Given a symmetric, Cartesian, monoidal category $(\mathcal{C}, \circ, id, \times, \mathtt{1}, \pi_1, \pi_2, \top, \sigma)$ with a strong monad (P, μ, η, τ) the natural transformations have the following types:

$$id : a \to a$$

$$\pi_1 : a_1 \times a_2 \to a_1$$
$$\pi_2 : a_1 \times a_2 \to a_2$$
$$\top : a \to 1$$
$$\sigma : a_1 \times a_2 \to a_2 \times a_1$$
$$\mu : P(P(a)) \to P(a)$$
$$\eta : a \to P(a)$$
$$\tau : a_1 \times P(a_2) \to P(a_1 \times a_2)$$

Define *comprehensions*, morphisms $[\cdots] : a \to P(b)$, as

- Given $t : a \to b$,
$$[t] \coloneqq \eta \circ t : a \to P(b)$$
- Given $t : b \to c$ and $u : a \to P(b)$,
$$[t|b \leftarrow u] \coloneqq P(t) \circ u : a \to P(c)$$
- Given $t : a_n \to b$ and $u_i : a_{i-1} \to P(a_i)$ for $i = 1, \ldots, n$,
$$[t|a_1 \leftarrow u_1, \ldots, a_n \leftarrow u_n]$$
$$\coloneqq$$
$$\mu \circ [[t|a_2 \leftarrow u_2, \ldots, a_n \leftarrow u_n] \,|a_1 \leftarrow u_1] : a_0 \to P(b)$$
- Given $u : a \to P(b)$,
$$[b|b \leftarrow u] \coloneqq [id|b \leftarrow u] : a \to P(b)$$
- Given $u : a \to P(b_1 \times b_2)$,
$$[b_1|b_1 \times b_2 \leftarrow u] \coloneqq [\pi_1|b_1 \times b_2 \leftarrow u] : a \to P(b_1)$$
$$[b_2|b_1 \times b_2 \leftarrow u] \coloneqq [\pi_2|b_1 \times b_2 \leftarrow u] : a \to P(b_2)$$
$$[b_2 \times b_1|b_1 \times b_2 \leftarrow u] \coloneqq [\sigma|b_1 \times b_2 \leftarrow u] : a \to P(b_2 \times b_1)$$
- Given $t : c \to d$ and $u : a \to P(b)$,
$$[t \times b|b \leftarrow u] \coloneqq \tau \circ (t \times u) : c \times a \to P(d \times b)$$
$$[b \times t|b \leftarrow u] \coloneqq P(\sigma) \circ \tau \circ (t \times u) \circ \sigma : a \times c \to P(b \times d)$$

Conjecture. *Extending to co-Cartesian categories will extend the comprehension calculus by pattern matching on sum types, much like π_1,π_2 perform pattern matching on product types.*