Given a symmetric, Cartesian, monoidal category $(\mathcal{C}, \circ, id, \times, \mathtt{1}, \pi_1, \pi_2, \top, \sigma)$ with a strong monad (P, μ, η, τ) the natural transformations have the following types:

$$id : a \to a$$

$$\pi_1 : a_1 \times a_2 \to a_1$$
$$\pi_2 : a_1 \times a_2 \to a_2$$
$$\top : a \to 1$$
$$\sigma : a_1 \times a_2 \to a_2 \times a_1$$
$$\mu : P\left(P\left(a\right)\right) \to P\left(a\right)$$
$$\eta : a \to P\left(a\right)$$
$$\tau : a_1 \times P\left(a_2\right) \to P\left(a_1 \times a_2\right)$$

Define *comprehensions*, morphisms $[\cdots] : a \to P\left(b\right)$, as

- Given $t : a \to b$,
$$[t] \coloneqq \eta \circ t : a \to P\left(b\right)$$
- Given $t : b \to c$ and $u : a \to P\left(b\right)$,
$$[t|b \leftarrow u] \coloneqq P\left(t\right) \circ u : a \to P\left(c\right)$$
- Given $t : a_n \to b$ and $u_i : a_{i-1} \to P\left(a_i\right)$ for $i = 1, \ldots, n$,
$$[t|a_1 \leftarrow u_1, \ldots, a_n \leftarrow u_n]$$
$$\coloneqq$$
$$\mu \circ \left[\left[t|a_2 \leftarrow u_2, \ldots, a_n \leftarrow u_n\right]|a_1 \leftarrow u_1\right] : a_0 \to P\left(b\right)$$
- Given $u : a \to P\left(b\right)$
$$[b|b \leftarrow u] \coloneqq u : a \to P\left(b\right)$$
- Given $u : a \to P\left(b_1 \times b_2\right)$,
$$[b_1|b_1 \times b_2 \leftarrow u] \coloneqq P\left(\pi_1\right) \circ u : a \to P\left(b_1\right)$$
$$[b_2|b_1 \times b_2 \leftarrow u] \coloneqq P\left(\pi_2\right) \circ u : a \to P\left(b_1\right)$$
$$[b_2 \times b_1|b_1 \times b_2 \leftarrow u] \coloneqq P\left(\sigma\right) \circ u : a \to P\left(b_2 \times b_1\right)$$
- Given $t : c \to d$ and $u : a \to P\left(b\right)$,
$$[t \times b|b \leftarrow u] \coloneqq \tau \circ \left(t \times u\right) : c \times a \to P\left(d \times b\right)$$
$$[b \times t|b \leftarrow u] \coloneqq P\left(\sigma\right) \circ \tau \circ \left(t \times u\right) \circ \sigma : a \times c \to P\left(b \times d\right)$$

Conjecture. *Extending to co-Cartesian categories will extend the comprehension calculus by pattern matching on sum types, much like* π_1,π_2 *perform pattern matching on product types.*