Lecture 2. Elementary Algebra

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Why elementary?

1. We quickly go through the elementary algebras today;
2. These are easy to understand and are easy to make mistakes;
3. Mastering the lecture today is essential to understand materials in the followings days.

Exponent and Polynomial

1. Definition
2. Rules of exponent:

$$x^a(x^b) = x^{a+b}; \qquad \frac{x^a}{x^b} = x^{a-b}; \qquad (x^a)^b = x^{ab};$$

$$(xy)^a = x^a y^a; \qquad (\frac{x}{y})^a = \frac{x^a}{y^a}; \qquad \frac{1}{x^a} = x^{-a};$$

$$\sqrt{x} = x^{1/2}; \qquad \sqrt[b]{x^a} = x^{a/b}; \qquad x^{(-a/b)} = \frac{1}{x^{a/b}}.$$

3. Polynomial
4. Polynomial regression: $y = \beta_0 + \beta_1 x + \beta_2 x^2 + ... + \epsilon$. This is one way of capturing non-linear relationship between x and y.

Logarithm

1. Definition
2. Logarithm laws:
   - Product: $\log_a xy = \log_a x + \log_b y$
   - Quotient:$\log_a x/y = \log_a x - \log_b y$
   - Power: $\log_a x^b = b \log_a x$
   - Root: $\log_a \sqrt[b]{x} = \frac{\log_a x}{b}$
   - Base change: $\log_a x = \frac{\log_k x}{\log_k a}$
3. Natural logarithm: $\ln x = \log_e x$
4. Common applications:
   (1) Elasticity
   (2) Linearize equation

Rules of algebra

1. Commutative property: $a + b = b + a$
2. Multiplication: $ab = ba$
3. Distributive property: $a(b + c) = ab + ac$
4. Identity property: $a + 0 = a$; $a \cdot 1 = a$
5. Inverse property: $a + (-a) = 0$; $a \cdot \frac{1}{a} = 1$
6. Properties of negation: -(-a) = a
7. Properties of equality
8. Properties of zero
9. Fractions
10. Simple rules sometimes provide interesting implications (an example).

Inequality

1. Inequality is frequently used in optimization problems;
2. Solving inequality;
3. Double inequalities;
4. Inequality with absolute value;
5*. How to find the solution for market equilibrium?

Summation and Product

1. Summation (or product) could be confusing sometimes. The best advice that I can give is to be careful and be patient. Watch out for subscripts.
2. Notations: $\sum$ (summation); $\prod$ (product).
3. Definitions:
$\sum_{i=1}^{n} x_i = x_1 + x_2 + ... + x_n$
$\prod_{i=1}^{n} x_i = x_1 \cdot x_2 \cdot ... \cdot x_n$
4. Double summation
5. Geometric sequence

Function

1. Economics research is 99% about function. How does X affect Y (casual effect)? What would be the value of Y (prediction)?

2. Mathematics definition: A function is a process or a relation that associates each element x of a set X, the domain of the function, to a **single** element y of another set Y, the target space of the function.

3. Functional forms;

4. Graph of a function: intercept, slope, curvature;

5*. Inverse function;

6*. Homogeneous and homothetic function.

End.