Lecture 3. Calculus I

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Derivative

Derivative is an essential concept in economics research: how y changes when x changes?

Definition Let $(x_0, f(x_0))$ be a point on the graph of $y = f(x)$. The derivative of f at x_0 is the slope of the tangent line to the graph of f at $(x_0, f(x_0))$. Analytically,

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

if the limit exists. When this limie does exist, we say that the function f is differentiable at x_0 with derivative $f'(x_0)$, or $\frac{df}{dx}(x_0)$, or $\frac{dy}{dx}(x_0)$.

Rules for computing derivatives

1. $(f \pm g)'(x_0) = f'(x_0) \pm g'(x_0)$
2. $(kf)'(x_0) = kf'(x_0)$
3. $(f \cdot g)'(x_0) = f'(x_0)g(x_0) + f(x_0)g'(x_0)$
4. $(\frac{f}{g})'(x_0) = \frac{f'(x_0)g(x_0) - f(x_0)g'(x_0)}{g(x_0)^2}$
5. $(f^n(x))' = n(f^{n-1}(x) \cdot f'(x)$
6. $(x^k)' = kx^{k-1}$

(Everybody in this room shall master these rules!)

Differentiability and continuity

1. A function is differentiable if it is differentiable at every at every point in its domain;

2. A function is continuous if its graph has no breaks;

3. A function is continuously differentiable if **its derivative** is a continuous function;

4. You should be familiar with the above concepts. You will hear a lot of "given a differentiable function, ... " in microeconomics.

Linear approximation

One useful implication of derivative is to approximate the value of a function. Recall the definition of first-order derivative,

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

So,

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0)}{h}$$

Equivalently,

$$f(x_0 + h) \approx f(x_0) + hf'(x_0)$$

You can approximate the value of f at a new point based on its original value and its derivative (See R). But, there are approximation errors, sometimes large.

Higher-order derivatives

1. Taking derivatives of the derivative functions.

2. Examples

3^*. Higher-order approximation (Taylor series approximation):

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \ldots$$

4. It is often the case that higher order approximation gives more accurate outputs.

5^*. In economics research, first- and second-order approximations are commonly applied such as for welfare decomposition. Here is an example.

Application

1. The sign of first-order derivative matters to the direction of slope. E.g., Increasing/decreasing function is a function with positive/negative derivative in the domain;

2. The sign of second-order derivative matters to the curvature of the curve (or the slope of the derivative). More on this tomorrow.

3. Examine the function features: cost/utility/revenue function;

4. Derivative can also be interpreted as local marginal effect, which can be used to recover elasticity;

5*. Numerical derivative is the process of finding the numerical value of a derivative of a given function at a given point based on the definition.

End.