Goal: Emulate the statistics of a chaotic system
1. Select a climate variable of interest
$\frac{\partial w}{\partial t} = \mathcal{N}(w, F) + \epsilon\xi,$
Option 1:
Option 2:
Section 2.2: Operator Framework for Emulators
2a. Emulate the full probability distribution
$p(w)$
2b. Emulate a statistical quantity (e.g. mean/variance)
$g(w)$
Emulated tas
CMIP6 Ensemble Mean tas
Year
Section 2.2.3: Conceptual Bridge (Duality)
3a. Approximate Fokker-Planck Operator
$\mathcal{F}(\cdot) = \frac{\partial}{\partial w}\left[D\frac{\partial}{\partial w}(\cdot) - \mathcal{N}(w, F)(\cdot)\right]$
Appendix A2: Technical Bridge (Adjoint)
4a. Approximate Koopman Operator
$\mathcal{K}(\cdot) = \mathcal{N}(w, F)\frac{\partial(\cdot)}{\partial w} + D\frac{\partial^2(\cdot)}{\partial w^2}$
Section 2.2.4: Connecting to Linear Response Theory
Appendix A6: Connecting response theory to the Fokker-Planck operator
Technical Bridge (GFDT)
Fluctuation Dissipation Theorem (FDT)
$\langle g(w)\rangle = \langle g(w)\rangle_0 + \int_{-\infty}^{t} e^{\mathcal{K}s}F(t-s)\,ds$
Machine Learning Emulators
Deep learning/diffusion models; fitting statistical distributions
Connect to GDFT
Section 2.2.4: Score-based response function
$R(\mathbf{x}, \mathbf{x}', t) = -\langle T(\mathbf{x}, t)s(T(\mathbf{x}', 0))\rangle$
Sections 2.3.1 & 2.3.3: Koopman Emulators
Method 3: EDMD
$\phi(T_{n+1}) = \tilde{\mathcal{K}}\phi(T_n) + \psi(F_n)$
Linear Basis
Method 2: DMD
$T_{n+1} = \mathcal{L}T_n + F_n$
Quasi-Equilibrium
Method 1: Pattern Scaling
$T(\mathbf{x}, t) = a(\mathbf{x})\overline{T}(t)$
Section 2.3.2: Impulse Response Emulators
Method 4: GFDT
$R(\mathbf{x}, \mathbf{x}', t) = \frac{\langle T_\delta(\mathbf{x}, t) - T_0(\mathbf{x}, t)\rangle}{|\delta(\mathbf{x}')|}$
No Access to Governing Eq.
Method 5: Deconvolution
$\mathbf{R} = \frac{\mathbf{F}^{-1}\mathbf{T}}{\Delta t}$
Exponential Decay
Method 6: Mode Fitting
$R(t) = \sum_{i=1}^{3} \alpha_i e^{\lambda_i t}$