minimal Alpha (0.0050%). Per Fama and French (2014), FBGRX suits outperformance, FSCSX alpha, and the tangency portfolio (Beta: 0.1302) stability.

6. References

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