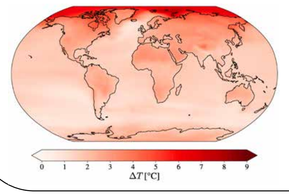
Goal: Emulate the statistics of a chaotic system

Section X: Climate variable of interest



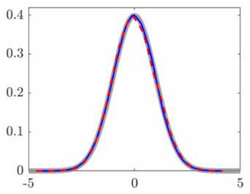
$$\frac{\partial w}{\partial t} = \mathcal{N}(w, F) + \epsilon\xi,$$

Option 1:

Option 2:

Section X: Emulate the full probability distribution

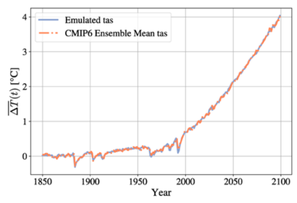
p(T):



Section X: Conceptual Bridge (Duality)

Section X: Emulate a statistical quantity (e.g. mean/variance)

g(T):



Section X: Koopman Formalism

Section X: Fokker-Planck Operator

$$\mathcal{F}(\cdot) = \frac{\partial}{\partial w}\left[D\frac{\partial}{\partial w}(\cdot) - \mathcal{N}(w, F)(\cdot)\right]$$

Appendix X: Technical Bridge (Adjoint)

Section X: Koopman Operator

$$\mathcal{K}(\cdot) = \mathcal{N}(w, F)\frac{\partial(\cdot)}{\partial w} + D\frac{\partial^2(\cdot)}{\partial w^2}$$

Section X: Linear Response Theory

Appendix X: Connecting response theory to the Fokker-Planck operator

Section X: Technical Bridge (GFDT)

Section X: Fluctuation Dissipation Theorem

$$\langle g(w)\rangle = \langle g(w)\rangle_0 + \int_{-\infty}^{t} e^{\mathcal{K}s} F(t-s)\, ds$$

Section X: Koopman Emulators

Section X: EDMD

$$\phi(T_{n+1}) = \tilde{\mathcal{K}}\phi(T_n) + \psi(F_n)$$

Linear Basis

Section X: DMD

$$T_{n+1} = \mathcal{L}T_n + F_n$$

Quasi-Equilibrium

Section X: Pattern Scaling

$$T(\mathbf{x}, t) = a(\mathbf{x})\overline{T}(t)$$

Section X: Response Function Emulators

Section X: GFDT

$$R(\mathbf{x}, \mathbf{x}', t) = \frac{\langle T_\delta(\mathbf{x}, t) - T_0(\mathbf{x}, t)\rangle}{|\delta(\mathbf{x}')|}$$

No Access to Governing Eq.

Section X: Deconvolution

$$\mathbf{R} = \frac{\mathbf{F}^{-1}\mathbf{T}}{\Delta t}$$

Exponential Decay

Section X: Mode Fitting

$$R(t) = \sum_{i=1}^{3} \alpha_i e^{\lambda_i t}$$