Title: AJAE Appendix for “Source Diversification and Import Price Risk”

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Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE).

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**Derivation of the Price, Variance, and Covariance Effects**

Recall from the text that imports are treated as intermediate goods that are used with domestic resources to produce one or several outputs. Let *R* denote the net revenue from outputs and domestic resources, and *pi* and *qi* denote the price and quantity of an imported commodity from the *i*th country (*i* = 1, 2 … *n*). The optimal allocation of **q** at time *t* is the solution to the following utility maximization problem (Wolak and Kolstad 1991):

.

Note that *E* and *V* are the expectation and variance operator, respectively. **q** and **p** are *n*-vectors containing quantities and prices, *Q* is total imports, and **ι** is an *n*-unit vector. Henceforth, the time subscripts (*t*) are omitted.

Let the conditional expectation and variance of price be denoted as **p** and **Ω**, respectively. The Lagrangian for the utility maximization problem is

(A.1) 

and the first order condition with respect to choice variable *qi* is

(A.2) .

From the optimization problem, we get an import demand function expressed in general form as follows:

(A.3) .

Derivation of the differential import allocation (DIA) model starts with the total differential of equation (A.3),

(A.4) .

With some manipulation, equation (A.4) can be restated in log-differential form,

(A.5) 

where *si* is the share of total imports from country *i* (*qi*/*Q*).

Following a similar procedure as Theil (1980) and Laitinen (1980), we derive the log-differential price effect , variance effect , and covariance effect at the optimal choice. In deriving these effects, the first step is to take the derivatives of the first order condition (A.2) with respect to the exogenous variables *Q*, *ph*, , and .



(A.6a) 

(A.6b) 

(A.6c) 

(A.6d) .

Note that the subscripts *g*, *h*, *j*, and *k* also denote the exporting source. and are Kronecker deltas equal to 1 if *h* = *i* and *g* = *i*, and 0 otherwise. Also note that



  .

To simply, let . Multiplying all terms in (A.6a), (A.6b), (A.6c), and (A.6d) by *Q*, *ph*, , and , respectively, and using the relationship results in the following:



(A.7a) 

(A.7b) 

(A.7c) 

(A.7d) .

Next, multiply the derivative terms by *qk*/*qk, qi*/*qi*, or *qj*/*qj* and use the relationship∂*q*/*q* = ∂log*q*,

(A.8a) 

(A.8b) 

(A.8c) 

(A.8d) .

and then multiply *U*11 and *U*12 by *U*1/ *U*1, and *U*21 and *U*22 by *U*2/ *U*2,

(A.9a)

(A.9b)

(A.9c)

(A.9d).

Dividing all terms by  yield the following:

(A.10a) 

(A.10b) 

(A.10c) 

(A.10d).

To simplify equations (A.10a) − (A.10d), assume the following relationships:







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*γi* is the marginal rate of substitution between the expected returns and variance (with respect to changes in *qi*); *κi* is the ratio of the marginal utility of total imports (*λ*) to the marginal utility of the variance; and *φi* and *φij* are component shares of the variance marginal utility. Using these relationships, and *Eh*= *phqh*, , and , equations (A.10a) – (A.10d) are restated as follows:



(A.11a) 

(A.11b) 

(A.11c) 

(A.11d) 

and rearranging terms,

(A.12a) 

(A.12b) 

(A.12c) 

(A.12d) .

Equations (A.12a) - (A.12d) can be expressed in matrix notation as follows:

(A.13a) 

(A.13b) 

(A.13c) 

(A.13d) 

 is a square matrix (*n*×*n*) where the *ij*th element is .  and . **σ2** is an n-vector containing the variances (), and **σσ** is an m-vector of the covariates (*σij*) where *m* = (*n2*‑*n*)/2, which is the number of unique covariates. **κ** is an *n*-vector with *κi* as the *i*th element; **Ψ**1 is an *n*-vector with *ψ*1,*i* as the *i*th element, and **Ψ**2 is an *n*-vector with *ψ*2,*i* as the *i*th element; **E** and **V** are *n*-vectors with *piqi* and  as the *i*th elements, respectively; and **CV** is a *m*-vector of covariance terms. **Γ** and **Φ***i* are diagonal matrices (*n*×*n*) with *γi* and *φi* along the diagonal, respectively, and **Φ***ih* is an *n*×*m* matrix of covariate component shares (*φgh*) where the *gh*th element is zero when *i* ≠ *g*. For instance, when *n* = 4



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Taking the derivative of the constraint () with respect to the exogenous variables *Q*, *ph*, , and , we get the following partial derivatives: , , , and. Note that the derivative of *Q* with respect to the other exogenous variables is zero. Next, multiply each partial by its corresponding exogenous variable, the *qk*’s by, and then divide both sides by *Q*. Defining the *k*th import share as, the partial derivatives can be expressed in log-differential form:



(A.14a) 

(A.14b) 

(A.14c) 

(A.14d) .

Defining **s** as an *n*-vector of import shares, equations (A.14a) - (A.14d) can also be expressed in matrix form:

(A.15a) 

(A.15b) 

(A.15c) 

(A.15d) .

Equations (A.13a) − (A.13d) and (A.15a) − (A.15d) form an eight-equation system with eight unknowns.

(A.16) .

Equation (A.16) is similar to Barten’s (1964) fundamental matrix equation in consumer theory (Theil and Clements 1987) and the fundamental matrix equation of the cost minimizing firm (Laitinen 1980). Note that the solution to the matrix equation requires the inverse of the leading matrix (Magnus and Neudecker 1999):



where , which is a scalar.

Given the inverse, the solution to (A.16) is

(A.17) 

and the total import, price, variance, and covariance effects are

(A.18a) 

(A.18b) 

(A.18c) 

(A.18d) .

The Lagrangian elasticities are

(A.19a) 

(A.19b) 

(A.19c) 

(A.19d) .

Using equations (A.18a) and (A.19a) – (A.19d), we can derive the following:



.

Note that these are the second terms in equations (A.18b), (A.18c), and (A.18d), respectively.

Thus, the price, variance, and covariance effects can be stated as follows:

(A.20a) 

(A.20b) 

(A.20c) 

and in scalar notation,

(A.21a) 

(A.21b) 

(A.21c) 

where *uij* denotes the *ij*th element of **U\***-1.

**Seasonality Estimates**

The seasonality estimates are reported in table A.1 and reflect the seasonal patterns in import demand holding total imports, prices, and price risk constant. Results show that seasonal changes in imports from one country are usually offset by imports from the other countries. For instance, increases in imports from Colombia in February correspond with decreases from Kenya, and increases in imports from Kenya and ROW in August are offset by decreases from Spain and Colombia. Overall, carnations from Spain and Colombia are the most seasonal where over half of the seasonality estimates are significant. Only four estimates are significant for the Netherlands and Kenya.

**Table A.1. Seasonality estimates for UK carnation imports**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Country | January | February | March | April | May | June | July | August | September | October | November | December |
| Spain | 0.052 | -0.006 | 0.055 | 0.045 | -0.007 | -0.057 | -0.033 | -0.049 | -0.005 | -0.046 | 0.039 | 0.031 |
|  | (0.021)b | (0.019) | (0.020)a | (0.019)b | (0.019) | (0.021)a | (0.018)c | (0.018)a | (0.017) | (0.019)b | (0.016)b | (0.022) |
| Netherlands | 0.017 | -0.003 | -0.077 | 0.012 | -0.068 | 0.133 | -0.004 | 0.009 | -0.015 | 0.000 | -0.020 | -0.057 |
|  | (0.020) | (0.019) | (0.021)a | (0.019) | (0.021)a | (0.021)a | (0.020) | (0.017) | (0.017) | (0.020) | (0.016) | (0.021)a |
| Kenya | 0.016 | -0.029 | 0.003 | -0.007 | 0.006 | 0.014 | 0.051 | 0.034 | 0.000 | 0.019 | -0.045 | -0.025 |
|  | (0.015) | (0.014)b | (0.014) | (0.014) | (0.016) | (0.014) | (0.013)a | (0.012)a | (0.012) | (0.014) | (0.012)a | (0.014)c |
| Colombia | -0.092 | 0.045 | -0.012 | -0.072 | 0.044 | 0.116 | 0.028 | -0.053 | -0.018 | 0.012 | -0.016 | 0.066 |
|  | (0.021)a | (0.018)b | (0.018) | (0.021)a | (0.020)b | (0.019)a | (0.017) | (0.017)a | (0.016) | (0.020) | (0.015) | (0.019)a |
| ROW | 0.007 | -0.008 | 0.031 | 0.022 | 0.025 | -0.206 | -0.043 | 0.058 | 0.038 | 0.015 | 0.042 | -0.015 |
|  | (0.017) | (0.016) | (0.017)c | (0.016) | (0.015) | (0.018)a | (0.016)a | (0.016)a | (0.014)a | (0.016) | (0.014)a | (0.017) |
| Asymptotic standard errors are in parentheses. ROW is the rest of the world.  a, b and c denote the 0.01, 0.05 and 0.10 significance level, respectively. | | | | | | | | | | | | |

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